QCD scales in finite nuclei

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(Received 6 December 1995)

The role of QCD scales and chiral symmetry in finite nuclei is examined. The Dirac-Hartree mean-field coupling constants of Nikolaus, Hoch, and Madland (NHM) are scaled in accordance with the QCD-based prescription of Manohar and Georgi. Whereas the 9 empirically based coupling constants of NHM span 13 orders of magnitude, the scaled coupling constants are almost all *natural*, being dimensionless numbers of order 1. We argue that this result provides good evidence that QCD and chiral symmetry apply to finite nuclei. [S0556-2813(96)04506-2]

PACS number(s): 24.85.+p, 12.39.Fe, 21.10.−k, 21.30.Fe

Although QCD is widely believed to be the underlying theory of the strong interaction, a direct description of nuclear properties in terms of the *natural* degrees of freedom of that theory, quarks and gluons, has proven elusive. The problem is that at sufficiently low energy, the *physical* degrees of freedom of nuclei are nucleons and (intranuclear) pions. Nevertheless, QCD can be mapped onto the latter Hilbert space and the resulting effective field theory is capable in principle of providing a dynamical framework for nuclear calculations. This framework is usually called chiral perturbation theory (χPT).

Two organizing principles govern this χPT: (1) (broken) chiral symmetry (which is manifest in QCD) and (2) an expansion in powers of (Q/Λ), where Q is a general intranuclear momentum or pion mass, and Λ is the generic QCD large-mass scale ~ 1 GeV, which in a loose sense indicates the transition region between the two alternative sets of degrees of freedom indicated above (that is, quark-gluon versus nucleon-pion transitions). Typically, one constructs Lagrangians (that is, interactions) that display (broken) chiral symmetry and retains only those terms with exponents less than or equal to some fixed power of $(1/\Lambda)$. The chiral symmetry itself provides a crucial constraint: a general term has the structure $\sim (Q/\Lambda)^N$ and $N \geqslant 0$ is mandated. This guarantees that higher-order constructions in perturbation theory (viz., loops) will have even higher (not lower) powers of (Q/Λ). The price one pays for this mapping from *natural* to *effective* degrees of freedom is an infinite series of interaction terms, whose coefficients are unknown and must be determined from experiment.

To date only a few nuclear calculations have been performed within this framework. The seminal work of Weinberg [1] highlighted the role of power counting and chiral symmetry in weakening N-body forces. That is, two-nucleon forces are stronger than three-nucleon forces, which are stronger than four-nucleon forces, ... This chain makes nuclear physics tractable.

Van Kolck and co-workers [2] developed a chiral nuclear potential model, including one-loop (two-pion exchange) contributions. Friar and Coon [3] developed nonadiabatic two-pion-exchange forces, while van Kolck, Friar, and Goldman [4] examined isospin violation in the nuclear force. Park, Min, and Rho [5] were the first to treat external electromagnetic and weak interactions with nuclei. Nuclear photopion production was calculated by Beane, Lee, and van Kolck [6]. Threshold π^0 production in pp collisions has been recently treated in [7]. Most of this work was focused on few-nucleon systems, where computational techniques are sophisticated. Only was the work of Lynn [8] on (nuclear) chiral liquids specifically directed at heavier nuclei and, more recently, Gelmini and Ritzi [9] have calculated nuclear matter properties using lowest-order nonlinear chiral effective Lagrangians.

Is there any evidence for chiral symmetry or QCD scales in finite nuclei? The tractability and astonishing success of the recent few-nucleon calculations of ^{2}H, ^{3}H, ^{3}He, ^{4}He, ^{5}He, ^{6}He, ^{6}Li, and ^{6}Be with only a weak three-nucleon force and no four-nucleon force [10] confirms Weinberg's power-counting prediction [1] in light nuclei and yields strong but indirect evidence for chiral symmetry. The work of Lynn [8] established a procedure for going beyond few-nucleon systems. Nuclear (N-body) forces either have zero range or are generated by pion exchange. Following Manohar and Georgi [11] we can scale a generic Lagrangian component as

$$\mathcal{L} \sim -c_{lmn}\left[\frac{\bar{\psi}\psi}{f_\pi^2\Lambda}\right]^l \left[\frac{\vec{\pi}}{f_\pi}\right]^m \left[\frac{\partial^\mu, m_\pi}{\Lambda}\right]^n f_\pi^2\Lambda^2, \qquad (1)$$

where ψ and $\vec{\pi}$ are nucleon and pion fields, respectively, f_π and m_π are the pion decay constant, 92.5 MeV, and pion mass, 139.6 MeV, respectively, $\Lambda \sim 1$ GeV has been discussed above, and (∂^μ, m_π) signifies either a derivative or the pion mass. Dirac matrices and isospin operators (we use $\vec{t}$ here rather than $\vec{\tau}$) have been ignored. Chiral symmetry demands [12]

$$\Delta = l + n - 2 \geqslant 0. \qquad (2)$$

Thus the series contains only *positive* powers of $(1/\Lambda)$. If the theory is *natural* [8,11,13], the dimensionless coefficients c_{lmn} are of order 1. Thus, all information on scales ultimately resides in the c_{lmn}. If they are natural, scaling works. Our limited experience with nuclear-force models suggests that natural coefficients are the rule [2,4].

