g boson and systematics of the $M1$ scissors mode

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We discuss systematics of the $M1$ scissors mode within the interacting boson model when the g-boson degree of freedom is included explicitly and microscopically motivated choices of model parameters are adopted. We try to relate the $M1$ centroid energy to the energetics of deformation. We conclude that, with the introduction of a hexadecapole-hexadecapole interaction and a g-boson admixture in the ground state of only a few percent, we can obtain reasonable estimates of the $M1$ centroid energy, without invoking a Majorana interaction. If one takes seriously variations in microscopic estimates of boson g factors, then the summed $M1$ strength near midshell can be interpreted in terms of boson occupation numbers which saturate. [S0556-2813(96)05206-5]

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One of the triumphs of the interacting boson model (IBM) is its ability to account for the properties of the $M1$ scissors mode discovered after its introduction [1]. There are, however, disquieting features: First, there is the fact that to reproduce the excitation energies of 1^+ scissors states the somewhat artificial Majorana interaction is apparently required [2]; second, the summed strength does not appear to be consistent with the saturation of the groundstate d-boson occupation number expected near midshell [3] (the ''$M1$ saturation'' problem). In this paper, we present possible resolutions to these problems. Our primary result is that, if one includes a g-boson degree of freedom (in addition to the usual s and d bosons), then one can dispense with the use of the Majorana interaction while still adhering to microscopically motivated values of the model parameters. If one takes seriously variations in microscopic estimates of boson g factors near midshell, then the summed strength does admit interpretation in terms of boson occupation numbers which saturate.

In earlier work [4], we identified a deformation contribution $E_c^{\rm def}$ to the centroid energy arising from the dependence in the action of the (standard) quadrupole interaction $-\kappa\hat{Q}_p\cdot\hat{Q}_n$ on the neutron-proton (or F-spin) symmetry of states. The magnitude of this deformation contribution would have been inadvertantly underestimated in [2] because of the nonstandard (and microscopically implausible) F-spin scalar quadrupole interaction $-\kappa(\hat{Q}_p+\hat{Q}_n)\cdot(\hat{Q}_p+\hat{Q}_n)$ adopted. For the deformed Sm isotopes, we found that $E_c^{\rm def}$ almost certainly could account for a substantial fraction of the centroid energy (80% or so for the choice of model parameters made in [4]). In this paper, we attempt to understand the energetics of the $M1$ scissors state solely in terms of the deformation contribution $E_c^{\rm def}$. We believe this approach to be natural: Within the sdIBM-2, the $M1$ scissors mode may be viewed in the intrinsic frame as an F-spin isovector quadrupole excitation [5]; within the sdgIBM-2 (which we consider below), the scissors mode is a superposition of F-spin isovector quadrupole and hexadecapole excitations.

In most IBM studies of the $M1$ scissors mode, it is customary to work within the sdIBM-2 [6,7]. We, however, are lead to adopt the sdgIBM-2 for a variety of reasons. First, there is the specific finding that the influence of g bosons on global properties of the $M1$ mode can be substantial, because the contribution of a boson type is weighted by its spin squared. Second, we want to explore the impact on $E_c^{\rm def}$ of the hexadecapole-hexadecapole interaction between neutron and proton bosons. Third, there is the general consideration that the microscopic foundations of the sdgIBM-2 are more transparent than those of the sdIBM-2. It is possible within the sdgIBM-2 to reproduce a spherical-to-deformed ground state shape transition in an isotopic chain with essentially *constant* Hamiltonian parameters (not the case within the sdIBM) which are microscopically reasonable [8]. The fact that one can work with essentially constant parameters suggests that reliable microscopic estimates should be possible even in the regime of deformed nuclei.

