$M1$ properties of tungsten isotopes in the interacting boson model-2

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(Received 20 December 1995)

The $M1$ properties of even $^{182-186}$W isotopes are investigated in the interacting boson model-2 (IBM-2). The $E2/M1$ mixing ratios, g factors, and summed $M1$ strength are calculated. A least-squares fit of the excitation energies is used to fix the IBM-1 projected Hamiltonian parameters, while the F-spin-breaking terms are adjusted to reproduce the $M1$ properties of low-lying states. The influence of F-spin mixing on the summed $M1$ strength is studied using the coherent state technique in perturbation theory. When the standard boson g factors are used, the $M1$ properties of the low-lying states are described satisfactorily, but the summed $M1$ strengths are found to be larger than present experimental values. Possible g factor adjustment, which reconciles the calculated and experimental $M1$ strength, is discussed. [S0556-2813(96)03306-7]

PACS number(s): 21.60.Fw, 23.20.Gq, 23.20.Js, 27.70.+q

I. INTRODUCTION

Since the introduction of the interacting boson model (IBM) [1], there has been a considerable interest in studying the $M1$ properties of even-even nuclei in this model [2–5]. Whereas in the IBM-1 version of the model no $M1$ transitions are allowed, unless more than a one-body transition operator is used, the IBM-2 version allows such transitions among the low-lying states and, moreover, predicts the existence of a new class of states, which should have strong $M1$ transitions to the low-lying states. Such states, in particular the so-called scissors mode 1^+ states, have been found experimentally [6]. It should be noted that $M1$ transitions among the low-lying states in the IBM-2 are possible only if the F-spin symmetry of the Hamiltonian is broken. Quantities, such as $E2/M1$ mixing ratios, are very sensitive to the particular way, in which this symmetry is broken, and so they may provide restrictions on the parameters used in the model, which cannot be obtained from fitting the excitation energies and the strongest $E2$ transitions.

If a one-body $M1$ transition operator is employed, then one is left with two free parameters, the proton and neutron g factors. As the collective-state $M1$ transitions are believed to have orbital character, one expects $g_\pi \approx 1\mu_N$ and $g_\nu \approx 0\mu_N$, which is also supported from different microscopic calculations [2,7,8]. However, when the 2_1^+ state g factors are calculated, the maximal F-spin value $[(N_\pi/N)g_\pi + (N_\nu/N)g_\nu]$ often fails to give agreement with experiment, when the standard boson g factors are used [9]. To resolve this discrepancy, F-spin mixing can be called upon again. It has been demonstrated that, in particular, the difference between the proton and neutron single-d-boson energies affects the 2_1^+ state g factors considerably [8,10–12].

A recent, prominent $M1$ characteristic of deformed nuclei is the summed $M1$ strength measured for rare-earth nuclei [13]. When calculated in the IBM-2, it is found to be proportional to $(g_\pi - g_\nu)^2$. If the Hamiltonian is F-spin invariant, the summed $M1$ strength is given by the Ginocchio sum rule [14] and is proportional to the average number of d bosons in the ground state. On the other hand, F-spin breaking may affect the summed $M1$ strength [15,16]. Therefore, once one decides to study $M1$ properties using the IBM-2, as many characteristics as possible should be considered simultaneously.

The tungsten isotopes were studied previously using the IBM [17]; however, in that investigation mainly the excitation spectra and $E2$ properties were explored. In this paper we calculate the spectra, $E2$ transitions, $E2/M1$ mixing ratios, 2_1^+ g factors, and summed $M1$ strength for the tungsten isotopes ^{182,184,186}W. A least squares fit of the excitation energies is used to obtain the IBM-1-projected Hamiltonian parameters. The 2_1^+ state g factors and the delta $E2/M1$ mixing ratios are fitted by adjusting the F-spin-breaking parameters, mainly the difference of the d-boson energies $(\varepsilon_\pi - \varepsilon_\nu)$ and the quadrupole operator parameters $(\chi_\pi - \chi_\nu)$. This is a similar approach to that used successfully by Kuyucak and co-workers to reproduce the Pt and Os g factors and mixing ratios [11,12]. Next the summed $M1$ strength is evaluated. To understand the departure from the Ginocchio sum rule, calculated using the obtained ground-state mean value of n_d, we employ the coherent state approach and perturbation theory. In this way we are able to derive an analytical formula which explains the contributions of different F-spin-breaking terms to the $M1$ strength. We show that the F-spin breaking, obtained from the fit to the low-lying state $M1$ characteristics, improves the summed $M1$ strength agreement with the trend given by the measured points. However, the absolute values of the summed $M1$ strength obtained in the present calculation are greater than the experimental values, when the standard boson g factors are used. To get to the experimental points, g_π must be reduced by more than 10%.

