# Roper excitation in $\alpha$ -proton scattering

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We study the Roper excitation in the  $(\alpha, \alpha')$  reaction. We consider all processes which may be relevant in the Roper excitation region, namely, Roper excitation in the target, Roper excitation in the projectile, and double  $\Delta$  excitation processes. The theoretical investigation shows that the Roper excitation in the proton target mediated by an isoscalar exchange is the dominant mechanism in the process. We determine an effective isoscalar interaction by means of which the experimental cross section is well reproduced. This should be useful to make predictions in related reactions and is a first step to construct eventually a microscopic  $NN \rightarrow NN^*$  transition potential, for which the present reaction does not offer enough information.

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# I. INTRODUCTION

We investigate theoretically the  $(\alpha, \alpha')$  reaction on a proton target at intermediate energies in order to obtain new information on the reaction mechanism and the properties of hadron resonances, especially the Roper resonance. The fact that the  $\alpha$  particle has isospin T=0 is particularly useful, since, due to isospin conservation, it reduces the number of reaction mechanisms which contribute to the reaction and allows an easier interpretation of the results.

The experimental study of the  $(\alpha, \alpha')$  reaction on the proton target was done in Ref. [1]. Two clear peaks were observed there: a large one, which was associated in Ref. [1] with  $\Delta$  excitation in the projectile (DEP), and a small one, at higher excitation energies, which was attributed in Ref. [1] to the Roper excitation in the target. This latter assumption requires the Roper resonance to be excited by the mediation of an isoscalar interaction which stimulated the authors of Ref. [1] to interpret the Roper resonance as a monopole excitation of the nucleon.

The idea of the DEP mechanism was suggested theoretically in Ref. [2] in connection with the (<sup>3</sup>He, *t*) reaction on nucleons and nuclei. It was found there that this mechanism produced small changes in the (<sup>3</sup>He, *t*) reaction on proton targets with respect to the dominant mechanism of  $\Delta$  excitation in the target (DET), but the changes were important in the reaction on neutron targets. Thanks to this mechanism, the excitation function of the (<sup>3</sup>He, *t*) reaction on deuteron targets [3] was well reproduced [4]. However, the clearest proof of the DEP mechanism was found in the experiment of Ref. [1] since, for reasons of isospin conservation, the DET mechanism is forbidden and all the strength for  $\Delta$  excitation comes from the DEP mechanism. A theoretical study was done in Ref. [5] along these lines and the large peak corresponding to  $\Delta$  excitation was nicely reproduced.

Another interesting aspect of the work of Ref. [5] is that it provides an accurate tool to evaluate the "background" of the  $(\alpha, \alpha')$  reaction which is necessary in order to obtain the strength for the Roper excitation. Given the fact that this background is much larger than the Roper signal, the precise determination of the background is important in order to assess the magnitude of the Roper excitation. In Ref. [1] some approximations and assumptions were done to determine the shape of the  $\Delta$  projectile contribution, and the strength was fitted to reproduce the peak. In Ref. [5] a more elaborate microscopic evaluation was done and both the shape and magnitude of the cross section were determined. As a consequence there are some differences (not too large) in the  $\Delta$  background evaluated in Refs. [1] and [5], and the strength of the Roper resonance at its peak is about 20% larger if the background of [5] is subtracted instead of the one in [1].

In the present paper we study the different mechanisms that can lead to the Roper excitation in the  $(\alpha, \alpha')$  reaction on the proton. However, instead of extracting the Roper signal by subtracting the  $\Delta$  background from the experimental cross section, we use the theoretical model of Ref. [5], which provides the  $\Delta$  excitation, and add to it the new mechanisms that excite the Roper resonance. This includes also the interference term between the target Roper and projectile  $\Delta$  excitations, which are found to be the dominant mechanisms. With this global model we obtain cross sections which are compared to the data in order to extract new information on the Roper resonance. We find that the reaction provides the strength of an effective isoscalar exchange for the  $NN \rightarrow NN^*$  transition.

In Sec. II we calculate all processes which may be relevant in the energy region of Ref. [1], namely, Roper excitation in the target, Roper excitation in the projectile, and double  $\Delta$  excitation processes. We compare the calculated results with experimental data in Sec. III. We summarize this paper in Sec. IV.

### II. MODEL FOR THE $(\alpha, \alpha')$ REACTION

In this section we consider a theoretical model of the  $(\alpha, \alpha')$  reaction on the proton target in the  $\Delta$  and Roper energy region. The reaction mechanisms which we consider here are summarized in Fig. 1. We include all processes which may be important in this energy region. In Fig. 1(a), we show the  $\Delta$  excitation in the projectile. Since the  $\Delta$  cannot be excited in the target [5], this is the only process to excite the single  $\Delta$  in the reaction. We can find the detailed description of the calculation and the results for this channel

