Isospin invariance and the spin structure of the amplitude for the process $^3H+^3He\rightarrow d^2H+^4He$

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We analyzed the general form of all polarization effects in the $^3H+^3He\rightarrow d^2H+^4He$ reaction. For this aim, we parametrized the spin structure of the full amplitude in terms of the transverse amplitudes which describe the different transitions between initial and final states, which in turn are differed by projections of nuclei spins along the normal to the reaction plane. The isospin invariance of the strong interaction generates definite symmetry properties of the transverse amplitudes relative to the change $\cos\theta\rightarrow-\cos\theta$, where θ is the deuteron production angle in the c.m.s. We found all symmetry relations for the polarization observables in the $^3H+^3He\rightarrow d^2H+^4He$ reaction, which are the generalized form of the well-known Barshay-Temmer theorem. We consider the one-, two-, and triple-spin correlations in $^3H+^3He\rightarrow d^2H+^4He$. For analysis of the triple-spin correlations, we focus on the structure functions formalism and show that a set of 41 structure functions describes all such correlations. Isospin invariance simplifies the spin structure of the full amplitude at $\theta=90°$; therefore, we calculated all polarization observables for this special kinematics. [S0556-2813(96)02106-1]

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I. INTRODUCTION

The $^3H+^3He\rightarrow d^2H+^4He$ reaction lies in a group of processes which are ideal for testing the validity of isospin invariance (II) in various polarization experiments, because both colliding particles (^3H, ^3He) belong to the same isospin multiplet with different I_z values. If the charge symmetry breaking (CSB) effects are neglected, the alpha particle and deuteron are found to be in the pure $I=0$ state. In an isospin invariant theory, the total isospin of the colliding particles and the total isospin of the final state are equal. The consequence of such a case manifests itself in the simple isospin structure of the amplitudes. On the other hand, the spin structure of the amplitude of this process is rich enough to exhibit a variety of possible polarization effects, which on first sight are independent on isospin structure of amplitude.

The spin and isospin structures of collision amplitudes for the processes $n+p\rightarrow d+\eta$ and $^3He+^3H\rightarrow d^2H+^4He$ are similar (but are not equivalent due to different p parities of η and ^4He). But from an experimental point of view, the reaction $^3H+^3He\rightarrow d^2H+^4He$ has evident preferences and characteristics: all particles (in initial and final states) are charged and stable;[1] the ^3He beam has (after acceleration) definite momentum and polarization; the inverse reaction, namely, $d^2H+^4He\rightarrow ^3H+^3He$ can be realized; the ^3H beam in the initial state has definite momentum.

The nuclear reactions in which ^3H and ^3He participate allow examination of the violation of isospin invariance. On the other hand, calculation of the Coulomb repulsion of the two protons in the ^3He nucleus indicate that this energy cannot completely account for the difference between the binding energies of ^3He and ^3H.

Sauer [1] claimed that the Coulomb energy anomaly ($\Delta n=81\mp29$ KeV) has to be mainly attributed to CSB in the two-nucleon interaction. In light of the recent measurement of charge form factors of both ^3He and ^3H, the same anomaly has been estimated as $\Delta n=71\mp19\mp5$ KeV [2,3]. More recent analysis based on the solution of the Coulomb-modified Faddeev equations gives $\Delta n=70\mp7$ KeV [4].

Measurements [5–7] of the ratio of differential cross sections of $\pi^{\pm}$-mesons scattering on ^3He and ^3H targets indicates charge symmetry breaking, which manifests itself in the difference between the neutron radius in ^3H [$r_n(^3H)$] and the proton radius in ^3He [$r_p(^3He)$] in which $r_n(^3H)-r_p(^3He)=(-30\pm8)10^{-3}$ fm [8,9].

The differential cross sections (for all unpolarized particles), vector, and tensor analyzing powers have been measured for the reaction $d^2H+^4He\rightarrow ^3H+^3He$ in the energy interval (32–50) MeV. These measurements demonstrate a significant violation of symmetry properties with respect to $\theta=90°$ [θ is the production angle of final nuclei in the center-of-mass system (c.m.s.) of the reaction] [10,11]. The same is true in low energy collisions as well [12–14]. A detailed examination of various possible mechanisms for the $^3H+^3He\rightleftharpoons d^2H+^4He$ reaction provides a theoretical explanation of symmetry violations [15–20].

Such symmetry properties of polarization observations have general nature as a result of isospin invariance of strong interaction. They are correct for some class of special processes. To explain this, the colliding particles in the $n+p\rightarrow n+p$, $n+p\rightarrow d+\pi^0(\eta)$, $^3H+^3He\rightleftharpoons d^2H+^4He$ reactions are in the same isospin multiplet, but the third components of their isospins are different. Furthermore, total isospin of the interacting particles has a single value. In such

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[1]Of course, ^3H is an unstable nucleus, but it decays through the weak interaction.

cases, there is a definite correlation between space-time and internal symmetry properties of the reaction amplitudes. In this present study, we employ the isospin invariance of the strong interaction as the internal symmetry. This correlation appears as a definite symmetry of different polarization observables. The most simple example of such space-time symmetry is given as

$$\frac{d\sigma}{d\Omega}(\theta)=\frac{d\sigma}{d\Omega}(\pi-\theta),$$

a relation between the differential cross sections of the above-mentioned reactions. This relation is a result of the known Barshay-Temmer theorem [21]. Isospin invariance allows us to generalize this theorem in the case of different polarization observables for such processes [22,23]. The validity of these relations is based only on the general symmetry properties of the fundamental interactions such as isospin invariance and P invariance, and therefore they must be valid for any mechanism.

The reaction $n+p\to n+p$ differs from $n+p\to d+\pi^0(\eta)$, because in this case we have two possible values of the total isospin. But isospin invariance meanwhile introduces one, but very important, restriction on the spin structure of the total amplitude, namely, it diminishes the total number of independent amplitudes from 6 to 5 [24]. Of course, this lowering of the number of amplitudes results in definite properties of the polarization observables for the $n+p\to n+p$ process [25–31].

In this paper, we analyze the polarization effects in the $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction in a complete way. We present the properties of the polarization phenomena which are induced by isospin invariance. For this purpose, it is necessary to parametrize the general spin structure of the full amplitude which is valid for any mechanism of the reaction. We focus in particular on such a parametrization which simplifies the expressions for the polarization observables in the terms of scalar amplitudes. The most suitable choice is the set of transverse amplitudes. This choice allows us to analyze the problem of the full experiment. In a similar way the same problem was analyzed for the processes $\gamma+N\to N+\pi$, $N+N\to N+N$, $p+d\to d+p$, etc. [32].

Isospin invariance dictates a definite symmetry of the amplitude of the $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction with respect to the $\cos\theta\to-\cos\theta$ change. At $\theta=90°$, the spin structure of the amplitudes can be simplified. Therefore $\theta=90°$ is an appropriate angle for such reactions.

This problem was analyzed previously [33–35] in detail for the $p+p\to d+\pi^+$ and $n+p\to d+\pi^0$ reactions. We present a generalization of this analysis for the reaction $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$. The spin structure of the amplitudes for $n+p\to d+\pi^0$ and $^3\mathrm{H}+{}^3\mathrm{He}\to{}^4\mathrm{He}+d^2\mathrm{H}$ must be different due to the difference in P parities and the isospin of the interacting particles.

In Sec. II, we present the general spin structure of the full amplitude for the $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction in terms of six independent so-called scalar amplitudes. We employ the set of transverse amplitudes [36–39]. In Sec. III, we consider the production of polarized deuterons through the collisions of unpolarized nuclei. We obtain here the exact expressions for the differential cross section with all unpolarized particles (in the final and initial state), for the transverse component (to the reaction plane) of the vector deuteron polarization, and for the different components of the tensor deuteron polarization. We establish symmetry properties of all the observables which are valid for any mechanism of the reaction $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ obeying isospin invariance. The polarization phenomena which are induced by the polarization of one of the colliding nuclei, namely $^3\vec{\mathrm{H}}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ and $^3\mathrm{H}+{}^3\overrightarrow{\mathrm{He}}\to d^2\mathrm{H}+{}^4\mathrm{He}$, are analyzed in Sec. IV. These polarization effects include the dependence of the vector and tensor deuteron polarizations on the vector polarizations of colliding nuclei. We obtain the expressions for all corresponding polarization observables in terms of the scalar amplitudes and establish their symmetry properties which follow from isospin invariance. The collision of two polarized nuclei with any direction of the polarization vectors is analyzed in Sec. V. The most general triple-spin correlations are analyzed in terms of the so-called structure functions (SF). We show in Sec. VI that a set of 41 SF's gives the complete information about such correlations. In Sec. VII, we consider the special case of transverse kinematics with $\theta=90°$ and calculate all polarization observables in terms of four independent amplitudes.

