Proton (neutron) spin rotation in a polarized nuclear target: Method for investigating nuclear interactions

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The nuclear interaction of a proton (neutron) beam of energy ~1 GeV with a polarized nuclear target of length l results in spin rotation of the incident particles through an angle $\theta=(10^{-3}-10^{-4})l(\mathrm{cm})$. Using the spin density matrix method, it is shown that there are two physically different mechanisms which lead to the spin rotation effect. The first mechanism, coherent spin rotation, has a quasioptical nature and depends directly on the real part of a spin dependent proton-proton (pp) and proton-neutron (pn) forward scattering amplitude. It manifests itself in a broad energy range from a few hundredths of an eV [but for (pp) interaction, from tens of MeV] to hundreds of GeV. The second mechanism, diffractive spin rotation, is caused by Coulomb-nuclear interference in (pp) scattering and is of the same order as coherent spin rotation in an energy region of about tens of MeV. The diffractive spin rotation angle decreases with the incident beam energy, and, at about 1 GeV, it represents only 1% of the value of the coherent spin rotation angle. Experimental measurement of the spin rotation angle makes it possible to reconstruct directly the real part of the forward scattering proton-proton and proton-neutron amplitudes. Spin rotation is proposed to be used for the investigation of threshold effects and of resonant baryon states in the intermediate energy region.

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I. INTRODUCTION

There are two physical principles, unitarity and analyticity, which, physicists believe, hold for the description of strong interactions. The unitarity principle gives the direct relation between a total cross section of colliding particles and the imaginary part of the forward scattering amplitude. It is well known in experimental particle physics how to measure a total spin dependent cross section of proton-proton (pp) and proton-neutron (pn) interactions (for a review, see [1]). The corresponding physical program has been carried out for a long time in the broad energy range of colliding particles [for the (nN) interactions, for example, from a few hundredths of an eV to hundreds of GeV].

Through analyticity we can get dispersion relations between the real and imaginary parts of the forward scattering amplitude. These relations are very valuable for analyzing strong interactions, especially if we know both the real and imaginary parts of the forward scattering amplitude in a broad energy range through independent experimental measurements. Moreover, knowledge of the real part of the forward scattering amplitude is important for the investigation of dibaryon resonances [2–5], the measurement of (pp) spin observables under small angle scattering [6,7], phase shift analysis [8], and the calculation of parity violation in (pN) and (nN) interactions [9].

However, up to the present, the usual method for calculating the real part of the spin dependent amplitude has been through a dispersion relation from the imaginary part. Note also that unlike the case of the spin independent part of the amplitude, where the method of dispersion relations is a well-determined procedure, the calculation of the spin dependent part of the amplitude using the dispersion integral is not very reliable, because, in this case, the contribution of the nonphysical region of the NN cutoff is very complicated to take into account [10].

There are also several experimental possibilities for the indirect measurement of the real part of the forward scattering amplitude [1]. Since no scattering experiment is possible in the forward direction, the determination of the real part of the forward amplitudes has always consisted in the measurement of well-chosen elastic scattering observables at small angles and then in the extrapolation of these observables towards zero angle [1]. In addition, there has been one experiment in which the real parts of n-p forward scattering amplitudes were calculated by using isospin symmetry relations [1] through the measurement of n-p scattering amplitudes at 180° c.m. [11,12]. All of these methods, however, contain discrete ambiguities in the reconstruction of the forward scattering matrix which can be removed only by new independent measurements. Consequently, what is needed is a direct reconstruction of the real part of the forward scattering matrix such as we have in the case of the imaginary part through the measurement of a total cross section.

It has been shown in [13–18] that there is an unambiguous method which makes the direct measurement of the real part of the spin dependent forward scattering amplitude possible. This technique is based on the effect of proton (neutron) beam spin rotation in a polarized nuclear target and uses the measurement of a proton (neutron) spin rotation angle under the conditions of a transmission experiment—the so-called spin rotation experiment. The analogous phenomenon for thermal neutrons was theoretically predicted by Podgoretsky and one of the authors (V.B.) [14] and experimentally measured by the Abragam [15] and Forte [17] groups (the phenomenon of nuclear precession of neutron spin in the quasimagnetic nuclear field of a polarized nuclear target).

The nature of the spin rotation effect can be easily understood using a simple quasioptical model of the nuclear interaction of an incident beam with the target. In this case the difference in the interaction of protons (neutrons) polarized parallel and antiparallel to the polarization of the nuclear target is described by two effective indices of refraction, $n_{\uparrow\uparrow}$ and $n_{\uparrow\downarrow}$. These refractive indices characterize the interaction of the incident beam with the target as a whole. This is different from the refractive index introduced in the usual optical model of nuclear interactions [19,20] which describes the scattering process on separate target nuclei. As is usual with two indices of refraction, the quasioptical phenomena of spin rotation and dichroism of absorption arise when the polarized protons (neutrons) penetrate the polarized nuclear target. Since the quasioptical description holds under arbitrary energies of the incident particles, the phenomenon of spin rotation is manifest both in the low and in the superhigh energy regions.

This very simple picture of the spin rotation phenomenon holds only for the forward scattered protons (neutrons) [i.e., for the coherent proton (neutron) wave]. However, the electromagnetic interaction of incident particles with the Coulomb field of the target results in fast damping of the coherent wave (especially for the polarized proton beam). It seems that this effect should lead to practically total suppression of coherent proton spin rotation in the polarized nuclear target, but this is not the case. We will show that a scattered proton also "senses" the quasimagnetic nuclear field in which its spin is rotated. Coulomb scattering in the nuclear target results in only insignificant depolarization of the beam and does not influence coherent spin rotation. Moreover, spin rotation for the scattered protons is enhanced, for, besides coherent spin rotation in the quasimagnetic nuclear field, there is incoherent spin rotation arising in single scattering events on the target nuclei. This incoherent spin rotation is conditioned by the interference between the nuclear and the electromagnetic interaction (diffractive spin rotation).

This paper, then, is organized as follows. In Sec. II we discuss the basic properties of the spin dependent N-N scattering matrix. In order to describe the dynamics of a polarized proton beam traveling through a polarized nuclear target under the conditions of a transmission experiment, we use, in Sec. III, the formalism of the spin density matrix. Then, based on this formalism, we describe both coherent and diffractive proton spin rotation in polarized matter. The expressions for the polarization of the incident beam which take into account both the processes of depolarization and diffusion of the beam over the solid angle are obtained in Sec. IV. It is seen that, in the GeV region, diffractive spin rotation is strongly suppressed, and spin rotation is mostly due to the coherent part. Accordingly, spin rotation can be described by two indices of refraction as discussed in Sec. V. In Sec. VI we consider various beam and target spin configurations for a spin rotation experiment. Such an experiment can be used to investigate threshold effects as well as to observe signals of possible dibaryon resonances.

