Off-mass-shell deformation of the nucleon structure function

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The off-mass-shell behavior of the nucleon structure function F_2^N is studied within an approach motivated by the Sullivan model. Deep inelastic scattering on the nucleon is considered to second order in the pion-nucleon coupling constant, corresponding to the dressing of the bare nucleons by the mesonic cloud. The inclusive and semi-inclusive deep inelastic processes on the deuteron involving off-shell nucleons are considered. A deformation of the mesonic cloud for the off-mass-shell nucleon, compared to the free one, generates observable effects in deep inelastic scattering. In particular, it leads to the breakdown of the convolution model, i.e., the deuteron structure functions are not expressed through the free nucleon structure function. Analysis of the semi-inclusive deep inelastic scattering on the deuteron, in the spectator approximation, shows that this reaction opens new possibilities to study the role of the off-shell effects in determining the nucleon structure function.

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I. INTRODUCTION

The study of the deep inelastic scattering (DIS) of leptons by nuclei has created a special problem, the description of DIS on off-mass-shell nucleons. By itself this problem is not something unique or new in nuclear physics, since the same problem of the off-mass-shell amplitudes arises in investigations of reactions of any kind (see, e.g., [1]).

Historically, off-mass-shell effects in DIS on the nucleons are divided into two classes. First are the so-called ''binding effects'' [2,3]. It was noticed that compared to the free nucleon structure function $F_2^N(x_N,Q^2)$, the structure function of the off-mass-shell nucleon, the ''bound nucleon,'' has a shift in the kinematical variable:

$$x=\frac{Q^2}{2pq}=\frac{x_N m}{p_0+p_z}, \tag{1}$$

where $Q^2=-q^2$, q is the four-momentum transfer in the reaction, m is the nucleon mass, p is the virtual nucleon momentum, and $x_N=-Q^2/(2q_0 m)$ is the Bjorken scaling variable for a free nucleon at rest. Since $p_0<m$ and averaging on the nucleus results in $\langle p_0+p_z\rangle<m$, the structure function of the bound nucleon is ''shifted'' to a smaller value of x_N. The exception is the region of high x_N, $x_N\gtrsim 1$, where due to the admixture of the $p_0+p_z>m$, the ''Fermi motion,'' the bound nucleon structure function is extended beyond single nucleon kinematics. When only such kinematical effects are taken into account, the bound-nucleon contribution to the nuclear structure function is given as a convolution of the free nucleon structure function with an effective distribution function for the nucleons [4,5]. Taking account of the binding effects in this distribution function leads to a description of the European Muon Collaboration (EMC) effect in DIS [2,3].

The second group of off-mass-shell effects includes all possible phenomena, other than the binding effects, which make the off-mass-shell structure function different from the on-mass-shell one. Such effects are related to the fact that the nucleon has an internal structure and the structure of the off-mass-shell nucleon requires a different description than just a kinematical shift in x. A systematic study of these off-mass-shell effects started recently [6,7], though some effects which do not reduce to the binding effects were discussed earlier [8]. Following the tradition of these papers [6,7] we call, throughout this paper, the effects of the second kind ''off-shell'' effects, distinguishing them from the binding effects.

In Ref. [6] the general form of the truncated nucleon tensor [see Fig. 1(a)] is studied. The structure functions of physical states are then presented as Feynman diagrams with insertion of the truncated nucleon tensor. For the transverse unpolarized structure functions the form of the insertion is found to be

$$m\hat{W}_T(p,q)=\hat{q}\chi_1(p,q)+\hat{p}\chi_2(p,q)+m\chi_3(p,q), \tag{2}$$

where χ_i are three different scalar functions, while for the scattering on the pointlike fermion only the term $\sim\hat{q}$ exists [5]. Calculation of the nuclear structure functions, keeping

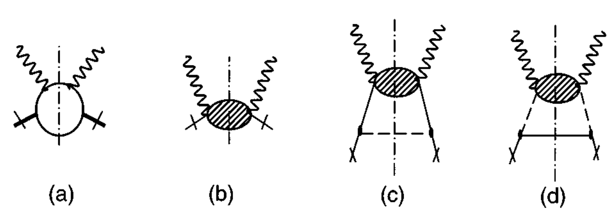

FIG. 1. The truncated nucleon tensor in the pion-nucleon model. Graph for the dressed nucleon tensor (a) is the sum of graphs (b)–(d), where virtual photon scatters on the bare nucleons and mesons.

only the $\sim\hat{q}$ term, leads to the existence of binding effects only and the convolution formula [9,10], which is a reasonable approximation, since the entire nonrelativistic nuclear physics works with pointlike nucleons. Nonuniform deformation off the mass shell of each of the three terms in Eq. (2) and the nontrivial spin structure of $\hat{W}_T(p,q)$ lead to new effects in the processes involving off-mass-shell nucleons.

