Reaction cross section in ultrarelativistic nuclear collisions

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The inelastic particle production process in ultrarelativistic heavy ion collision is modeled in terms of an effective scalar field produced by the colliding objects. The logarithmic increase of the inelastic cross section with the incident energy is obtained. It is also shown that this increase is intimately related to the size and surface diffuseness of the colliding objects: the larger the size and surface diffuseness are, the earlier the increase of the cross section starts.

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I. INTRODUCTION

It is well known that proton-proton (or antiproton) total and inelastic cross sections increase with energy for incident momenta above 200 GeV/c in the laboratory system. In analogy to that, it has been suggested [1] that the nuclear cross section in nucleus-nucleus reactions might also increase with energy in the ultrarelativistic region, apart from the Coulomb contribution. The physical reason for this is that, for a relativistic colliding system, the profile function could increase rapidly with the incident energy at a given impact parameter. As a consequence, the opacity of the surface region would become greater, as if the nucleus becomes effectively larger for inelastic channels. This picture has already been suggested in the analysis of nucleon-nucleon high energy collisions [2]. In Ref. [1], it was shown that in fact the particle production cross section calculated with a classical field model for pions presents this behavior; the larger the radius and surface diffuseness of the system are, the more enhanced the effect becomes. In this work, we exploit this idea further. We show that the above scenery for nuclear cross section is still valid within the context of a simple quantum field theoretical model for inelastic ultrarelativistic collisions of nuclei.

It may be argued that the logarithmic increase of the nuclear interaction cross section at high energy can be understood in terms of the exchange of vector mesons, such as ω, in analogy to the virtual-quantum method of Weizsäcker and Williams for the electromagnetic interaction [3]. In fact, we can show that the number of virtual quanta grows with $\ln\sqrt{s}$. If this number is proportional to the cross section as in the case of Weizäcker-Williams method for electromagnetic excitation, we would get a logarithmic increase of the cross section. However, if we take into account the nuclear shadow effect, interpreting the number of virtual mesons as being proportional to the eikonal function, then the increase in energy of the cross section is reduced to $\ln(\ln\sqrt{s})$, provided that the eikonal is Gaussian in the impact parameter. So this extension of the Weizsäcker-Williams method would lead to an increase of the cross section much slower than $\ln\sqrt{s}$.

As was pointed out in Ref. [1], the existence of many channels of inelastic excitations plays an essential role for the increase of the cross section. In order to simulate such excitations in the framework of a field theory, we introduce effective scalar fields whose mass spectrum is associated with the virtual excitation of quark-antiquark pairs. We calculate the eikonalized inelastic cross section corresponding to the production of quanta of these fields and show that it increases with $\ln\sqrt{s}$ as in Ref. [1].

We conclude that the usually assumed near constancy of the nuclear reaction cross section in ultrarelativistic heavy ion collisions is not a sound hypothesis. It is not impossible that, even at kinetic energies of 200A GeV, already attained at CERN, the nuclear reaction cross section for heavy nuclei may be significantly larger than the geometrical values.

II. EFFECTIVE SCALAR FIELD MODEL

The description of a hadronic collision process at very high energies requires nonperturbative QCD dynamics, involving the process of multiple particle production. As for the produced particle spectra, many calculations based on a string fragmentation mechanism or hydrodynamic models reproduce quite well the experimental data. In these phenomenological models, however, the total (inelastic) cross section of the reaction is usually introduced as an input quantity.

If we are interested only in the total cross section, the detailed dynamics of the hadronization process is not relevant. To the total cross section, what is important is how the multitude of inelastic channels are excited by incident particles. In hadronic collisions these inelastic channels are mainly due to quark-antiquark pair excitations caused by exchange of soft gluons. Therefore, one possible way to discuss the excitation of inelastic channels is to express these pairs by an effective scalar field with mass corresponding to the excitation energy of the pair.

For proton-proton collision process, Doi [4] developed an eikonal approximation to calculate the total and production cross section in a field theoretical model in which a vector meson exchange produces a massive scalar field. As can easily be shown, his model leads to a slowly increasing total cross section as a function of incident energy. França and Hama [5] obtained a similar result using a more simple model, substituting the vector particle exchange process by a classical geometric source term. This is in fact equivalent to the local interaction limit of Doi's model. One of the advantages of this field theoretical model is that it is exactly soluble for any external classical source term. In this work, we adopt this classical source model to describe the dynamics of the effective scalar field, to which we attribute the amplitude of the quark-antiquark excitation.