II. SPIN STRUCTURE OF THE AMPLITUDE OF THE $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ PROCESS

Utilizing the P invariance of strong and electromagnetic interactions which are responsible for the $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction, the general expression for the spin structure of amplitude is given in the following form:

$$\mathcal{F}=\tilde{\chi}_2\sigma_2F\chi_1,$$

$$\begin{aligned}F=&\,i(1+\vec{\sigma}\cdot\hat{\vec{n}})\vec{U}^*\cdot\hat{\vec{n}}f_1+i(1-\vec{\sigma}\cdot\hat{\vec{n}})\vec{U}^*\cdot\hat{\vec{n}}f_2\\&+(\vec{\sigma}\cdot\hat{\vec{m}}+i\vec{\sigma}\cdot\hat{\vec{\kappa}})(\vec{U}^*\cdot\hat{\vec{m}}f_3+\vec{U}^*\cdot\hat{\vec{\kappa}}f_4)\\&+(\vec{\sigma}\cdot\hat{\vec{m}}-i\vec{\sigma}\cdot\hat{\vec{\kappa}})(\vec{U}^*\cdot\hat{\vec{m}}f_5+\vec{U}^*\cdot\hat{\vec{\kappa}}f_6),\end{aligned}\qquad(1)$$

where $\hat{\vec{\kappa}}=\vec{\kappa}/|\vec{\kappa}|$, $\hat{\vec{n}}=\vec{\kappa}\times\vec{q}/|\vec{\kappa}\times\vec{q}|$, $\hat{\vec{m}}=\hat{\vec{n}}\times\hat{\vec{\kappa}}$; $\vec{\kappa}$ and $\vec{q}$ are three-momenta of $^3\mathrm{H}$ and $^4\mathrm{He}$ (in c.m.s.), respectively; $\vec{U}$ is the deuteron polarization vector; $\chi_1(\chi_2)$ is the two-component spinor of $^3\mathrm{H}(^3\mathrm{He})$; $f_i=f_i(s,\cos\theta)$, $i=1-6$, are so-called scalar amplitudes which depend on the total energy s of the interacting particles and $\cos\theta$; and θ is the deuteron production angle in the c.m.s. Due to the deuteron positive P parity, $\vec{U}$ is an axial vector.

Any theoretical model which represents the internal structure of the interacting nuclei and the mechanisms of the discussed reaction does not have any effect on the general form of Eq. (1). Therefore, the spin structure of the amplitude depends only on the P parities and the spins of the particles. Only the weak interaction, whose effect is negligible in this process, necessitates effects a small change in Eq. (1).

Isospin invariance of the strong interaction results in definite symmetry properties for the scalar amplitudes $f_i(s,\cos\theta)$ relative to a $\cos\theta\to-\cos\theta$ (or $\theta\to\pi-\theta$) change:

TABLE I. The products of special combinations of σ matrixes, $A_{\alpha\beta}$.

$\alpha\downarrow\backslash\beta\rightarrow$	$1+\vec{\sigma}\cdot\vec{n}$	$1-\vec{\sigma}\cdot\vec{n}$	$\vec{\sigma}\cdot\hat{\vec{m}}+i\vec{\sigma}\cdot\hat{\vec{\kappa}}$	$\vec{\sigma}\cdot\hat{\vec{m}}-i\vec{\sigma}\cdot\hat{\vec{\kappa}}$
$1+\vec{\sigma}\cdot\vec{n}$	$2(1+\vec{\sigma}\cdot\vec{n})$	0	0	$2(\vec{\sigma}\cdot\hat{\vec{m}}-i\vec{\sigma}\cdot\hat{\vec{\kappa}})$
$1-\vec{\sigma}\cdot\vec{n}$	0	$2(1-\vec{\sigma}\cdot\vec{n})$	$2(\vec{\sigma}\cdot\hat{\vec{m}}+i\vec{\sigma}\cdot\hat{\vec{\kappa}})$	0
$\vec{\sigma}\cdot\hat{\vec{m}}+i\vec{\sigma}\cdot\hat{\vec{\kappa}}$	$2(\vec{\sigma}\cdot\hat{\vec{m}}+i\vec{\sigma}\cdot\hat{\vec{\kappa}})$	0	0	$2(1-\vec{\sigma}\cdot\vec{n})$
$\vec{\sigma}\cdot\hat{\vec{m}}-i\vec{\sigma}\cdot\hat{\vec{\kappa}}$	0	$2(\vec{\sigma}\cdot\hat{\vec{m}}-i\vec{\sigma}\cdot\hat{\vec{\kappa}})$	$2(1+\vec{\sigma}\cdot\vec{n})$	0

$$f_1(s,-\cos\theta)=f_1(s,+\cos\theta),$$
$$f_2(s,-\cos\theta)=f_2(s,+\cos\theta),$$
$$f_3(s,-\cos\theta)=f_5(s,+\cos\theta),$$
$$f_4(s,-\cos\theta)=-f_6(s,+\cos\theta), \quad (2)$$
$$f_5(s,-\cos\theta)=f_3(s,+\cos\theta),$$
$$f_6(s,-\cos\theta)=-f_4(s,+\cos\theta).$$

These relations are obtained from Eq. (1) by taking into account isospin conservation in $^3H+^3He\rightarrow d^2H+^4He$ reaction. The total isospin of the initial system must be zero and the amplitude F does not change sign, one can obtain from the generalized Pauli principle that

$$\hat{\vec{\kappa}}\rightarrow-\hat{\vec{\kappa}}, \quad \chi_1\leftrightarrow\chi_2. \quad (3)$$

In the framework of isospin invariance, ^3He and ^3H nuclei are identical Fermi particles.

Equations (2) lead to numerous relations between polarization observables of the $^3H+^3He\rightarrow d^2H+^4He$ reaction. From Eqs. (2), it follows immediately that under special kinematical conditions, namely for $\theta=90°$, the number of independent amplitudes separable from Eq. (1) reduces from 6 to 4.

After some trivial transformations, the parametrization given in Eq. (1) corresponds to the so-called spin structure products of amplitudes for $1/2+1/2\rightarrow0+0$ and $0+0\rightarrow1+0$ processes. Indeed, Eq. (1) can be written in the form

$$F=i\hat{\vec{n}}\cdot\vec{U}^*(g_1+\vec{\sigma}\cdot\vec{n}g_2)+\vec{U}^*\hat{\vec{m}}\cdot(g_3\vec{\sigma}\cdot\hat{\vec{m}}+g_4\vec{\sigma}\cdot\hat{\vec{\kappa}})$$
$$+\vec{U}^*\cdot\hat{\vec{\kappa}}(g_5\vec{\sigma}\cdot\hat{\vec{m}}+g_6\vec{\sigma}\cdot\hat{\vec{\kappa}}), \quad (4)$$

where the new amplitudes are connected with amplitudes f_i by

$$g_1=f_1+f_2, \quad g_2=f_1-f_2, \quad g_3=f_3+f_5,$$
$$g_4=i(f_3-f_5), \quad g_5=f_4+f_6, \quad g_6=i(f_4-f_6). \quad (5)$$

Equations (2) and (5) give

$$g_1(s,-\cos\theta)=+g_1(s,+\cos\theta), \quad (6)$$
$$g_2(s,-\cos\theta)=+g_2(s,+\cos\theta),$$
$$g_3(s,-\cos\theta)=+g_3(s,+\cos\theta),$$
$$g_4(s,-\cos\theta)=-g_4(s,+\cos\theta),$$
$$g_5(s,-\cos\theta)=-g_5(s,-\cos\theta),$$
$$g_6(s,-\cos\theta)=+g_6(s,+\cos\theta).$$

At $\theta=90°$, we obtain $g_4(s,0)=g_5(s,0)=0$.

Every parenthetical expression in Eq. (4) corresponds to spin structure amplitudes of the $1/2+1/2\rightarrow0+0$ processes (but with different parities of colliding or produced particles), and three products $\hat{\vec{n}}\cdot U^*$, $U^*\cdot\hat{\vec{m}}$, and $U^*\cdot\hat{\vec{\kappa}}$ describe the spin structure of the amplitude of the $0+0\rightarrow1+0$ processes (again with appropriate change of parity).

Parametrization of Eq. (1) involves the introduction of transverse amplitudes to the theory, which provides an appropriate method for analyzing polarization effects in the ^3H and ^3He process and, especially, for the study of a complete experiment. One can see that combinations of $(1\pm\vec{\sigma}\vec{n})/2$ are the projection operators for the ^3H and ^3He spin states where the direction of $\hat{\vec{n}}$ is chosen as the quantization spin axis. Then, in this basis, the combinations $\vec{\sigma}\cdot\hat{\vec{m}}\pm\vec{\sigma}\cdot\hat{\vec{\kappa}}$, being lowering and raising spin operators, have nonzero matrix elements only between states of ^3H and ^3He with different signs of spin projections. Such parametrization of amplitudes diminishes possible interference effects of the f_i amplitudes in different simple polarization observables for the $^3H+^3He\rightarrow d^2H+^4He$ process.

Let us write the connection between the amplitudes f_i and the helicity amplitudes commonly used for analyzing the polarization effects. Denoting the helicity amplitudes as $F_{\lambda_1\lambda_2,\lambda_d}$, where λ_1,λ_2 are the helicities of particles with χ_1 and χ_2 spinors, and λ_d is the deuteron helicity, one can find the following relations:

$$F_{++,0}=-\sin\theta(f_3-f_5)-\cos\theta(f_4-f_6),$$
$$F_{+-,0}=i\sin\theta(f_3+f_5)+i\cos\theta(f_4+f_6), \quad (7)$$
$$\sqrt{2}F_{++,+}=f_1-f_2-i\cos\theta(f_3+f_5)+i\sin\theta(f_4+f_6),$$
$$\sqrt{2}F_{--,+}=i(f_1+f_2)+\cos\theta(f_3-f_5)-\sin\theta(f_4-f_6),$$
$$\sqrt{2}F_{-+,+}=f_1-f_2+i\cos\theta(f_3+f_5)-i\sin\theta(f_4+f_6),$$
$$\sqrt{2}F_{+-,+}=-f_1+f_2+i\cos\theta(f_3+f_5)-i\sin\theta(f_4+f_6).$$

If parametrization of Eq. (1) for the amplitudes is valid for direct and inverse processes of $^3H+^3He\rightleftharpoons d^2H+^4He$, the relations between the helicity and scalar amplitudes for these two processes must be different. Indeed, for the inverse process, $d^2\mathrm{H}+{}^4\mathrm{He}\to{}^3\mathrm{H}+{}^3\mathrm{He}$, with helicity amplitudes $F_{\lambda_d,\lambda_1\lambda_2}$, one can obtain instead of Eq. (7)

$$F_{0,++}=-f_4+f_6,\quad F_{0,-+}=i(f_4+f_6), \tag{8}$$

$$\sqrt{2}F_{+,++}=i\cos\theta(f_1+f_2)-\sin\theta(f_1-f_2)-f_3+f_5,$$

$$\sqrt{2}F_{+,--}=-i\cos\theta(f_1+f_2)+\sin\theta(f_1-f_2)-f_3+f_5,$$

$$\sqrt{2}F_{+,-+}=i\sin\theta(f_1+f_2)+\cos\theta(f_1-f_2)+i(f_3+f_5),$$

$$\sqrt{2}F_{+,+-}=i\sin\theta(f_1+f_2)+\cos\theta(f_1-f_2)+i(f_3-f_5).$$

The relations between the helicity amplitudes (in transversity formalism) and the amplitudes f_i are given in the Appendix.