The investigations of Refs. [6–8] are based on models for nucleon structure, motivated by the constituent quark model, and they involve unknown quark-nucleon amplitudes which are parametrized to fit the nucleon structure function on the mass shell. The off-shell effects are found to be only a few percent of the absolute value of the structure function of nuclei. Apparently the magnitude and behavior of these corrections are dependent on the form of the parametrizations of the quark-nucleon vertices with parameters being fixed to describe the observables (structure functions) for the nucleon on the mass shell, whereas the continuation to the off-mass-shell region must be ruled by dynamics (the equations of motion for participating fields) and fixed by constraints, such as the Ward-Takahashi relations, so that the conservation laws are satisfied and at the same time the consistency of the model with the calculation of any other observables is provided. On the other hand, the smallness of the corresponding corrections makes it difficult to test models experimentally in the usual deep inelastic experiments on nuclei.

The circumstances mentioned above motivated us to continue investigations of the off-shell effects in the nucleon structure function. In particular, we aim (i) to consider a dynamical model, other than the quark models [6–8], for the nucleon structure function, so as to compare results of independent approaches, and (ii) to discuss possible experiments, other than inclusive DIS, which can open new perspectives to study off-shell effects.

In the present paper we consider the pion-nucleon model for the structure function of the nucleon, which is motivated by the well-known Sullivan model [12–14]. We argue that the model is relevant for the case and discuss model ambiguities involved in the calculations. Then we calculate off-shell effects in inclusive and semi-inclusive DIS on the deuteron. The second reaction provides new opportunities for an experimental investigation of the off-shell effects in the nucleon structure function.

II. THE PION-NUCLEON MODEL FOR THE NUCLEON STRUCTURE FUNCTION

The role of the pion cloud or, more generally, mesonic cloud in the formation of the peripheral structure of the nucleon structure function has been discussed widely [12–14,16]. We will discuss only the pion cloud, since its contribution significantly dominates the contributions of the heavier mesons. The conventional analysis is based on corrections to the scattering on the nucleon, calculated to lowest order in the pion-nucleon coupling constant $\sim g^2$ [see Figs. 1(b)–1(d)] with pseudoscalar coupling [15]:

$$\mathscr{L}_{\rm int} = -ig\bar{\psi}(x)\gamma_5\boldsymbol{\tau\phi}\psi(x). \qquad (3)$$

In this case the diagrams (c) and (d) of Fig. 1 in the pion-nucleon model are logarithmically divergent. The formal logic of the field theory requires renormalization by introducing structure functions of the "bare" nucleons and mesons and some counterterms in such a way as to provide the correct value of the calculated structure functions where the natural normalization point for the counterterms is the nucleon mass shell. This is enough if we are going to consider off-mass-shell behavior of the nucleon structure function; however, this is not sufficient if we are intending to reach conclusions about contributions of the mesons to the structure function of the free nucleon.

The physically motivated Sullivan model provides a possibility to estimate the contribution of the pion cloud to the free nucleon structure function. In the modern form [13,16] the model is based on the graphs (b)–(d) in Fig. 1, and ingredients of the model include elementary structure functions of bare nucleons and mesons as well as meson-nucleon vertex form factors in diagrams (c) and (d). The vertex form factors provide a cutoff for the "unphysical" high momenta of the pions and make the contribution of diagrams (c) and (d) finite.

The physical picture corresponding to DIS on the nucleon in the Sullivan model is as follows. The nucleon is presented as a superposition of two states, the bare nucleon state and the nucleon-plus-one-pion state. This corresponds to the truncation of the Fock space so as to provide calculation to the second order in the pion-nucleon constant [13]. As a result the physical nucleon is a bare nucleon ("the nucleon core") surrounded by an extended pion cloud. The bare nucleons and mesons are the structureless, pointlike, objects from the point of view of the meson-nucleon model. The extended structure of the physical nucleon is then generated as a result of the dressing of the bare nucleon. On the other hand, the bare nucleons and mesons are composed of quarks and gluons, which can be detected in the reactions with large momentum transfer, $Q^2 \gg m^2$, much higher than typical transfer in the meson-nucleon interactions, $Q^2 \lesssim m^2$. In DIS the virtual photon interacts with a quark or antiquark inside the exchange meson or bare nucleon component of the physical nucleon. In this picture the quark-gluon degrees of freedom are "hidden" in the effective hadron degrees of freedom and their presence is displayed through the elementary structure functions of the bare nucleons and mesons and the hadronic interactions. This model is relevant to a study of nuclear effects in the nucleon structure function, since the NN potential can be successfully defined in the same g^2 approximation, i.e., the one-boson-exchange potential, as the dressing diagrams of Fig. 1. At the same time the off-mass-shell behavior of the structure functions is governed by the meson-nucleon dynamics and, therefore, is consistent with the dynamics of the deuteron which, we assume, is also described in the meson-nucleon model [17–20].