II. THE NUCLEON-NUCLEON SCATTERING MATRIX

In this section we review the basic properties of the N-N scattering matrix. By assuming parity conservation, time-reversal invariance, and the Pauli principle, we can write the general form of the scattering matrix for colliding nucleons [21] as

$$M(\hat{k},\hat{k}')=\{(a+b)+(a-b)(\vec{\nu}\hat{\sigma}_1)(\vec{\nu}\hat{\sigma}_2)+(c+d)(\vec{m}\hat{\sigma}_1)\times(\vec{m}\hat{\sigma}_2)+(c-d)(\vec{l}\hat{\sigma}_1)(\vec{l}\hat{\sigma}_2)+e[\vec{\nu}(\hat{\sigma}_1+\hat{\sigma}_2)]+f[\vec{\nu}(\hat{\sigma}_1-\hat{\sigma}_2)]\}/2, \tag{1}$$

where $\hat{\sigma}_1$ and $\hat{\sigma}_2$ are the Pauli matrices acting on the first and second nucleon wave function,

$$\vec{m}=\frac{\hat{k}-\hat{k}'}{|\hat{k}-\hat{k}'|},\quad \vec{l}=\frac{\hat{k}+\hat{k}'}{|\hat{k}+\hat{k}'|},\quad \vec{\nu}=\frac{(\hat{k}\times\hat{k}')}{|\hat{k}\times\hat{k}'|}, \tag{2}$$

$\hat{k}$ and $\hat{k}'$ are unit vectors in the direction of the incident and scattered particles, respectively, and a, b, c, d, e, and f are six complex invariant amplitudes dependent on the angle θ between $\hat{k}$ and $\hat{k}'$ and on the particle energy E. If we assume also isospin invariance of the scattering matrix then the amplitude f vanishes.

A direct transition of the scattering matrix (1) to the case of zero scattering angle is impossible because of the uncertainty in the directions of $\vec{m}$ and $\vec{l}$ when $\hat{k}'=\hat{k}$. However, the requirement of uniqueness of the forward scattering matrix $M(\hat{k},\hat{k})$ leads to the following relations between amplitudes:

$$e(0)=0,$$

$$f(0)=0, \tag{3}$$

$$a(0)-b(0)=c(0)+d(0),$$

which are used to get

$$\hat{M}(0)=\tfrac{1}{2}[(a+b)+(c+d)(\hat{\sigma}_1\hat{\sigma}_2)-2d(\hat{\sigma}_1\hat{k})(\hat{\sigma}_2\hat{k})]. \tag{4}$$

As follows from Eq. (4), there are only three out of six nonzero independent complex parameters which completely describe the forward scattering amplitudes.

Recall also [19,20] that the proton-proton and proton-neutron scattering matrix may be written as

$$M(\hat{k},\hat{k}')=M_{\rm el}(\hat{k},\hat{k}')+M_n(\hat{k},\hat{k}'), \tag{5}$$

where we separated the contributions to the scattering amplitudes conditioned by the electromagnetic and strong interactions. Note that at energy ~10 MeV the nuclear part of the p-p scattering matrix $M_n(\hat{k},\hat{k}')$ is strongly modified by the electromagnetic interaction [19,20].

The scattering matrix $M_{\rm el}(\hat{k},\hat{k}')$ of a proton on the Coulomb center is obtained in the Appendix:

$$M_{\rm el}=a_{\rm el}(\theta)\left(1-\frac{ib_g}{2}(\hat{\sigma}_1\vec{\nu})\theta+(b_g\theta)^2/8\right), \tag{6}$$

where $b_g=(g-2)(\gamma^2-1)/(2\gamma)+(\gamma-1)/\gamma$, and γ and g are the Lorentz factor and gyromagnetic ratio of the proton, respectively. The amplitude $a_{\rm el}(\theta)$ is a maximum at $\theta=0$ and falls off as $1/\theta^2$ as the scattering angle increases. This is quite different from the smooth behavior of the nuclear part of $M_n(\theta)$ near zero angle,

$$M_n(\theta \ll 1) \approx M_n(0) = A + B(\hat{\sigma}_1 \hat{\sigma}_2) + C(\hat{\sigma}_1 \hat{k})(\hat{\sigma}_2 \hat{k}), \quad (7)$$

where the coefficients A, B, and C are complex functions of energy. Taking into account (5), (6), and (7), this allows us to write the matrix $M(\theta)$ for small scattering angles as

$$M(\theta \ll 1) \approx M_{\rm el}(\theta) + M_n(0). \quad (8)$$

Comparing formulas (4) and (8) it is possible to find relations between the coefficients in (2) and (7);

$$A = [a(0) + b(0)]/2 - a_{\rm el}(0),$$
$$B = [c(0) + d(0)]/2, \quad (9)$$
$$C = -d(0).$$

The calculation of the imaginary parts of amplitudes A, B, and C is possible through the optical theorem [22,23]

$$\sigma_{\rm tot} = \frac{4\pi}{k} \, {\rm Im}\{[a(0)+b(0)]/2 + [c(0)+d(0)](\vec{P}_b \vec{P}_t)/2 - d(0)(\vec{P}_b \hat{k})(\vec{P}_t \hat{k})\}, \quad (10)$$

where $\vec{P}_b$ and $\vec{P}_t$ are the beam and target polarization and $\sigma_{\rm tot}$ is the total cross section of the N-N interaction. The total cross section in turn is directly determined by measuring the final intensity of the n (p) beam that has passed through the polarized nuclear target under the conditions of a transmission experiment. However, besides intensity, the n (p) beam possesses another experimentally measurable characteristic, polarization. Polarization, together with intensity, changes when the n (p) beam traverses the polarized nuclear target. We will show that the change in beam polarization is connected with the real part of the spin dependent amplitudes B and C by a simple algebraic relation. Hence, an experiment which measures not only transmission but also final polarization of the beam, a spin rotation experiment, opens the possibility for the complete experimental reconstruction of the amplitudes B and C.