Let ϕ be such an effective scalar field. We assume that the dynamics of this scalar field is governed by

$$(\Box+\mu^2)\phi(x)=J(x), \tag{1}$$

where μ represents the excitation energy of the q-$\bar{q}$ pair. $J(x)$ is a classical scalar source term which describes the colliding objects.

For a given external classical source term $J(x,\mathbf{b})$, the production cross section of ϕ quanta of mass μ can be written [6] in terms of the eikonal function given by

$$\chi(\mathbf{b},\mu)=\frac{1}{2}\int\frac{d^3\mathbf{k}}{2k_0}|j(k,\mathbf{b})|^2, \tag{2}$$

where $j(k,\mathbf{b})$ is the (four-dimensional) Fourier transform of $J(x,\mathbf{b})$ and $\mathbf{b}$ the impact parameter. As stated above, we identify ϕ with a q-$\bar{q}$ pair of invariant mass μ. Assuming that excitations of different values of μ are independent, we should sum over all values of μ. Let $f(\mu)$ be the spectrum of the excitation modes. Then, the total inelastic cross section is described by the eikonal function integrated over μ,

$$\chi_{\text{tot}}(\mathbf{b})=\int d\mu f(\mu)\chi(\mathbf{b},\mu), \tag{3}$$

as

$$\sigma_R=\int d^2\mathbf{b}(1-e^{-2\chi_{\text{tot}}(\mathbf{b})}). \tag{4}$$

It has been suggested [5] that the source term may be chosen as

$$J(x,\mathbf{b})=g|\beta_P-\beta_T|\rho_P(x,\mathbf{b})\rho_T(x,\mathbf{b}), \tag{5}$$

where ρ_P and ρ_T represent (boosted-)hadronic densities of projectile and target, β_P and β_T are the center of mass velocities of these colliding particles, and g is a coupling constant. With this form, we are assuming that the effective scalar field is excited according to the geometric overlap of the colliding hadronic matter. Note that $J(x,\mathbf{b})$ is a scalar under the Lorentz transformation in the direction of the incident beam. The collision geometry in the center of mass frame is shown in Fig. 1.

It has been shown [5] that for the collision of two identical sources with Gaussian distribution $\rho_{\text{rest}}(\mathbf{r})=ce^{-\alpha r^2}$, the eikonal in Eq. (2) can be calculated as

$$\chi(\mathbf{b},\mu)=\frac{\pi}{32}\frac{g^2c^4}{4\alpha^4}e^{-\alpha b^2}\int d^2\mathbf{k}_\perp e^{-k_\perp^2/4\alpha} \times\exp\left[-\frac{k_\perp^2+\mu^2}{8\alpha\gamma^4\beta^2}\right]K_0(\omega[k_\perp^2+\mu^2]), \tag{6}$$

where $\omega=(1+\beta^2)/8\alpha\gamma^2\beta^2$, γ and β are the Lorentz factor and velocity of the colliding objects in the center of mass frame, and K_0 is the modified Bessel function. From this, the asymptotic behavior of χ can be calculated as $\chi(\mathbf{b},\mu)\sim\ln(\sqrt{s}/\mu)$, and consequently the cross section for fixed mass μ increases as a function of energy as

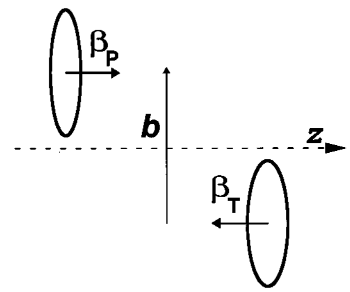

FIG. 1. Geometry of the collision in the center of mass frame. β_P (β_T) is the projectile (target) velocity, **b** is the impact parameter, and z is the collision axis.

$$\sigma(\mu)\sim\ln[\ln(\sqrt{s})]. \tag{7}$$

If the q-$\bar{q}$ pair excitation spectrum $f(\mu)$ has a power form $\propto\mu^\delta$ with $\delta>-1$, we have, from Eq. (4)

$$\sigma_R\sim\frac{1}{2}(\delta+1)\ln(s). \tag{8}$$

That is, the high energy behavior of the reaction cross section increases logarithmically provided $\delta>-1$. More precisely, including the next-leading term, the cross section becomes

$$\sigma_R\simeq\frac{\pi(\delta+1)}{2\alpha}\ln s+\frac{\pi}{\alpha}\ln\left[\frac{\pi^2g^2c^4e^{\gamma_E}\Gamma^2\left(\frac{\delta+1}{4}\right)}{128\alpha^3}\left(\frac{2\alpha}{m^2}\right)^{\frac{\delta+1}{2}}\right], \tag{9}$$

where γ_E is the Euler constant, $\Gamma(x)$ the gamma function, m the nucleon mass, and $\sqrt{s}$ is the center of mass energy per pair of nucleons.