The vertex form factor plays a crucial role in the Sullivan model of the nucleon structure function [13,16]. Without form factors the one-loop diagrams (c) and (d) of Fig. 1 are divergent, and so the cutoff parameters control the magnitude of the contribution of these diagrams to the nucleon structure function. On the other hand, for the purpose of analysis of the off-shell behavior of the nucleon structure function, we can work without form factors, attributing all divergences to the renormalization of the structure functions of the bare nucleons and mesons. However, since we intend to make a connection with other calculations, we perform all numerical estimates within the Sullivan model with form factors. The form of the vertex form factor can be chosen as [17]

$$F(p,p')=f((p-p')^2)h(p^2)h(p'^2), \tag{4}$$

where p and p' are incoming and outgoing nucleon momenta, respectively, and $(p-p')$ is the pion momentum. The form factors are normalized so that

$$f(\mu^2)=1, \quad h(m^2)=1, \tag{5}$$

where μ is the pion mass. So for the on-mass-shell nucleon structure function the diagram (c) is regularized by the single form factor $h(p')$ and diagram (d) by $f((p-p')^2)$.

We accept the pointlike spinor structure $\sim\hat{q}$ of the bare nucleons, while the other structures [cf. Eq. (2)] are generated by the extended pion cloud, dressing diagrams (c) and (d). Calculation of the diagrams (b)–(d) gives (we consider the structure function F_2)

$$\hat{F}_2(x,p^2)=\int_0^1 dy\left[\hat{f}^N(y,p^2)\tilde{F}_2^N\left(\frac{x}{y}\right)+\hat{f}^\pi(y,p^2)\tilde{F}_2^\pi\left(\frac{x}{y}\right)\right], \tag{6}$$

with

$$\hat{f}^N(y,p^2)=\frac{\hat{q}}{2pq}[\delta(1-y)+f_1^N(y,p^2)]+\hat{p}f_2^N(y,p^2) + mf_m^N(y,p^2), \tag{7}$$

$$\hat{f}^\pi(y,p^2)=\frac{\hat{q}}{2pq}f_1^\pi(y,p^2)+\hat{p}f_2^\pi(y,p^2)+mf_m^\pi(y,p^2), \tag{8}$$

where $\hat{q}=q_\mu\gamma^\mu$, etc., and $\tilde{F}_2^{N(\pi)}$ is the bare nucleon (pion) structure function. The term $\delta(1-y)$ arises from the diagram (b), f_i^N from (c), and f_i^π from (d) $(i=1,\ldots,3)$. The explicit form of the functions $f_i^{N,\pi}(y,p^2)$ is presented in the Appendix. If we neglect the form factors, i.e., put $F(p,p')=1$, direct calculations with formulas (A2)–(A7) lead to conditions

$$f_i^N(y,p^2)=f_i^\pi(1-y,p^2) \quad (i=1,\ldots,3). \tag{9}$$

The result (9) also can be obtained heuristically as a consequence of the probabilistic interpretation of the structure function [13]. However, inclusion of the form factors [different for the diagrams (c) and (d)] breaks relation (9), leading to a violation of the charge and/or momentum conservation in the process [13]. The underlying reason for this is that in covariant calculations it is impossible to introduce form factors which are "symmetric" with respect to the nucleon and meson momenta.

The other topic is the choice of the cutoff parameters in the form factors. It is known [13,16] that to have a reasonable physical interpretation of the calculated structure functions the cutoff masses in the form factor (4) should be significantly smaller than are found from an analysis of the meson-exchange potentials [17–20]. To regulate the divergent diagram (c), a form factor is chosen [13],

$$f(k^2)=\left(\frac{\Lambda_\pi^2-\mu^2}{\Lambda_\pi^2-k^2}\right)^2, \quad \Lambda_\pi=1 \text{ GeV}, \tag{10}$$

which corresponds to the form factor for diagram (d):

$$h(p^2)=\left(\frac{\Lambda_N^2-m^2}{\Lambda_N^2-p^2}\right)^2, \quad \Lambda_N=1.475 \text{ GeV}, \tag{11}$$

where parameter Λ_N is fixed to preserve the baryon charge (not the momentum) conservation. On the other hand, there is no reason to put a smaller cutoff mass in the form factor corresponding to the external nucleon line for the diagrams (c),(d), where the cutoff mass Λ_N should be $\sim$1.5–2.0 GeV [17].

We have to stress once again that for the analysis of off-mass-shell effects in the nucleon structure functions we do not necessarily need the regulating form factors in the divergent diagrams. We are introducing these form factors only to relate to the well-known and widely discussed Sullivan model for the nucleon structure function. The dependence of our results on the cutoff masses $\Lambda_{\pi(N)}$ in the loops of diagrams (c) and (d) is weak compared to the $\Lambda_{\pi(N)}$ dependence of the pion (nucleon) contribution to the free nucleon structure function. Moreover, to restore symmetry between nucleon and pion contributions in Eqs. (6)–(8) we use Eq. (9) to define the nucleon contribution through the contribution of pions. This way to proceed is supported by analysis [13] using time-ordered perturbation theory, where it is shown that Eq. (9) is valid, and introducing the form factors in the covariant calculations damages significantly only the nucleon contribution.