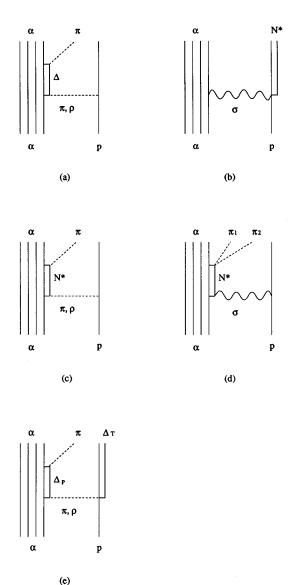




FIG. 1. Diagrams for the  $(\alpha, \alpha')$  reaction which we consider in this paper. They are (a)  $\Delta$  excitation in the projectile calculated in Ref. [5], (b) Roper excitation in the target, (c) Roper excitation in the projectile with decay into  $\pi N$ , (d) Roper excitation in the projectile with decay into  $\pi \pi N$ , and (e) double  $\Delta$  excitation. The  $\sigma$ exchange must be interpreted as an effective interaction in the T=0exchange channel (see text).

in Ref. [5]. All the other channels are new and they are discussed below.

We consider the diagrams for the Roper resonance excitation depicted in Figs. 1(b)–1(d). In Fig. 1(b) the Roper resonance is excited in the target by the exchange of some isoscalar objects between the  $\alpha$  and the proton. Because of isospin conservation of the  $\alpha$ , the isovector mesons ( $\pi$  and  $\rho$ ) do not contribute in this process. The cross section for this process is given by

 $\frac{d^2\sigma}{dE_{\alpha'} d\Omega_{\alpha'}} = \frac{p_{\alpha'}}{(2\pi)^3} \frac{2M_{\alpha}^2 M}{\lambda^{1/2}(s, M^2, M_{\alpha}^2)} \overline{\Sigma}\Sigma |T|^2 |G^*(s^*)|^2 \Gamma^*(s^*), \quad (1)$ 

where  $\lambda(\dots)$  is the Kallen function and  $G^*(s)$  is the propagator of the Roper resonance defined as

$$G^{*}(s) = \frac{1}{\sqrt{s} - M^{*} + \frac{i}{2}\Gamma^{*}(s)},$$
(2)

where  $M^*$  is the mass of the  $N^*$ ,  $M^* = 1440$  MeV, and  $\Gamma^*(s)$  is the energy-dependent Roper width [6],

$$\Gamma^*(s) = \Gamma^*(s = M^{*2}) \frac{q_{\text{c.m.}}^3(s)}{q_{\text{c.m.}}^3(M^{*2})},$$
(3)

with  $\Gamma^*(s=M^{*2})=350$  MeV and  $q_{c.m.}(s)$  the  $\pi$  momentum in the center of mass frame of  $\pi N$  system with the energy  $\sqrt{s}$ . Equation (3) assumes for the *s* dependence that the dominant decay channel is  $N^* \rightarrow \pi N$ . We will modify the width in the next section as described in the Appendix in order to be more consistent with the experimental data. In what follows, for simplicity, we construct a model assuming  $\sigma$  exchange alone as responsible for the isoscalar part of the  $NN \rightarrow NN^*$  transition. Further on we shall reinterpret the meaning of this phenomenologically derived " $\sigma$ " exchange. The spin sum and average of  $|T|^2$  can be written as

$$\overline{\Sigma}\Sigma|T|^2 = 16F_{\alpha g}^2 g_{\sigma NN^*}^2 g_{\sigma NN}^2 |D_{\sigma}(q)F_{\sigma}^2(q)|^2, \qquad (4)$$

where we are assuming couplings of the  $\sigma$  to the *N* and  $N^*$  of the type  $g_{\sigma NN}\bar{\psi}\psi\phi$  and  $g_{\sigma NN^*}\bar{\psi}_{N^*}\psi\phi$ +H.c. In Eq. (4),  $D_{\sigma}(q)$  is the propagator of the  $\sigma$  meson defined as

$$D_{\sigma}(q) = \frac{1}{q^{02} - \vec{q}^2 - m_{\sigma}^2},$$
(5)

with  $m_{\sigma} = 550$  MeV, and  $F_{\sigma}(q)$  is the  $\sigma$  form factor [7],

$$F_{\sigma}(q) = \frac{\Lambda_{\sigma}^2 - m_{\sigma}^2}{\Lambda_{\sigma}^2 - q^2},\tag{6}$$

with  $\Lambda_{\sigma} = 1700$  MeV. In Eq. (4),  $F_{\alpha}$  is the  $\alpha$ - $\alpha'$  transition form factor which includes the distortion effects and depends on the momentum transfer between  $\alpha$  and  $\alpha'$ . The form factor is the same as that explained in Ref. [5] and accounts for the distortion of the nucleon wave plus the distortion of a pion wave from the point of production of the pion. It thus implicitly assumes that the resonance will decay into the  $\pi N$  system. The pion distortion is slightly changed here. We use the same eikonal form as in Ref. [5] but take Im $\Pi = -p_{\pi}\sigma\rho$  with  $\sigma$  the  $\pi N$  experimental cross section and  $\rho$  the nuclear density. This is appropriate at the higher energies met in the present problem where the model of Ref. [5] is not meant to be applied. The  $\sigma NN$  coupling constant is taken from the Bonn potential [7],  $g_{\sigma NN}^2/4\pi = 5.69$ , and the  $\sigma NN^*$  coupling constant  $g_{\sigma NN^*}$  is an unknown parameter which we shall determine from the experimental data. We should, however, bear in mind that we are constructing an effective isoscalar interaction and those couplings have not to be taken literally as the meson baryon couplings of a microscopic model like in [7]. Yet it is useful to take  $g_{\sigma NN}$  as in the Bonn model since it already provides the appropriate scale of the interaction strength.