The influence of the g boson on the $M1$ mode is illustrated by the generalization within the sdgIBM-2 of the Ginocchio sum rule for $M1$ strength [3]. Under the approximations that the neutron and proton boson g factors g_n and g_p are independent of the boson spin (supported by microscopic estimates [9]) and that the ground state is a state of maximal F spin (apparently accurate to within a few percent for deformed nuclei), the summed $M1$ strength [4]

$$\sum_i B(M1,0_1^+\to 1_i^+)=\frac{3}{4\pi}(g_p-g_n)^2\frac{P}{N-1}\sum_{l\,\rm even} l(l+1)n_l^{\rm g.s.}, \tag{1}$$

where $n_l^{\rm g.s.}$ is the ground-state occupation number of all bosons (both neutron and proton) of spin l and $P\equiv N_pN_n/N$ (as usual). In line with the assertion of the previous paragraph, the boson occupation numbers are weighted

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by the corresponding spin squared. This multipolarity weighting enhances the contribution to the summed $M1$ strength of any (presumably small) g-boson admixture in the ground state by a factor of 10/3 relative to the contribution from the d-boson admixture. Thus, even a small g-boson admixture of 5% or so can account for some (20–30%) percent of the summed $M1$ strength [4]. Multipolarity-weighting factors reappear below in Eq. (6) for E_c^{def}.

The unwieldiness of the sdgIBM-2 in comparison to the sdIBM-2 would seem a high price to pay for a more microscopically acceptable description of the $M1$ scissors mode. However, to discuss the summed strength and centroid energy of the $M1$ scissors mode, it is enough to evaluate ground-state expectation values. Furthermore, provided we confine ourselves to well-deformed nuclei, we can legitimately make two simplifying approximations about the character of the ground state: first, that it is of maximal F spin, and second, that ground state expectation values can be reliably estimated within the Hartree-Bose approximation. In this way, we reduce the problem at hand to one of manageable proportions. The requisite calculational techniques were developed in [10] and the viability of this approach has been confirmed by application to the Sm isotopes [4].

Within our approach, the summed $M1$ strength evaluates to [4]

$$\sum_i B(M1,0_1^+\rightarrow 1_i^+)=\frac{3}{4\pi}(g_p-g_n)^2 P\mathcal{S}_{M1}, \qquad (2)$$

where, to leading order in N (consistent with our neglect of angular momentum projection), $\mathcal{S}_{M1}$ concides with the average angular momentum squared

$$l_c^2\equiv\sum_{l\ \mathrm{even}} l(l+1)x_l^2 \qquad (3)$$

of a boson in the axially symmetric Hartree-Bose condensate (x_l is the probability amplitude that a condensate boson has spin l). For the sdgIBM-2 Hamiltonian $\hat{H}=\hat{H}_0+\hat{V}$ with the single d- and g-boson energies ϵ_l^ρ accommodated in

$$\hat{H}_0=\sum_{l,\rho}\epsilon_l^\rho \hat{n}_l^\rho \qquad (4)$$

[ρ distinguishes between neutrons ($\rho=n$) and protons ($\rho=p$)] and an attractive multipole neutron-proton interaction

$$\hat{V}=-\sum_k \kappa_k \hat{T}_p^{(k)}\cdot\hat{T}_n^{(k)} \qquad (5)$$

containing quadrupole ($k=2$) and hexadecapole ($k=4$) terms, the deformation contribution to the $M1$ centroid energy E_c is given by

$$E_c^{\mathrm{def}}/N=\frac{1}{2}\sum_k k(k+1)\kappa_k\mathcal{O}_p^{(k)}\mathcal{O}_n^{(k)}/\mathcal{S}_{M1}, \qquad (6)$$

where $\mathcal{O}_{p/n}^{(2)}$ ($\mathcal{O}_{p/n}^{(4)}$) is the expectation value of the quadrupole (hexadecapole) moment operator for a single proton/neutron condensate boson:

$$\mathcal{O}_\rho^{(k)}\equiv\sum_{j_1,j_2}\langle j_1 0 j_2 0|k0\rangle(t_\rho^{(k)})_{j_1j_2}x_{j_1}x_{j_2}, \qquad (7)$$

the $(t_\rho^{(k)})_{j_1j_2}$'s being the set of sdgIBM-2 parameters appearing in the multipole operator $\hat{T}_\rho^{(k)}$ and normalized so that $(t_\rho^{(k)})_{0k}\equiv 1\equiv(t_\rho^{(k)})_{k0}$. (Below, we shall drop the distinction between the parameters of $\hat{T}_n^{(k)}$ and $\hat{T}_p^{(k)}$.) Equation (6) is the generalization of the result of [4] for E_c on inclusion of a hexadecapole-hexadecapole interaction.