The free nucleon structure function is defined by inserting operator (6) between Dirac spinors. The final expression coincides with the the result of the Sullivan model [13,16]:

$$F_2^N(x,p^2=m^2)=N_N\int_0^1 dy\left[f^N(y,m^2)\tilde{F}_2^N\left(\frac{x}{y}\right) + f^\pi(y,m^2)\tilde{F}_2^\pi\left(\frac{x}{y}\right)\right], \tag{12}$$

where N_N is the normalization factor defined by the conservation of baryon number:

$$N_N^{-1}=1+\int_0^1 dy f^N(y,m^2). \tag{13}$$

A direct calculation shows that (13) actually preserves the baryon number in the one-loop approximation, with or without $(\Lambda_\pi\to\infty)$ form factors. We take the form factor (10) [13] as a basis for the calculations. In this case $N_N^{-1}-1\approx 0.24$. Therefore, for bare structure functions of the nucleons and mesons we utilize a fit of the empirical structure functions of the free nucleons and pions. This gives good agreement between the calculated structure function (12) and the experimental data, since the corrections (c) and (d) are not too large.

III. NUCLEON CONTRIBUTION TO THE DEUTERON STRUCTURE FUNCTIONS

Now we are in a position to calculate the nucleon contribution to the deuteron structure function. We start with a consideration of inclusive DIS on the deuteron [Fig. 2(a)]. The structure function for the deuteron, $F_2^D(x_D)$, is defined as a matrix element of the operator (6). A consistent way is to use the covariant amplitude for the deuteron, the Bethe-Salpeter amplitude [20,11], or relativistic wave functions [17], with the normalization of the amplitude, based on the deuteron charge, in the one-loop approximation. This will be done elsewhere. In the present work, to test the method, we utilize the usual nonrelativistic wave function of the deuteron. The nonrelativistic reduction of the covariant operator (6) is similar to earlier approaches [9,10,7]. Here we reduce the 4×4 Dirac structure of the operator (6) to the 2×2 Pauli operators, keeping all kinematics in the relativistic form.

The deuteron structure function then is defined by

$$F_2^D(x_N)=N_D\int d^4p\int_0^{M_D/m}d\xi\,\delta\left(\xi-\frac{p_+}{m}\right)|\Psi_D(p)|^2\xi\left\{\frac{\langle\hat{q}\rangle}{2pq}\tilde{F}_2^N\left(\frac{x_N}{\xi}\right)+\frac{\langle\hat{q}\rangle}{2pq}F_2^{(1)}\left(\frac{x_N}{\xi},p^2\right)+\langle\hat{p}\rangle F_2^{(2)}\left(\frac{x_N}{\xi},p^2\right)+\langle m\rangle F_2^{(3)}\left(\frac{x_N}{\xi},p^2\right)\right\}, \tag{14}$$

where N_D is the renormalization constant preserving conservation of baryon number in the deuteron, M_D is the deuteron mass, $p_0=M_D-\sqrt{m^2+\mathbf{p}^2}$ is the off-mass-shell energy of the nucleon, $p_+=p_0+p_z$, and

$$F_2^{(i)}\left(\frac{x_N}{\xi},p^2\right)=\int_0^1 dy\left\{\tilde{F}_2^N\left(\frac{x_N}{y\xi}\right)f_i^N(y,p^2)+\tilde{F}_2^\pi\left(\frac{x_N}{y\xi}\right)f_i^\pi(y,p^2)\right\},\quad i=1,\ldots,3. \tag{15}$$

The brackets $\langle\cdots\rangle$ denote the nonrelativistic expression for the subsequent operators:

$$\xi\frac{\langle\hat{q}\rangle}{2pq}=\left(1+\frac{p_z}{m}\right),\quad \xi\langle\hat{p}\rangle=2p_+\left(p_0+\frac{\mathbf{p}^2}{\mathbf{m}}\right),\quad \xi\langle m\rangle=2mp_+. \tag{16}$$

It is anticipated that off-shell effects are small in the deuteron. This is a consequence of the fact that, on the average, the shift from the mass shell for the nucleon in the deuteron is small: $\langle p^2\rangle\approx m^2[1-(3\text{–}4)\times10^{-2}]$. In heavier nuclei this shift is apparently larger, up to $\sim10\%$ for a nucleus like iron, which can lead to a more significant effect. However, it is interesting to find other possibilities to investigate the off-shell behavior of the nucleon structure functions.