In order to extract some insight about the behavior of a heavy nucleus collision, we take the Gaussian used above, not normalized to A, but adjusted to fit the more realistic Woods-Saxon density

$$\rho_{\text{WS}}(r)=\frac{\rho_0}{1+\exp\left(\frac{r-R_0}{d}\right)},\quad R_0=r_0A^{1/3}, \tag{10}$$

at the nuclear surface. This can be done by imposing that its value and the derivative coincide with those obtained with the Woods-Saxon density at $r=R_0$. We obtain, in the high energy limit,

$$\sigma_R\simeq\pi R^2\left[1+2\frac{d}{R}\gamma_E+(\delta+1)\frac{d}{R}\ln\left(\frac{\varepsilon}{\varepsilon_0}\right)\right], \tag{11}$$

where ε is the kinetic energy of the projectile nucleus per nucleon in the laboratory frame, $R=R_P+R_T$, and

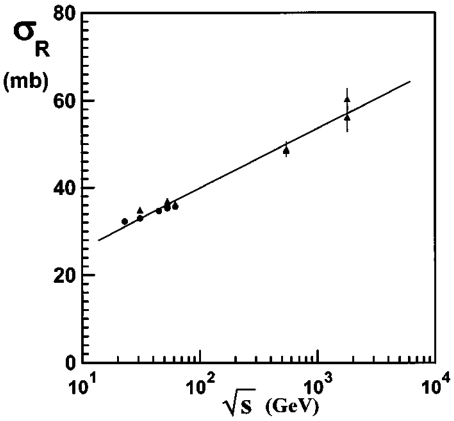

FIG. 2. Proton-proton (circles) and proton-antiproton (triangles) inelastic cross sections at high energies. The experimental data are from Ref. [7], and the solid line is our result [Eq. (9)], with $\delta = -0.56$ and $g = 3.62\ \mathrm{fm}^{(7+\delta)/2}$.

$$\varepsilon_0 = md\left[\frac{16}{\pi^2 g^2 \rho_0^4}\,\frac{R}{d^3}\,\frac{(R_P R_T)^{(\delta-7)/4}}{\Gamma^2\left(\frac{\delta+1}{4}\right)}\right]^{\frac{2}{\delta+1}}, \tag{12}$$

where the index $P(T)$ refers to the variables of the projectile (target).

These last two expressions reveal some characteristics of the behavior of the high energy reaction cross section in this model. The first two terms in Eq. (11) correspond to the usual geometric cross section $\sigma_{\mathrm{geo}} = \pi(R_P + R_T)^2$ of two nuclei and its surface correction. The last term indicates that, above a certain threshold value ε_0 of the kinetic energy, the reaction cross section increases logarithmically over the geometrical value. This threshold depends on the size of the nuclei — for a reasonable range of δ ($\delta < 5$), the bigger the nuclei, the lower the threshold energy, a result similar to that obtained with a classical field model in Ref. [1].

The quantitative results of our model depends on two parameters, the coupling constant g and the parameter for the mass spectrum δ. We may estimate these values by applying the same model to proton- (antiproton-) proton collisions. In Fig. 2 we show a fit of the experimental data on pp and $\bar{p}p$ inelastic cross section as a function of energy [7]. If the hadronic matter distribution of a nucleon is to be described by a Gaussian form with a rms radius of 0.8 fm, we get $\delta = -0.56$ and $g = 3.62\ \mathrm{fm}^{(7+\delta)/2}$.

High energy cross section data are also available for α-α collisions. The He nuclei can be described by a Gaussian form, with a rms radius of 1.63 fm, with a normalization to four nucleons, providing us with $\alpha = 0.565\ \mathrm{fm}^{-2}$ and $c = 0.305\ \mathrm{fm}^{-3}$ in the Gaussian. In Fig. 3, we show the result of the calculation together with the experimental data [8]. Here we have used the above values of δ and g. From this figure, we can see that at the available experimental energies, the reaction cross section still does not show any increase with energy.