Unfortunately, zero-range nuclear-force models are not widely used. However, a recent calculation has been performed using zero-range forces for an extended range of mass number A and this work provides significant new information on QCD and chiral symmetry in nuclei. Nikolaus, Hoch, and Madland (NHM) [14] used a series of zero-range interactions to perform Dirac-Hartree calculations in a mean-field approximation for a total of 57 nuclei. Their Lagrangian (using their notation) is given by

$$\mathcal{L}=\mathcal{L}_{\rm free}+\mathcal{L}_{4f}+\mathcal{L}_{\rm hot}+\mathcal{L}_{\rm der}+\mathcal{L}_{\rm em}, \tag{3}$$

where $\mathcal{L}_{\rm free}$ and $\mathcal{L}_{\rm em}$ are the kinetic and electromagnetic terms, respectively, and

$$\mathcal{L}_{4f}=-\tfrac{1}{2}\alpha_S(\overline{\psi}\psi)(\overline{\psi}\psi)-\tfrac{1}{2}\alpha_V(\overline{\psi}\gamma_\mu\psi)(\overline{\psi}\gamma^\mu\psi) -\tfrac{1}{2}\alpha_{TS}(\overline{\psi}\vec{\tau}\psi)\cdot(\overline{\psi}\vec{\tau}\psi)-\tfrac{1}{2}\alpha_{TV}(\overline{\psi}\vec{\tau}\gamma_\mu\psi)\cdot(\overline{\psi}\vec{\tau}\gamma^\mu\psi), \tag{4}$$

$$\mathcal{L}_{\rm hot}=-\tfrac{1}{3}\beta_S(\overline{\psi}\psi)^3-\tfrac{1}{4}\gamma_S(\overline{\psi}\psi)^4 -\tfrac{1}{4}\gamma_V[(\overline{\psi}\gamma_\mu\psi)(\overline{\psi}\gamma^\mu\psi)]^2, \tag{5}$$

$$\mathcal{L}_{\rm der}=-\tfrac{1}{2}\delta_S(\partial_\nu\overline{\psi}\psi)(\partial^\nu\overline{\psi}\psi)-\tfrac{1}{2}\delta_V(\partial_\nu\overline{\psi}\gamma_\mu\psi)(\partial^\nu\overline{\psi}\gamma^\mu\psi). \tag{6}$$

In these equations, ψ is the nucleon field, the subscripts S and V refer to the isoscalar-scalar and isoscalar-vector densities, respectively, and the subscripts TS and TV refer to the isovector-scalar and isovector-vector densities, respectively, containing the nucleon isospin operator $\vec{\tau}$. The nine coupling constants of the NHM Lagrangian were determined in a self-consistent procedure that solved the model equations for several nuclei simultaneously in a nonlinear least-squares adjustment algorithm with respect to measured ground-state observables (Table IV of Ref. [14]). The predictive power of the extracted coupling constants is quite good both for other finite nuclei and for the properties of saturated nuclear matter (see Tables VIII, IX, and XI of Ref. [14]).

$\mathcal{L}_{4f}$ contains four two-nucleon-force terms corresponding to $\Delta=0$, and the first term of $\mathcal{L}_{\rm hot}$ is a three-nucleon-force term corresponding to $\Delta=1$, whereas the remaining two terms are four-nucleon-force terms corresponding to $\Delta=2$. Finally, $\mathcal{L}_{\rm der}$ contains two nonlocal two-nucleon-force terms, also corresponding to $\Delta=2$. The derivative terms act on $\overline{\psi}\psi$, rather than on just one of the fields, because the latter would generate a factor $E\cong M$, the nucleon mass, whereas the former generates an energy difference that is considerably smaller. The latter terms would spoil the series in Eq. (1), since $M\cong\Lambda$. However, either by a transformation or by rearranging the series, this problem could in principle be eliminated [8].

The construction of the NHM Lagrangian was motivated by empirically based improvements to a Walecka-type scalar-vector model [15,16], but using contact (zero-range) interactions to allow treatment of the Fock (exchange) terms.

TABLE I. Optimized coupling constants for the NHM Lagrangian and corresponding dimensional power-counting coefficients and chiral expansion order.