Let us consider now semi-inclusive DIS on the deuteron. The spectator mechanism [21] for this reaction is presented in Fig. 2(b). Even for nonrelativistic momenta of the spectator nucleon the shift from the mass shell of the interacting nucleon can be much larger than in heavy nuclei, for instance, $\sim20\%$ for $p_s\sim300$ MeV/c and $\sim40\%$ for $p_s\sim500$ MeV/c. Thus semi-inclusive DIS provides a unique possibility to study off-shell behavior of the nucleon structure function. In particular, the off-shell effects can be studied as a function of the shift from the mass shell (or spectator momentum).

In the nonrelativistic approach, with disregard for off-shell effects, the structure function of the off-shell neutron, $F_2^n(x,p_s)$, measured with the detection of the proton-spectator, has the form

$$F_2^n(x,p_s)=\left(1+\frac{p_z}{m}\right)|\Psi(-\mathbf{p_s})|^2F_2^n\left(\frac{x}{\zeta}\right), \tag{17}$$

where $\zeta=p_+/m$, $p_0=M_D-\sqrt{m^2+\mathbf{p_s}^2}$, $p_z=-(\mathbf{p_s})_z$. Dividing the data by the flux factor $(1+p_z/m)$, which is known from kinematics, and plotting $F_2^n(x,p_s)$ as a function of $z=x/\zeta[z\in(0,1)]$, we should have a result proportional to the free neutron structure function $F_2^n(z)$, with coefficient $|\Psi(p_s)|^2$; i.e., at any fixed p_s the ratio of the measured structure function to the free neutron structure function should be constant. This conclusion has to be changed if there is a nontrivial p^2 dependence of the nucleon structure function, i.e., if off-shell effects exist. The structure function $F_2^n(x,p_s)$ in this case is defined by an equation similar to Eq. (14), only without integration over p and with the relevant isospin modifications.

IV. NUMERICAL RESULTS AND DISCUSSION

Prior to calculating the deuteron structure functions, let us qualitatively discuss the possible phenomena in the structure functions of the off-shell nucleon. Omitting details of the pseudoscalar coupling of the pions and nucleons we can es-

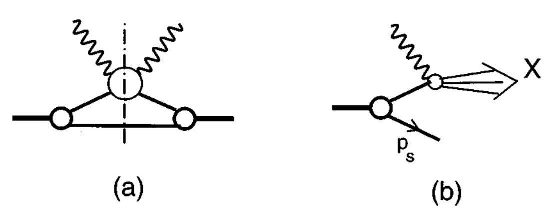

FIG. 2. Deep inelastic scattering on the deuteron: inclusive (a) and semi-inclusive (b).

timate the amplitude of the nucleon to emit the virtual meson as

$$\mathcal{M}(p,p') \propto \frac{1}{k^2-\mu^2}, \tag{18}$$

where the kinematics are defined by

$$p=(p_0,\mathbf{p}), \quad p'=\left(E',\frac{\mathbf{p}}{2}-\mathbf{k}\right), \quad E=\sqrt{m^2+\left(\frac{\mathbf{p}}{2}-\mathbf{k}\right)^2},$$

$$k=\left(k_0,\frac{\mathbf{p}}{2}+\mathbf{k}\right), \quad k_0=p_0-E'. \tag{19}$$

For simplicity let us compare amplitudes for a nucleon with momentum p_1 ($p_{01}=p_0<m$) and a "more virtual" nucleon with momentum p_2 [$p_{02}=(p_0-\Delta),\Delta>0$], and other components of p_1, p_2, and p' are kept the same. The sign of the combination

$$\mathcal{M}^{-1}(p_1,p')-\mathcal{M}^{-1}(p_2,p') \propto -\Delta^2+2\Delta(p_0-E')<0 \tag{20}$$

controls the relative magnitude of these two amplitudes. Since amplitudes (18) are negative, the relation (20) means that the absolute value of the amplitude $\mathcal{M}(p_2,p')$ is larger than $\mathcal{M}(p_1,p')$. Thus as a nucleon moves farther from the mass shell an increase in the emission of virtual pions may be expected. In accordance with Eq. (9) the role of virtual nucleons is also increased. On the other hand, Eq. (13) implies that the weight of the bare component is decreased. The maximums of both effective distributions $f_N(y)$ and $f_\pi(y)$ are at $y<1$, $y\sim 0.2$–0.3 for pions and $y\sim 0.7$–0.8 for nucleons; therefore, contributions of both components are concentrated at smaller x than for the bare component, where $f_{\text{bare}}(y)\propto\delta(1-y)$. As a result, for the off-shell structure function we expect an increase at small x and a decrease at large x, compared to the structure function of a nucleon moving closer to the mass shell. These conclusions may be affected by the pseudoscalar coupling, the presence of the vertex form factors, and the Fermi motion in the deuteron.