In order to get Eq. (1) we have replaced the energy conservation  $\delta$  function in terms of the Roper propagator and width as follows:

$$\delta(E_{\alpha} + E_N - E_{\alpha'} - E^*) \to \frac{\Gamma^*(s^*)}{2\pi} \frac{E^*}{M^*} |G^*(s^*)|^2, \quad (7)$$

so as to include all decay channels of the Roper resonance.

In the process shown in Fig. 1(c), the Roper resonance is excited in the projectile,  $\alpha$  particle, and decays into  $\pi N$ . The Roper resonance is excited by  $\pi$  and  $\rho$  exchange between the target and the projectile. We include both  $\pi^+$  and  $\pi^0$  for the final state. We can write the cross section as

$$\frac{d^2\sigma}{dE_{\alpha'} \ d\Omega_{\alpha'}} = \frac{p_{\alpha'}}{(2\pi)^5} \frac{M_{\alpha}^2 M^2}{\lambda^{1/2} (s, M^2, M_{\alpha}^2)} \int d^3p_{\pi} \frac{1}{E_{N'} \omega_{\pi}} \overline{\Sigma} \Sigma |T|^2 \delta(E_{\alpha} + E_N - E_{\alpha'} - E_{N'} - \omega_{\pi}).$$
(8)

The spin sum and average of  $|T|^2$  for this process is

$$\overline{\Sigma}\Sigma|T|^{2} = 48F_{\alpha}^{2} \left(\frac{f}{\mu}\right)^{2} \left(\frac{f'}{\mu}\right)^{4} |G^{*}(s^{*})|^{2} \{ [V_{l}^{\prime 2}(q) - V_{t}^{\prime 2}(q)](\vec{p}_{\pi \text{c.m.}} \cdot \hat{q})^{2} + V_{t}^{\prime 2}(q)\vec{p}_{\pi \text{c.m.}}^{2} \} \left(\frac{-q^{2}}{\vec{q}^{2}}\right), \tag{9}$$

where  $q = p_N - p_{N'}$ ,  $\vec{p}_{\pi c.m.}$  is the pion momentum in the Roper rest frame, and  $f^2/4\pi = 0.08$ , f' = 0.472 [6]. The factor  $(-q^2/\vec{q}^2)$  arises from the relativistic invariant  $\pi NN$  vertex [5].  $V'_l$ ,  $V'_t$  stand for the longitudinal and transverse parts of the  $NN \rightarrow NN^*$  interaction. We have taken

$$V_l'(q) = \left(\frac{\vec{q}^2}{q^{02} - \vec{q}^2 - \mu^2} F_{\pi}^2(q) + g'\right), \qquad (10)$$

$$V_t'(q) = \left(\frac{\vec{q}^2}{q^{02} - \vec{q}^2 - m_\rho^2}F_\rho^2(q)C_\rho + g'\right), \qquad (11)$$

where  $F_{\pi}(q)$  and  $F_{\rho}(q)$  are the pion and  $\rho$ -meson form factor in the form of Eq. (6) with  $\Lambda_{\pi} = 1300$  MeV and  $\Lambda_{\rho} = 1400$  MeV,  $C_{\rho} = 3.96$  [7], and g', the Landau-Migdal parameter, is taken to be 0.60. The momentum q in Eqs. (10), (11) is taken in the Roper rest frame [5]. The invariant mass  $\sqrt{s^*}$  of the Roper resonance is approximated to be

$$s^* = (q^0 + M)^2 - \left(\frac{\vec{q} + \vec{p}_{\pi}}{2}\right)^2,$$
 (12)

using the momentum variables in the  $\alpha$  rest frame [5]. In this approximation the momentum transfer is shared equally by the initial and final nucleon in the  $\alpha$ .

Now we consider the process of Fig. 1(d), the projectile Roper excitation which decays into the nucleon and the two pions in the T=0, S-wave channel, which carries a certain fraction of the Roper width [8]. We have again only the isoscalar exchange contribution because of isospin conservation, which is accounted for by means of the effective  $\sigma$ exchange used for diagram (b). The cross section can be expressed as

$$\frac{d^2\sigma}{dE_{\alpha'}\ d\Omega_{\alpha'}} = \frac{p_{\alpha'}}{2(2\pi)^8} \frac{M_{\alpha}^2 M^2}{\lambda^{1/2}(s, M^2, M_{\alpha}^2)} \int d^3p_{\pi_2} \frac{1}{\omega_{\pi_2}} \int d^3p_{\pi_1} \frac{1}{E_{N'}\omega_{\pi_1}} \overline{\Sigma}\Sigma |T|^2 \delta(E_{\alpha} + E_N - E_{\alpha'} - E_{N'} - \omega_{\pi_1} - \omega_{\pi_2}).$$
(13)