III. COLLISIONS OF UNPOLARIZED NUCLEI

We calculate now the simplest polarization observables which are connected with one or two polarized particles in initial or final states. The products of special combination of σ matrixes, which are necessary for such calculations are given in Table I.

Using the results of Table I one obtains

$$\begin{aligned}\tfrac{1}{2}FF^{\dagger}&=(1+\vec{\sigma}\cdot\hat{\vec{n}})\{|f_1|^2|\vec{U}\cdot\hat{\vec{n}}|^2+|f_5\vec{U}^*\cdot\hat{\vec{m}}+f_6\vec{U}^*\cdot\hat{\vec{\kappa}}|^2\}\\&=(1-\vec{\sigma}\cdot\hat{\vec{n}})\{|f_2|^2|\vec{U}\cdot\hat{\vec{n}}|^2+|f_3\vec{U}^*\cdot\hat{\vec{m}}+f_4\vec{U}^*\cdot\hat{\vec{\kappa}}|^2\}\\&\quad+2\vec{\sigma}\cdot\hat{\vec{m}}\ \mathrm{Im}\{-f_2(f_5^*\vec{U}\cdot\hat{\vec{m}}+f_6^*\vec{U}\cdot\hat{\vec{\kappa}})\vec{U}^*\cdot\hat{\vec{n}}+f_1^*(f_3\vec{U}^*\cdot\hat{\vec{m}}+f_4\vec{U}^*\cdot\hat{\vec{\kappa}})\vec{U}\cdot\hat{\vec{n}}\}\\&\quad+2\vec{\sigma}\cdot\hat{\vec{\kappa}}\ \mathrm{Re}\{-f_2(f_5^*\vec{U}\cdot\hat{\vec{m}}+f_6^*\vec{U}\cdot\hat{\vec{\kappa}})\vec{U}^*\cdot\hat{\vec{n}}+f_1^*(f_3\vec{U}^*\cdot\hat{\vec{m}}+f_4\vec{U}^*\cdot\hat{\vec{\kappa}})\vec{U}\cdot\hat{\vec{n}}\}.\end{aligned} \tag{9}$$

Utilizing Eq. (9), it is straightforward to calculate any polarization observable associated with the polarization of the nucleus ${}^3\mathrm{He}$ (with spinor χ_2). For the polarized nucleus being ${}^3\mathrm{H}$ (with spinor χ_1) it is necessary to use the product $F^{\dagger}F$:

$$\begin{aligned}\tfrac{1}{2}F^{\dagger}F&=(1+\vec{\sigma}\cdot\hat{\vec{n}})\{|f_1|^2|\vec{U}\cdot\hat{\vec{n}}|^2+|f_3\vec{U}^*\cdot\hat{\vec{m}}+f_4\vec{U}^*\cdot\hat{\vec{\kappa}}|^2\}\\&=(1-\vec{\sigma}\cdot\hat{\vec{n}})\{|f_2|^2|\vec{U}\cdot\hat{\vec{n}}|^2+|f_5\vec{U}^*\cdot\hat{\vec{m}}+f_6\vec{U}^*\cdot\hat{\vec{\kappa}}|^2\}+2\vec{\sigma}\cdot\hat{\vec{m}}\ \mathrm{Im}A+2\vec{\sigma}\cdot\hat{\vec{\kappa}}\ \mathrm{Re}A,\end{aligned} \tag{10}$$

where

$$A=f_2^*(f_3\vec{U}^*\cdot\hat{\vec{m}}+f_4\vec{U}^*\cdot\hat{\vec{\kappa}})\vec{U}\cdot\hat{\vec{n}}-f_1^*(f_5\vec{U}^*\cdot\hat{\vec{m}}+f_6\vec{U}^*\cdot\hat{\vec{\kappa}})\vec{U}\cdot\hat{\vec{n}}.$$

Using Eq. (9) or (10) one finds for the case of the collision of both unpolarized nuclei:

$$\begin{aligned}\tfrac{1}{4}\ \mathrm{Tr}FF^{\dagger}=\tfrac{1}{4}\ \mathrm{Tr}F^{\dagger}F&=|\hat{\vec{n}}\cdot\vec{U}|^2(|f_1|^2+|f_2|^2)+|\vec{U}\cdot\hat{\vec{m}}|^2(|f_3|^2+|f_4|^2)+|\vec{U}\cdot\hat{\vec{\kappa}}|^2(|f_4|^2+|f_6|^2)\\&\quad+(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{\kappa}}+\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{m}})\ \mathrm{Re}(f_3f_4^*+f_5f_6^*)+i(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{\kappa}}-\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{m}})\ \mathrm{Im}(f_3f_4^*+f_5f_6^*).\end{aligned} \tag{11}$$

Summing over deuteron polarizations one can obtain from Eq. (11) the following expression for the differential cross section:

$$\frac{d\sigma}{d\Omega}=\frac{1}{64\pi^2\,\mathrm{s}}\frac{|\vec{q}|}{|\vec{\kappa}|}\ \tfrac{1}{4}\ \mathrm{Tr}\overline{F^{\dagger}F}\approx\tfrac{1}{4}\ \mathrm{Tr}\overline{F^{\dagger}F}, \tag{12}$$

where the line above $F^{\dagger}F$ denotes summing over the deuteron polarizations $\overline{U_a^{\star}U_b}=\delta_{ab}$. We neglect here the three-momenta of the deuteron in comparison with its mass, namely $|\vec{q}|/M\ll 1$.

Then from Eq. (11), one finds

$$\frac{d\sigma^0}{d\Omega}\approx|f_1|^2+|f_2|^2+|f_3|^2+|f_4|^2+|f_5|^2+|f_6|^2. \tag{13}$$

This result shows that the differential cross section does not contain any amplitude interference terms.

The symmetry properties of the amplitudes $f_i(s,\cos\theta)$ [Eq. (2)] result in the definite θ dependence of the differential cross section, namely

$$\frac{d\sigma^0}{d\Omega}(\pi-\theta)=\frac{d\sigma^0}{d\Omega}(\theta). \tag{14}$$

This property of $d\sigma^0/d\Omega$ is valid only for exact isospin invariance of a strong interaction (the Barshay-Temmer theorem [21]).

Nonzero components of the tensor $Q_{ij}^{(0,0)}$ [which is defined as $U_iU_j^{\star}=(1/3)\delta_{ij}+Q_{ij}^{0,0}+i\epsilon_{ijk}S_k$ and describes in Cartesian coordinates the quadrupole or tensorial polarization of the deuteron] are given as

$$Q_{\kappa\kappa}^{(0,0)}\frac{d\sigma^0}{d\Omega}\approx-|f_1|^2-|f_2|^2+|f_4|^2+|f_6|^2,$$

$$Q_{mm}^{(0,0)}\frac{d\sigma^0}{d\Omega}\approx-|f_1|^2-|f_2|^2+|f_3|^2+|f_5|^2, \tag{15}$$

$$Q_{m\kappa}^{(0,0)}\frac{d\sigma^0}{d\Omega}=Q_{\kappa m}^{(0,0)}\frac{d\sigma^0}{d\Omega}\approx \mathrm{Re}(f_3f_4^*+f_5f_6^*),$$

where the upper index (0,0) indicates that both initial particles are unpolarized.

From Eq. (11), one can see that the produced deuteron also has the vector polarization $\vec{P}^{(0,0)}$ with the only a normal nonzero component:

$$P_n^{(0,0)}\frac{d\sigma^0}{d\Omega}\approx-\ \mathrm{Im}(f_3f_4^*+f_5f_6^*). \tag{16}$$

The deuteron polarization observables satisfy the following symmetry conditions:

$$Q_{mm}^{(0,0)}(s,-\cos\theta)=+Q_{mm}^{(0,0)}(s,+\cos\theta), \tag{17}$$

$$Q_{\kappa\kappa}^{(0,0)}(s,-\cos\theta)=+Q_{mm}^{(0,0)}(s,+\cos\theta),$$

$$Q_{m\kappa}^{(0,0)}(s,-\cos\theta)=-Q_{\kappa m}^{(0,0)}(s,+\cos\theta),$$

$$P_n^{(0,0)}(s,-\cos\theta)=-P_n^{(0,0)}(s,+\cos\theta),$$

and can be considered as a generalization of the Barshay-Temmer theorem [21] for the polarization properties of the produced deuteron. All these relations are consequences of isospin invariance. Existing experimental data for the inverse process $d^2\mathrm{H}+{}^4\mathrm{He}\rightarrow{}^3\mathrm{H}+{}^3\mathrm{He}$ also prove the validity of similar relations for the analyzing powers (due to the tensor and vector polarizations of a deuteron beam).