We now aim to evaluate E_c^{def}/N using a mixture of global microscopic estimates [11] of the sdgIBM-2 parameters required and empirical information [12] to fix $\mathcal{S}_{M1}$ and the Hartree-Bose wave function $\{x_l\}$. We believe that the global parameter estimates of [11] should suffice for a qualitatively reliable description of systematics. Estimates inferred from [11] of the difference $\epsilon_p-\epsilon_n$ in the energies of single proton and neutron bosons (of a given spin) seem to account at least qualitatively for the systematics of 2_1^+ g factors in heavy rare-earth nuclei [9]; as we now discuss, microscopic estimates in [9] of the difference $\delta g\equiv g_p-g_n$ in the g factors of proton and neutron bosons (of a given spin) seem to provide a resolution of the ''$M1$ saturation'' problem first pointed out in [3]. (We denote by g_p/g_n the average of the g factors of d and g proton/neutron bosons; microscopic estimates indicate that the dependence of boson g factors on the boson spin is small.)

The default choice of a microscopically plausible value for δg would be $\delta g\equiv 1$. However, the interpretation of values of $\mathcal{S}_{M1}$ extracted from summed $M1$ strength data is problematic with this choice of δg. Given its relation to the order parameter l_c^2 for deformation, one would expect $\mathcal{S}_{M1}$ to saturate as one approaches midshell (and deformation saturates): Instead, with the ''naive'' choice of $\delta g\equiv 1$, one finds that the extracted values of $\mathcal{S}_{M1}$ do *not* saturate as one approaches midshell, but show a marked decrease [cf. Fig. 1(a)]. (In the context of the theoretical analysis of this paper, this is the ''$M1$ saturation'' problem.)

This anomalous behavior in the values of $\mathcal{S}_{M1}$ extracted would seem to be an artifact of ignoring variations in δg. The global microscopic estimates of g_p and g_n in [9] suggest that, for heavy rare-earth nuclei, substantial variations in δg (of some 25% or so) from one isotope chain to the next or within some isotope chains are possible. When these estimates of g_p and g_n are used in the extraction of $\mathcal{S}_{M1}$ from summed $M1$ strength data *without* any fine adjustments whatsoever [the corresponding behavior of δg is plotted in Fig. 1(b)], we find that the values of $\mathcal{S}_{M1}$ obtained do appear to be consistent with saturation at a value of about $\mathcal{S}_{\mathrm{sat}}=2$ [cf. Fig. 1(a)].

Although we believe that the decreasing *trend* displayed by the estimates of δg in Fig. 1(b) is reliable, the value to which they converge near midshell is too high (the estimates in [9] omit the quenching of fermion g factors). We anticipate that a value of δg much closer to unity would emerge in calculations with more realistic input (e.g., quenched fermion g factors and a choice of surface-delta interaction consistent with a larger splitting in neutron and proton boson