In Sec. II we comment on a recent tungsten $E2/M1$ mixing ratio calculation by the Sussex group [18], in particular,

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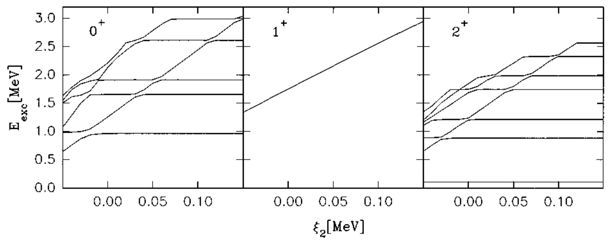

FIG. 1. Excitation energy dependence on the Majorana interaction parameter ξ_2. The lowest 0^+ and 2^+ states, as well as the 1_1^+ state are shown. The IBM parameters used are the same as those in Ref. [18], Fig. 5. More states are shown, however.

on the conclusion made there that a different energy dependence of the 1^+ and 2^+ mixed-symmetry states is obtained, when the Majorana interaction parameters are varied. In Sec. III we present our results for excitation energies, $E2$ values, $E2/M1$ mixing ratios, and the 2^+ g factors. The $M1$ strength results are presented in Sec. IV and conclusions are drawn in Sec. V.

II. DEPENDENCE OF MIXED-SYMMETRY STATE ENERGIES ON THE MAJORANA INTERACTION PARAMETERS

Recently an $E2/M1$ multipole mixing ratio calculation in ^{182,184,186}W was reported [18], in which good agreement with the experimental values was achieved by varying the Majorana strength parameters ξ_k. In the present calculation we take a more traditional approach of keeping the Majorana interaction fixed and changing other Hamiltonian parameters. We would like to comment, however, on a conclusion drawn in Ref. [18], namely, that a different dependence on ξ is obtained for the mixed-symmetry 1^+ state than for the mixed-symmetry 2^+ state. In particular, the conclusion was reached that the 2^+ mixed-symmetry state saturates and becomes constant at about 2 MeV, while the 1^+ state energy increases linearly with ξ_2. This aspect was discussed in Sec. III and demonstrated in Figs. 4 and 5 of Ref. [18]. In those figures, only the lowest four 2^+ states were shown. Using the same set of parameters as the Sussex group, we have recalculated the dependence given in Fig. 5 of Ref. [18] but for more excited states. Moreover, we have evaluated the maximal F-spin projector mean value for different states. The results of our calculation are shown in Fig. 1. Our findings disagree with those of Ref. [18]. The F-spin projection calculation confirms that the states for which the energy remains constant with the increasing ξ_2 are dominantly $F_{\max}$ states, whereas the states for which the energy increases linearly with ξ_2 are dominantly of mixed-symmetry nature. There is no saturation of the 2^+ mixed-symmetry state energy with increasing ξ. The energy dependence of different bands on the Majorana strength parameters can be understood using the coherent state technique, which we will apply in Sec. IV. Here, let us just mention that using a Majorana interaction defined as

$$M_{\pi\nu}=\xi_2(s_\nu^\dagger d_\pi^\dagger - s_\pi^\dagger d_\nu^\dagger)^{(2)}\cdot(s_\nu\tilde{d}_\pi - s_\pi\tilde{d}_\nu)^{(2)} -2\sum_{k=1,3}\xi_k(d_\nu^\dagger d_\pi^\dagger)^{(k)}\cdot(\tilde{d}_\nu\tilde{d}_\pi)^{(k)}, \tag{1}$$

one finds that the excitation energy dependence for the mixed-symmetry $K=0^+$ band is

$$E(F_{\max}-1,K=0^+)\approx N\xi_2$$

for the mixed-symmetry $K=1^+$ band is

$$E(F_{\max}-1,K=1^+)\approx\langle n_s\rangle\xi_2+\langle n_d\rangle(\tfrac{3}{5}\,\xi_1+\tfrac{2}{5}\,\xi_3),$$

and for the $K=2^+$ mixed-symmetry band is

$$E(F_{\max}-1,K=2^+)\approx\langle n_s\rangle\xi_2+\langle n_d\rangle\xi_3,$$

where $\langle n_s\rangle$ and $\langle n_d\rangle$ denote the average number of s and d bosons, respectively, in the coherent ground state. It should be noted that these dependences are reflected in Fig. 1.

III. $M1$ PROPERTIES OF LOW-LYING STATES

In the present calculation we use the standard IBM-2 Hamiltonian

$$H=\varepsilon_\pi n_{d\pi}+\varepsilon_\nu n_{d\nu}+k_{\pi\nu}Q_\pi\cdot Q_\nu+V_{\pi\pi}+V_{\nu\nu}+M_{\pi\nu}, \tag{2}$$

with

$$V_{\rho\rho}=\tfrac{1}{2}\,k_\rho Q_\rho\cdot Q_\rho+\sum_{L=0,2,4}\tfrac{1}{2}\,c_{L\rho}(d_\rho^\dagger d_\rho^\dagger)^{(L)}\cdot(\tilde{d}_\rho\tilde{d}_\rho)^{(L)}, \quad \rho\equiv\pi,\nu, \tag{3}$$

and

$$Q_\rho=(d_\rho^\dagger s_\rho+s_\rho^\dagger\tilde{d}_\rho)^{(2)}+\chi_\rho(d_\rho^\dagger\tilde{d}_\rho)^{(2)}, \quad \rho\equiv\pi,\nu. \tag{4}$$

The Majorana interaction $M_{\pi\nu}$ is given by Eq. (1).