IV. POLARIZATION PHENOMENA FOR THE $^3\vec{\mathrm{H}}+{}^3\mathrm{He}\rightarrow d^2\mathrm{H}+{}^4\mathrm{He}$ AND $^3\mathrm{H}+{}^3\overrightarrow{\mathrm{He}}\rightarrow d^2\mathrm{H}+{}^4\mathrm{He}$ COLLISION

Let us consider the case when one of the colliding nuclei (^{3}H or ^{3}He) is polarized. First, we analyze the scattering of a polarized beam (^{3}H) on an unpolarized target (^{3}He). In our notation, the ^{3}H beam is described by spinor χ_1.

(I) The results of the analysis of all three possible independent spin orientations of ^{3}H nuclei are given below.

(I.1) ^{3}H polarization vector is along $\vec{n}$:

$$\tfrac{1}{4}\ \mathrm{Tr}F^\dagger F\vec{\sigma}\cdot\hat{\vec{n}}=|\vec{U}\cdot\hat{\vec{n}}|^2(|f_1|^2-|f_2|^2)+|\vec{U}\cdot\hat{\vec{m}}|^2(|f_3|^2-|f_5|^2)+|\vec{U}\cdot\hat{\vec{\kappa}}|^2(|f_4|^2-|f_6|^2)$$
$$+(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{\kappa}}+\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{m}})\ \mathrm{Re}(f_3^*f_4-f_5f_6^*)+i(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{\kappa}}-\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{m}})\ \mathrm{Im}(f_3^*f_4-f_5^*f_6). \tag{18}$$

A nonzero analyzing power $A^{(n,0)}$ characterizes the production of an unpolarized deuteron:

$$A^{(n,0)}\frac{d\sigma^0}{d\Omega}\approx(|f_1|^2-|f_2|^2+|f_3|^2+|f_4|^2-|f_5|^2-|f_6|^2). \tag{19}$$

Such a polarization observable depends only on the absolute values of the amplitudes f_i.

The tensor polarization of produced deuterons is characterized by the following nonzero components of the tensor $Q_{ij}^{(n,0)}$ [the upper index $(n,0)$ indicates that we have collisions of a polarized ^{3}H beam (in the $\vec{n}$ direction) with an unpolarized ^{3}He target]:

$$\frac{d\sigma^0}{d\Omega}Q_{mm}^{(n,0)}\approx-|f_1|^2+|f_2|^2+|f_3|^2-|f_5|^2,$$

$$\frac{d\sigma^0}{d\Omega}Q_{\kappa\kappa}^{(n,0)}\approx-|f_1|^2+|f_2|^2+|f_3|^2-|f_6|^2, \tag{20}$$

$$\frac{d\sigma^0}{d\Omega}Q_{m\kappa}^{(n,0)}\approx Q_{\kappa m}^{(n,0)}\frac{d\sigma^0}{d\Omega}\approx \mathrm{Re}(f_3f_4^*-f_5f_6^*).$$

The vector deuteron polarization must be transverse:

$$P_n^{(n,0)}\frac{d\sigma}{d\Omega}\approx-\ \mathrm{Im}(f_3f_4^*-f_5f_6^*). \tag{21}$$

(I.2) ^{3}H polarization vector is along $\vec{m}$:

$$\tfrac{1}{4}\ \mathrm{Tr}F^\dagger F\vec{\sigma}\cdot\hat{\vec{\kappa}}=(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{n}}+\vec{U}^*\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\ \mathrm{Re}(f_2^*f_3-f_1f_5^*)+(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{n}}+\vec{U}^*\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\ \mathrm{Im}(f_2^*f_4-f_1f_6^*)$$
$$+i(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{n}}-\vec{U}^*\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\ \mathrm{Re}(f_2f_3^*+f_1f_5^*)+i(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{n}}-\vec{U}^*\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\ \mathrm{Re}(f_2f_4^*+f_1f_6^*). \tag{22}$$

It follows that there are nonzero polarization observables given by

$$Q^{(m,0)}_{mn}\frac{d\sigma}{d\Omega}=Q^{(m,0)}_{nm}\frac{d\sigma}{d\Omega}\approx-\ \mathrm{Im}(f_1f_5^*+f_2f_3^*),$$

$$Q^{(m,0)}_{\kappa n}\frac{d\sigma}{d\Omega}=Q^{(m,0)}_{n\kappa}\frac{d\sigma}{d\Omega}\approx-\ \mathrm{Im}(f_1f_6^*+f_2f_4^*), \tag{23}$$

$$P^{(m,0)}_{\kappa}\frac{d\sigma}{d\Omega}\approx-\ \mathrm{Re}(f_2f_3^*+f_1f_5^*),\quad P^{(m,0)}_{m}\frac{d\sigma}{d\Omega}\approx \mathrm{Re}(f_2f_4^*+f_1f_6^*).$$

(I.3) ^{3}H polarization vector is along $\vec{\kappa}$:

$$\tfrac{1}{4}\ \mathrm{Tr}F^{\dagger}F\vec{\sigma}\cdot\hat{\vec{\kappa}}=(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{n}}+\vec{U}^*\cdot\hat{\vec{n}}\vec{U}\cdot\hat{\vec{m}})\ \mathrm{Re}(f_2^*f_3-f_1f_5^*)+(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{n}}+\vec{U}^*\cdot\hat{\vec{n}}\vec{U}\cdot\hat{\vec{\kappa}})\ \mathrm{Re}(f_2^*f_4-f_1f_6^*)$$
$$-i(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{n}}-\vec{U}^*\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\ \mathrm{Im}(f_2^*f_3+f_1^*f_5)-i(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{n}}-\vec{U}^*\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\ \mathrm{Im}(f_1f_6^*+f_2^*f_4). \tag{24}$$

The nonzero polarization observables can be found as

$$Q^{(\kappa,0)}_{mn}\frac{d\sigma}{d\Omega}=Q^{(\kappa,0)}_{nm}\frac{d\sigma}{d\Omega}\approx \mathrm{Re}(f_2f_3^*-f_1f_5^*),$$

$$Q^{(\kappa,0)}_{\kappa n}\frac{d\sigma}{d\Omega}=Q^{(\kappa,0)}_{n\kappa}\frac{d\sigma}{d\Omega}\approx \mathrm{Re}(f_2f_4^*-f_1f_6^*), \tag{25}$$

$$P^{(\kappa,0)}_{\kappa}\frac{d\sigma}{d\Omega}\approx \mathrm{Im}(f_2^*f_4+f_1f_6^*),\quad P^{(\kappa,0)}_{m}\frac{d\sigma}{d\Omega}\approx-\ \mathrm{Im}(f_1f_5^*-f_2f_3^*).$$

(II) In the case of scattering an unpolarized beam of ^{3}H on a polarized ^{3}He target, we also consider all three possible orientations of the ^{3}He polarization vector.

(II.1) Target is polarized along $\vec{n}$:

$$\tfrac{1}{4}\ \mathrm{Tr}FF^{\dagger}\vec{\sigma}\cdot\hat{\vec{n}}=|\vec{U}\cdot\hat{\vec{m}}|^2(|f_5|^2-|f_3|^2)+|\vec{U}\cdot\hat{\vec{n}}|^2(|f_1|^2-|f_2|^2)+|\vec{U}\cdot\hat{\vec{\kappa}}|^2(|f_6|^2-|f_4|^2)$$
$$+(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{\kappa}}+\vec{U}^*\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{\kappa}})\ \mathrm{Re}(f_5^*f_6-f_3f_4^*)-i(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{\kappa}}-\vec{U}^*\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{\kappa}})\ \mathrm{Im}(f_5^*f_6-f_3^*f_4). \tag{26}$$

Nonzero polarization observables are as follows:

$$A^{(0,n)}\frac{d\sigma}{d\Omega}\approx|f_1|^2-|f_2|^2-|f_3|^2-|f_4|^2+|f_5|^2+|f_6|^2,$$

$$Q^{(0,n)}_{mm}\frac{d\sigma}{d\Omega}\approx-|f_1|^2+|f_2|^2-|f_3|^2+|f_5|^2,$$

$$Q^{(0,n)}_{\kappa\kappa}\frac{d\sigma}{d\Omega}\approx-|f_1|^2+|f_2|^2-|f_4|^2+|f_6|^2, \tag{27}$$

$$Q^{(0,n)}_{m\kappa}\frac{d\sigma}{d\Omega}\approx Q^{(0,n)}_{\kappa m}\frac{d\sigma}{d\Omega}\approx \mathrm{Re}(-f_3f_4^*+f_5f_6^*),$$

$$P^{(0,n)}_{n}\frac{d\sigma}{d\Omega}\approx \mathrm{Im}(-f_3f_4^*+f_5f_6^*).$$

(II.2) Target is polarized along $\vec{m}$:

$$\frac{1}{4}\,\mathrm{Tr}FF^{\dagger}\vec{\sigma}\cdot\hat{\vec{m}}=-(\vec{U}\cdot\hat{\vec{m}}\vec{U}^{*}\cdot\hat{\vec{n}}+\vec{U}^{*}\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Im}(f_1^{*}f_3+f_2f_5^{*})-(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^{*}\cdot\hat{\vec{n}}+\vec{U}^{*}\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Im}(f_1^{*}f_4+f_2f_6^{*})$$
$$+i(\vec{U}\cdot\hat{\vec{m}}\vec{U}^{*}\cdot\hat{\vec{n}}-\vec{U}^{*}\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Re}(f_1f_3^{*}+f_2f_5^{*})+i(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^{*}\cdot\hat{\vec{n}}-\vec{U}^{*}\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Re}(f_1^{*}f_4+f_2f_6^{*}). \quad (28)$$