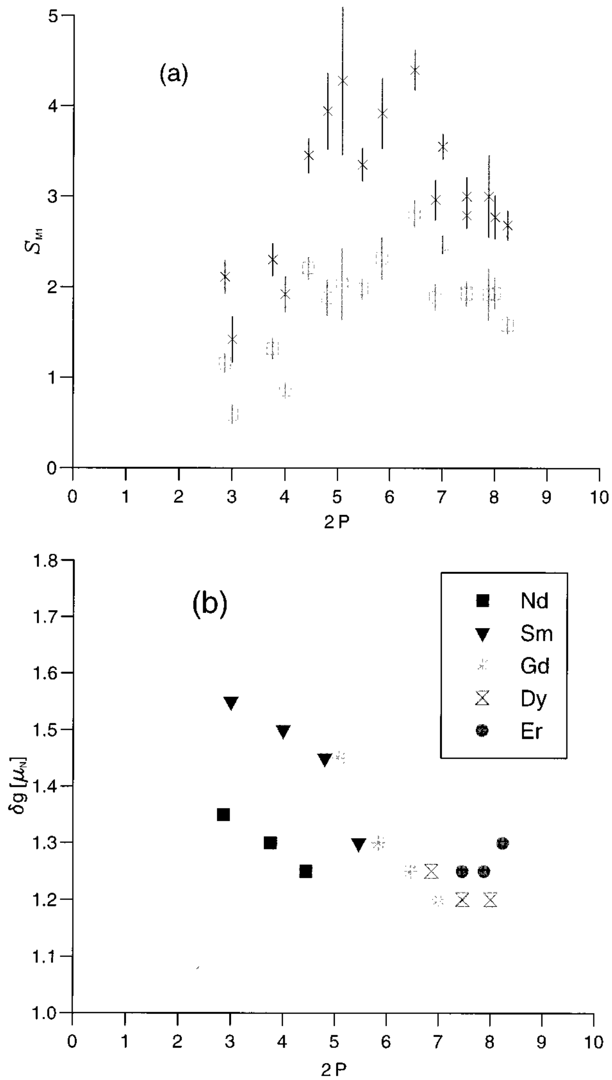

FIG. 1. Empirical values of $\mathcal{S}_{M1}$. (a) Values of $\mathcal{S}_{M1}$ extracted from summed $M1$ strength when a fixed value of $\delta g=1$ is used (crosses) and when varying microscopic estimates of δg are used (boxes). The error bars reflect only the experimental uncertainties in the summed $M1$ strength. (b) The varying microscopic estimates of δg used in (a) (inferred from Fig. 2 in [9]).

energies), implying a substantially higher saturation value of $\mathcal{S}_{M1}$. We shall take $\mathcal{S}_{\rm sat}\simeq 3$ (corresponding to a value of $\delta g \simeq 1$ near midshell).

Recently, a ''parmeter-free'' approach to estimating summed M1 strength based on the Ginocchio sum rule (within the sdIBM-2) has been shown to be quite successful, with perhaps even predictive power [13]. In this work, variations in δg are ignored [like Eq. (1), the Ginocchio sum rule implies that the summed strength is proportional to $(\delta g)^2$]; δg is set equal to unity from the outset. Our considerations above on the influence of variations in boson g factors suggest that it may be important to include these at least for $P<P_{\rm sat}$ ($\simeq 2.25$), i.e., the Nd and Sm isotopes. The probable outcome would be to worsen the agreement between theoretical and empirical estimates of the average quadrupole boson number for these isotopes, although any change may be compensated by a more careful treatment of the $B(E2)$ saturation parameter λ introduced in Eq. (4) of [13] (also set equal to unity). In any event, our findings on variations in boson g factors do not seem to be in conflict with the main thrust of the work in [13].

The saturation in the value of $\mathcal{S}_{M1}$ in Fig. 1(a) is an important result. We take it as evidence that the ground state wave function $\{x_l\}$ itself ''saturates'' for $P>P_{\rm sat}$. The values $x_2^{\rm sat}$ and $x_4^{\rm sat}$ at which x_2 and x_4, respectively, saturate can be related to $\mathcal{S}^{\rm sat}$ if one introduces the fraction p_g of the summed $M1$ strength due to the g-boson admixture in the ground state: In the limit of large N, $x_2^{\rm sat}=\sqrt{(1-p_g)\mathcal{S}_{\rm sat}/6}$ and $x_4^{\rm sat}=\sqrt{p_g\mathcal{S}_{\rm sat}/20}$. (The saturation value $x_0^{\rm sat}$ of the s-boson amplitude is obtained via the normalization condition $x_0=\sqrt{1-x_2^2-x_4^2}$.) We leave open for the moment the choice of p_g. In the absence of any g bosons ($p_g=0$), the saturation value $\mathcal{S}_{\rm sat}=3$ implies a d-boson content of well-deformed ground states of 50% $[(x_2^{\rm sat})^2=0.5]$, consistent with sdIBM phenomenology.

The sensitivity of $E_c^{\rm def}/N$ to p_g (and $\mathcal{S}_{\rm sat}$) will be investigated below. As regards the other input required for the evaluation of $E_c^{\rm def}/N$, the following general comments apply.