The parametrizations of the nucleon structure functions from [22] and pion structure function from [23] are used as input. The Bonn potential wave function for the deuteron [19] is utilized throughout. The form factor for the external nucleon line of diagrams (c) and (d) of Fig. 1, is taken to be of the form [17]

$$h(p^2)=\frac{2(\tilde{\Lambda}_N^2-m^2)^2}{2(\tilde{\Lambda}_N^2-m^2)(\tilde{\Lambda}_N^2-p^2)+(m^2-p^2)^2}. \tag{21}$$

Since the role of this form factor in the diagrams for the reaction with an external probe is uncertain, results are presented both with ($\tilde{\Lambda}_N=1.65$ GeV) and without ($\tilde{\Lambda}_N\to\infty$) the form factor. The second case is our choice for the basic set of parameters.

The results for the deuteron structure function $F_2^D(x)$ are presented in Fig. 3. The dotted line presents the calculation disregarding the off-shell effects, the convolution model. The solid curve is a result of calculations with full formulas (6)–(8) with our basic set of parameters. The dashed curve shows

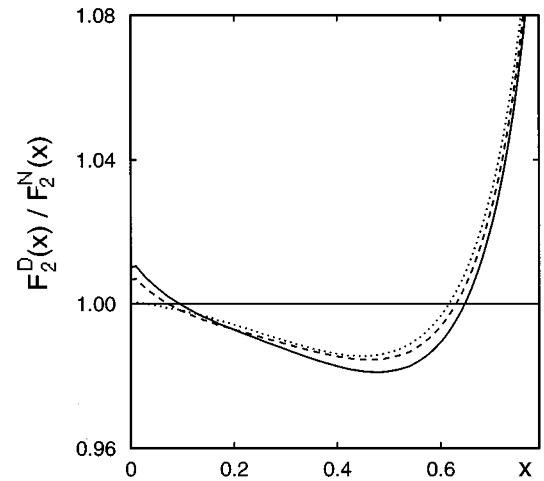

FIG. 3. The ratio of the deuteron structure function to the free nucleon structure function. Curves: solid curve, basic set of parameters; dashed curve, with additional form factor for the off-mass-shell nucleon ($\tilde{\Lambda}_N=1.65$ GeV); dotted curve, convolution model.

the effect of the extra form factor for the external line of the virtual nucleon. The additional form factor slightly decreases the off-shell effects, since it "holds" the nucleon closer to the mass shell. These results confirm our estimates presented at the beginning of the present section. They are also in qualitative agreement with earlier results [7,8] for $x>0.3$, where the structure function of the nucleon in the deuteron suffers additional suppression in comparison with the usual convolution model. This mechanism, indeed, can be complimentary to the binding effects in explaining the EMC effect.

Corresponding effective distribution functions of the pions are presented in Fig. 4. The "mean value" distribution for the deuteron (dotted curve) differs only slightly from the free nucleon distribution (solid line). At the same time the deuteron distribution is very similar to the distribution from

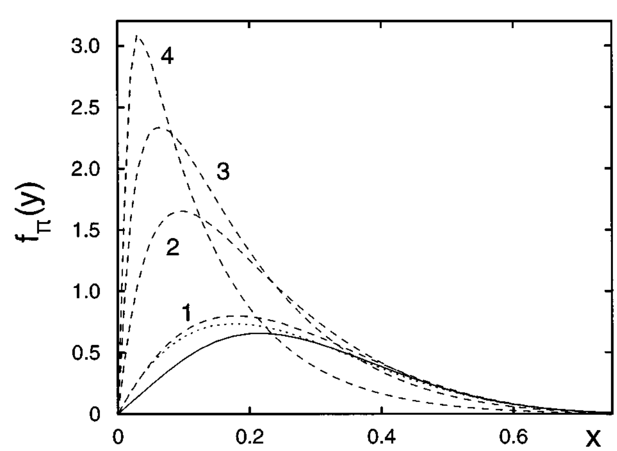

FIG. 4. The effective distribution of the pion in the nucleon (the basic set of parameters). Curves: solid curve, free nucleon; dotted curve, nucleon in the deuteron; dashed curve, nucleon in the semi-inclusive reaction with different spectator momentum, p_s (1, $p_s=0.1$ GeV/c; 2, $p_s=0.3$ GeV/c; 3, $p_s=0.5$ GeV/c; 4, $p_s=0.9$ GeV/c).

