$^{12}C+^{12}C$ and $^{16}O+^{12}C$ potentials by inversion

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In the light of the recent discovery that different forms of the S matrix produce equally good fits to heavy-ion scattering data, we use the simple Ericson parametrization of the phase shifts to analyze the experimental data and apply the Glauber approximation to evaluate the corresponding optical potential. [S0556-2813(96)02206-6]

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The recent literature shows a revival of interest in the determination of the optical potential describing elastic scattering of heavy ions. This is a consequence of the availability of techniques for solving the inverse scattering problem at fixed energy as well as advanced computer codes [1]. In this ''phenomenology by inversion,'' the choice of a smoothly l-dependent scattering matrix is used as basic input. The five-parameter McIntyre form [2] provides the best fit to the data. The S matrix, thus specified, is used to determine the local, l-dependent optical potential by an inversion procedure. The inversion is performed either with a full quantal calculation [3] or by means of a semiclassical WKB inversion scheme [4]. However, the problem of parameter ambiguity in the determination of the S matrix by fitting the experimental angular distribution has been a subject of several debates [5]. In particular, Steward *et al.* [6] have recently shown that elastic-scattering cross-section data of 1449-MeV ^{12}C on ^{208}Pb and 1503-MeV ^{16}O on ^{12}C can be equally well fitted by assuming several different forms for the dependence of the S matrix on angular momentum. The corresponding extracted optical potentials would have a diverse behavior which would have ''a decisive influence on conclusions about the physics involved.'' Indeed, different parametrizations of the S matrix have already been applied in the diffraction-model analysis of experimental data [7].

In this work, we hope to draw attention towards a simple parametrization suggested by Ericson [8] in the early stages of development of the diffraction model. It has the additional advantage of providing an analytical expression for the elastic-scattering cross section. This parametrization involves only three parameters, each one reflecting a specific aspect of the data. Therefore, the parameter values are ''unique'' in the sense that a slight variation in any of them produces a different quality of agreement with the experimental data. The inversion procedure applied to Ericson's parametrized phase shifts can be performed analytically using Glauber's expression [9] for the scattering amplitude, which has been successfully applied to analyze similar data. This formalism is used to deduce local potentials describing the angular distributions of the elastic scattering of ^{12}C by ^{12}C at incident energies ranging from 139.6 up to 2400 MeV [10] and of ^{16}O by ^{12}C at an incident energy of 608 MeV [11]. These data are interesting, since, as emphasized by Brandan, Fricke, and McVoy [12], they are extensive, offer a smooth scattering matrix as a function of l according to previous analyses, and display both the Fraunhofer and Fresnel diffraction patterns.

We start the proposed version of ''phenomenology by inversion of phase shifts'' (PIPS) with Ericson's parametrization of the nuclear S matrix [8]:

$$\exp(2i\delta_l)=\left[1+\exp\left(\frac{l_0-l-i\lambda}{\Delta}\right)\right]^{-1}, \tag{1}$$

where l_0 is the angular momentum of the surface grazing trajectory, Δ measures the smoothness of transition from complete absorption to complete transmission, and λ characterizes the reflection and refraction in the nuclear surface region. It has been shown [13] that the ratio λ/Δ is proportional to the ratio of the real to imaginary parts of the optical potential at the nuclear surface.

The advantages of the Ericson parametrization are the following.

(i) The S matrix given by Eq. (1) satisfies the unitarity condition $|S_l|\leq 1$ for all values of l as far as $\lambda/\Delta\leq\pi/2$. Therefore, there is no need to add a spline correction as done in Ref. [10].