III. SPIN DENSITY MATRIX FORMALISM

The most natural quantum mechanical description of a polarized proton (neutron) beam traveling through a polarized target utilizes the spin density matrix w. In the nonrelativistic approximation which holds in the energy region up to tens of MeV, the density matrices of the polarized proton (neutron) beam and polarized target are

$$w(b) = I(\vec{k})[1 + \hat{\sigma}\vec{P}_b(\vec{k})]/2,$$
$$w(t) = (1 + \hat{\sigma}\vec{P}_t)/2, \quad (11)$$

where $\hat{\sigma}_i$ are the Pauli matrices, $I(\vec{k})$ and $\vec{P}_b(\vec{k})$ are the intensity and polarization of the proton beam, $\vec{k}$ is the incident particle momentum, and $\vec{P}_t$ is the target polarization. In our case, we treat the polarized proton target as a thermal reservoir with an infinite set of degrees of freedom which we then average. Neglecting the influence of the polarized proton beam on the target we can treat the polarization $\vec{P}_t$ as a constant vector for the duration of the experiment. Therefore the density matrix for the system, $w(b,t)$, consisting of the polarized target plus polarized proton beam, can be written as a tensor product of the matrices $w(b)$ and $w(t)$;

$$w(b,t) = w(b) \otimes w(t). \quad (12)$$

The master equation for the spin density matrix of this system has the standard form

$$\frac{dw(b,t)}{dt} = -\frac{i}{\hbar}(\hat{H}w - w\hat{H}) + \left(\frac{dw}{dt}\right)_{sct} \quad (13)$$

with the Hamiltonian $\hat{H} = -\mu_p \hat{\sigma}_t \vec{B} - \mu_p \hat{\sigma}_b \vec{B} - \iota\Gamma/2$. Here $\mu_p = \hbar g e/(4mc)$ is the proton (neutron) magnetic moment, e and m are the proton charge and mass, and g is the Landé factor. The Hamiltonian includes the magnetic interaction between protons of the beam and target and the holding magnetic field $\vec{B}$ of the polarized nuclear target and also the decay part of the Hamiltonian, Γ, which is responsible for inelastic nuclear interactions. The term $(dw/dt)_{sct}$ describes the change in the density matrix due to elastic collisions in the target and will be considered more thoroughly.

Let us introduce the S matrix $\hat{S}$ describing the transitions from the initial state before scattering to the final state after particle scattering by the target nucleus. The density matrix after the collision can be written as

$$w' = \hat{S} w \hat{S}^+. \quad (14)$$

An element of the S matrix $\hat{S}$ is

$$S_{s's}(\vec{k}', \vec{k}) = \langle \vec{k}' s' | \hat{S} | \vec{k} s \rangle, \quad (15)$$

where $\vec{k}$ and $\vec{k}'$ are the initial and final momentum of the scattered particle, and s and s' are the spin variables of the system target plus beam before and after the collision, respectively. We represent $S_{s's}(\vec{k}',\vec{k})$ in the form

$$S_{s's}(\vec{k}',\vec{k}) = \delta(\vec{k}',\vec{k})\,\delta_{s's} + 2\pi i\left(\frac{2\pi\hbar^2}{\mu L^3}\right) \times \delta(E - E')\, M_{ss'}(\vec{k}',\vec{k}), \quad (16)$$

where we introduced the scattering matrix $M_{s's}(\vec{k}',\vec{k})$ (1) as is usually done in scattering theory [19]. Here L is the element of length used for normalization, E is the energy, and μ is the reduced mass of the colliding particles. We assume that the density matrix of the system under consideration has the form

$$\langle \vec{k}' s' | w | \vec{k} s \rangle = w_{ss'}\,\delta(\vec{k}',\vec{k}). \quad (17)$$

Here we neglect the nondiagonal elements of the density matrix in momentum space [24,25]. The spin density matrix after collision can then be written as

$$w_{s\bar{s}'}(\vec{k}) = \sum_{s's''} \sum_{\vec{k}'\vec{k}''} \langle \vec{k} s | \hat{S} | \vec{k}' s' \rangle \langle \vec{k}' s' | w | \vec{k}'' s'' \rangle \langle \vec{k}'' s'' | \hat{S} | \vec{k} \bar{s} \rangle. \quad (18)$$

To sum over $\vec{k}$ we use the relation

$$\sum_{\vec{k}} \to L^3 \int \frac{d^3\vec{k}}{(2\pi\hbar)^3} = \frac{L^3}{(2\pi)^3} \int \left(\frac{2mE}{\hbar^2}\right)^{1/2} \frac{m}{\hbar^2}\, dE\, d\Omega. \tag{19}$$

Then, substituting the expression (16) in (18), we can get

$$w_{s\bar{s}'}(\vec{k}) = w_{s\bar{s}}(\vec{k}) + \Delta t\left(\frac{\hbar k}{mL^3}\right)\Bigg[\frac{2\pi i}{k}\Bigg(\sum_{s'} M_{ss'}(\vec{k},\vec{k}) w_{s'\bar{s}}(\vec{k}) - \sum_{s''} w_{ss''}(\vec{k}) M^{+}_{s''\bar{s}}(\vec{k},\vec{k})\Bigg) + \int d\Omega \sum_{s's''} M_{ss'}(\vec{k}',\vec{k}) w_{s's''}(\vec{k}) M^{+}_{s''\bar{s}}\Bigg], \tag{20}$$

where Δt is the characteristic scattering time. We are interested in the quantity

$$\frac{w'-w}{\Delta t} = \frac{\Delta w}{\Delta t}, \tag{21}$$

where we compare the density matrices at the times t and $t+\Delta t$. The time interval Δt should be chosen so that it is much more than the characteristic correlation time of the system τ but still small enough to fulfill the condition that the difference Δw is linear over Δt. We can then replace expression (21) by the derivative $(d/dt)_{sct}$. Note that we can use this derivative only for the description of the change in $w(t)$ for a time interval more than τ.

Thus the master equation for the density matrix describing the behavior of the polarized proton (neutron) beam in polarized nuclear matter can finally be written as

$$\frac{dw}{dt} = -\frac{i}{\hbar}[\hat{H},w] + v\rho\, \mathrm{Tr}_t\Bigg[\frac{2\pi i}{k}[M(\theta=0)w(\vec{k}) - w(\vec{k})M^{+}(\theta=0)] + \int d\Omega\, M(\vec{k}')w(\vec{k}')M^{+}(\vec{k}')\Bigg], \tag{22}$$

where $\vec{k}'=\vec{k}+\vec{q}$, $\vec{q}$ is the momentum transfer from the incident particle to the target nucleus, v is the speed of incident particles, and ρ is the nuclei density in the target. In (22) we take the trace over the spin states of the target nuclei.

IV. THE DYNAMICS OF A POLARIZED PROTON BEAM IN A POLARIZED NUCLEAR TARGET

Now we can obtain the differential equations for the intensity and polarization of a proton beam passing through the polarized nuclear target.