The spin sum and average of  $|T|^2$  is now

$$\overline{\Sigma}\Sigma|T|^{2} = \frac{3}{2}64F_{\alpha}^{2}C^{2}g_{\sigma NN}^{2}g_{\sigma NN*}^{2}|G^{*}(s^{*})|^{2}|D_{\sigma}(q)F_{\sigma}^{2}(q)|^{2},$$
(14)

where C is the coupling constant of the  $N^* \rightarrow N + 2\pi$  decay and  $C = -2.66\mu^{-1}$  [6]. The variable  $s^*$  is obtained in a similar way as in Eq. (12),

$$s^* = (q^0 + M)^2 - \left(\frac{\vec{q} + \vec{p}_{\pi_1} + \vec{p}_{\pi_2}}{2}\right)^2, \qquad (15)$$

with the momenta in the  $\alpha$  rest frame.

We omit details of the effective Lagrangians and couplings used for the different vertices. All of them are compiled in Appendixes A and B of Ref. [6] and we follow them strictly. The factor  $\frac{3}{2}$  in front of Eq. (14) is an isospin factor which sums the contribution of the  $\pi^+\pi^-$  decay channel and the  $\pi^0\pi^0$  decay channel (which has the factor  $\frac{1}{2}$  of symmetry).

In addition to this decay channel we could add the  $N^* \rightarrow \pi \Delta$  channel which carries a fraction of (20-30)% of the  $N^*$  decay width. However, as we shall see, the projectile Roper excitation mechanism with the dominant  $N^*$  decay

channel,  $N^* \rightarrow \pi N$  [Fig. 1(c)], which we have studied before, gives a negligible contribution, basically because of the small  $\pi NN^*$  coupling. Since in this case one has again the exchange of  $\pi$  and  $\rho$  mesons as in Fig. 1(c), and the fraction of the  $N^* \rightarrow \pi \Delta$  decay is smaller than that of  $N^* \rightarrow \pi N$ , this mechanism should give even a smaller contribution and we do not evaluate it here.

Finally we consider the double  $\Delta$  excitation process as shown in Fig. 1 (e). We have  $\pi$  and  $\rho$  meson exchange in this process and we have two  $\Delta$  resonances: One is in the target and the other one in the projectile. The cross section is

$$\frac{d^2\sigma}{dE_{\alpha'} \ d\Omega_{\alpha'}} = \frac{p_{\alpha'}}{(2\pi)^6} \frac{M_{\alpha}^2 M}{\lambda^{1/2}(s, M^2, M_{\alpha}^2)} \\ \times \int \frac{d^3 p_{\pi}}{\omega_{\pi}} \overline{\Sigma} \Sigma |T|^2 |G_{\Delta_T}(s_{\Delta_T})|^2 \Gamma_{\Delta_T}(s_{\Delta_T}),$$
(16)

where the propagator and the width of the  $\Delta$  are defined as

$$G_{\Delta}(s) = \frac{1}{\sqrt{s} - M_{\Delta} + \frac{i}{2}\Gamma_{\Delta}(s)}$$
(17)

and

$$\Gamma_{\Delta}(s) = \frac{2}{3} \frac{1}{4\pi} \left(\frac{f^*}{\mu}\right)^2 \frac{M}{\sqrt{s}} q_{\text{c.m.}}^3,$$
(18)

with  $M_{\Delta} = 1232$  MeV,  $f^{*2}/4\pi = 0.36$ , and  $q_{c.m.}$  the  $\pi$  momentum for  $\Delta$  decay at rest with mass  $\sqrt{s}$  in the  $\pi N$  system. The index  $\Delta_T$  indicates the  $\Delta$  resonance in the target. Here we replaced the energy conservation  $\delta$  function in terms of the  $\Delta$  propagator and the width in the target in the same way as Eq. (7). The sum and average over spin of  $|T|^2$  is given as

$$\overline{\Sigma}\Sigma|T|^{2} = \left(\frac{16}{9}\right)^{2} \frac{4}{3} F_{\alpha}^{2} \left(\frac{f^{*}}{\mu}\right)^{6} |G_{\Delta_{p}}(s_{\Delta_{p}})|^{2} \{ [V_{l}^{\prime 2}(q) - V_{t}^{\prime 2}(q)](\vec{p}_{\pi \text{c.m.}} \cdot \hat{q})^{2} + V_{t}^{\prime 2}(q)\vec{p}_{\pi \text{c.m.}}^{2} \},$$
(19)

where  $V'_l$ ,  $V'_t$  are defined in Eqs. (10) and (11). The index  $\Delta_P$  indicates the  $\Delta$  resonance in the projectile. The magnitude of  $s_{\Delta_P}$  is defined as Eq. (12). Equation (19) already accounts for the possibility of  $\pi^0, \pi^+, \pi^-$  decay of the projectile  $\Delta$  and all isospin channels of the target  $\Delta$ .