(ii) Using the Watson-Sommerfeld transformation [8,14] and keeping only the two poles of the S matrix nearest to the real axis in the complex l plane, one obtains the following analytical expression for the elastic scattering differential cross section [15]:

$$\sigma(\theta)=N\ \csc\ \theta\ \exp(-2\pi\Delta\theta)\left\{\cos^2\left[\left(l_0+\frac{1}{2}\right)\theta+\frac{\pi}{4}+\arctan\frac{\pi\Delta}{l_0+\frac{1}{2}}\right]+\sinh^2(\pi\Delta\theta_c-\lambda\theta)\right\}, \tag{2}$$

where

$$N=8\pi\left(\frac{\Delta}{k}\right)^2\sqrt{(l_0+1/2)^2+(\pi\Delta)^2}\ \exp(2\lambda\theta_C),$$

k is the incident wave number, and θ_C is the Coulomb deflection angle:

$$\theta_C=2\arctan\frac{n}{l_0+1/2},\quad n=Z_1Z_2e^2/\hbar v$$

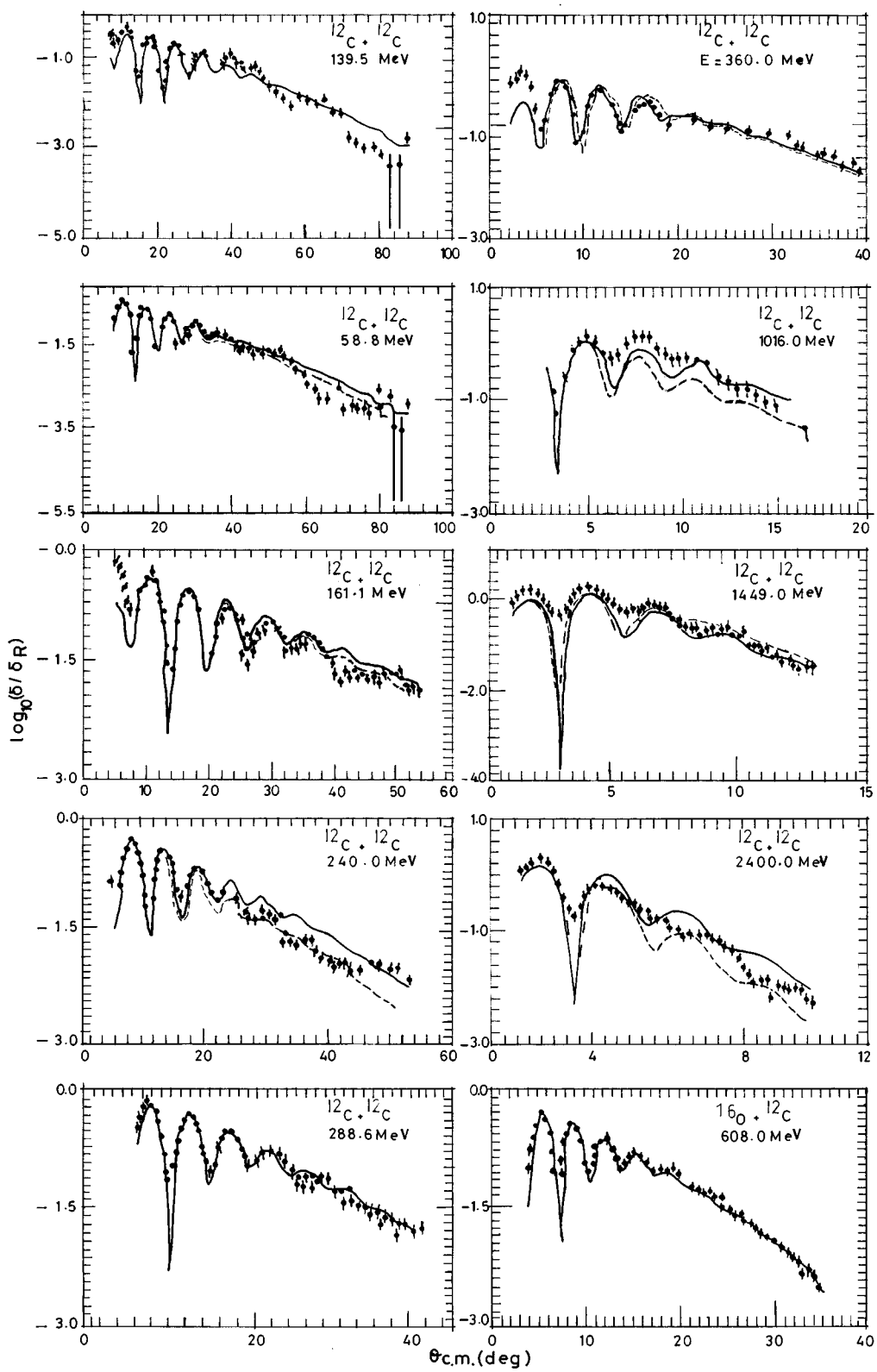

FIG. 1. Angular distributions of elastic scattering of ^{12}C and ^{16}O by ^{12}C. The dots are the data of Refs. [10,11]. The dashed curves are calculated using Eq. (2) with the best-fit parameter sets (columns 2–4 of Table I). The solid curves are calculated using the average parameter sets (columns 10–12 of Table I).

with $Z_{1,2}$ being the projectile and target atomic numbers, v their relative velocity, and e the electron charge. Comparison between the values of $\sigma(\theta)$ calculated by means of this expression with those obtained by the exact summation of partial waves with $S(l)$ given by Eq. (1) suggests [13] that this expression is accurate for scattering angles $\theta > \theta_C$.

(iii) The approximate expression for $\sigma(\theta)$ given by Eq. (2) has the following features which allow a ''unique'' de-