The intensity of the beam is

$$I = \mathrm{Tr}_b \mathrm{Tr}_t w, \tag{23}$$

where the spin matrix w is determined by Eq. (12). Using (23) and (12) and taking the trace over the polarization states of the incident protons, the change in the beam intensity is written as

$$\frac{dI}{dt} = v\rho\, \mathrm{Tr}_t \mathrm{Tr}_b\Bigg[\frac{2\pi i}{k}[M(\theta=0)w(\vec{k}) - w(\vec{k})M^{+}(\theta=0)] + \int d\Omega\, M(\vec{k}')w(\vec{k})M(\vec{k}')\Bigg] - \Gamma I. \tag{24}$$

Polarization of the proton beam is determined as

$$\vec{P} = \frac{\mathrm{Tr}_b \mathrm{Tr}_t w\hat{\sigma}}{\mathrm{Tr}_b \mathrm{Tr}_t w} = \frac{\mathrm{Tr}_b \mathrm{Tr}_t w\hat{\sigma}}{I}, \tag{25}$$

where we defined $\vec{P}_b = \vec{P}$. To find the differential equation for the beam polarization at a distance z in the polarized nuclear target we consider the expression

$$\Delta\vec{P} = \vec{P}(z+dz) - \vec{P}(z), \tag{26}$$

where $dz \ll z$. Expanding the spin density matrix $w(z+dz)$ into a Taylor series

$$w(z+dz) = w(z) + \frac{dw(z)}{dz}\, dz + \cdots \tag{27}$$

and substituting it in (26) we get

$$\frac{d\vec{P}(z)}{dz} = \frac{\mathrm{Tr}_b \mathrm{Tr}_t (dw/dz)\hat{\sigma}}{I(z)} - \vec{P}(z)\, \frac{\mathrm{Tr}_b \mathrm{Tr}_t dw/dz}{I(z)}. \tag{28}$$

Equations (24) and (28) completely describe all observable characteristics of the proton beam in the polarized nuclear target.

To solve these equations we will use the fact that under the conditions of a spin rotation or transmission experiment we are interested only in those protons, which, after passing the target, fall into the small angular range $\theta_m \ll 1$ near the initial direction of the beam $\theta < \theta_m$. In this case, instead of the exact scattering matrix $M(\vec{k}',\vec{k})$ in Eqs. (24) and (28), we will use its approximation (8) which holds true for small scattering angles. Note also that existing polarized nuclear targets, as a rule, are much smaller than the nuclear length. This means that a particle scattered through an angle $\theta > \theta_m$, because of the nuclear interaction, cannot return to the selected angular range and, therefore, drops out of consideration. This process, the decrease of particle number from the incident beam because of nuclear scattering through large angles, can be described by Eqs. (24) and (28) if we integrate over $d\Omega$ up to the angle θ_m.

To simplify calculations we consider in this section the concrete geometry of a spin rotation experiment when the initial polarization of incident protons is perpendicular and the beam axis is parallel to the polarization of the target. The general experimental geometry will be treated in Sec. V.

Since in the small angle range the electromagnetic interaction is much larger than the nuclear interaction, let us first consider only the contribution of the electromagnetic scattering and neglect the nuclear. The corresponding amplitude is given by Eq. (6). Substituting the spin density matrix w (12) in (24), we get the differential equation for the intensity of the proton beam

$$\frac{dI(\vec{k})}{dz}=\rho\int d\Omega'\ \frac{d\sigma_{\mathrm{Col}}}{d\Omega'}\left[I(\vec{k}+\vec{q})-I(\vec{k})+\frac{b_g^2\theta'^2}{2}\ I(\vec{k}+\vec{q})\right]-\Gamma I(\vec{k}), \tag{29}$$

where $d\sigma_{\mathrm{col}}/d\Omega'$ is the Coulomb cross section. Here we used the optical theorem (10). Applying the same procedure to Eq. (28) we have

$$\frac{d\vec{P}(\vec{k})}{dz}=\frac{2\mu_p[\vec{B}\times\vec{P}(\vec{k})]}{v}+\rho\int d\Omega'\ \frac{d\sigma_{\mathrm{Col}}}{d\Omega'}\left[\frac{I(\vec{k}+\vec{q})[\vec{P}(\vec{k}+\vec{q})-\vec{P}(\vec{k})]}{I(\vec{k})}-(b_g\theta')^2\ \frac{I(\vec{k}+\vec{q})\vec{P}(\vec{k}+\vec{q})}{I(\vec{k})}\bigg/4\right]. \tag{30}$$

Using the approximation of small angle scattering and expanding the intensity and polarization of the beam into powers of the momentum transfer, we get

$$I(\vec{k}+\vec{q})\approx I(\vec{k})+q_i\ \frac{\partial I(\vec{k})}{\partial k_i}+\frac{1}{2}\ q_iq_j\ \frac{\partial^2 I(\vec{k})}{\partial k_i\partial k_j}, \tag{31}$$

$$\vec{P}(\vec{k}+\vec{q})\approx\vec{P}(\vec{k})+q_i\ \frac{\partial\vec{P}(\vec{k})}{\partial k_i}+\frac{1}{2}\ q_iq_j\ \frac{\partial^2\vec{P}(\vec{k})}{\partial k_i\partial k_j}. \tag{32}$$

We can then rewrite Eqs. (29) and (30) in the form

$$\frac{dI(\theta,z)}{dz}=\frac{\overline{\theta^2}}{4}\ \Delta_\theta\ I(\theta,z)+\frac{b_g^2\overline{\theta^2}}{2}\ I(\theta,z)-\Gamma I(\theta,z), \tag{33}$$

$$\frac{d\vec{P}(\theta,z)}{dz}=\frac{2\mu_p(\vec{B}\times\vec{P})}{v}+\frac{\overline{\theta^2}}{4}\left(\Delta_\theta\vec{P}+\frac{2}{I}\ \frac{\partial I}{\partial\theta_i}\ \frac{\partial\vec{P}}{\partial\theta_i}\right)-\frac{b_g^2\overline{\theta^2}}{4}\ \vec{P}, \tag{34}$$

where

$$\overline{\theta^2}=\rho\int d\Omega'\ \frac{d\sigma_{\mathrm{el}}}{d\Omega'}\ \theta'^2=16\pi\left(\frac{Z\alpha}{E}\right)^2\rho\ \ln(183Z^{-1/3}) \tag{35}$$

is the mean square-root scattering angle conditioned by the electromagnetic interaction, α is the fine-structure constant, and Z is the charge of the target nuclei. The initial conditions for Eqs. (33) and (34) are $I(\theta,z=0)=I_0\delta(\theta)$, $P_y(\theta,z=0)=P_0$, $P_x(\theta,z=0)=0$, and $\vec{P}_z(\theta,z=0)=0$, where $\delta(\theta)$ is the delta function.