The Hamiltonian parameters are fitted to obtain the excitation energies and the electromagnetic properties in the following way. A least-squares fit to the excitation energies of each isotope was attempted in the full IBM-2 calculation. Only six parameters, however, were varied in the fit, namely,

$$\varepsilon=\frac{N_\pi}{N}\varepsilon_\pi+\frac{N_\nu}{N}\varepsilon_\nu, \tag{5a}$$

$$\chi=\frac{N_\pi}{N}\chi_\pi+\frac{N_\nu}{N}\chi_\nu, \tag{5b}$$

$$k=k_{\pi\nu}, \tag{5c}$$

$$c_L=[N_\pi(N_\pi-1)c_{L\pi}+N_\nu(N_\nu-1)c_{L\nu}]/N(N-1), \quad L=0,2,4, \tag{5d}$$

while the differences

$$\Delta\varepsilon=\varepsilon_\pi-\varepsilon_\nu, \tag{6a}$$

TABLE I. IBM-2 Hamiltonian parameters used in the calculation for the ^{186,184,182}W isotopes. The boson numbers are $N_\pi=4$ and $N_\nu=7,8,9$, respectively. The Majorana interaction parameters are $\xi_1=\xi_2=\xi_3=\xi$. The χ parameters are dimensionless. The other parameters are given in MeV.

	^{186}W	^{184}W	^{182}W
ε_π	0.622	0.617	0.585
ε_ν	0.322	0.367	0.685
$k_{\pi\nu}$	-0.094	-0.100	-0.122
χ_π	-0.627	-0.730	-0.669
χ_ν	-0.327	-0.480	-0.819
$c_{0\pi}$	-0.437	-0.383	-0.289
$c_{0\nu}$	-0.357	-0.343	-0.289
$c_{2\pi}$	-0.260	-0.225	-0.202
$c_{2\nu}$	-0.180	-0.185	-0.202
$c_{4\pi}$	0.105	0.049	-0.060
$c_{4\nu}$	0.025	0.009	-0.060
ξ	0.12	0.10	0.09

$$\Delta\chi=\chi_\pi-\chi_\nu, \tag{6b}$$

$$\Delta c_L=c_{L\pi}-c_{L\nu}, \quad L=0,2,4, \tag{6c}$$

were kept constant throughout the fit and the Majorana interaction parameters were restricted by the condition $\xi_1=\xi_2=\xi_3$ and adjusted to yield the 1^+ state around 3 MeV. Further, we set $k_\rho=0$, $\rho\equiv\pi,\nu$. The $E2/M1$ mixing ratios and the 2_1^+ g factors are particularly sensitive to the differences (6). We tried to find the best combinations in order to reproduce the experimental data.

We use the $E2$ operator

$$T(E2)=e_\pi Q_\pi+e_\nu Q_\nu, \tag{7}$$

with the quadrupole operators given by Eq. (4), and the $M1$ operator

$$T(M1)=\sqrt{\frac{3}{4\pi}}(g_\pi L_\pi+g_\nu L_\nu), \tag{8}$$

with $L_\rho=\sqrt{10}(d_\rho^\dagger\tilde{d}_\rho)^{(1)}$, $\rho\equiv\pi,\nu$. The $E2/M1$ mixing ratio is defined by

$$\delta(E2/M1)=0.835E_\gamma[\mathrm{MeV}]\frac{\langle f||T(E2)||i\rangle}{\langle f||T(M1)||i\rangle}, \tag{9}$$

with the $E2$ matrix element given in e b and the $M1$ in μ_N. The g factor of a state $|k\rangle$ is obtained from

$$g_k=\frac{\langle k||T(M1)||k\rangle}{\langle k||(\sqrt{3/4\pi})(L_\pi+L_\nu)||k\rangle}. \tag{10}$$

The IBM-2 parameters obtained for ^{186,184,182}W are summarized in Table I. The boson numbers used are $N_\pi=4$ and $N_\nu=7,8,9$, respectively. The corresponding calculated and experimental energy spectra are shown in Figs. 2, 3, and 4. The $E2$ transitions with the quadrupole moments are shown in Table II and the g factors together with the $E2/M1$ mixing ratios are presented in Table III. For this calculation we used the standard boson g factors $g_\pi=1\mu_N$ and $g_\nu=0$. The $E2$ boson charges were chosen to reproduce the ground-state–band transitions. The adopted proton charge is $e_\pi=0.15$ e b, while the neutron charges are $e_\nu=0.13,0.12,0.11$ e b for ^{186,184,182}W, respectively.