TABLE I. Three-parameter sets of parameters used to fit the angular distribution data for scattering of ^{12}C and ^{16}O by ^{12}C in Fig. 1. Columns 2–4 are the best-fit parameters extracted as explained in items (a)–(c) in the text. Columns 5–7 give the values of the strong absorption radius R_0, refraction coefficient ρ, and surface diffuseness a as given by Eqs. (4). Columns 8 and 9 give the average values of R_0 and a calculated using Eqs. (5). Columns 10–12 report the corresponding three-parameter sets obtained by substitution in Eqs. (4). Columns 13–16 give the values l_0 and Δ obtained by McEwan *et al.* [10] and by Charagi and Gupta [16].

E (MeV)	$l_0+1/2$	λ	Δ	R_0	ρ	a	$\overline{R_0}$	$\overline{a}$	$\overline{l_0+1/2}$	$\overline{\lambda}$	$\overline{\Delta}$	l_g	Δ_g	l^{eff}	Δ^{eff}
							$^{12}C+^{12}C$								
139.5	26.50	2.60	2.00	6.29	0.58	0.45	6.28	0.45	26.48	2.69	2.02	27.56	3.25	26.0	1.75
158.8	28.20	2.79	2.20	6.22	0.58	0.46	6.23	0.46	28.25	2.87	2.18	29.64	3.25	27.6	1.92
161.1	28.50	3.00	2.30	6.23	0.62	0.48	6.23	0.46	28.48	2.90	2.21	28.98	3.47		
240.0	34.50	3.63	3.02	6.08	0.62	0.51	6.06	0.48	34.38	3.53	2.80	34.37	3.21	33.1	2.56
288.6	38.40	3.98	3.25	5.96	0.60	0.49	5.94	0.49	38.25	3.98	3.25	38.10	5.55	35.7	2.74
360.0	39.50	4.60	3.60	5.62	0.64	0.50	5.85	0.50	41.10	4.33	3.50	38.29	6.03	39.1	3.26
1016	60.00	7.00	6.40	5.00	0.58	0.53	5.06	0.59	60.72	7.27	7.13	59.29	7.23	56.9	6.25
1449	69.70	8.40	8.80	4.85	0.58	0.61	4.68	0.63	67.27	8.68	9.12	68.56	9.05	64.4	7.99
2400	72.00	10.5	14.3	3.89	0.56	0.77	4.02	0.71	74.47	11.18	13.2	75.80	13.8		
							$^{16}O+^{12}C$								
608.0	53.50	6.40	4.80	5.10	0.60	0.45									

termination of the parameters l_0, λ, and Δ involved in the Ericson model.