As a result, we have the following relations for nonzero polarization characteristics of tensor and vector polarization of deuterons:

$$Q_{mn}^{(0,m)}\frac{d\sigma}{d\Omega}=Q_{nm}^{(0,m)}\frac{d\sigma}{d\Omega}\approx-\,\mathrm{Im}(f_1f_3^{*}+f_2f_5^{*}),$$
$$Q_{\kappa n}^{(0,m)}\frac{d\sigma}{d\Omega}=Q_{n\kappa}^{(0,m)}\frac{d\sigma}{d\Omega}\approx-\,\mathrm{Im}(f_1f_4^{*}+f_2f_6^{*}), \quad (29)$$
$$P_{m}^{(0,m)}\frac{d\sigma}{d\Omega}\approx\mathrm{Re}(f_1f_4^{*}+f_2f_6^{*}),\qquad P_{m}^{(0,m)}\frac{d\sigma}{d\Omega}\approx-\,\mathrm{Re}(f_1f_3^{*}+f_2f_5^{*}).$$

(II.3) Target is polarized along $\vec{\kappa}$:

$$\frac{1}{4}\,\mathrm{Tr}FF^{\dagger}\vec{\sigma}\cdot\hat{\vec{\kappa}}=(\vec{U}\cdot\hat{\vec{m}}\vec{U}^{*}\cdot\hat{\vec{n}}+\vec{U}^{*}\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Re}(f_1^{*}f_3-f_2f_5^{*})+(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^{*}\cdot\hat{\vec{n}}+\vec{U}^{*}\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Re}(f_1^{*}f_4-f_2f_6^{*})$$
$$+i(\vec{U}\cdot\hat{\vec{m}}\vec{U}^{*}\cdot\hat{\vec{n}}-\vec{U}^{*}\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Im}(f_1f_3^{*}-f_2f_5^{*})+i(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^{*}\cdot\hat{\vec{n}}-\vec{U}^{*}\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Im}(f_1^{*}f_4-f_2f_6^{*}). \quad (30)$$

The following formulas are valid for the nonzero polarization observables of the deuteron:

$$Q_{mn}^{(0,\kappa)}\frac{d\sigma}{d\Omega}=Q_{nm}^{(0,\kappa)}\frac{d\sigma}{d\Omega}\approx\mathrm{Re}(f_1f_3^{*}-f_2f_5^{*}),$$
$$Q_{\kappa n}^{(0,\kappa)}\frac{d\sigma}{d\Omega}=Q_{n\kappa}^{(0,\kappa)}\frac{d\sigma}{d\Omega}\approx\mathrm{Re}(f_1f_4^{*}-f_2f_6^{*}), \quad (31)$$
$$P_{m}^{(0,\kappa)}\frac{d\sigma}{d\Omega}\approx\mathrm{Im}(f_1f_4^{*}-f_2f_6^{*}),$$
$$P_{\kappa}^{(0,\kappa)}\frac{d\sigma}{d\Omega}\approx-\,\mathrm{Re}(f_1f_3^{*}-f_2f_5^{*}).$$

Taking into account these formulas for the polarization observables and the symmetry properties of scalar amplitudes, Eq. (6), we can obtain numerous relations between these observables.

(A) Polarization of ^{3}H or ^{3}He along $\vec{n}$:

$$Q_{\kappa\kappa}^{(n,0)}(s,-\cos\theta)=+Q_{\kappa\kappa}^{(0,n)}(s,+\cos\theta),$$
$$Q_{mm}^{(n,0)}(s,-\cos\theta)=+Q_{mm}^{(0,n)}(s,+\cos\theta),$$
$$Q_{m\kappa}^{(n,0)}(s,-\cos\theta)=-Q_{m\kappa}^{(0,n)}(s,+\cos\theta), \quad (32)$$
$$P_{n}^{(n,0)}(s,-\cos\theta)=-P_{n}^{(0,n)}(s,+\cos\theta),$$
$$A^{(n,0)}(s,+\cos\theta)=+A^{(0,n)}(s,-\cos\theta).$$

(B) Polarization of ^{3}H or ^{3}He along $\vec{m}$:

$$Q_{mn}^{(m,0)}(s,-\cos\theta)=+Q_{mn}^{(0,m)}(s,+\cos\theta),$$
$$Q_{\kappa n}^{(m,0)}(s,-\cos\theta)=-Q_{\kappa n}^{(0,m)}(s,+\cos\theta),$$
$$P_{\kappa}^{(m,0)}(s,-\cos\theta)=+P_{\kappa}^{(0,m)}(s,+\cos\theta), \quad (33)$$
$$P_{m}^{(m,0)}(s,-\cos\theta)=-P_{m}^{(0,m)}(s,+\cos\theta).$$

(C) Polarization of ^{3}H or ^{3}He along $\vec{n}$:

$$Q_{mn}^{(\kappa,0)}(s,-\cos\theta)=-Q_{mn}^{(0,\kappa)}(s,+\cos\theta),$$
$$Q_{\kappa n}^{(\kappa,0)}(s,-\cos\theta)=+Q_{\kappa n}^{(0,\kappa)}(s,+\cos\theta),$$
$$P_{\kappa}^{(\kappa,0)}(s,-\cos\theta)=-P_{\kappa}^{(0,\kappa)}(s,+\cos\theta), \quad (34)$$
$$P_{m}^{(\kappa,0)}(s,-\cos\theta)=+P_{m}^{(0,\kappa)}(s,+\cos\theta).$$

We see that the isospin invariance relates the polarization observables for $^3\vec{\mathrm{H}}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ and $^3\mathrm{H}+{}^3\overrightarrow{\mathrm{He}}\to d^2\mathrm{H}+{}^4\mathrm{He}$ for the same spin orientations of initial particles but at different angles, namely θ and $\pi-\theta$.

Therefore, all relations with (–) sign predict the zero values for the corresponding polarization observables at $\theta=90°$. Such observables must change sign at $\theta=90°$, so this angle is especially appropriate for testing the validity of (II) and examining the effects of (II) violation. The latter effects must correspond to a change of position for the zero crossing of corresponding observables. Similar changes can be measured with high accuracy [26–31].

IV. COLLISIONS OF THE TWO POLARIZED NUCLEI

We now consider the dependence of the differential cross-section of the $^3\vec{\mathrm{H}}+{}^3\overrightarrow{\mathrm{He}}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction on the vector polarizations of both colliding particles. Using the P invariance we obtain the most general form of such dependence:

$$\frac{d\sigma}{d\Omega}(\vec{P}_1,\vec{P}_2)=\frac{d\sigma^0}{d\Omega}[1+A^{(mm)}\hat{\vec{m}}\cdot\vec{P}_1\hat{\vec{m}}\cdot\vec{P}_2+A^{(nn)}\hat{\vec{n}}\cdot\vec{P}_1\hat{\vec{n}}\cdot\vec{P}_2+A^{(\kappa\kappa)}\hat{\vec{\kappa}}\cdot\vec{P}_1\hat{\vec{\kappa}}\cdot\vec{P}_2+\hat{\vec{m}}\cdot\vec{P}_1\hat{\vec{\kappa}}\cdot\vec{P}_2A^{(m,\kappa)}+\hat{\vec{m}}\cdot\vec{P}_2\hat{\vec{\kappa}}\cdot\vec{P}_1A^{(\kappa,m)}]. \tag{35}$$

The analyzing powers $A^{(ij)}$ are proportional with

$$\tfrac{1}{4}\ \mathrm{Tr}\ \overline{F\vec{\sigma}\cdot\vec{P}_1F^{\dagger}\vec{\sigma}\cdot\vec{P}_2}, \tag{36}$$

where the line denotes summation over deuteron polarizations, and yields:

$$A^{(n,n)}\frac{d\sigma}{d\Omega}\approx|f_1|^2+|f_2|^2,$$

$$A^{(\kappa,\kappa)}\frac{d\sigma}{d\Omega}\approx 2\ \mathrm{Re}(f_1f_2^*-f_3f_5^*-f_4f_6^*),$$

$$A^{(m,m)}\frac{d\sigma}{d\Omega}\approx 2\ \mathrm{Re}(f_1f_2^*+f_3f_5^*+f_4f_6^*), \tag{37}$$

$$A^{(\kappa,m)}\frac{d\sigma}{d\Omega}\approx -2\ \mathrm{Im}(f_1f_2^*+f_3f_5^*+f_4f_6^*),$$

$$A^{(m,\kappa)}\frac{d\sigma}{d\Omega}\approx 2\ \mathrm{Im}(f_1f_2^*-f_3f_5^*-f_4f_6^*).$$