It is apparent that the calculated spectra are in good agreement with the experimental ones. A characteristic feature of the present calculation is the appearance of nonzero c_L terms. Excluding those terms from the fit and setting them to zero would lead to a substantially worse description of the spectra. From Table I we observe that ε_π and χ_π remain almost constant for all the isotopes, while ε_ν and $|\chi_\nu|$ increase from ^{186}W to ^{182}W. The c_L parameters get reduced on average with increasing N_ν. We also get a reasonable description of the $E2$ transition and the quadrupole moments. Note that the 2_2^+ states in all the isotopes are the 2_γ^+ states,

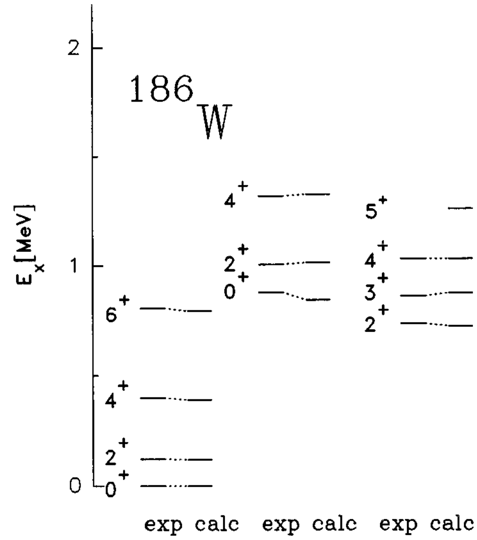

FIG. 2. Experimental and calculated excitation energies in MeV of ^{186}W. The Hamiltonian parameters are presented in Table I.

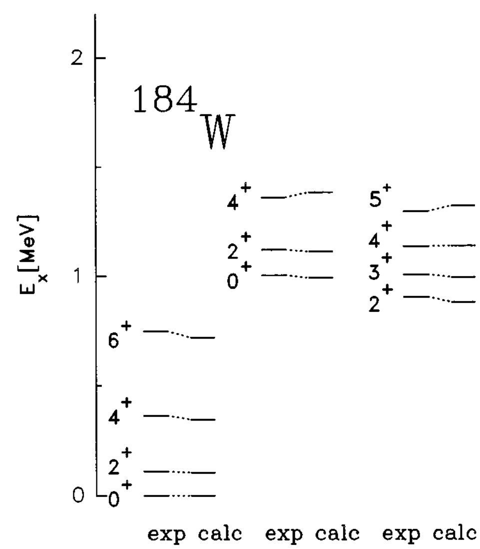

FIG. 3. The same as in Fig. 2 for ^{184}W.

TABLE II. Experimental and calculated $B(E2)$, in $e^2\,b^2$, and quadrupole moment, in e b, values for ^{186,184,182}W. The boson charges used are $e_\pi=0.15$ e b, and $e_\nu=0.13,0.12,0.11$ e b for ^{186,184,182}W, respectively. The experimental values are taken from Refs. [19–23].

	^{186}W		^{184}W		^{182}W	
$B(E2)$	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.
$2_1^+\to0_1^+$	0.705(16)	0.666	0.740(19)	0.746	0.839(18)	0.850
$4_1^+\to2_1^+$	0.914(63)	0.941	0.995(87)	1.051	1.201(61)	1.199
$6_1^+\to4_1^+$	1.123(82)	1.011	1.138(62)	1.127	1.225(135)	1.290
$2_2^+\to0_1^+$	0.056(5)	0.023	0.0274(14)	0.0132	0.021(1)	0.006
$2_2^+\to2_1^+$	0.127(6)	0.060	0.0522(25)	0.0274	0.041(1)	0.010
$2_2^+\to4_1^+$	-	0.0043	0.0030(2)	0.0023	0.00021(1)	0.00021
$2_3^+\to0_1^+$	-	0.0000	0.0013(2)	0.0003	0.006(1)	0.001
$2_3^+\to0_2^+$	-	0.464	-	0.564	1.225(368)	0.486
$2_3^+\to2_1^+$	-	0.0000	-	0.0004	0.0039(5)	0.0023
Q	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.
2_1^+	-1.57(30)	-1.623	$-1.98^{+0.06}_{-0.04}$	-1.738	$-2.00^{+0.04}_{-0.08}$	-1.864
2_2^+	1.3(3)	1.489	$2.36^{+0.10}_{-0.05}$	1.594	$1.94^{+0.10}_{-0.04}$	1.185

while the 2_3^+ states are the 2_β^+ states, both experimentally and in the calculation. We were able to reproduce the 2_1^+ g factors as well as most of the $E2/M1$ mixing ratios. In particular, all the signs are reproduced correctly. It should be noted that a sign change appears in both the $2_\gamma^+\to2_1^+$ and $2_\beta^+\to2_1^+$ transition mixing ratios, when going from ^{184}W to ^{182}W. Moreover, in ^{182}W there is an opposite sign between the $2_\gamma^+\to2_1^+$ mixing ratio and the $3_\gamma^+\to2_1^+$ and $3_\gamma^+\to4_1^+$ mixing ratios. We were able to reproduce all of these features in the calculation. Mainly, the sign change of $\Delta\varepsilon$ and $\Delta\chi$ for ^{182}W in comparison to ^{184}W is responsible for this effect. We also calculated the admixtures of lower F-spin states in the ground state. They are 1.4%,2.0%,1.1% for ^{186,184,182}W, respectively.