As we shall see later on, the diagrams of Figs. 1(c), 1(d), 1(e) are negligible and the two important mechanisms are given by the diagrams of Figs. 1(a), 1(b). When we compare our calculated results with the data [1], we include the interference of the two processes. Obviously we must select only the  $N^* \rightarrow \pi N$  decay channel in Fig. 1(b) and evaluate the amplitude for this process explicitly in order to have the same final state as in Fig. 1(a) and thus have some interference. The interference contribution is given by Eq. (8), replacing  $\overline{\Sigma}\Sigma |T|^2$  by

$$\overline{\Sigma}\Sigma(T_{N^*}^*T_{\Delta} + T_{\Delta}^*T_{N^*}) = 2 \operatorname{Re}\left[\frac{64}{3}F_{\alpha}^2 \left(g_{\sigma NN}F_{\sigma}D_{\sigma}g_{\sigma NN^*}F_{\sigma}G^*\frac{f'}{\mu}\right)^* \left(\frac{f^*}{\mu}G_{\Delta}\frac{f^*}{\mu}[(V_l' - V_l')(\vec{p}_{\pi\Delta}\cdot\hat{q})(\vec{p}_{\pi*}\cdot\hat{q}) + V_l'(\vec{p}_{\pi\Delta}\cdot\vec{p}_{\pi*})]\frac{f}{\mu}\sqrt{\frac{-q^2}{\dot{q}^2}}\right)\right],$$
(20)

where  $T_{N*}$  is the *T* matrix of the target Roper process followed by  $\pi N$  decay,  $T_{\Delta}$  is that of the projectile  $\Delta$  process, and  $\vec{p}_{\pi\Delta}$  is the pion momentum in the  $\Delta$  rest frame and  $\vec{p}_{\pi*}$  is in the *N*\* rest frame. This last expression sums the contribution from the production of a  $\pi^0$  and a  $\pi^+$ .

We should note that the interference between the T=1/2and T=3/2 excitations (with the simultaneous different spin excitation) has appeared because they occur on different nucleons, one in the target and the other one in the projectile. Should these excitations have occurred both on the target nucleon, there would have been no interference. In our case the  $\Delta$  excitation in the target is forbidden but it would have appeared if we had a <sup>3</sup>He projectile instead of <sup>4</sup>He, and there would be no interference between  $\Delta$  excitation and Roper excitation on the target.

#### **III. NUMERICAL RESULTS**

We should mention first the gross features of the data. As can be seen in Ref. [1], the observed cross section has a peak around  $\omega = 550$  MeV after subtracting the contribution of the projectile  $\Delta$  excitation [Fig. 1(a)] of Ref. [1], which indicates the Roper excitation [1]. The data of the energy integrated cross section of this  $N^*$  peak are also available at several angles [1]. The data of Ref. [1] has been reanalyzed with a more precise background subtraction [9]. With these corrections the height at the  $\Delta$  peak is about 15% lower than in Ref. [1]. In Fig. 2 we show the new spectrum [9] with the appropriate normalization deduced from the scales in the energy-integrated cross section of Ref. [1] and the correction in Ref. [9]. By subtracting the  $\Delta$  background evaluated in Ref. [5] we can see that the strength of the Roper excitation

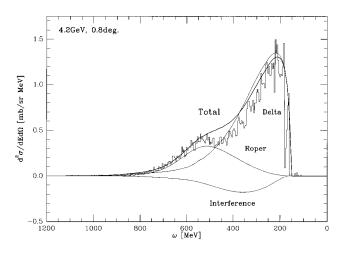


FIG. 2. Calculated cross sections of the target Roper process and the projectile  $\Delta$  process [5] at  $E_{\alpha} = 4.2$  GeV and  $\theta = 0.8^{\circ}$ . The variable  $\omega$  is the energy transfer defined as  $\omega = E_{\alpha} - E_{\alpha'}$ . The thick line indicates the sum of all contributions. Experimental data are taken from Ref. [10]. Here we used  $g_{\alpha NN*}^2/4\pi = 2.35$ .

# at its peak is of the order of 0.25 (mb/sr MeV).

We evaluate the cross section with the mechanisms discussed in the former section and show the results in Fig. 3. As we said, in the diagrams Fig. 1(c) and Fig. 1(e) all the couplings are known. Hence, we can calculate the cross section from these diagrams, which we show in the figure. As we can see there, their strength is very small and by no means can they account for the strength in the Roper region. This leaves diagrams Fig. 1(b) and 1(d) to do the job. The cross sections for these two processes are both proportional to  $g_{\sigma NN^*}^2$ . Even without knowing anything about this coupling, we can determine the ratio of the cross sections for these two mechanisms. We found that the target Roper process is much more important than the projectile Roper process followed by  $\pi\pi N$  decay by about a factor of 100. The cross section of the projectile process is suppressed because of the final state phase space which involves two pions.