It follows from Eq. (2) and Eq. (37), that the definite symmetry properties must be valid for A_{ij}:

$$A^{(n,n)}(s,-\cos\theta)=+A^{(n,n)}(s,+\cos\theta),$$

$$A^{(\kappa,\kappa)}(s,-\cos\theta)=+A^{(\kappa,\kappa)}(s,+\cos\theta),$$

$$A^{(m,m)}(s,-\cos\theta)=+A^{(m,m)}(s,+\cos\theta), \tag{38}$$

$$A^{(\kappa,m)}(s,-\cos\theta)=+A^{(\kappa,m)}(s,+\cos\theta),$$

$$A^{(m,\kappa)}(s,-\cos\theta)=+A^{(m,\kappa)}(s,+\cos\theta).$$

V. TRIPLE POLARIZATION CORRELATIONS

We need the specific structure functions formalism in order to study the correlation of the polarizations of all three particles in the ${}^3\mathrm{H}+{}^3\mathrm{He}\to d{}^2\mathrm{H}+{}^4\mathrm{He}$ reaction. To explain this, let us consider the following tensor:

$$T_{ij}=\tfrac{1}{4}\ \mathrm{Tr}F_i\vec{\sigma}\cdot\vec{P}_1F_j^{\dagger}\vec{\sigma}\cdot\vec{P}_2 \tag{39}$$

with

$$F_i=\vec{U}^*\cdot\vec{F}=U_i^*F_i\,.$$

Taking into account the P invariance, T_{ij} can be written in terms of polarization vectors $\vec{P}_1$ and $\vec{P}_2$ of the colliding particles as

$$T_{ij}=\hat{\vec{m}}\cdot\vec{P}_1\hat{\vec{m}}\cdot\vec{P}_2S_{ij}(t_1-t_5)+\hat{\vec{n}}\cdot\vec{P}_1\hat{\vec{n}}\cdot\vec{P}_2S_{ij}(t_6-t_{10})+\hat{\vec{\kappa}}\cdot\vec{P}_1\hat{\vec{\kappa}}\cdot\vec{P}_2S_{ij}(t_{11}-t_{15})+\hat{\vec{m}}\cdot\vec{P}_1\hat{\vec{\kappa}}\cdot\vec{P}_2S_{ij}(t_{16}-t_{20})+\hat{\vec{\kappa}}\cdot\vec{P}_1\hat{\vec{m}}\cdot\vec{P}_2S_{ij}(t_{21}-t_{25})+\hat{\vec{m}}\cdot\vec{P}_1\hat{\vec{n}}\cdot\vec{P}_2A_{ij}(t_{26}-t_{29})+\hat{\vec{n}}\cdot\vec{P}_1\hat{\vec{m}}\cdot\vec{P}_2A_{ij}(t_{30}-t_{33})+\hat{\vec{\kappa}}\cdot\vec{P}_1\hat{\vec{n}}\cdot\vec{P}_2A_{ij}(t_{33}-t_{37})+\hat{\vec{n}}\cdot\vec{P}_1\hat{\vec{\kappa}}\cdot\vec{P}_2A_{ij}(t_{38}-t_{41}), \tag{40}$$

where we make use of the following notations:

$$S_{ij}(t_1-t_5)=\hat{m}_i\hat{m}_jt_1+\hat{n}_i\hat{n}_jt_2+\hat{\kappa}_i\hat{\kappa}_jt_3+\{\hat{m},\hat{\kappa}\}_{ij}t_4+i[\hat{m},\hat{\kappa}]_{ij}t_5,$$

$$A_{ij}(t_{26}-t_{29})=\{\hat{m},\hat{n}\}_{ij}t_{26}+\{\hat{\kappa},\hat{n}\}_{ij}t_{27}+i[\hat{m},\hat{n}]_{ij}t_{28}+i[\hat{\kappa},\hat{u}n]_{ij}t_{29}, \tag{41}$$

$$\{\hat{m},\hat{\kappa}\}_{ij}=\hat{m}_i\hat{\kappa}_j+\hat{m}_j\hat{\kappa}_i\,,$$

$$[\hat{m},\hat{\kappa}]_{ij}=\hat{m}_i\hat{\kappa}_j-\hat{m}_j\hat{\kappa}_i\,,$$

and $t_i(s,\cos\theta)$ are the real structure functions.

The symmetrical part of T_{ij} defines the tensorial deuteron polarization (in its dependence on $\vec{P}_1$ and $\vec{P}_2$), and the antisymmetrical part of T_{ij}, the vector polarization of the produced deuteron.

Therefore P-even correlations of initial vector polarizations such as

$$\hat{\vec{m}}\cdot\vec{P}_1\hat{\vec{m}}\cdot\vec{P}_2,$$

$$\hat{\vec{\kappa}}\cdot\vec{P}_1\hat{\vec{\kappa}}\cdot\vec{P}_2,$$

$$\hat{\vec{n}}\cdot\vec{P}_1\hat{\vec{n}}\cdot\vec{P}_2,$$

$$\hat{\vec{m}}\cdot\vec{P}_1\hat{\vec{\kappa}}\cdot\vec{P}_2,$$

and

$$\hat{\vec{\kappa}}\cdot\vec{P}_1\hat{\vec{m}}\cdot\vec{P}_2$$

result in the following three nonzero components of the quadrupole deuteron polarization tensor Q_{ij},:Q_{mm},$Q_{\kappa\kappa}$, and $Q_{m\kappa}=Q_{\kappa m}$, and the normal component of vector deuteron polarization, P_n.

As an example, one can obtain

$$Q_{mm}^{(m,m)}\approx t_1-t_2,$$

$$Q_{\kappa\kappa}^{(m,m)}\approx t_3-t_2,$$

$$Q_{m\kappa}^{(m,m)}\approx t_4,$$

$$P_n^{(m,m)}\approx t_5.$$

P-odd correlations of initial $\vec{P}_1$ and $\vec{P}_2$ polarization vectors, namely,

$$\hat{\vec{m}}\cdot\vec{P}_1\hat{\vec{n}}\cdot\vec{P}_2,$$

$$\hat{\vec{n}}\cdot\vec{P}_1\hat{\vec{m}}\cdot\vec{P}_2,$$

$$\hat{\vec{\kappa}}\cdot\vec{P}_1\hat{\vec{n}}\cdot\vec{P}_2,$$

and

$$\hat{\vec{n}}\cdot\vec{P}_1\hat{\vec{\kappa}}\cdot\vec{P}_2$$

produce two other (nondiagonal) components of the Q_{ij} tensor, i.e., Q_{mn} and $Q_{\kappa n}$, and two components of the vector polarization of the produced deuteron in the plane of reaction: P_m and P_κ.

On the other hand,

$$Q_{mn}^{(m,n)}\frac{d\sigma}{d\Omega}\approx t_{26},\quad Q_{\kappa n}^{(m,n)}\frac{d\sigma}{d\Omega}\approx t_{27},$$

$$P_m^{(m,n)}\frac{d\sigma}{d\Omega}\approx -t_{28},\quad P_\kappa^{(m,n)}\frac{d\sigma}{d\Omega}\approx t_{29}.$$

Using Eqs. (39) and (40) one finds the following expressions for all 41 structure functions in terms of the amplitudes f_i:

$$t_1=-t_{11}=2\ \mathrm{Re}f_3f_5^*, \tag{42}$$

$$t_2=t_{12}=2\ \mathrm{Re}f_1f_2^*,$$

$$t_3=-t_{13}=2\ \mathrm{Re}f_4f_6^*,$$

$$t_4=-t_{14}=\ \mathrm{Re}(f_3f_6^*+f_4f_5^*),$$

$$t_5=-t_{15}=\ \mathrm{Im}(f_3f_6^*-f_4f_5^*),$$

$$t_6=-|f_3|^2-|f_5|^2,$$

$$t_7=|f_1|^2+|f_2|^2,$$

$$t_8=|f_4|^2+|f_6|^2,$$

$$t_9=-\ \mathrm{Re}(f_3f_4^*+f_5f_6^*),\quad t_{10}=\ \mathrm{Im}(f_3f_4^*+f_4f_6^*),$$

$$t_{16}=t_{21}=-2\ \mathrm{Im}f_3f_5^*,\quad t_{17}=-t_{22}=-2\ \mathrm{Im}f_1f_2^*,$$

$$t_{18}=t_{23}=-2\ \mathrm{Im}f_4f_6^*,\quad t_{19}=t_{24}=-2\ \mathrm{Im}(f_3f_6^*+f_4f_5^*),$$

$$t_{20}=t_{25}=-\ \mathrm{Re}(f_3f_6^*-f_4f_5^*),$$

$$t_{26}=-\ \mathrm{Im}(f_1f_5^*-f_2f_3^*),\quad t_{27}=-\ \mathrm{Im}(f_1f_6^*-f_2f_4^*),$$

$$t_{28}=\ \mathrm{Re}(f_1f_5^*-f_2f_3^*),\quad t_{29}=\ \mathrm{Re}(f_1f_6^*-f_2f_4^*),$$

$$t_{30}=-\ \mathrm{Im}(f_1f_3^*-f_2f_5^*),\quad t_{31}=-\ \mathrm{Im}(f_1f_4^*-f_2f_6^*),$$

$$t_{32}=\ \mathrm{Re}(f_1f_3^*-f_2f_5^*),\quad t_{33}=\ \mathrm{Re}(f_1f_4^*-f_2f_6^*),$$

$$t_{34}=-\ \mathrm{Re}(f_1f_5^*+f_2f_3^*),\quad t_{35}=-\ \mathrm{Re}(f_1f_6^*+f_2f_4^*),$$

$$t_{36}=-\ \mathrm{Im}(f_1f_5^*-f_2f_3^*),\quad t_{37}=-\ \mathrm{Im}(f_1f_6^*-f_2f_4^*),$$

$$t_{38}=\ \mathrm{Re}(f_1f_3^*+f_2f_5^*),\quad t_{39}=\ \mathrm{Re}(f_1f_4^*+f_2f_6^*),$$

$$t_{40}=\ \mathrm{Im}(f_1f_3^*+f_2f_5^*),\quad t_{41}=\ \mathrm{Im}(f_1f_4^*+f_2f_6^*).$$

All these functions have definite symmetry properties relative to change of $\cos\theta\rightarrow-\cos\theta$, namely,

$$t_i(s,-\cos\theta)=t_i(s,+\cos\theta)$$

$$\text{for}\ \ i=1,2,3,6,7,8,11,12,13,17,\text{ and }\ 22,$$

$$t_i(s,-\cos\theta)=-t_i(s,+\cos\theta)$$

$$\text{for}\ \ i=4,5,9,10,14,15,16,18,19,20,21,23,24,25,$$

$$\begin{aligned}
t_{26}(s,-\cos\theta)&=t_{30}(s,+\cos\theta),\\
t_{27}(s,-\cos\theta)&=-t_{31}(s,+\cos\theta),\\
t_{28}(s,-\cos\theta)&=t_{32}(s,+\cos\theta),\\
t_{29}(s,-\cos\theta)&=-t_{33}(s,+\cos\theta),\\
t_{34}(s,-\cos\theta)&=-t_{38}(s,+\cos\theta),\\
t_{35}(s,-\cos\theta)&=t_{39}(s,+\cos\theta),\\
t_{36}(s,-\cos\theta)&=-t_{40}(s,+\cos\theta),\\
t_{37}(s,-\cos\theta)&=t_{41}(s,+\cos\theta).
\end{aligned}\tag{43}$$

Of course all these relations could be easily transformed into definite relations between polarization observables of the $^3\vec{\mathrm{H}}+{}^3\overrightarrow{\mathrm{He}}\rightarrow\vec{d}\,{}^2\mathrm{H}+{}^4\mathrm{He}$ reaction.