In Fig. 4 we show the reaction cross section for the collisions of ^{16}O with ^{208}Pb, with the nuclei described by Woods-Saxon matter distributions. The experimental data are taken from Ref. [9], in which the cross sections at 60A and

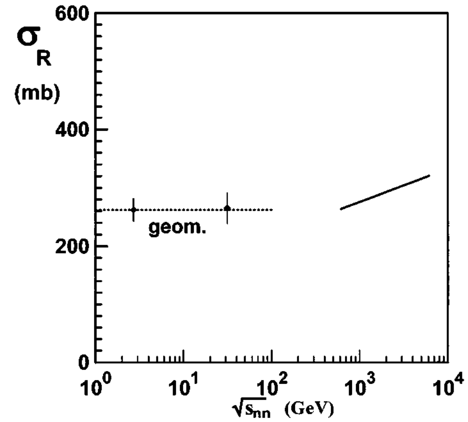

FIG. 3. Alpha-alpha reaction cross section as a function of center of mass energy per nucleon. The experimental data are from Ref. [8], and the solid line is the result of our calculation with a Gaussian parametrization to the nuclear matter distribution with a rms radius of 1.63 fm and with the same constants g and δ as in the proton-proton case. The dashed line is the geometrical cross section, considered to be given by the lower energy experimental point.

200A GeV do not show a strong dependence with energy but are higher than the geometrical value. The solid line shows the results of our calculation with g and δ fixed above, and the increase in the cross section is seen to begin earlier than in lighter nuclei.

Although we have fixed the parameters g and δ, actually we have no *a priori* reasons to believe that the values that fit the proton- (antiproton-) proton collision data should stay the same for the nucleus-nucleus case, since the dynamics of the quark degrees of freedom inside a nucleus would be different inside a nucleon. The coupling constant g and excitation spectrum parameter δ somehow reflect the correlations in the incident channel between quark-antiquark pairs at high energies. If the nuclear system is an aggregation of independent nucleons, such correlations can be excited only through the collision of nucleon pairs, and the cross section should be calculated in the usual Glauber formula, as in the "soft

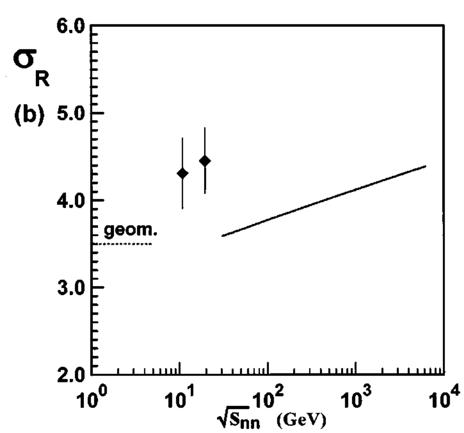

FIG. 4. ^{16}O-^{208}Pb reaction cross section as a function of center of mass energy per nucleon. The experimental data are from Ref. [9]. The solid line is the result of our calculation with the same constants g and δ as in the proton-proton case.

sphere model'' [10]. In this case, the increase of cross section is proportional to $\ln\ln\sqrt{s}$ for Gaussian matter distribution and the threshold of the increase is much higher than the one shown in Fig. 4. On the other hand, the presence of a nuclear mean field, mediated by virtual mesons inside the nucleus, may originate correlated particle-antiparticle pairs which can be excited from the incident channel. If this happens, the increase of the total cross section starts much earlier, especially for heavy nuclei. The quantitative result shown in Fig. 4 is obtained assuming that the quark-antiquark correlation is the same as that of nucleons scaled to the nuclear size and density.

III. CONCLUSIONS

At high energies, one usually assumes that the nuclear part of the heavy ion reaction cross section is given by the geometry of the system on the basis of the argument of the short range nature of the strong interactions. There is, however, no *a priori* reason to assume that this cross section should remain constant in the ultrarelativistic energy domain. We expect, instead, that in a relativistic bound state system the effects associated with the virtual particle-antiparticle pairs — which behave almost as real particles during the interaction — will become more important as the energy increases. This should lead to an increase in the cross section with incident energy for nucleus-nucleus collisions, in a similar manner to high energy proton-proton (antiproton) collisions.

In this work, we considered a model in which the effect of particle-antiparticle excitations is represented by an effective scalar field. Its mass spectrum is related to the excitation spectrum of a correlated quark-antiquark pair in the medium. We assumed a Klein-Gordon equation with source term specified classically by the time-dependent overlap of the two colliding objects. These assumptions might be supported if the incident channel couples to the quark-antiquark pair via exchange of a vector field. It is shown that the reaction cross section increases logarithmically with energy, $\sigma_R \propto \ln\sqrt{s}$, for the case of Gaussian distribution of the source matter.

The most important result of the present work is that the onset of the logarithmic growth depends on the size of the nucleus: As far as the q-$\bar{q}$ pair excitation spectrum does not increase too fast, the larger the colliding objects are, the earlier the increase starts. Quantitatively, the rate and onset of increase depend on how the correlated quark-antiquark pair coupled to the incident channel, namely, how intense the virtual meson field in the nuclear medium is. It is thus interesting to observe such an increase experimentally.

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