(1) $E_c^{\rm def}/N$ is insensitive to the precise values of the multipole parameters $\{t_\rho^{(k)}\}$ provided they are drawn from within the range of physically reasonable choices. This insensitivity has its origin in the fact that, for physically reasonable choices of the multipole parameters $\{t_\rho^{(k)}\}$, the dominant contribution to the condensate moment $\mathcal{O}_\rho^{(k)}$ comes from the term $2\,x_0x_k$.

(2) For $P>P_{\rm sat}$ (where the saturated wave function $\{x_l^{\rm sat}\}$ applies), $E_c^{\rm def}/N$ is a linear function of a neutron-proton interaction strength κ_k; $E_c^{\rm def}/N$ is more sensitive to the strengths κ_k than to the multipole parameters $\{t_\rho^{(k)}\}$.

(3) With the study of systematic trends in mind, we characterize the multipole parameters and neutron-proton interaction strengths to be used as functions of P. In fact, we are able to represent these parameters as linear functions of P (see below for details). Our estimate of $E_c^{\rm def}/N$ then emerges as a smooth function of P.

We now discuss our choices of multipole parameters and neutron-proton interaction strengths in more detail.

In our choice of the multipole parameters $\{t_\rho^{(k)}\}$, we take advantage of the insensitivity of $E_c^{\rm def}/N$. For simplicity, we drop the distinction between proton ($\rho=p$) and neutron ($\rho=n$) parameters. Instead, we work with the (F-scalar) averages $t_{j_1j_2}^{(k)}\equiv 1/2[(t_p^{(k)})_{j_1j_2}+(t_n^{(k)})_{j_1j_2}]$ of the global estimates in [11] of $\{t_p^{(k)}\}$ and $\{t_n^{(k)}\}$. In the domain of interest ($P\gtrsim P_{\rm sat}$), these averages can be represented approximately as linear functions of P [cf. Fig. 2]. We adopt the best straight-line fits in our subsequent evaluations of $E_c^{\rm def}/N$.

Following [11], the neutron-proton interaction strength κ_k is related to the strength $F^{(k)}$ of the corresponding shell-model interaction by the multiplicative renormalization $\kappa_k=\alpha_{kp}\alpha_{kn}F^{(k)}$ (in the notation of [11], this relation reads $K_{\pi\nu}^{(k)}=\alpha_{k\nu}\alpha_{k\pi}F_{\pi\nu}^{(k)}$). Perhaps surprisingly, greater uncertainties surround the effective shell-model strengths $F^{(k)}$ in the region of interest (the deformed region) than the multipole renormalization constants $\alpha_{k\rho}$, estimates of which are given in [11].