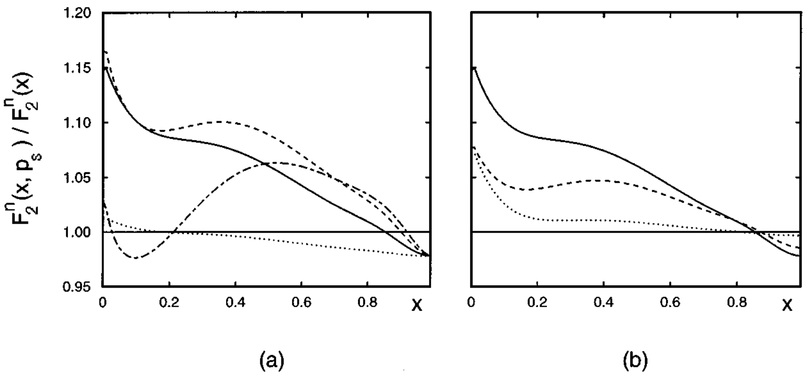

FIG. 5. The ratio of the neutron structure function, measured in semi-inclusive deep inelastic scattering on the deuteron, to the free neutron structure function. (a) Dependence on the spectator proton momentum. Curves are calculated with a basic set of parameters: dotted curve, $p_s = 0.1$ GeV/c; solid curve, $p_s = 0.3$ GeV/c; dashed curve, $p_s = 0.5$ GeV/c; dot-dashed curve, $p_s = 0.9$ GeV/c. (b) Sensitivity to the model assumptions. Curves for $p_s = 0.3$ GeV/c: solid curve, basic set of parameters; dashed curve, with additional form factor for the off-mass-shell neutron ($\tilde{\Lambda}_N = 1.65$ GeV); dotted curve, calculation with the distributions (23)–(25) with nucleon form factor inside the loop ($\Lambda_N = 1.475$ GeV).

the semi-inclusive reaction at the spectator momentum $p_s = 100$ MeV/c, because the mean value of the nucleon momentum in the deuteron is ~ 100–150 MeV/c, depending on the potential model.

Results for the semi-inclusive reaction with the proton spectator are shown in Fig. 5. Calculations for the ratio of the neutron structure function in the semi-inclusive reaction to the free neutron structure function are presented. This ratio is obtained after exclusion of the (i) flux factor $(1 + p_z/m)$ and (ii) weight $|\Psi(p)|^2$ from the total structure functions. The first one is a procedure well defined by the kinematics of the reaction; however, the second is rather ambiguous, since the wave function of the deuteron is strongly model dependent. We do this for methodological purposes, since it is worthwhile to compare the relative effects at different p_s. If there is no off-shell effect, such a ratio would be just a constant $\sim |\Psi(p)|^2$. Otherwise it will have a slope as shown in Fig. 5. All curves in Fig. 5 are scaled for comparison. Note also that the flux factors in the three matrix elements (16) are slightly different, and so after dividing by the factor $(1 + p_z/m)$, the structure functions remain dependent on the angle θ_s of the spectator momentum relative to $\mathbf{q}$. This dependence is too weak to discuss in relation to the possible experiments and, furthermore, it is not clear if this angular dependence is just an artifact of the nonrelativistic reduction. Here we choose $\theta_s = \pi/4$.

The dependence of the off-shell effects on the spectator proton momentum is shown in Fig. 5(a). There are two competing mechanisms, the increase of the structure function at small x and decrease at medium x. To understand such a behavior let us consider the effective distribution functions of the pions in the nucleon for the semi-inclusive reaction (Fig. 4), a steady increase of the pion distribution function at small x with an increase of the shift of the nucleon farther from mass shell. At the same time at medium and large x these distributions have a tendency to vanish. However, the calculations at very large spectator momentum $p_s \sim 1$ GeV/c, are probably beyond or very close to the boundary of the applicability of the pion-nucleon model and the potential model for deuteron.

Figure 5(b) shows estimates of some of the model ambiguities involved in our calculations. In particular, all calculations give qualitatively the same behavior of the off-shell effects. However, manipulation of the form factors may lead to a suppression of the magnitude of the effect. (Note that calculations for the dotted curve are made for the sake of illustration, since they break the energy-momentum conservation in the reaction.)

V. CONCLUSIONS AND COMMENTS

We have presented model calculations of the off-shell effects in deep inelastic scattering on bound nucleons.

(1) The truncated nucleon tensor has been calculated in the pion-nucleon model, motivated by the Sullivan model. The formulas explicitly contain the p^2 dependence and allow an analysis of off-shell effects in the nucleon structure functions.

(2) The nucleon contribution in the deuteron structure function F_2^D has been calculated using a nonrelativistic wave function. The off-shell corrections are found to be rather small, but they can be complimentary to the binding corrections in the explanation of the EMC effect.

(3) Semi-inclusive deep inelastic scattering on the deuteron has been considered in the spectator approximation with the proton spectator in the final state. It is found that this reaction provides new opportunities to study off-shell effects in the nucleon structure function. Even at nonrelativistic momenta of the spectator, the off-shell effects for the struck nucleon are larger than in inclusive deep inelastic scattering on heavy nuclei and order of magnitude larger than on the deuteron. This type of experiment would help to select models relevant to describe the structure functions of nucleons and, therefore, the nuclei.

We did not consider here the contributions of meson-exchange currents, which should be part of a consistent analysis of DIS on nuclei as a system of interacting nucleon and meson fields [9–11,24]. However, when the internal degrees of freedom of the nuclear constituents are "defrozen," such an analysis becomes a nontrivial problem, since it is not clear how the internal dynamics of the constituents interferes with the dynamics of the system. In any case, the physics here can be extremely interesting and there is much to learn about how to build a composite system from its composite constituents.