(a) The cosine function in Eq. (2) is responsible for the oscillation pattern of the differential cross section. The ''average'' period of oscillation is equal to $\Theta=\pi/(l_0+1/2)$.

(b) The heights of the diffraction minima are proportional to the quantity $\sinh^2(\pi\Delta\theta_C-\lambda\theta)$. The deepest pair of minima thus occurs near the angle $\theta_{\min}=\pi\Delta\theta_C/\lambda$.

(c) At large values of θ, the contribution of the oscillating cosine function decreases compared to that of the monotonously increasing hyperbolic sine, which can be approximated by an exponential function. Therefore, when $\theta\gg\pi\Delta\theta_C/\lambda$,

$$\sigma(\theta)\sin\theta\approx\frac{N}{4}\exp(-2\pi\Delta\theta_C)\exp[2(\lambda-\pi\Delta)\theta]. \quad (3)$$

Thus, the slope of the exponential decay of the differential cross section at large angles is approximately equal to $2(\lambda-\pi\Delta)$.

We have used expression (2) to analyze the elastic scattering of ^{12}C by ^{12}C at incident energies ranging from 139.6 up to 2400 MeV [10] and of ^{16}O by ^{12}C at an incident energy of 608 MeV [11]. The quality of agreement of this expression with the experimental data of Refs. [10,11] is illustrated in Fig. 1. The parameters used in the fitting are listed in Table I. These parameters are also expressed in terms of the strong-absorption radius R_0, surface diffuseness a, and refraction coefficient ρ by means of the following semiclassical relations [7]:

$$kR_0=n+\sqrt{n^2+(l_0+1/2)^2},$$

$$ka=\Delta, \quad (4)$$

$$k\rho=\frac{\lambda}{\sqrt{n^2+(l_0+1/2)^2}}.$$

We see from the table that while ρ is varying very little around an average value of $\overline{\rho}=0.6$ fm, the parameters R_0 and a show a systematic dependence upon the incident energy. This behavior encouraged us to introduce the following unified ''three-parameter set'' that would fit the $^{12}C+^{12}C$ scattering angular distribution at all energies:

$$\overline{R_0}=7.0-0.16k \text{ fm},$$

$$\overline{a}=0.37+0.018k \text{ fm}, \quad (5)$$

$$\rho=0.6 \text{ fm}.$$

Figure 1 also shows the quality of fits achieved with those parameters obtained by substituting the three-parameter set (5) into Eqs. (4) and reported in Table I. We have then compared our values of $l_0+1/2$ and $\overline{a}$ with the corresponding values obtained by McEwan *et al.* (Tables 3 and 4 in [10]) and by Charagi and Gupta [16] by applying a Coulomb-modified Glauber model and using the strength of the effective nucleon-nucleon interaction as fitting parameters. Table I shows that our average parameters are in close agreement with those obtained by means of both the PIPS method of Ref. [10] and by the Coulomb-modified Glauber model [16].

Now, we consider the $S(l)\rightarrow V(r)$ inversion procedure using Glauber's eikonal approximation [9,17]. In the Glauber theory, the scattering amplitude is given by

$$f(\theta)=-ik\int_0^\infty J_0(2kb\sin\theta)[\exp\{2i\chi(b)\}-1]bdb, \quad (6)$$

where $J_0(x)$ is the Bessel function, b the impact parameter, and $\chi(b)$ the thickness profile defined by

$$\chi(b)=\frac{1}{\hbar v}\int_0^\infty\frac{V(r)rdr}{\sqrt{r^2-b^2}}. \quad (7)$$

Here $V(r)$ is the optical potential and v the relative velocity. Using the semiclassical relation $l\approx kb$ and regarding l as a