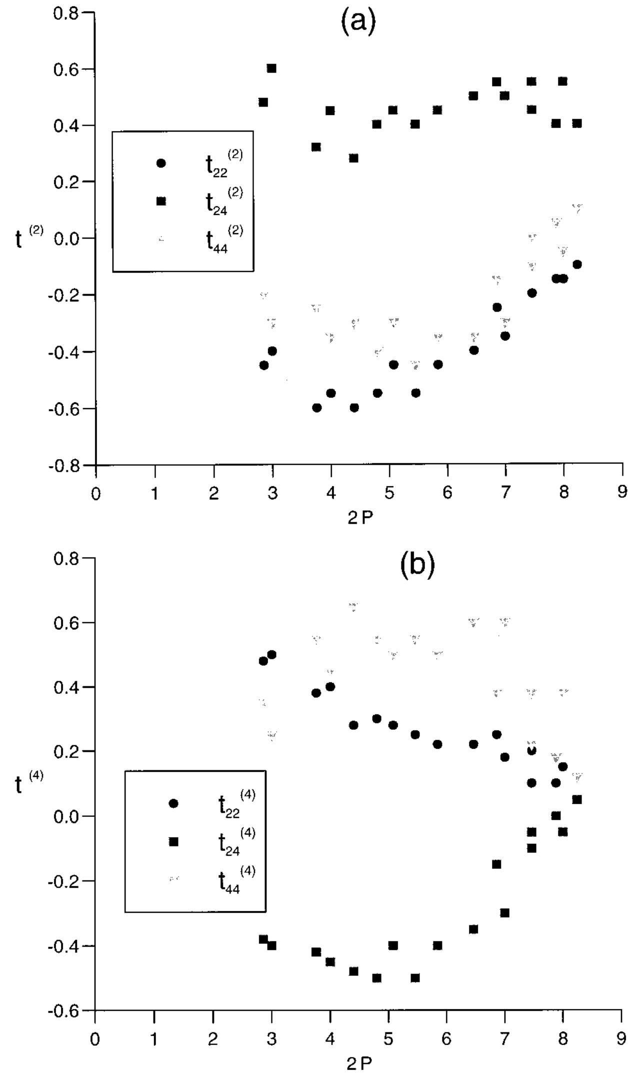

FIG. 2. Global microscopic estimates of the multipole parameters $\{t^{(k)}\}$. (a) Quadrupole parameters. (b) Hexadecapole parameters.

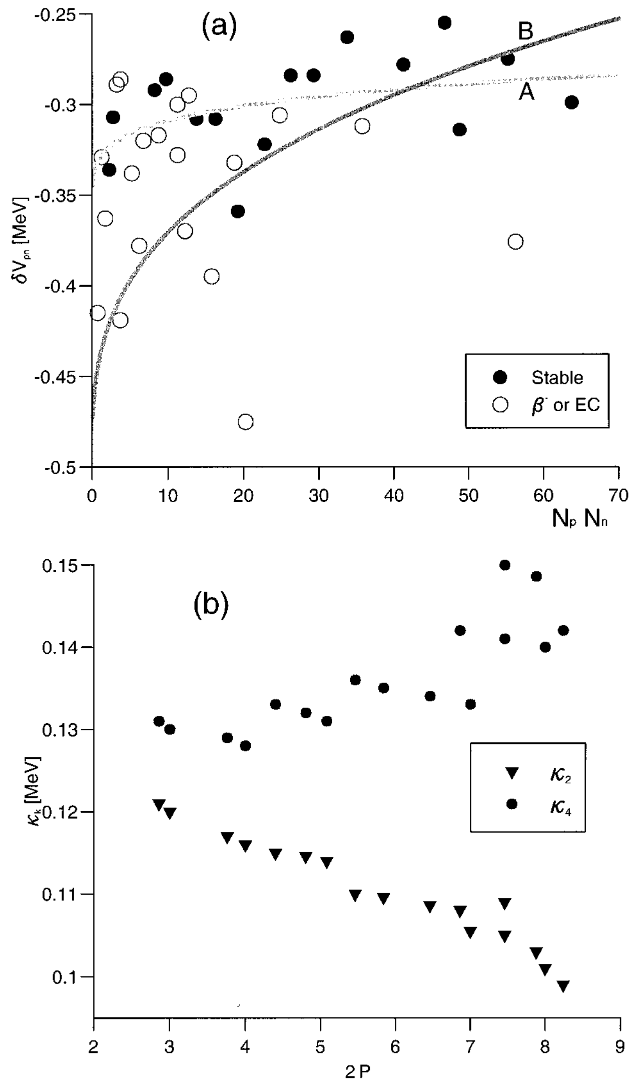

FIG. 3. Variation of interaction strengths. (a): The empirical neutron-proton interaction energy δV_{pn}. In distinguishing between values obtained from binding energy data on stable nuclei (solid circles) and unstable nuclei (open circles), we lump nuclei which undergo α decay together with stable nuclei (EC in the legend denotes that electron capture is the primary decay mode). See text for the significance of curves A and B. (b) The sdgIBM-2 interaction strengths κ_k.

Variations in effective shell-model parameters like $F^{(k)}$ through a major shell are anticipated on general grounds. More specifically, one expects the hexadecapole-hexadecapole interaction strength to be substantially weaker than the quadrupole-quadrupole interaction strength and both to decrease as one approaches midshell (although not necessarily in the same way). In lieu of input from microscopic shell-model studies (which are, of course, not tractable for the complex nuclei of interest), we draw on the empirical approach typified by the study in [14]. The extent of the variation in the interaction strengths $F^{(k)}$ can be gauged from the empirical interaction energy δV_{pn} between the last pair of protons and the last pair of neutrons (we adopt the notation of [14]): The *changes* observed in δV_{pn} reflect, in the first instance, changes in the quadrupole-quadrupole interaction strength (there is an equally important monopole-monopole contribution to δV_{pn} which is essentially constant and, presumably, a weak changing hexadecapole-hexadecapole contribution). For $N_pN_n>10$ (the domain of interest to us), a modest variation in the empirical neutron-proton interaction δV_{pn} is found [cf. Fig. 3(a) based on Table I in [14]], implying that changes in the quadrupole-quadrupole strength $F^{(2)}$ are weak.