Hence, the diagram of Fig. 1(b) for Roper excitation in the target stands as the only likely mechanism to explain the

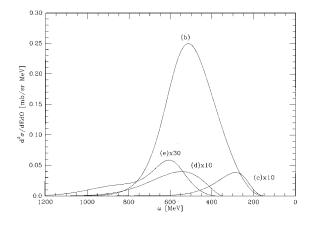


FIG. 3. Calculated cross sections  $d\sigma/d\Omega \ dE$  for  $(\alpha, \alpha')$  on the proton at  $E_{\alpha} = 4.2$  GeV and  $\theta = 0.8^{\circ}$ . The variable  $\omega$  is the energy transfer defined as  $\omega = E_{\alpha} - E_{\alpha'}$ . Each line indicates the contribution from the process shown in Fig. 1. Here we used  $g_{\sigma NN*}^2/4\pi = 1.79$ .

data. Thus we fix for the moment the strength of  $g_{\sigma NN}^*$ , the only unknown in the theory, in order to reproduce a strength of the peak of about 0.25 (mb/sr MeV). The value of the coupling constant that we get is  $g_{\sigma NN}^2/4\pi = 1.79$ . With this coupling we can now evaluate the diagram Fig. 1(d) and we find, as shown in the figure, a very small contribution.

We can explain the reasons why those terms are so small here. The cross section of the projectile Roper excitation can be compared with that of the projectile  $\Delta$  excitation [Fig. 1(a)] in Ref. [5] directly. They have the same phase space and the same *T* matrix except for some factors. We found that the cross section is so small simply because of the small coupling constants. The cross section of the projectile Roper excitation can be evaluated from that of projectile  $\Delta$  excitation using a ratio of the coupling constants,  $(f'/f^*)^4$ =  $2.4 \times 10^{-3}$ .

For the double  $\Delta$  process the reasons are the following: First, the peak position of the target  $\Delta$  excitation is different from that of the projectile  $\Delta$  excitation because of the kinematics [2,4]. Hence, the cross section is the result of a small overlap of two different resonance peaks. Second, the resonant strength associated with  $\Delta$  excitation in the projectile, which peaks at small excitation energies, is now considerably reduced because the phase space available is very restricted when one forces another  $\Delta$  to be excited simultaneously in the target. To confirm our results we try to evaluate the result of the double  $\Delta$  process using the available ones, from that of the projectile  $\Delta$  process. The T matrix is the same in both processes except for some factors. The phase space is now different due to the different final states. To simulate the double  $\Delta$  process we increase the final nucleon mass of the projectile  $\Delta$  process. We found that the projectile  $\Delta$  process with 940+250 (MeV) final nucleon mass has a peak at the same position of that of the double  $\Delta$  process, and its height is around 1/100 of the original projectile  $\Delta$  process because of the phase space differences. In addition the peak height of the double  $\Delta$  process must be even lower than this peak because of the  $\Delta$  width in the target. Hence, we can reconfirm qualitatively the small contribution of the double  $\Delta$  process.

All these things considered, the Roper excitation in the target of Fig. 1(b) is the only mechanism which is left to explain the data. All other processes [Figs. 1(c)–(e)] provide typically two orders of magnitude smaller cross section than the experimental data. As we can see in the figure, we need only the target Roper excitation and we neglect all the other processes hereafter, except for the projectile  $\Delta$  excitation which is large and has already been evaluated [5].

We show the target Roper contribution together with the projectile  $\Delta$  contribution [5] and their interference in Fig. 2 and compare them to the data. Here we take the  $g_{\sigma NN*}^2/4\pi = 2.35$ . We found that the Roper excitation produces a wide peak around  $\omega = 520$  MeV. The interference has a negative contribution to the cross section and peaks around  $\omega = 350$  MeV. The calculated cross section provides a fair account of the cross section but the dip region between  $N^*$  and  $\Delta$  excitation is poorly reproduced. We have chosen a particular sign for  $g_{\sigma NN*}$ , the same as  $g_{\sigma NN}$ , which leads to destructive interference. If the opposite sign is chosen, the

constructive interference leads to a cross section in large disagreement with the data.

In order to obtain a better agreement with the data we change the expression of the width of the Roper resonance in Eq. (3). Experiments tell us that the Roper resonance decays not only into  $\pi + N$  (65%) but also into  $\pi + \pi + N$  (35%) [8]. We describe in the Appendix how we take into account the  $2\pi + N$  decay. The Roper width  $\Gamma^*$  in Eqs. (1) and (2) is replaced by this new form and the distortion effects of final  $2\pi$  are also considered in  $F_{\alpha}$ . Then we take the freedom to change the Roper mass and width in the range of their uncertainties [8] and try to obtain a best fit to the data by changing  $M^*, \Gamma^*(s=M^{*2})$ , and  $g_{\sigma NN^*}$ . The calculated results depend generally on these parameters in the following way: The peak moves to a lower  $\omega$  value for larger width and smaller mass, the peak is higher for smaller width and larger  $g_{\sigma NN^*}$ , the peak is steeper for smaller width, and the interference is relatively more important for smaller  $g_{\sigma NN}*$ . The result for our best fit is shown in Fig. 4, where we see that the data are well reproduced. The best fit parameters have been:  $M^* = 1430$  MeV,  $\Gamma^*(s = M^{*2}) = 300$  MeV, and  $g_{\sigma NN*}^2/4\pi = 1.33.$ 