As follows from (33) and (34), electromagnetic scattering does not have a significant effect on the polarization of the beam. Because of the small angle nature of this scattering, the protons of the beam are distributed diffusely over the solid angle θ with diffusion coefficient $\overline{\theta^2}$. This diffusion does not influence the direction of the beam polarization. It only leads to a decrease in the magnitude of the polarization, the so-called depolarization process described in [26]. According to Eq. (34), the depolarization rate of the proton beam in the target equals

$$\eta=1-\theta_{\mathrm{dep}}^2/4, \tag{36}$$

where

$$\theta_{\mathrm{dep}}^2=b_g^2\overline{\theta^2}l, \tag{37}$$

and l is the target length, a result which is in accord with [26]. For the ammonia target of length ~10 cm and an energy of the proton beam of about 10 MeV, the depolarization rate η is about $1–10^{-7}$. This means that depolarization is a very small effect even for low energy protons and can be omitted.

Let us consider the results of the joint effect of the electromagnetic and nuclear interactions. Substituting expression (8) in the equation for the intensity (24) and polarization (28) and using the same procedure as above we get

$$\frac{dI}{dz}=\frac{\overline{\theta^2}}{4}\ \Delta_\theta I+\frac{b_g^2\theta^2}{2}\ I-\Gamma I-\frac{4\pi\rho I}{k}\ \mathrm{Im}A+2\rho I\int d\Omega'\ \mathrm{Re}a_{\mathrm{el}}(\theta')\mathrm{Re}A, \tag{38}$$

$$\frac{d\vec{P}(\vec{k})}{dz}=\frac{2\mu_p(\vec{B}\times\vec{P})}{v}+\frac{\overline{\theta^2}}{4}\left(\Delta_\theta\vec{P}+\frac{2}{I}\ \frac{\partial I}{\partial\theta_i}\ \frac{\partial\vec{P}}{\partial\theta_i}\right)-\frac{b_g^2\overline{\theta^2}}{4}\ \vec{P}+\frac{4\pi\rho}{k}\ (\mathrm{Re}B+\mathrm{Re}C)(\vec{P}_t\times\vec{P})-\frac{4\pi\rho}{k}\ (\mathrm{Im}B+\mathrm{Im}C)\vec{P}_t-2\rho\int d\Omega\ \mathrm{Re}a(\theta)[(\vec{P}_t\times\vec{P})(\mathrm{Im}C+\mathrm{Im}B)+\vec{P}_t(\mathrm{Re}B+\mathrm{Re}C)]. \tag{39}$$

In formulas (38) and (39) under the integral we left only the terms which describe the influence of Coulomb-nuclear interference on the dynamics of the beam intensity and polarization. In the small angle approximation these terms dominate nuclear scattering. Equations (38) and (39) completely describe the behavior of the proton beam polarization in a polarized nuclear target located in the supporting magnetic field $\vec{B}$.

In Eq. (39) the terms which are proportional to the vector $(\vec{P}_t\times\vec{P})$ result in proton beam spin rotation. The terms proportional to $\vec{P}_t$ lead to spin dichroism of the polarized nuclear target. In direct analogy with [14,16] we may say that there arises a quasimagnetic nuclear field of the polarized nuclear target with strength

$$\vec{G}=\frac{2\pi\hbar^2\rho}{m\Delta\mu}\,\vec{P}_t\bigg[(\mathrm{Re}B+\mathrm{Re}C)-\frac{k\alpha}{E}\ln\bigg(\frac{1}{k\theta_{\mathrm{Col}}r_s}\bigg)(\mathrm{Im}B+\mathrm{Im}C)\bigg], \tag{40}$$

where r_s is the atomic screening radius and $\Delta\mu$ is the proton anomalous magnetic moment. Spin rotation of the incident proton beam takes place in the effective magnetic field $\vec{B}_{\mathrm{eff}}$ which equals the sum of the magnetic and quasimagnetic nuclear fields.

Also, as follows from formulas (39) and (40), there are two physically different mechanisms for spin rotation and dichroism of a proton beam passing through a polarized nuclear target. The first is connected with coherent processes of proton scattering by the target nuclei and is determined by the real and imaginary parts of the forward nuclear scattering amplitude. The nature of the other mechanism is connected with incoherent processes whenever an incident proton is scattered through a nonzero angle. Indeed, because of the spin dependence of the nuclear interaction, the incident proton can flip its spin in a single scattering event on the target nucleus. The structure of Coulomb-nuclear interference is such that this incoherent scattering process leads to the accumulation of the spin rotation angle while the beam passes the polarized target. Unlike coherent spin rotation which is manifest during the interaction of the incident beam with the target as a whole, we will call the process of incoherent accumulation of spin rotation angle diffractive spin rotation.

Let us compare the coherent and diffractive spin rotation angles in the polarized nuclear target. As follows from (40), the diffractive spin rotation angle is

$$\theta_{\mathrm{dif}}=4\pi\rho P_t l(\mathrm{Im}B+\mathrm{Im}C)\,\frac{\alpha}{E}\,\ln\bigg(\frac{1}{k\theta_{\mathrm{Col}}r_s}\bigg), \tag{41}$$

where P_t is the magnitude of the target nuclei polarization, and for an energy $E=100$ MeV it is approximately ten times less than the coherent spin rotation angle

$$\theta_{\mathrm{coh}}=\frac{4\pi\rho l P_t}{k}\,(\mathrm{Re}B+\mathrm{Re}C)\approx 10^{-2}\ \ \mathrm{rad}. \tag{42}$$

In the region of energy $E\sim 10$ MeV these two mechanisms complement one another, which leads to an increase of the total spin rotation angle. With a further increase of the incident proton energy, the ratio between the diffractive and coherent spin rotation angles tends to the value α, and, starting with an energy $E\sim 1$ GeV, the spin rotation angle is determined, to a 1% accuracy, by coherent spin rotation alone. This allows us to neglect the electromagnetic interaction for these energies and to consider the interaction of relativistic protons and neutrons by the introduction of a refractive index for the polarized nuclear target.