Let us also mention one more interesting point. There is a second excited $K=0^+$ band in ^{186}W (not shown in the figure) with the 0^+ energy 1.153 MeV and 2^+ energy 1.286 MeV. In the calculation we get a $K=0^+$ band with 0_3^+ at 1.200 MeV and 2_4^+ at 1.364 MeV. It is a candidate for the above-mentioned experimental band; e.g., the calculated $E2/M1$ mixing ratio $\delta(E2/M1;2_4^+\to2_1^+)=+6.3$ compares well with the experimental value of $+13^{+70}_{-6}$.

Recently, the properties of low-lying states in the tungsten isotopes have also been calculated within the context of the dynamic deformation model (DDM) [24]. In Ref. [24] the authors have mainly focused on an analysis of quadrupole moments, for which reasonable agreement is also obtained in the present IBM-2 calculations. Note that we obtain a reasonable value for the branching ratio $B(E2,2_\gamma^+\to2_1^+)/B(E2,2_\gamma^+\to4_1^+)$ in ^{182}W, which is considerably underestimated in the DDM. Unfortunately, only one $E2/M1$ mixing ratio is given in Ref. [24] (-43 for the $2_\gamma^+\to2_1^+$ transition in ^{186}W to be compared with the experimental value -11^{+3}_{-4} and the IBM-2 result -15.4). One should, however, keep in mind that the DDM approach is more microscopically motivated than the present phenomenologically oriented IBM-2 analysis.

TABLE III. Experimental and calculated $E2/M1$ mixing ratios and the g factors in μ_N for the ^{186,184,182}W isotopes. The standard boson g factors $g_\pi=1\mu_N$, $g_\nu=0$ were used. The $E2$ charges are the same as used in Table II. The experimental values are taken from Refs. [19–22].

	^{186}W		^{184}W		^{182}W	
$\delta(E2/M1)$	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.
$2_2^+\to2_1^+$	-11^{+3}_{-4}	-15.4	-16.9(6)	-17.1	$+30^{+6}_{-4}$	+20.3
$2_3^+\to2_1^+$	-	+2.2	+2.3(6)	+3.0	-9^{+3}_{-6}	-2.1
$3_1^+\to2_1^+$	-	-18.9	-13.2(9)	-20.3	-33^{+6}_{-9}	-19.4
$3_1^+\to4_1^+$	-	-7.1	-8.5(8)	-8.7	-8.9^{+18}_{-21}	-10.8
$4_2^+\to4_1^+$	-	-8.2	-6.3^{+32}_{-20}	-8.5	$+5.6^{+13}_{-10}$	+35.7
$4_3^+\to4_1^+$	-	+1.9	-	+1.5	-2.8(10)	-1.3
g	Expt.	Calc.	Expt.	Calc.	Expt.	Calc.
2_1^+	0.308(12)	0.314	0.288(7)	0.289	0.263(7)	0.251
2_2^+	0.20(4)	0.245	0.12(4)	0.189	-	0.175

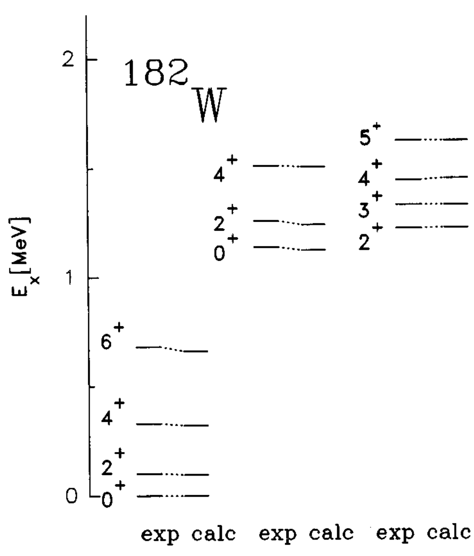

FIG. 4. The same as in Fig. 2 for ^{182}W.

IV. SUMMED $M1$ STRENGTH AND THE F-SPIN MIXING

In the previous section we studied how F-spin-breaking terms of the IBM-2 Hamiltonian influence g factors and $E2/M1$ mixing ratios. The Hamiltonian parameters were adjusted to obtain the best description of those quantities. If the IBM-2 Hamiltonian is F-spin invariant, then the summed $M1$ strength is given by the Ginocchio sum rule [14]

$$\sum_i B(M1,0_1^+\to 1_i^+)=\frac{18}{4\pi}(g_\pi-g_\nu)^2\langle n_d\rangle\frac{N_\pi N_\nu}{N(N-1)}. \quad (11)$$

It was noted already that F-spin breaking may influence the summed $M1$ strength in a non-negligible manner [15,16]. The summed $M1$ strength for the tungsten isotopes was recently measured [25]. An interesting feature of the summed $M1$ strength for the tungsten isotope is its reduction in comparison to other rare-earth nuclei, in particular those below the middle shell. In this section we report our results of a summed $M1$ strength calculation. Moreover, we discuss how the F-spin breaking influences the summed strength in the coherent state formalism, using the angular-momentum-projected $1/N$ expansion [26–28] and perturbation theory in an analogous way as it was used recently for the g factors [8] and $E2/M1$ characteristics [29].