VI. POLARIZATION EFFECTS IN CASE OF $\theta=90°$ DEUTERON PRODUCTION

As it was mentioned in the Introduction, the spin structure of the total amplitude of the process $^3\mathrm{He}+{}^3\mathrm{H}\rightarrow d\,{}^2\mathrm{H}+{}^4\mathrm{He}$ is simplified for special kinematical conditions, namely for deuteron production at $\theta=90°$ in the c.m.s. of the considered reaction. This simplification is a result of the isospin invariance of the strong interaction for the specific initial states with both particles belonging to the same isospin multiplet. We can consider this simplification as a generalization of collinear two-particle collisions. In the last case the spin structure of the amplitude is simplified due to the conservation of the total spin projection. Therefore this symmetry property is transformed for the $^3\mathrm{He}+{}^3\mathrm{H}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction for $\theta=90°$ to a similar simplification of the spin structure of the total amplitude. As a result, we obtain definite and specific predictions for polarization experiments.

The polarization effects on the deuteron in the $p+p\to d+\pi^+$ reaction at $\theta=90°$ has been studied in Ref. [34].

In this paper, we employ the parametrization of the amplitude for the $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction, namely, the transverse spin structure. Naturally, the analysis of the complete experiment for such a kinematical condition is simplified.

Let us analyze this question in more detail. The amplitude F contains in this case, namely at $\theta=90°$, a contribution from only four independent amplitudes:

$$F=(1+\vec{\sigma}\cdot\hat{\vec{n}})\vec{U}^*\cdot\hat{\vec{n}}f_1+(1-\vec{\sigma}\cdot\hat{\vec{n}})\vec{U}^*\cdot\hat{\vec{n}}f_2+i\vec{\sigma}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{m}}f_3+\vec{\sigma}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{\kappa}}f_4, \tag{44}$$

where the amplitude $f_{1-4}(s,\cos 90°)=f_{1-4}(s,0)=f_{1-4}(s)$ depends now only on the energy of colliding particles. Of course, all four amplitudes are complex functions of the variable s.

As in the general case, for the analysis of polarization effects in $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$, at $\theta=90°$, it is necessary to calculate the following two products:

$$\begin{aligned}FF^\dagger=&\,2(1+\vec{\sigma}\cdot\hat{\vec{n}})|\vec{U}^*\cdot\hat{\vec{n}}|^2|f_1|^2+2(1-\vec{\sigma}\cdot\hat{\vec{n}})|\vec{U}^*\cdot\hat{\vec{n}}|^2|f_2|^2+\vec{U}^\star\cdot\hat{\vec{m}}|^2|f_3|^2+|\vec{U}^\star\cdot\hat{\vec{\kappa}}|^2|f_4|^2\\&+2\vec{\sigma}\cdot\hat{\vec{m}}\ \mathrm{Im}\{(f_1+f_2)f_3^\star\vec{U}\cdot\hat{\vec{m}}\vec{U}^\star\cdot\hat{\vec{n}}-(f_1-f_2)f_4^\star\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{n}}\}\\&+2\vec{\sigma}\cdot\hat{\vec{\kappa}}\ \mathrm{Re}\{(f_1+f_2)f_4^\star\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^\star\cdot\hat{\vec{n}}-(f_1-f_2)f_3^\star\vec{U}\cdot\hat{\vec{m}}\vec{U}^\star\cdot\hat{\vec{n}}\}+2\vec{\sigma}\cdot\hat{\vec{n}}\ \mathrm{Re}f_3f_4^\star\vec{U}^\star\cdot\hat{\vec{m}}\vec{U}^\star\cdot\hat{\vec{\kappa}},\end{aligned} \tag{45}$$

and

$$\begin{aligned}F^\dagger F=&\,2(|f_1|^2+|f_2|^2)|\vec{U}\cdot\hat{\vec{n}}|^2+2(|f_1|^2-|f_2|^2)|\vec{U}\cdot\hat{\vec{n}}|^2\vec{\sigma}\cdot\hat{\vec{n}}+|f_3|^2|\vec{U}\cdot\hat{\vec{m}}|^2+|f_4|^2|\vec{U}\cdot\hat{\vec{\kappa}}|^2+2\vec{\sigma}\cdot\hat{\vec{m}}\ \mathrm{Im}\{(f_1+f_2)f_3^\star\vec{U}\cdot\hat{\vec{m}}\vec{U}^\star\cdot\hat{\vec{n}}\\&+(f_1-f_2)f_4^\star\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^\star\cdot\hat{\vec{n}}\}+2\vec{\sigma}\cdot\hat{\vec{\kappa}}\ \mathrm{Re}\{(f_1+f_2)f_4^\star\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^\star\cdot\hat{\vec{n}}-(f_1-f_2)f_3^\star\vec{U}\cdot\hat{\vec{m}}\vec{U}^\star\cdot\hat{\vec{n}}\}-2\vec{\sigma}\cdot\hat{\vec{n}}\ \mathrm{Re}f_3f_4^\star\cdot\vec{U}^\star\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{\kappa}}.\end{aligned} \tag{46}$$

Formulas (45) and (46) allow us to calculate all polarization effects in the $^3\mathrm{He}+{}^3\mathrm{H}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction, namely, $^3\mathrm{He}+{}^3\mathrm{H}\to\vec{d}^2\mathrm{H}+{}^4\mathrm{He}$, $^3\overrightarrow{\mathrm{He}}+{}^3\mathrm{H}\to d^2\mathrm{H}+{}^4\mathrm{He}$, $^3\mathrm{He}+{}^3\vec{\mathrm{H}}\to d^2\mathrm{H}+{}^4\mathrm{He}$ (only one particle is polarized in the initial or final state), $^3\overrightarrow{\mathrm{He}}+{}^3\vec{\mathrm{H}}\to d^2\mathrm{H}+{}^4\mathrm{He}$, $^3\overrightarrow{\mathrm{He}}+{}^3\mathrm{H}\to d^2\mathrm{H}+{}^4\mathrm{He}$, $^3\mathrm{He}+{}^3\vec{\mathrm{H}}\to\vec{d}^2\mathrm{H}+{}^4\mathrm{He}$ (any pair of the particles in the initial or final state is polarized) and $^3\overrightarrow{\mathrm{He}}+{}^3\vec{\mathrm{H}}\to\vec{d}^2\mathrm{H}+{}^4\mathrm{He}$ (all three nuclei with nonzero spin are polarized).

Indeed, the differential cross section of collision of the both unpolarized nuclei ^{3}He and ^{3}H with production of the polarized deuterons is defined as

$$\begin{aligned}\frac{d\sigma}{d\Omega}\approx&\ \tfrac{1}{4}\ \mathrm{Tr}FF^\dagger=\tfrac{1}{4}\ \mathrm{Tr}F^\dagger F\\&=(|f_1|^2+|f_2|^2)|\vec{U}\cdot\hat{\vec{n}}|^2+|\vec{U}\cdot\hat{\vec{m}}|^2|f_3|^2\\&+|\vec{U}\cdot\hat{\vec{\kappa}}|^2|f_4|^2.\end{aligned} \tag{47}$$

Equation (47) covers the description of the following cases.

(a) The differential cross section of the production of the unpolarized deuterons contains only the squares of the amplitude moduli:

$$\frac{d\sigma^0}{d\Omega}\approx|f_1|^2+|f_2|^2+|f_3|^2+|f_4|^2. \tag{48}$$

(b) The tensor polarization of the produced deuterons is characterized by the following nonzero components:

$$\begin{aligned}Q_{mm}^{(0,0)}\frac{d\sigma^0}{d\Omega}&\approx-|f_1|^2-|f_2|^2+|f_3|^2,\\Q_{\kappa\kappa}^{(0,0)}\frac{d\sigma^0}{d\Omega}&\approx-|f_1|^2-|f_2|^2+|f_4|^2.\end{aligned} \tag{49}$$

(c) The vector polarization of the produced deuterons is equal to zero exactly (for any mechanism of the considered reaction).

The next step is the calculation of scattering asymmetries (or the analyzing power) in $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ which are induced by the vector polarization of one of the colliding nuclei.

(I) The scattering of a polarized ^{3}H beam by the ^{3}He unpolarized target $^3\vec{\mathrm{H}}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$.