V. THE REFRACTION OF HIGH ENERGY PROTONS (NEUTRONS) IN A POLARIZED NUCLEAR TARGET

Let a relativistic polarized proton (neutron) beam of energy E be incident on a target with polarized nuclei. The wave function of Dirac particles in vacuum is described by the plane wave

$$\Psi_i=\exp(ikz)u_k, \tag{43}$$

where $\vec{k}$ is the wave vector of the incident particle and u_k is the bispinor of the free particle defining its spin state.

According to [14,18], the wave function of a relativistic particle in a medium can also be described by a plane wave. The wave function has the form

$$\Psi_f(l)=\frac{1}{\sqrt{2E}}\begin{cases}\sqrt{E+m}\ \exp(ik\hat{n}l)\hat{W}\\ \sqrt{E-m}(\hat{\sigma}\hat{k})\exp(ik\hat{n}l)\hat{W}.\end{cases} \tag{44}$$

Here

$$\hat{n}=1+\frac{2\pi\rho}{k^2}\,\hat{f}(0) \tag{45}$$

is the operator index of refraction of the particle in polarized matter, $\hat{W}$ is the spinor, ρ is the density of scatterers in the target, and $\hat{f}(0)$ is the forward elastic scattering amplitude of the particles by the polarized nuclei. This amplitude acts as an operator in the spin space of the incident particles and is averaged over the spin states of the target particles. Thus the influence of the medium on the incident beam propagating in the initial direction consists finally in a change in the phase and amplitude of the wave function ($\hat{n}$ has both a real and an imaginary part).

The scattering operator of particles with spin 1/2 in the relativistic case is defined by a two-dimensional matrix acting on the spinor $\hat{W}$ which describes the behavior of the wave function in the rest system of the particle. Therefore the general form of the forward scattering amplitude is analogous to the nonrelativistic amplitude (4). We note that, according to the results of the previous section, we only consider that part of the scattering amplitude which is conditioned by the nuclear interaction of the beam and target.

Taking into account (44), the spinor $\hat{W}(l)$ of the particle that has passed through the target of length l can be written as

$$\hat{W}(l)=\exp(in_0kl)\exp(i\vec{G}\hat{\sigma}l)\hat{W}(0), \tag{46}$$

where

$$n_0=1+\frac{2\pi\rho}{k^2}\,A, \tag{47}$$

$$\vec{G}=\frac{2\pi\rho}{k}\,[B\vec{P}_t+C(\vec{P}_t\hat{k})]=G_1\hat{g}_1+iG_2\hat{g}_2, \tag{48}$$

$\hat{g}_1$ and $\hat{g}_2$ are the unit vectors, and G_1 and G_2 are real numbers.

Next we find the change in the particle polarization in the rest system while traveling through the polarized target of length $z+dz$ such that $dz\ll z$. In this case we can write the expression

$$\vec{P}(z+dz)=\frac{\int\hat{W}^{+}(z+dz)\hat{\sigma}\hat{W}(z+dz)d^3x}{\int\hat{W}^{+}(z+dz)\hat{W}(z+dz)d^3x}. \tag{49}$$

Using the expansion

$$W(z+dz)=W(z)[1+i(n_0k+\vec{G}\hat{\sigma})dz], \tag{50}$$

we transform expression (49) into the differential equation

$$\frac{d\vec{P}(z)}{dz}=2G_1[\hat{g}_1\times\vec{P}(z)]-2G_2\{\hat{g}_2-\vec{P}(z)[\hat{g}_2\vec{P}(z)]\}. \tag{51}$$

By acting analogously we can get the differential equation for the change in the intensity of the polarized beam in a polarized nuclear target:

$$\frac{dI(z)}{dz}=-2\,\mathrm{Im}(n_0)kI(z)-2G_2[\hat{g}_2\vec{P}(z)]I(z). \tag{52}$$

Equations (51) and (52) should be solved together under the initial conditions $\vec{P}(0)=\vec{P}_i$, $I(0)=I_i$. According to (51), the polarization of the incident particles passing the polarized nuclear target undergoes rotation through the angle

$$\theta=2G_1l \tag{53}$$

about the vector $\hat{g}_1$. Comparing Eq. (51) with the Bargmann-Michel-Telegdi equation [27] for the spin motion of a charged particle in a magnetic field, we can introduce, as in Sec. III, the quasimagnetic field of the polarized nuclear target using the relation

$$2G_1\hat{g}_1=\frac{e}{2m}\left(g-2+2\frac{1}{\gamma}\right)\vec{B}_n+\frac{e}{2m}\,(g-2)\,\frac{E}{E+m}\,\vec{v}(\vec{v}\vec{B}_n). \tag{54}$$

In the nonrelativistic energy range, the quasimagnetic nuclear field $\vec{B}_n$, defined by relation (54), transforms into an expression for the quasimagnetic nuclear field [14,16]. Also note that, for the geometry considered in Sec. III, differential equation (51) is analogous to the equation for polarization (39) when we neglect the effects of diffusion, depolarization, diffractive spin rotation, and dichroism. Thus we get an equation describing the change in the polarization of relativistic particles because of the coherent scattering by the target nuclei at arbitrary directions of the vectors $\vec{P}_i$, $\hat{g}_1$, and $\hat{g}_2$.

VI. EXPERIMENTAL POSSIBILITIES FOR THE INVESTIGATION OF THE SPIN ROTATION EFFECT IN A POLARIZED NUCLEAR TARGET

Let us consider the necessary elements of an experimental setup to carry out the spin rotation experiment. First, there is a polarized proton (neutron) beam whose polarization may be rotated in any direction. Further, a (preferably) long ($\sim$10 cm) nuclear target in a frozen spin mode, which is to have at least two spin directions, ideally any spin direction. The beam extracted from the accelerator must be monitored by a very accurate polarimeter and the final spin state of the transmitted beam is to be determined by another polarimeter, also very accurate. The beam intensity must be carefully monitored and the corresponding monitors must be independent of beam polarization.

Assume that the target polarization $\vec{P}_t$ is parallel (this is the case of longitudinal polarization) or orthogonal (transversal polarization) to the momentum of the incident particle $\vec{k}$. Then $\hat{g}_1=\hat{g}_2=\vec{P}_t/P_t$ and Eqs. (51) and (52) are reduced to a simple form. Consider two specific cases for which the initial polarization of the incident beam is (a) $\vec{P}_i \| \vec{P}_t$ or (b) $\vec{P}_i \perp \vec{P}_t$, which encompass, anyway, all sets of the effects which can arise.