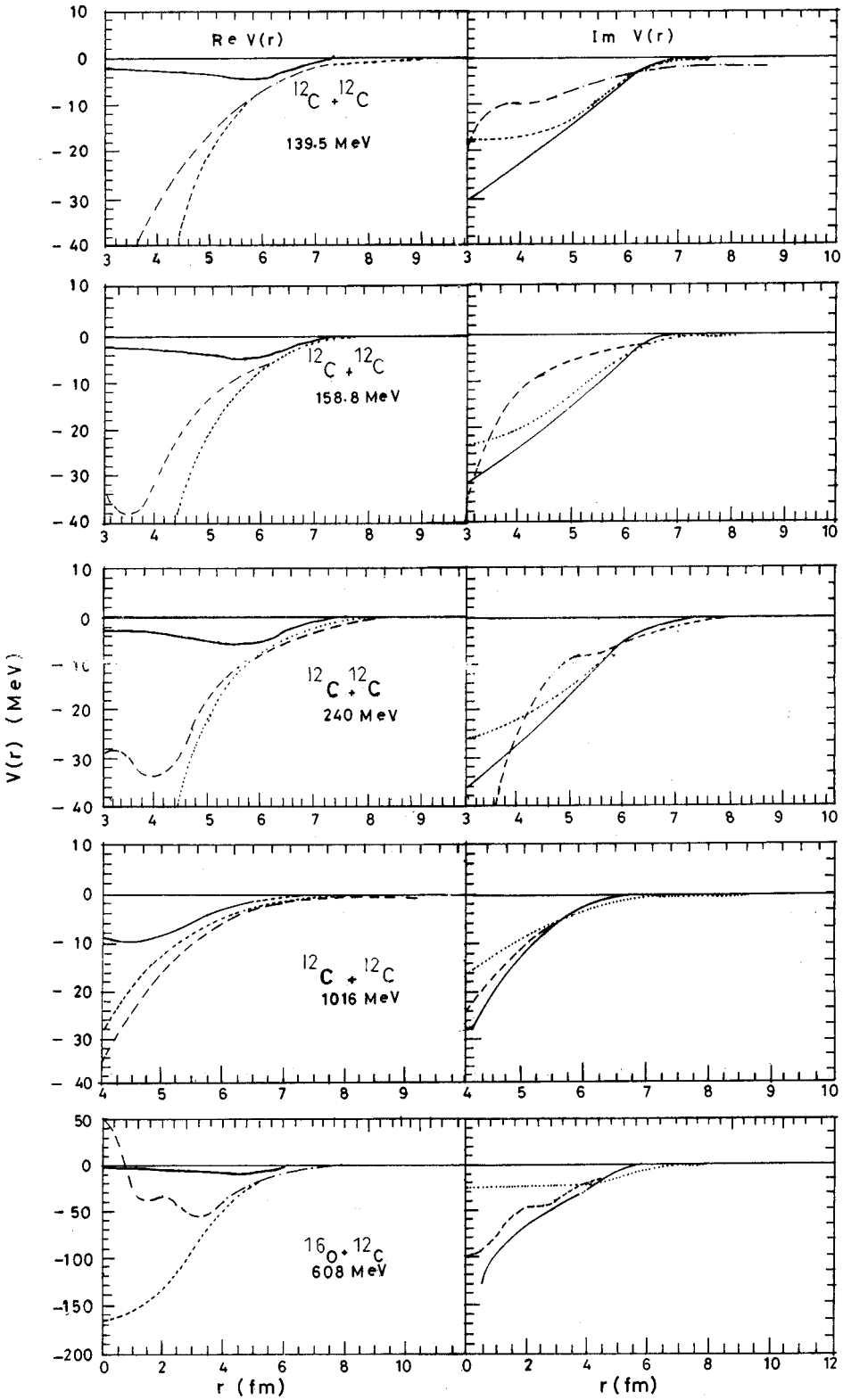

FIG. 2. Real (left column) and imaginary (right column) parts of the optical potential. The solid lines are calculated using Eq. (9). The dashed and dotted lines are taken from Refs. [10,11] and from Ref. [19], respectively.