The summed $M1$ strength is given generally by

$$\sum_i B(M1,0_1^+\to 1_i^+)=\frac{3}{16\pi}(g_\pi-g_\nu)^2\langle \text{g.s.},J=0^+|(L_\pi-L_\nu)^{(1)}\cdot(L_\pi-L_\nu)^{(1)}|\text{g.s.},J=0^+\rangle. \quad (12)$$

The Ginocchio sum rule (11) can be obtained up to the $1/N$ corrections in the intrinsic state formalism with the axially symmetric ground-state band projected from

$$|\text{g.s.},F_{\max},K=0^+\rangle=(N_\pi!N_\nu!)^{-1/2}(\Gamma_\pi^\dagger)^{N_\pi}(\Gamma_\nu^\dagger)^{N_\nu}|0\rangle, \quad (13)$$

where

$$\Gamma_\rho^\dagger=x_0 s_\rho^\dagger+x_2 d_{0\rho}^\dagger,\quad \rho\equiv\pi,\nu,\quad x_0^2+x_2^2=1. \quad (14)$$

In the case of F-spin breaking, the ground state (13) mixes with lower-F-spin $K=0^+$ states. There are two such states which contribute significantly to the summed strength, namely,

$$|F_{\max}-1,K=0^+\rangle=N^{-1/2}\left[\sqrt{\frac{N_\nu}{N_\pi}}\,\tilde{\Gamma}_\pi^\dagger\Gamma_\pi-\sqrt{\frac{N_\pi}{N_\nu}}\,\tilde{\Gamma}_\nu^\dagger\Gamma_\nu\right]|\text{g.s.},F_{\max},K=0^+\rangle, \quad (15)$$

where

$$\tilde{\Gamma}_\rho^\dagger=-x_2 s_\rho^\dagger+x_0 d_{0\rho}^\dagger,\quad \rho\equiv\pi,\nu, \quad (16)$$

and

$$|F_{\max}-2,K=0^+\rangle=[(N-1)(N-2)]^{-1/2}[\sqrt{N_\nu(N_\nu-1)}d_{1\pi}^\dagger d_{-1\pi}^\dagger|\Gamma_\pi^{N_\pi-2}\Gamma_\nu^{N_\nu}\rangle-\sqrt{(N_\pi-1)(N_\nu-1)}$$
$$\times(d_{1\pi}^\dagger d_{-1\nu}^\dagger+d_{-1\pi}^\dagger d_{1\nu}^\dagger)|\Gamma_\pi^{N_\pi-1}\Gamma_\nu^{N_\nu-1}\rangle+\sqrt{N_\pi(N_\pi-1)}d_{1\nu}^\dagger d_{-1\nu}^\dagger|\Gamma_\pi^{N_\pi}\Gamma_\nu^{N_\nu-2}\rangle]. \quad (17)$$

The state (15) is orthogonal to the β-band intrinsic state

$$|\beta,F_{\max},K=0^+\rangle=N^{-1/2}[\tilde{\Gamma}_\pi^\dagger\Gamma_\pi+\tilde{\Gamma}_\nu^\dagger\Gamma_\nu]|\text{g.s.},F_{\max},K=0^+\rangle. \quad (18)$$

On the other hand, the states that complete the two Tamm-Dancoff boson space of (17) are unphysical and are proportional to the rotations of the ground state and the $K=1^+$ state. For example, it can be checked that the projection of this $F_{\max}-1$ state on $J=0$ is zero. The situation here is analogous to the discussion by Leviatan in Ref. [30].

The mixing amplitude of the $F_{\max}-1$ state (15) into the $F_{\max}$ state (13) can be estimated perturbatively,

$$\frac{1}{E_{\text{g.s.}}-E_{F_{\max}-1}}\langle F_{\max}-1,K=0^+|H|\text{g.s.},F_{\max},K=0^+\rangle,$$

and similarly for the $F_{\max}-2$ state (17).

Up to $1/N$ corrections and terms linear in mixing amplitudes, the final formula estimating the departure from the Ginocchio sum rule due to the F-spin breaking is obtained after some manipulation,

$$\sum_i B(M1,0_1^+\to 1_i^+)=\frac{18}{4\pi}(g_\pi-g_\nu)^2\langle n_d\rangle\frac{N_\pi N_\nu}{N(N-1)}\left[1+2\frac{1}{E_{\text{g.s.}}-E_{F_{\max}-1}}\frac{x_0}{x_2}\frac{N_\nu-N_\pi}{N-2}H_{g1}+2\frac{1}{E_{\text{g.s.}}-E_{F_{\max}-2}}\frac{(N_\nu-1)(N_\pi-1)}{N-2}H_{g2}\right], \tag{19}$$