The case (a) is a standard transmission experiment wherein we observe the process of absorption in the polarized target without a change in the direction of the initial beam polarization. The absorption is different for particles polarized parallel and antiparallel to the target polarization. The initial intensity I_0 of the polarized beam then changes according to

$$I(l)=I_0\exp(-\sigma_{\pm}\rho l), \tag{55}$$

where

$$\sigma_{\pm}=\frac{4\pi}{k}\,[\mathrm{Im}(A)+\mathrm{Im}(B)(\vec{P}_t\vec{P}_i)+\mathrm{Im}(C)(\vec{P}_t\hat{k})(\hat{k}\vec{P}_i)]. \tag{56}$$

In the case (b) the coherent scattering on the polarized nuclei results in spin rotation of the incident particles about the target polarization $\vec{P}_t$

$$\vec{P}_f=\vec{P}_i\cos(2G_1l)-(\vec{P}_i\times\vec{P}_t)\sin(2G_1l)+\tanh(\sigma_s\rho l)\vec{P}_t. \tag{57}$$

The spin rotation angle θ equals

$$\theta=\frac{4\pi\rho P_t l}{k}\,\mathrm{Re}[B+C(\vec{P}_t\hat{k})]. \tag{58}$$

As follows from formula (58), the spin rotation angle is directly connected with the real part of the forward scattering amplitudes. The values ReB and ReC can be determined separately by measuring spin rotation angles for two cases when the target spin is parallel and antiparallel to the beam direction $\hat{k}$. This means that by measuring the final intensity and polarization of the beam in cases (a) and (b) we can directly reconstruct the spin dependent forward scattering matrix.

Let us consider one more configuration of the target and the beam in a transmission experiment intended to observe proton (neutron) spin rotation. We assume that the incident particle momentum is directed at some angle (which does not equal $\pi/2$) with respect to the target polarization, and that the incident beam polarization is perpendicular to the plane formed by the vectors $\vec{P}_t$ and $\hat{k}$. In this case the effect of proton (neutron) spin rotation about the vector $\hat{g}_1$ combined with absorption dichroism, determined by the vector $\hat{g}_2$, will cause absorption asymmetry of the polarized beam. Indeed, in the first approximation to the quantities G_1l and G_2l we can get from Eqs. (51) and (52) that

$$I(l)=I(0)\exp(-\sigma_0\rho l)\left[1-4(G_2 l)^2+2\left(\frac{2\pi\rho l}{k}\right)^2 \times(\mathrm{Re}B\ \mathrm{Im}C-\mathrm{Re}C\ \mathrm{Im}B)[\vec{P}_t\times\hat{k}(\hat{k}\vec{P}_t)]\vec{P}_i\right], \tag{59}$$

which indicates the possibility of measuring the combination of the real and imaginary parts of the forward scattering amplitudes. Unlike a spin rotation experiment, this transmission experiment does not allow us to determine Re B and Re C separately. A maximum asymmetry in this experiment,

$$\mathrm{Asym}=\frac{I(1)-I(2)}{I(1)+I(2)}, \tag{60}$$

can be reached for two configurations of the vectors $\vec{P}_t$ and $\hat{k}$ when the angle between these two vectors is $\pm\pi/4$. Note that this transmission experiment does not require the measurement of the final polarization of the beam. But we need to rotate the target spin about the direction $\hat{k}$.

Among the applications enumerated in the Introduction, we will emphasize and discuss in detail the possibility of investigating baryon exotic states. Indeed, a great number of experimental studies of (pp) and (pn) interactions in the energy region up to 3 GeV, demonstrate a series of pronounced peculiarities, which may be a manifestation of dibaryon resonances [3,4]. For example, the data on spin dependent cross sections σ_T and σ_L and also loop behavior of the curves on Argand diagrams are indicative of a dibaryon resonance. However, there are works which explain the manifestation of these peculiarities by introducing new inelastic channels [5].

In this situation direct evidence in favor of the existence (or the lack) of resonances might be supplied by the simultaneous measurement of both the imaginary and real parts of the forward spin dependent scattering amplitudes in (pp) and (pn) interactions. Let us now assume that the dibaryon resonance exists at an energy E_r. In this case the scattering amplitude near the resonance has the form [20]

$$\mathrm{Re}[f(0)]=\mathrm{Re}[f_{\mathrm{nr}}(0)]+\xi\,\frac{(E-E_r)\Gamma_r/2}{(E-E_r)^2+\Gamma_r^2/4}, \tag{61}$$

where $f_{\mathrm{nr}}(0)$ is the nonresonant part of the forward scattering amplitude which has a smooth energy dependence, Γ_r is the resonant width, and ξ is a real coefficient. As follows from (61), the resonant part of the real forward scattering amplitude changes sign when the energy E intersects the resonant energy E_r. This effect then leads to the fact that the contribution to the spin rotation angle conditioned by the resonant part of the forward scattering amplitude also changes sign. This effect can be a direct signal of resonance existence. Note that the experimental determination of the energetic dependence of the spin rotation angle allows us to separate the resonance state from the background of newly opening inelastic channels and also allows the investigation of threshold phenomena in (pn) interactions. Indeed, near the threshold of a reaction, the (pn) forward scattering amplitude has the universal form [20]

$$\mathrm{Re}[f(0,E)]=\mathrm{Re}[f_{\mathrm{th}}(0)]-\frac{k_{\mathrm{th}}}{4\pi}A_m\sqrt{E_{\mathrm{th}}-E}\,\cos(2i\delta_0),\qquad E<E_{\mathrm{th}}, \tag{62}$$

$$\mathrm{Re}[f(0,E)]=\mathrm{Re}[f_{\mathrm{th}}(0)]-\frac{k_{\mathrm{th}}}{4\pi}A_m\sqrt{E-E_{\mathrm{th}}}\,\sin(2i\delta_0),\qquad E>E_{\mathrm{th}}, \tag{63}$$

where k_{th} and E_{th} are the threshold momentum and energy, $f_{\mathrm{th}}(0)$ is the scattering amplitude at $E=E_{\mathrm{th}}$, and A_m and δ_0 are real numbers. As follows from (62) and (63), the existence of a threshold leads to the characteristic energy dependence on the spin rotation angle $\theta\sim\sqrt{E-E_{\mathrm{th}}}$, which is different from the energetic dependence near the resonance,

$$\theta\sim\frac{(E-E_r)\Gamma/2}{k[(E-E_r)^2+\Gamma^2/4]}. \tag{64}$$

In addition, the measurement of the spin rotation angle of protons (neutrons) makes it possible to determine the functions $B(E)$ and $C(E)$ separately. This is very important for the investigation of the origin of dibaryon resonance [10].