We show the calculated angular distribution of the Roper excitation in Fig. 5. The interference contribution is not included in this distribution. The data are from Ref. [1] and they should be corrected by the new background subtraction [9]. We should also notice that the fact that the interference term between the projectile  $\Delta$  and target Roper mechanism is not small does not allow a clean experimental separation of these mechanisms. With this caveat, the comparison of our results with the experimental data should only be taken as qualitative. The main point we want to stress here is that the monotonous fall down of the cross section is reproduced and, in our theoretical analysis, it is mostly a consequence of the  $(\alpha, \alpha')$  transition form factor and not a property tied to the Roper resonance itself. We found that our results reproduce the trend of the data well.

Finally we want to comment on the  $\pi N$  scattering amplitude of  $P_{11}$  channel. In this channel the observed amplitude [10, 11] has a different form than the standard Breit-Wigner form of the Roper resonance due to the coupling to the nucleon. In the energy region which we consider in this paper, the differences are as follows: First the real part of the observed amplitude has the opposite sign to the Breit-Wigner form at  $\sqrt{s} \le 1.2$  GeV and second the shape of the real part of the observed scattering amplitude is steeper than the Breit-Wigner form at  $1.2 \le \sqrt{s} \le 1.3$  GeV because of the off-shell nucleon effect. In order to see the effect of these differences we calculated the  $\alpha$  spectrum with a modified Roper propagator which has a steeper real part at  $1.2 \le \sqrt{s} \le 1.3$  GeV according to the data of the scattering amplitude. We have checked that including these modifications in the "Roper" excitation changes only a bit the results of Fig. 4 in the region of the dip, reducing moderately the cross section there. Theoretically the inclusion of the nucleon pole term in addition to the Roper pole would help in producing the shape in the  $P_{11}$  channel.

Now we would like to comment on the meaning of the " $\sigma$ " exchange interaction obtained. In a more microscopic description of the  $NN \rightarrow NN^*$  transition along the lines of the

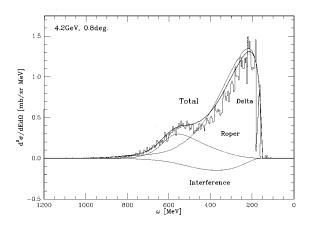


FIG. 4. Same as in Fig. 2. Here we used  $g_{\sigma NN*}^2/4\pi = 1.33$ ,  $M^* = 1430$  MeV,  $\Gamma^*(s = M^{*2}) = 300$  MeV, and the Roper width discussed in the Appendix.

boson exchange model, in the isoscalar channel which we have investigated we would also have a contribution from  $\omega$  exchange and from uncorrelated  $2\pi$  exchange. It is easy to see that assuming a similar scaling here for  $\omega$  exchange and the uncorrelated  $2\pi$  exchange, with respect to  $\sigma$  exchange, as one has in the NN potential [7], the effects of  $\omega$  and uncorrelated  $2\pi$  exchange are very important and one finds a large cancellation between  $\sigma$  and  $\omega$  exchange. In addition one should use this as input for a transition potential and initial and final state interactions of the NN or NN\* systems (correlations) should also be taken into account. For all these reasons the " $\sigma$ " exchange potential which we have obtained should not be interpreted as a  $\sigma$  exchange for the  $NN \rightarrow NN^*$  transition along the lines of a one-boson exchange model. It is simply an effective interaction which accounts for all the ingredients in the T=0 exchange channel  $(\sigma, \omega, \text{ and correlations})$ . One may wonder, why use the explicit  $\sigma$  mass in the exchange? There is certainly no justification for it, except that a posteriori one finds that the mass of the object exchanged is irrelevant in the description of the cross section and it can be equally reproduced using any other mass. Hence the " $\sigma$ " exchange obtained stands only as a useful and intuitive parametrization of the effective interaction in the T=0 channel. With this easy interaction one can make predictions for analogous reactions using other nuclei, one can evaluate cross sections at other energies of the beam, etc.

Obviously, although the limited information of the present reaction does not allow one to extract enough information to construct a one-boson exchange model for the  $NN \rightarrow NN^*$ transition, the job done here, separating the  $\Delta$  projectile excitation from the Roper excitation, provides some partial, but useful information, on the  $NN \rightarrow NN^*$  transition to be used in the future in attempts to construct a microscopical model for this interaction. Some steps in this direction, by looking at the role of uncorrelated  $2\pi$  exchange, have been given in Ref. [12].