The empirical approach also suggests that the neutron-proton interaction has a simple dependence on the product N_pN_n. For the sake of definiteness, we adopt the functional form $F^{(k)}=F_0^{(k)}[1-\alpha(N_pN_n)^{1/\delta}]$ advocated in [11]. (For simplicity, we tacitly assume that variations in the hexadecapole-hexadecapole interaction strength $F^{(4)}$ resemble those in the quadrupole-quadrupole interaction strength $F^{(2)}$.) In [11], $\alpha=0.12$ and $\delta=3$. We also set $\alpha=0.12$, but, on the basis of an eyeball comparison with the systematics of variations in the empirical neutron-proton interaction for $N_pN_n>10$ [cf. curve A in Fig. 3(a)], we employ $\delta=6$ [curve B in Fig. 3(a) depicts the variation associated with the alternative choice of $\delta=3$]. No significance should

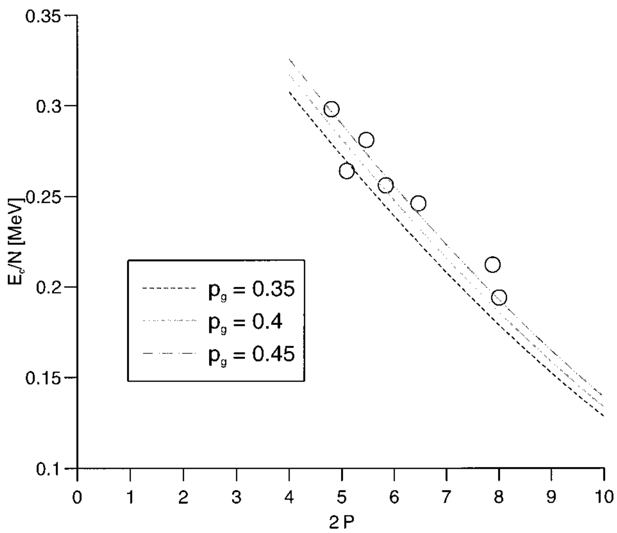

FIG. 4. Variation of $E_c^{\rm def}/N$ with P ($>P_{\rm sat}$). The curves are obtained by evaluating Eq. (6) for three choices of p_g listed in the legend; $\mathcal{S}_{\rm sat}=3$ throughout. Empirical values of E_c/N are plotted as open circles.

be attached to these particular values of δ and α: They serve to guarantee that the variation in $F^{(k)}$ is slight for $N_pN_n>10$. Our choice of the scale factors $F_0^{(k)}$ also differs from that adopted in [11]. We use $F_0^{(2)}=0.05$ MeV and $F_0^{(4)}=0.00177$ MeV. With these scale factors, the values of the κ_k's for ^{148}Sm, ^{150}Nd, and ^{196}Pt are comparable with those found in the detailed microscopically motivated parameter fits reported in [11].

With the above *Ansätze* for the $F^{(k)}$'s and the estimates in [11] of the multipole renormalization constants $\alpha_{k\rho}$, the neutron-proton interaction strengths κ_k are approximately linear functions of P for $P>1$ [cf. Fig. 3(b)]. As was the case with the multipole parameters $\{t^{(k)}\}$, we adopt the best straight-line fits when evaluating $E_c^{\rm def}$. Most of the variation seen in Fig. 3(b) is due to the product $\alpha_{kp}\alpha_{kn}$ of multipole renormalization constants. Observe that the *boson* interaction strengths κ_k are comparable despite the fact that the *fermion* hexadecapole-hexadecapole interaction strength $F^{(4)}$ is more than an order of magnitude smaller than the *fermion* quadrupole-quadrupole interaction strength $F^{(2)}$.