with $\langle n_d\rangle=x_2^2N$, and

$$H_{g1}=x_0x_2(\varepsilon_\pi-\varepsilon_\nu)-x_2^2N\sqrt{\tfrac{1}{14}}k_{\pi\nu}(\chi_\pi-\chi_\nu)-k_{\pi\nu}(N_\pi-N_\nu)x_2\left[2x_0^3-2x_2^2x_0+(\chi_\pi+\chi_\nu)\sqrt{\tfrac{1}{14}}\,x_2(x_2^2-3x_0^2)+\tfrac{2}{7}\chi_\pi\chi_\nu x_2^2x_0\right]+\tfrac{1}{2}k_\pi\left(x_0^2-x_2^2-\chi_\pi\sqrt{\tfrac{2}{7}}\,x_2x_0\right)\left[2(N_\pi-1)\left(2x_2x_0-\chi_\pi\sqrt{\tfrac{2}{7}}\,x_2^2\right)-\chi_\pi\sqrt{\tfrac{2}{7}}\right]-\tfrac{1}{2}k_\nu\left(x_0^2-x_2^2-\chi_\nu\sqrt{\tfrac{2}{7}}\,x_2x_0\right)\left[2(N_\nu-1)\times\left(2x_2x_0-\chi_\nu\sqrt{\tfrac{2}{7}}\,x_2^2\right)-\chi_\nu\sqrt{\tfrac{2}{7}}\right]+x_0x_2^3\left[(N_\pi-1)\left(\tfrac{1}{5}c_{0\pi}+\tfrac{2}{7}c_{2\pi}+\tfrac{18}{35}c_{4\pi}\right)-(N_\nu-1)\left(\tfrac{1}{5}c_{0\nu}+\tfrac{2}{7}c_{2\nu}+\tfrac{18}{35}c_{4\nu}\right)\right], \tag{20a}$$

$$H_{g2}=2k_{\pi\nu}\left[x_0^2-(\chi_\pi+\chi_\nu)\sqrt{\tfrac{1}{14}}x_0x_2+\chi_\pi\chi_\nu\tfrac{1}{14}x_2^2\right]-k_\pi\left(x_0^2-2\chi_\pi\sqrt{\tfrac{1}{14}}x_0x_2+\chi_\pi^2\tfrac{1}{14}x_2^2\right)-k_\nu\left(x_0^2-2\chi_\nu\sqrt{\tfrac{1}{14}}\,x_0x_2+\chi_\nu^2\tfrac{1}{14}x_2^2\right)+x_2^2\left[-\tfrac{1}{5}(c_{0\pi}+c_{0\nu})-\tfrac{1}{7}(c_{2\pi}+c_{2\nu})+\tfrac{12}{35}(c_{4\pi}+c_{4\nu})\right]. \tag{20b}$$

Equation (19) shows that the contribution to the summed $M1$ strength from the F-spin mixed components interferes with the main contribution from the $F_{\max}$ state. From Eq. (19) we can also deduce how different Hamiltonian F-spin-breaking terms affect the summed $M1$ strength. The dominant contribution comes from the quadrupole-quadrupole interaction, particularly if $k_\pi=k_\nu=0$. We can see that the $F_{\max}-2$ contribution is positive for $k_{\pi\nu}<0$. It leads to enhancement of the $M1$ strength. On the other hand, the c_L terms with values like those given in Table I dominate the $F_{\max}-1$ contribution and lead to a decrease of the $M1$ strength. Apparently, if $N_\pi\approx N_\nu$, the $F_{\max}-1$ contribution becomes unimportant. In general, the $\Delta\varepsilon$ and $\Delta\chi$ breaking has a small effect on the summed $M1$ strength.

Results of our numerical calculation, as described in Sec. III with the parameters given in Table I, of the summed $M1$ strength for ^{186,184,182}W are shown in Fig. 5. The solid line represents the calculation with the standard choice of the g factors $g_\pi=1\mu_N$, $g_\nu=0$, and the dashed line shows the Ginocchio sum rule (11) values calculated using the ground-state expectation $\langle n_d\rangle$ taken from our calculation. We get $\langle n_d\rangle/N=0.501,0.567,0.567$ for ^{186,184,182}W, respectively. We observe that the calculated $M1$ strength is higher than the experimental one. This aspect can be addressed in two ways: by reduction of $\langle n_d\rangle/N$ or by reduction of $g_\pi-g_\nu$. The mean value $\langle n_d\rangle/N$ may be reduced slightly if the quadrupole-quadrupole interaction among like nucleons is introduced to the Hamiltonian. One can obtain almost the same fit to the spectra and the same $E2$ transitions, using this interaction. However, the $M1$ properties are influenced by it. Choosing, e.g., $k_\pi=k_\nu=k_{\pi\nu}/4$ still preserves the correct $E2/M1$ mixing ratio signs. However, we were not able to get an overall description as good as the one without this interaction. Moreover, the $M1$ strength was not significantly reduced. Using the second option, reduction of $g_\pi-g_\nu$, one gets the points connected by the dotted line in Fig. 5. The new g factors $g_\pi=0.88\mu_N$ and $g_\nu=0.06\mu_N$ were chosen in such a way that the summed $M1$ strength of ^{182}W is reproduced and, simultaneously, the 2_1^+ g factors are almost unchanged. With this choice, the $E2/M1$ mixing ratios presented in Table III should be divided by 0.82.