continuous variable, we identify $\chi(b)$ with the phase shifts δ_l. Once a functional form is adopted for the phase shift, and we do this by means of Ericson's parametrization (1), and simplifying Eqs. (4) for the parameters to yield $l_0 \approx kR_0$ and $\lambda \approx k\rho$, the function $\chi(b)$ becomes

$$\chi(b) = \frac{i}{2} \ln\left[l + \exp\left(\frac{R_0 - b - i\rho}{a} \right) \right]. \tag{8}$$

Equation (7) can then be regarded as an Abel integral equation [18] whose solution is given by

$$V(r)=\frac{\hbar v}{2\pi}\frac{1}{r}\frac{d}{dr}\int_r^\infty \frac{\chi(b)bdb}{\sqrt{b^2-r^2}}=\frac{\hbar v}{\pi a i}\int_0^\infty \frac{du}{\sqrt{r^2+u^2}}\left[1+\exp\left(\frac{\sqrt{r^2+u^2}-R_0+i\rho}{a}\right)\right]^{-1}. \quad (9)$$

The integration in Eq. (9) has been carried out numerically using the parameters reported in Table I. A comparison of the real and imaginary parts of $V(r)$ with those of the Woods-Saxon potentials obtained in optical-model analyses and the potentials previously deduced using the PIPS model is shown in Fig. 2. We notice the following.

(1) The potentials obtained in the present paper agree with those obtained in Refs. [10,11] as well as the phenomenological Woods-Saxon potentials [19] near the strong-absorption radius. This can only be understood by recognizing the strong-absorption nature of the reactions under consideration, which renders the scattering insensitive to the value of the potential at small impact parameters. Satchler [5], for example, was able to fit the S-matrix data with both shallow and deep real optical potentials.

(2) While the real potentials obtained in Refs. [10,11] agree better with the real parts of the phenomenological Woods-Saxon potentials, our potential provides a better agreement with the imaginary parts.

(3) From the previous arguments, one may suggest that Eq. (9) provides a reasonable description for the imaginary optical potential beyond the strong-absorption radius. At $r\geqslant R_0+a$, the integral in Eq. (9) can be carried out approximately, and one obtains

$$\mathrm{Im}\ V(r)\approx-\frac{\hbar v\ \cos(\rho/a)\exp(R_0/a)}{4a\sqrt{2\pi}}\frac{\exp(-r/a)}{\sqrt{r/a}}. \quad (10)$$

This expression suggests the strength of the imaginary potential increases with energy roughly as $\sqrt{E}$ while its radial dependence enters through the ratio r/a. The latter suggestion leads to the identification of the diffuseness of the imaginary optical potential with the diffuseness parameter of the diffraction model as given by Eq. (4).

We finally note that the potential obtained in Eq. (9) is derived from Glauber's expression for the phase shifts. It is well known that the Glauber approximation for potential scattering can be obtained by using the WKB approximation with the additional assumption that $|V(r)|\ll E$. Both approximations are fairly valid beyond the strong-absorption radius and both break down for small values of r which influence low partial waves and where the phenomenological optical potentials are comparable in magnitude with the incident energy.

In summary, the present formalism calculates the optical potential of strongly absorbed projectiles starting with a parametrized S matrix as already done by the authors of Refs. [10,11]. However, it presents a method of determining the parameters of the phase shifts ''uniquely.'' The analytical formula (2) for the elastic-scattering differential cross section deduced by assuming Ericson's parametrization of the S matrix allows one to couple each parameter with a specific feature of the angular distribution. The average ''three-parameter set'' (5) could fairly well fit all the nine curves of the $^{12}C+^{12}C$ scattering angular distribution. The inversion procedure $S(l)\rightarrow V(r)$ is then carried out analytically, yielding a simple expression for the local optical potential in terms of these uniquely defined parameters.

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