Another phenomenon not considered here and which can have an affect on the deuteron structure functions at very small x, say, $x<0.1$, is so-called nuclear shadowing [25,24]. These corrections would cancel (or partially cancel) the enhancement of the deuteron structure function (see Fig. 3), $x\to 0$, generated by the pions [24,26].

We would like also to make some comments about the pion (meson) physics in DIS on nuclei. This topic has as long a history as studies of nuclear effects in DIS, starting from the famous EMC effect [27]. At one point it was concluded that there are no excess pions in nuclei. It was based on simple estimates of the pionic contribution to the nuclear structure functions and probably a more correct conclusion has to be that something has been overlooked in these calculations. This point was recently reexamined in an interesting work [28]. Without going into details we would like to note that the physics here can be even more intricate. For instance, off-shell effects in the pion structure function can be significant as is found in [29] (see also [30]).

Our last comment is related to the state of experiments in this area. There is an interesting potential in the study of DIS on the deuteron in a semi-inclusive setup. In particular, the possibility to study the off-mass-shell behavior of the nucleon structure function is really unique. (Other interesting physics could be studied as well [21].) Such experiments, for instance, at CEBAF [31], would be beneficial both for the theory of nuclear effects in DIS and, perhaps, for the more fundamental theories (models) of the structure of the nucleon.

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APPENDIX

The light cone variables are

$$p=(p_0,\mathbf{p})=(p_+,p_-,\mathbf{p}_\perp), \quad p_\pm=p_0\pm p_z,$$

$$p^2=p_+p_--p_\perp^2, \quad pp'=\tfrac{1}{2}(p_+p'_-+p_-p'_+)-\mathbf{p}_\perp\mathbf{p}'_\perp \tag{A1}$$

$$d^4p=\tfrac{1}{2}dp_+dp_-d\mathbf{p}_\perp, \quad d\mathbf{p}_\perp=p_\perp dp_\perp d\alpha.$$

The explicit expressions for the functions $f_i^{N(\pi)}(y,p^2)$, $i=1,\ldots,3$, are

$$f_1^N(y,p^2)=-\frac{g^2h^2(p^2)}{16\pi^3}\frac{y}{1-y}\int_0^\infty dp'_\perp p'_\perp\int_0^{2\pi}d\alpha \times\frac{h^2(p'^2)}{(p'^2-m^2)^2}\left\{2yp^2+\frac{1}{y}[p'^2-m^2]-2pp'\right\}, \tag{A2}$$

$$f_2^N(y,p^2)=-\frac{g^2h^2(p^2)}{16\pi^3}\frac{y}{1-y}\int_0^\infty dp'_\perp p'_\perp\int_0^{2\pi}d\alpha \times\frac{-yh^2(p'^2)}{(p'^2-m^2)^2}, \tag{A3}$$

$$f_3^N(y,p^2)=-\frac{g^2h^2(p^2)}{16\pi^3}\frac{y}{1-y}\int_0^\infty dp'_\perp p'_\perp \times\int_0^{2\pi}d\alpha\frac{h^2(p'^2)}{(p'^2-m^2)^2}, \tag{A4}$$

$$f_1^\pi(y,p^2)=\frac{g^2h^2(p^2)}{16\pi^3}\frac{y}{1-y}\int_0^\infty dk_\perp k_\perp \times\int_0^{2\pi}d\alpha\frac{f^2(k^2)}{(k^2-\mu^2)^2}\{2yp^2-2pk\}, \tag{A5}$$

$$f_2^\pi(y,p^2)=\frac{g^2h^2(p^2)}{16\pi^3}\frac{y}{1-y}\int_0^\infty dk_\perp k_\perp\int_0^{2\pi}d\alpha\frac{(1-y)f^2(k^2)}{(k^2-\mu^2)^2}, \tag{A6}$$

$$f_3^\pi(y,p^2)=-\frac{g^2h^2(p^2)}{16\pi^3}\frac{y}{1-y}\int_0^\infty dk_\perp k_\perp\int_0^{2\pi}d\alpha\frac{f^2(k^2)}{(k^2-\mu^2)^2}, \tag{A7}$$

where $h(p^2)$ and $f(k^2)$ are the model form factors [see Eqs. (4)–(11)]; the $\pm$ components of p' and k are defined as follows:

$$p'_+=yp_+, \quad k_+=yp_+,$$

$$p'_-=\frac{1}{p_+(1-y)}[p^2-\mu^2-p_\perp'^2-yp_-p_++2p_\perp p'_\perp\cos\alpha], \tag{A8}$$

$$k_-=\frac{1}{p_+(1-y)}[p^2-m^2-k_\perp^2-yp_-p_++2p_\perp k_\perp\cos\alpha].$$

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