Constant	Magnitude	Dimension	c_{lmn}	Order
α_S	-4.508×10^{-4}	MeV^{-2}	-1.93	Λ^0
α_{TS}	7.403×10^{-7}	MeV^{-2}	0.013	Λ^0
α_V	3.427×10^{-4}	MeV^{-2}	1.47	Λ^0
α_{TV}	3.257×10^{-5}	MeV^{-2}	0.56	Λ^0
β_S	1.110×10^{-11}	MeV^{-5}	0.27	Λ^{-1}
γ_S	5.735×10^{-17}	MeV^{-8}	8.98	Λ^{-2}
γ_V	-4.389×10^{-17}	MeV^{-8}	-6.87	Λ^{-2}
δ_S	-4.239×10^{-10}	MeV^{-4}	-1.81	Λ^{-2}
δ_V	-1.144×10^{-10}	MeV^{-4}	-0.49	Λ^{-2}

It was not motivated either by power counting or by chiral symmetry. The pion degrees of freedom are ignored and the Lagrangian is not complete; additional operators in each order of $(1/\Lambda)$ are possible. Specifically, the NHM Lagrangian, Eqs. (4)–(6), has four operators in order $(1/\Lambda)^0$, one operator in order $(1/\Lambda)^1$, and four operators in order $(1/\Lambda)^2$, constituting an incomplete mix of three different orders in $(1/\Lambda)$.

Nevertheless, a meaningful comparison can be made of the generic chiral Lagrangian given by Eqs. (1) and (2) and the NHM Lagrangian given by Eqs. (4)–(6), precisely because our test of naturalness does not care whether a particular c_{lmn} coefficient has the value 0.5 or 2.0 or some other value near 1. Changing (refining) the model by adding terms would change *all* of the c_{lmn}, but the same test of naturalness still applies. Adding new terms would simply change a specific coefficient by an amount $\sim$1 (or less). That is, testing naturalness is largely and uniquely independent of the details, such as adding pions or performing more sophisticated nuclear calculations, provided the framework is given by Eqs. (1) and (2) while the physics is introduced via the measured observables of nuclei.

The nine coupling constants of the NHM Lagrangian are shown in Table I, both in dimensional and dimensionless form, the latter obtained by equating Eqs. (1) and (4)–(6), with $\Lambda=1$ GeV, using isospin operators $\vec{t}$ in Eq. (1), and solving for c_{lmn} in terms of α, β, γ, and δ. In the former form they span more than 13 orders of magnitude, while in the latter form 6 of the 9 coupling constants can be regarded as natural. Only the very small α_{TS} and large γ_S and γ_V are unnatural. However, the sum of the latter appears to be natural, and we speculate that the difference may not be well determined in the least-squares adjustments to the measured observables. The unnaturally small α_{TS}, if correct, would presuppose a symmetry to preserve its small value.

Other unpublished calculations by NHM with fewer coupling constants led to somewhat different numerical results for those constants, but application of the procedure above shows that they are mostly natural. This illustrates an important point: the various terms in the Lagrangian $\mathcal{L}$ are sensitive to different nuclear properties. Thus, small changes in the larger coefficients do not drastically alter the smaller ones. Another example of this is that Hartree-Fock calculations with contact interactions are nothing more than Hartree calculations with slightly modified coefficients, obtained by use of Fierz identities [17]. The α, β, γ, and δ terms do not mix between types, but only within each type. The results of [17] show that the exchange mixing within the α terms will not by itself destroy naturalness.

Having obtained these results using Dirac-Hartree calculations in a mean-field approximation with contact interactions, it is useful to compare them with corresponding calculations that use meson exchanges. Three of the four order $(1/\Lambda)^0$ coupling constants of Table I can be compared with the latter calculations by using the (approximate) relations

$$\alpha_S = -\left(\frac{g_\sigma}{m_\sigma}\right)^2, \tag{7}$$

$$\alpha_V = \left(\frac{g_\omega}{m_\omega}\right)^2, \tag{8}$$

$$\alpha_{TV} = \left(\frac{g_\rho}{m_\rho}\right)^2, \tag{9}$$

where (g_σ, m_σ), (g_ω, m_ω), and (g_ρ, m_ρ) are, respectively, the coupling constant and mass for the (fictitious) σ, ω, and ρ meson exchange. The three coupling constants and m_σ were determined from measured observables of finite nuclei in the calculation, whereas m_ω and m_ρ have their nominal values. Reinhard [18] has compiled the results of 12 such calculations. Evaluating Eqs. (7)–(9) for the 12 and taking their averages yields $\alpha_S = -3.93\times10^{-4}$, $\alpha_V = 2.78\times10^{-4}$, and $\alpha_{TV} = 3.65\times10^{-5}$. Clearly, these values compare well with the corresponding results in Table I, yielding another example of naturalness in finite nuclei.

Although these results were not obtained to test chiral symmetry and QCD scales and hence are imperfect, they are conversely completely unbiased. They are very indicative of the role of chiral symmetry and QCD in finite nuclei, and complement the work on chiral suppression of many-body forces in light nuclei. A systematic study of the former approach is clearly indicated.

The work of J.L.F. and D.G.M. was performed under the auspices of the U.S. Department of Energy.

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