We can see from Fig. 5 that the F-spin-breaking contribution improves the trend of the $M1$ strength in comparison with the Ginocchio sum rule values. Using our coherent state analysis, described above, we observe that the dominance of the c_L terms in ^{186,184}W leads to a decrease of the strength, whereas in ^{182}W the quadrupole-quadrupole interaction contribution becomes dominant due to the increase of $k_{\pi\nu}$, and, together with the sign change of c_4, contributes to the fact that the $F_{\max}-2$ contribution becomes positive for this isotope and enhances the strength. We checked the formula (19) by using the parameters and the energies of the $F_{\max}-1$ and

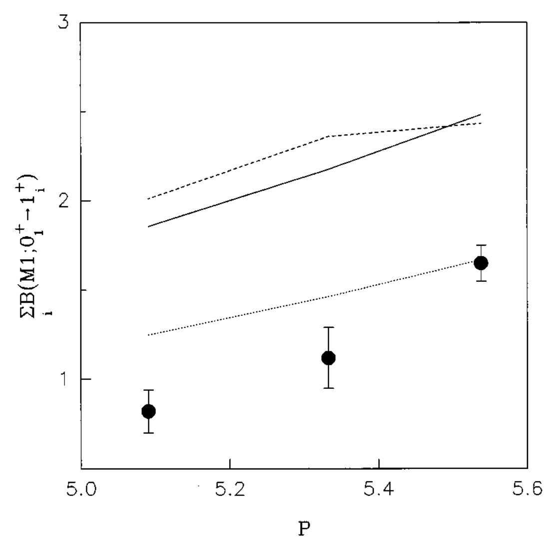

FIG. 5. Experimental and calculated summed $M1$ strength in μ_N^2 for ^{182,184,186}W as a function of the parameter $P=2N_\pi N_\nu/N$. The solid line corresponds to the calculation with $g_\pi=1\mu_N$, $g_\nu=0$, and parameters from Table I. The dashed line represents the Ginocchio sum rule result with the average d-boson number taken from the calculation. The dotted line corresponds to the calculation with $g_\pi=0.88\mu_N$ and $g_\nu=0.06\mu_N$.

$F_{max}-2$ states from the numerical calculations. For example, in the case of ^{182}W the $F_{max}-1$ 0^+ state corresponding to (15) is the 7th state with excitation energy 3.67 MeV, while the $F_{max}-2$ 0^+ state corresponding to (17) is the 22nd state with excitation energy 5.79 MeV. In the present calculations Eq. (19) reproduced the numerical calculation results up to 1%.

V. CONCLUSIONS

In this study we calculated $M1$ properties of the tungsten isotopes $^{182-186}$W in the IBM-2. The Hamiltonian parameters that are essential for obtaining correct excitation spectra were fitted by the least-squares method in a full IBM-2 calculation. On the other hand, the parameters to which the low-lying state g factors and $E2/M1$ mixing ratios are the most sensitive, such as $\Delta\varepsilon$, $\Delta\chi$, were adjusted so as to obtain the best results. Using this approach, we were able to reproduce satisfactorily the $M1$ properties of the low-lying states. At the same time we were able to make predictions for the 2^+_γ state g factors. A similar success was achieved recently by Kuyucak and co-workers in an analogous approach for the Pt and Os nuclei [11,12]. This may indicate that the IBM-2 is capable of describing the $M1$ properties of even-even rare-earth nuclei. We note that, unlike the excitation spectra and the strong $E2$ transitions, the $M1$ properties restrict significantly the Hamiltonian parameters.

We also calculated the summed $M1$ strength and studied its dependence on F-spin breaking using the coherent state formalism. The $M1$ strength magnitude is correlated with the deformation, and so F-spin breaking cannot affect it dramatically. We found, however, that the F-spin admixtures can change the summed $M1$ strength from the Ginocchio sum rule by about 10% with the ground state having 98% of the maximal F-spin component. The F-spin breaking essential for the correct description of the low-lying $M1$ properties improved the agreement of the summed $M1$ strength trend in comparison with the Ginochio sum rule. However, the calculated magnitude is too large when the standard boson g factors are used. To obtain the experimental value for ^{182}W we had to change the boson g factors to $g_\pi=0.88\mu_N$ and $g_\nu=0.06\mu_N$. With this choice, the calculated 2^+_1 g factors given in Table III remain almost unchanged. This change of the boson g factors may still be acceptable from the microscopic point of view; however, we do not have an immediate explanation for it. It appears that the reduction of the summed $M1$ strength for rare-earth nuclei above midshell is rather systematic. Clearly, more calculations, like the one presented here, are needed for other isotopes to see if the IBM-2 can explain the $M1$ properties consistently, or if some other mechanism beyond the scope of the model must be taken into account.

ACKNOWLEDGMENTS

This work was supported by the NSF Grant No. PHY93-21668 and by the Czech Republic Grants Nos. GA ASCR A1048504 and GA CR No. 202/93/2472.

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