Let us next estimate the proton spin rotation angle when polarized particles travel through a target consisting of polarized hydrogen using the results of an experiment [28]. In this experiment the amplitude and phase of the scattering matrix were reconstructed using the total set of the experimental observables. For energies $E<3$ GeV the magnitude of the real part of the spin dependent part of the forward (pp) scattering amplitude, determined from [28], is equal to $\mathrm{Re}[B+C(\vec{P}_t\hat{k})]\sim10^{-13}$ cm. Thus, for example, for a polarized proton beam with an energy $E<3$ GeV incident on an ammonia target ($\rho=0.83$ g/cm^3) with a hydrogen percentage $F=18\%$ and which has a polarization $P_{\mathrm{H}}=80\%$, we get an estimate $\theta=(10^{-3}$–$10^{-4})l$(cm). In the case when the initial beam polarization $P_x(0)=P_z(0)=0$, $P_y(0)=P$, and the target polarization is directed along the incident beam momentum, the components of the beam polarization are

$$P_x(0)=P\ \sin(\theta),\qquad P_y(0)=P\ \cos(\theta). \tag{65}$$

The measurement of the spin rotation angle in the polarized target of length $\sim$10 cm on a level 10^{-2}–10^{-3} rad is possible when we eliminate from consideration the larger component of polarization P_y. This can be accomplished by the orientation of the beam polarimeter in the plane perpendicular to the x axis. With this orientation, the polarimeter will measure only the P_x polarization component. Let us then estimate the data acquisition time for the measurement of the spin rotation angle when a polarized proton beam at the SATURNE II accelerator (polarized proton beam intensity equals 2×10^8 sec^{-1} [28,29]) travels in a polarized solid ammonia target (NH_3) of length $l=4.4$ cm [29]. Then, using the data from [29], for the proton energy $E=1$ GeV, we get a spin rotation angle $\theta\approx2\times10^{-4}$ rad and a data acquisition time of 100 h. For protons with energy 500 MeV we get

$\theta \approx 5 \times 10^{-4}$ rad and an acquisition time of 10 h, which meets the requirements of a standard transmission (spin rotation) experiment.

Summing up, we point out that the interaction of a polarized proton (neutron) beam with a polarized nuclear target results in quasioptical effects of spin rotation of the proton (neutron) beam and of spin dichroism of the target. These effects can be observed in a broad energy range. The electromagnetic interaction of the protons with the nuclei does not destroy the coherent effects of spin rotation and dichroism and, moreover, results in additional diffractive spin rotation in the energy range ~10 MeV. Experimental measurement of the spin rotation angle is of most interest when the incident proton (neutron) beam energy is ~1 GeV, where the effect opens new possibilities, namely, the investigation of threshold effects and of possible resonance baryon states.

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APPENDIX

Let us consider the scattering process of a fast particle with the spin 1/2 on a Coulomb center when the scattering angle $\theta_0 \ll 1$. In this case the main contribution into the scattering amplitude is from the region of large impact parameters $\zeta \gg \hbar / k\theta_0$ where the potential energy is much less than the kinetic energy of the particle. This means that we can use the eikonal approximation and the scattering amplitude can be written as

$$M_{\rm el}(\theta) = -\frac{ik}{2\pi} \int \zeta \, d\zeta \{\exp[iS(\zeta)/\hbar] - 1\} \times \int_0^{2\pi} \exp(-ik\theta_0 \zeta \cos\psi) d\psi. \tag{A1}$$

Here

$$S(\vec{\zeta}) = \frac{1}{v} \int_{-\infty}^{\infty} u(\vec{\zeta}, z) dz, \tag{A2}$$

$u(\vec{\zeta}, z)$ is the Coulomb potential, and ψ is the angle between the vector $\vec{\zeta}$, which is perpendicular to the particle momentum, and the scattering plane. Let us take into account the particle spin precession in the field of the Coulomb center. According to [30], this can be done by multiplying the function $\exp[iS(\vec{\zeta})/\hbar]$ by the rotation operator

$$R[\theta(\zeta)] = \exp[-i(\hat{s}\vec{\nu}_1)\theta(\zeta)], \tag{A3}$$

where $\vec{\nu}_1 = (\vec{k} \times \vec{\zeta}) / |\vec{k} \times \vec{\zeta}|$, and $\theta(\zeta)$ is the spin rotation angle corresponding to the motion of the charged particle along the classical trajectory with the impact parameter ζ, and it equals

$$\theta(\vec{\zeta}) = b_g \theta_0(\vec{\zeta}). \tag{A4}$$

On the other hand, $\theta_0(\vec{\zeta})$ can be determined by the action function

$$\theta_0(\vec{\zeta}) = \frac{1}{\hbar k} \frac{d}{d\zeta} S(\vec{\zeta}). \tag{A5}$$

Thus to calculate $M_{\rm el}(\theta_0)$ we have to substitute in (A1) instead of $\exp[iS(\vec{\zeta})/\hbar]$ the expression

$$\exp[iS(\zeta)/\hbar] \hat{R}[\theta(\vec{\zeta})] = \exp[iS(\zeta)/\hbar]\{\cos[\theta(\zeta)/2] - i(\hat{s}\vec{\nu}_1)\sin[\theta(\zeta)/2]\}. \tag{A6}$$

Then, with the accuracy of θ_0^3, Eq. (A1) can be written as

$$M_{\rm el} = a_{\rm el}(\theta_0) + b_g(\hat{s}\vec{\nu}) \int J_1(k\theta_0\zeta)\left(\frac{d}{d\zeta}\exp[S(\zeta)/\hbar]\right)\zeta \, d\zeta + \frac{ib_g^2}{8k}\int_0^\infty \zeta \, d\zeta \, J_0(k\theta_0\zeta)\exp[iS(\zeta)/\hbar]\left(\frac{dS(\zeta)}{d\zeta}\right)^2, \tag{A7}$$

where $\vec{\nu}$ is the unit vector which is orthogonal to the scattering plane. Using the well-known relations for the Bessel functions

$$\frac{d}{dx}[xJ_1(x)] = xJ_0(x), \qquad \frac{d}{dx}J_0(x) = -J_1(x), \tag{A8}$$

and also the relation

$$\frac{dS}{d\zeta} = \frac{2e^2}{kv\zeta}, \tag{A9}$$

which holds for the Coulomb potential, we get with an accuracy of θ_0^3

$$M_{\rm el}(\theta_0) = a_{\rm el}(\theta_0)[1 - ib_g(\hat{s}\vec{\nu})\theta_0 + (b_g\theta_0)^2/8]. \tag{A10}$$

Note that the second term in formula (A10) describes the spin rotation around the vector $\vec{\nu}$ through the angle $b_g\theta_0$ when a charged particle is scattered by a Coulomb center.

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