#### **IV. SUMMARY**

We have studied the Roper excitation in the  $(\alpha, \alpha')$  reaction on the proton target. All processes which may be rel-

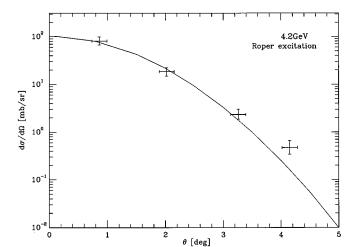


FIG. 5. Calculated differential cross sections  $d\sigma/d\Omega$  of the target Roper process as a function of the scattering angle in the laboratory frame. The parameters are the same as in Fig. 4. The experimental data are taken from Ref. [1]. See warnings in the text about the interpretation of the results.

evant in this energy region were investigated. We found that the experimental  $\alpha'$  spectrum can be reproduced by two processes, the projectile  $\Delta$  excitation and the target Roper process. The target Roper process is mediated by an isoscalar exchange between the  $\alpha$  and the proton and we have determined from the experiment the effective isoscalar  $NN \rightarrow NN^*$  transition t matrix.

We could find a good reproduction of the data with values of  $M^*$  and  $\Gamma^*$  close to the average values quoted in the particle data table 8. We found a good agreement with the data with  $M^*=1430$  MeV,  $\Gamma^*(s=m^{*2})=300$  MeV, and a certain choice of the parameters of the effective interaction.

The experimental dependence of the cross section on the  $\alpha'$  angle was qualitatively reproduced and found to be tied to the  $\alpha$  form factor, not to the properties of the Roper resonance.

The present work also lays the ground for extension of studies of  $N^*$  excitation in nuclei in order to study the modification of the  $N^*$  properties in a nuclear medium. The excitation of the  $N^*$  with the  $(\alpha, \alpha')$  reaction, because of the large strength and clean signature, would be probably one of the ideal tools for such studies.

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## APPENDIX: DECAY WIDTH OF THE ROPER RESONANCE

In this appendix we will explain our model of the widths of the Roper resonance. We include the  $N^* \rightarrow \pi + N$  and  $N^* \rightarrow \pi + \pi + N$  decay channels. Writing the decay width of each channel by  $\Gamma^*_{\pi}$  and  $\Gamma^*_{\pi\pi}$ , we define the total decay width as

$$\Gamma^*(s) = \Gamma^*_{\pi}(s) + \Gamma^*_{\pi\pi}(s). \tag{A1}$$

The width of the  $\pi + N$  decay channel has the same form as that of Ref. [6],

$$\Gamma_{\pi}^{*}(s) = \Gamma_{\pi}^{*}(s = M^{*2}) \frac{q_{\text{c.m.}}^{3}(s)}{q_{\text{c.m.}}^{3}(M^{*2})}, \qquad (A2)$$

where  $\Gamma_{\pi}^{*}(s=M^{*2})=P_{\pi}\Gamma^{*}(s=M^{*2})$ , with  $\Gamma^{*}(s=M^{*2})$ the experimental Roper width and  $P_{\pi}$  the  $\pi N$  decay branching ratio. The magnitude  $q_{\text{c.m.}}(s)$  is the  $\pi$  momentum in the center of mass frame of the  $\pi N$  system with energy  $\sqrt{s}$ .

For the width of the  $\pi + \pi + N$  decay channel, we assume the  $N^* \rightarrow \pi + \Delta$  as an intermediate state in this paper and express the width as follows:

$$\Gamma_{\pi\pi}^{*}(s) = \int \frac{d^{3}p_{\pi}}{(2\pi)^{3}} \frac{d^{3}p_{\Delta}}{(2\pi)^{3}} \frac{M_{\Delta}}{E_{\Delta}} \frac{1}{2\omega_{\pi}} \overline{\Sigma} \Sigma |T|^{2} (2\pi)^{4} \\ \times \delta^{4}(p^{*} - p_{\pi} - p_{\Delta}),$$
(A3)

where  $p^{*\mu}$  is the four-momenta of the Roper resonance and is  $(\sqrt{s}, \vec{0})$  in the Roper rest frame. The  $\pi \Delta N^*$  coupling is taken to be of the same form as that of  $\pi N \Delta$  with the coupling strength  $f_{\pi \Delta N^*}$  [6]. After replacing the energy conservation  $\delta$  function into the  $\Delta$  propagator and width as in Eq. (7) in the text, we find the  $\Gamma_{\pi\pi}^*$  is described as

$$\Gamma_{\pi\pi}^*(s) = \frac{1}{3\pi^2} \left( \frac{f_{\pi\Delta N^*}}{\mu} \right)^2 \int dp_{\pi} \frac{p_{\pi}^4}{\omega_{\pi}} |G_{\Delta}(s_{\Delta})|^2 \Gamma_{\Delta}(s_{\Delta}),$$
(A4)

which has included all the isospin channels, where  $G_{\Delta}$  and  $\Gamma_{\Delta}$  are defined in Eqs. (17) and (18), respectively. The coupling constant  $f_{\pi\Delta N^*}$  is determined by the normalization condition  $\Gamma^*_{\pi\pi}(s=M^{*2})=P_{\pi\pi}\Gamma^*(s=M^{*2})$  with  $\Gamma^*(s=M^{*2})$  the experimental Roper width and  $P_{\pi\pi}$  the  $\pi\pi N$  decay branching ratio. We obtain  $f_{\pi\Delta N^*}=2.47$  for  $M^*=1440$  MeV,  $\Gamma^*(s=M^{*2})=350$  MeV, and  $P_{\pi\pi}=0.35$  [8].

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