We now turn to the comparison for $P>P_{\rm sat}$ between $E_c^{\rm def}$ and empirical data on the $M1$ centroid energy E_c (taken from [15]). Barring p_g, all the inputs for $E_c^{\rm def}$ are fixed by the choices discussed above. Our previous work indicates that the saturation value for p_g should be non-negligible: more than 0.3 (cf. Table II in [4]). We find excellent agreement between $E_c^{\rm def}$ and empirical data on E_c (taken from [15]) for a value of p_g of slightly more than 0.4 (cf. Fig. 4). The hexadecapole interaction term in Eq. (6), which we have been at pains to include in this work, is important: Its contribution is never less than 40% of $E_c^{\rm def}$ for $p_g\simeq 0.4$. The corresponding g-boson admixture in the ground state is compatible with that found in previous studies [4,8]: $(x_4^{\rm sat})^2=0.06$.

The d-boson content of the ground state implied by the choice of $\mathcal{S}_{\rm sat}\simeq 3$ and $p_g\simeq 0.4$ is 30%, which is arguably a little low (in other sdgIBM-2 studies, the d-boson content saturates at a little more than 40%). In part, this low value is an artifact of our neglect of angular momentum projection, which has the consequence that, with a given wave function $\{x_l\}$, we overestimate $\mathcal{S}_{M1}$ and underestimate $E_c^{\rm def}$, or, if we use $\mathcal{S}_{M1}$ and $E_c^{\rm def}$ to constrain the wave function (as we have done), we underestimate x_2 (and presumably also x_4). This low value of $x_2^{\rm sat}$ may also reflect that our choice of $\mathcal{S}_{\rm sat}$ is inappropriate. Improved estimates of boson g factors would be desirable.

Our tentative conclusion is that we can account for the $M1$ centroid energy solely in terms of the deformation contribution $E_c^{\rm def}$ once we introduce a hexadecapole-hexadecapole interaction. It is clear that a stringent test of our attempt to account for the $M1$ centroid energy in terms of the deformation contribution $E_c^{\rm def}$ would be provided by independent input on reasonable choices of $\mathcal{S}_{\rm sat}$ and p_g. With this in mind, we are currently investigating whether the choices of $\mathcal{S}_{\rm sat}$ and p_g indicated by the present work permit a satisfactory description of ground-state–band properties of well-deformed nuclei (moments of inertia, static moments, and intraband transition probabilities). However, given the uncertainties inherent in the IBM parameters used in our estimates of $E_c^{\rm def}$, we still have some latitude (in particular, the sensitivity of the hexadecapole condensate moments $\mathcal{O}_\rho^{(4)}$ to the hexadecapole parameters $\{t_\rho^{(4)}\}$ may require more careful treatment). Moreover, we find it difficult to believe that the level of agreement between $E_c^{\rm def}$ and empirical data on E_c seen in Fig. 4 is fortuitous: Not only is our estimate of $E_c^{\rm def}$ the same order of magnitude as E_c, but it also apparently reproduces the systematic variation with P. In fact, a simple new phenomenological parametrization of the $M1$ centroid energy E_c is suggested by the comparison in Fig. 4: A polynomial fit to our estimate of $E_c^{\rm def}$ for $p_g=0.4$ yields the quadratic approximation $E_c/N = 0.484\ \text{MeV}-(0.0919\ \text{MeV})P + (0.00436\ \text{MeV})P^2$, applicable when $P\gtrsim P_{\rm sat}$. (To the naked eye, this quadratic approximation coincides with our estimate of $E_c^{\rm def}/N$, which is nominally a cubic in P.) If our optimism about our description of the energetics of the $M1$ scissors mode should prove unfounded, this approximate expression will, hopefully, still prove useful.

In previous sdgIBM studies it has been usual (presumably for the sake of simplicity) to include only a quadrupole-quadrupole interaction in the Hamiltonian. We believe our work shows that a hexadecapole-hexadecapole interaction should also be included. We claim that the need within the sdgIBM-2 [8] for Majorana-type interactions in the description of 1^+ states should then fall away. In line with the microscopic considerations of [16], the Majorana-type interactions in the sdIBM-2 compensate for the omission of the g-boson degree of freedom.

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