(I-1) The ^{3}H beam is polarized in the $\hat{\vec{m}}$ direction:

$$\tfrac{1}{4}\,\mathrm{Tr}F^{\dagger}F\vec{\sigma}\cdot\hat{\vec{m}}=\tfrac{1}{2}(\vec{U}\cdot\hat{\vec{m}}\vec{U}^{*}\cdot\hat{\vec{n}}+\vec{U}\cdot\hat{\vec{n}}\vec{U}^{*}\cdot\hat{\vec{m}})\,\mathrm{Im}(f_1+f_2)f_3^{\star}+\tfrac{1}{2}(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^{*}\cdot\hat{\vec{n}}+\vec{U}\cdot\hat{\vec{n}}\vec{U}^{*}\cdot\hat{\vec{\kappa}})\,\mathrm{Re}(f_1-f_2)f_4^{\star}$$
$$+\frac{i}{2}(-\vec{U}\cdot\hat{\vec{m}}\vec{U}^{*}\cdot\hat{\vec{n}}+\vec{U}\cdot\hat{\vec{n}}\vec{U}^{*}\cdot\hat{\vec{m}})\,\mathrm{Re}(f_1+f_2)f_3^{\star}-\frac{i}{2}(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^{*}\cdot\hat{\vec{n}}-\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^{*}\cdot\hat{\vec{n}})\,\mathrm{Re}(f_1-f_2)f_4^{\star}. \quad (50)$$

(a) The asymmetry of the production of unpolarized deuterons must be equal to zero.

(b) The nonzero components of the vector polarization of the produced deuterons can appear in the scattering plane only:

$$P_{\kappa}^{(m,0)}\frac{d\sigma}{d\Omega}\approx \mathrm{Re}(f_1+f_2)f_3^{\star},$$

$$P_{m}^{(m,0)}\frac{d\sigma}{d\Omega}\approx -\,\mathrm{Re}(f_1-f_2)f_4^{\star}. \quad (51)$$

(c) The tensor polarization of the produced deuterons is characterized by the following nonzero components:

$$Q_{mn}^{(m,0)}\frac{d\sigma}{d\Omega}=Q_{nm}^{(m,0)}\frac{d\sigma}{d\Omega}\approx\tfrac{1}{2}\,\mathrm{Im}(f_1+f_2)f_3^{\star},$$

$$Q_{\kappa n}^{(m,0)}\frac{d\sigma}{d\Omega}=Q_{n\kappa}^{(m,0)}\frac{d\sigma}{d\Omega}\approx\tfrac{1}{2}\,\mathrm{Im}(f_1-f_2)f_4^{\star}. \quad (52)$$

(I-2) The beam of ^{3}H is polarized in the $\hat{\vec{n}}$ direction:

$$\tfrac{1}{4}\,\mathrm{Tr}F^{\dagger}F\vec{\sigma}\cdot\hat{\vec{n}}=(|f_1|^2-|f_2|^2)|\vec{U}\cdot\hat{\vec{n}}|^2$$
$$-\tfrac{1}{2}(\vec{U}\cdot\hat{\vec{m}}\vec{U}^{*}\cdot\hat{\vec{\kappa}}+\vec{U}^{\star}\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{\kappa}})\,\mathrm{Re}f_3f_4^{\star}$$
$$+\frac{i}{2}(\vec{U}\cdot\hat{\vec{m}}\vec{U}^{*}\cdot\hat{\vec{\kappa}}-\vec{U}^{\star}\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{\kappa}}\,\mathrm{Im}f_3f_4^{\star}. \quad (53)$$

(a) The asymmetry of the production of the unpolarized deuterons is nonzero:

$$A^{(n,0)}\frac{d\sigma}{d\Omega}\approx|f_1|^2-|f_2|^2. \quad (54)$$

(b) The vector polarization of the produced deuterons has only the normal component which is defined as

$$P_{n}^{(n,0)}\frac{d\sigma}{d\Omega}\approx-\tfrac{1}{2}\,\mathrm{Im}f_3f_4^{\star}. \quad (55)$$

(c) The nonzero components of the tensor deuteron polarization are given by

$$Q_{nn}^{(n,0)}\approx|f_1|^2-|f_2|^2,$$

$$Q_{m\kappa}^{(n,0)}\frac{d\sigma}{d\Omega}=Q_{\kappa m}^{(n,0)}\frac{d\sigma}{d\Omega}\approx-\tfrac{1}{2}\,\mathrm{Re}f_3f_4^{\star}. \quad (56)$$

(I-3) The ^{3}H beam is polarized in the $\hat{\vec{\kappa}}$ direction:

$$\tfrac{1}{4}\,\mathrm{Tr}F^{\dagger}F\vec{\sigma}\cdot\hat{\vec{\kappa}}=\tfrac{1}{2}(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^{*}\cdot\hat{\vec{n}}+\vec{U}^{\star}\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Re}(f_1+f_2)f_4^{\star}+\frac{i}{2}(\vec{U}\cdot\hat{\vec{m}}\vec{U}^{*}\cdot\hat{\vec{n}}+\vec{U}^{\star}\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Re}(f_1-f_2)f_3^{\star}$$
$$+\frac{i}{2}(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^{*}\cdot\hat{\vec{n}}-\vec{U}^{\star}\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Im}(f_1+f_2)f_4^{\star}+\frac{i}{2}(\vec{U}\cdot\hat{\vec{m}}\vec{U}^{*}\cdot\hat{\vec{n}}-\vec{U}^{\star}\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Im}(f_1-f_2)f_3^{\star}. \quad (57)$$

(a) The analyzing power for producing the unpolarized deuterons must be zero (for this spin orientation):

$$A^{(\kappa,0)}=0. \quad (58)$$

(b) The vector polarization of the produced deuterons is characterized by two nonzero components in the reaction plane which are determined as

$$P_{m}^{(\kappa,0)}\frac{d\sigma}{d\Omega}\approx \mathrm{Im}(f_1+f_2)f_4^{\star},$$

$$P_{\kappa}^{(\kappa,0)}\frac{d\sigma}{d\Omega}\approx -\,\mathrm{Im}(f_1-f_2)f_3^{\star}. \quad (59)$$

(c) The nonzero components of the tensor polarization of the produced deuterons are given by

$$Q^{(\kappa,0)}_{\kappa n}\frac{d\sigma}{d\Omega}=Q^{(\kappa,0)}_{n\kappa}\frac{d\sigma}{d\Omega}\approx\tfrac{1}{2}\,\mathrm{Re}(f_1+f_2)f_4^\star,$$

$$Q^{(\kappa,0)}_{mn}\frac{d\sigma}{d\Omega}=Q^{(\kappa,0)}_{nm}\frac{d\sigma}{d\Omega}\approx\tfrac{1}{2}\,\mathrm{Re}(f_1-f_2)f_3^\star. \tag{60}$$

(II) The analyzing powers (or asymmetries) for the scattering of the unpolarized beam ^{3}H by the polarized target ^{3}He can be calculated in a similar way. All three possible spin orientations of the polarized target and their consequences are given below.

(II-1) The target is polarized along the $\hat{\vec{m}}$ direction:

$$\tfrac{1}{4}\,\mathrm{Tr}FF^\dagger\vec{\sigma}\cdot\hat{\vec{m}}=(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{n}}+\vec{U}^\star\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Im}(f_1+f_2)f_3^\star-(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{n}}+\vec{U}^\star\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Im}(f_1-f_2)f_4^\star$$
$$-i(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{n}}-\vec{U}^\star\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Re}(f_1+f_2)f_3^\star+i(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{n}}-\vec{U}^\star\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Re}(f_1-f_2)f_4^\star. \tag{61}$$

This expression contains the following information about the polarization properties of the deuterons produced in the reaction $^3\mathrm{H}+{}^3\overrightarrow{\mathrm{He}}\to d^2\mathrm{H}+{}^4\mathrm{He}$.

(a) The analyzing power corresponding to production of the unpolarized deuterons is equal to zero exactly (for this spin orientation)

$$A^{(0,m)}=0. \tag{62}$$

This result is the consequence of the P invariance of the strong (and electromagnetic also) interaction.

(b) The vector polarization of the produced deuterons is characterized by two nonzero components in the decay plane, namely:

$$P^{(0,m)}_\kappa\frac{d\sigma}{d\Omega}\approx\mathrm{Re}(f_1+f_2)f_3^\star,$$

$$P^{(0,m)}_m\frac{d\sigma}{d\Omega}\approx\mathrm{Re}(f_1-f_2)f_4^\star. \tag{63}$$

(c) The tensor polarization of the produced deuterons is characterized by the following nonzero components:

$$Q^{(0,m)}_{m,n}\frac{d\sigma}{d\Omega}=Q^{(m,0)}_{nm}\frac{d\sigma}{d\Omega}\approx\tfrac{1}{2}\,\mathrm{Im}(f_1+f_2)f_3^\star,$$

$$Q^{(0,m)}_{\kappa,n}\frac{d\sigma}{d\Omega}=Q^{(0,m)}_{n\kappa}\frac{d\sigma}{d\Omega}\approx\tfrac{1}{2}\,\mathrm{Im}(f_1-f_2)f_4^\star. \tag{64}$$

These nonzero components of $Q^{(0,m)}_{ij}$ are the consequence of P invariance.

(II-2) The target is polarized along the $\hat{\vec{n}}$ direction: It can be shown that all polarization characteristics of the produced deuterons coincide with corresponding characteristics for the scattering of the polarized ^{3}H beam (with the polarization in the $\hat{\vec{n}}$ direction). Only the deuteron vector polarization has different signs for the $^3\mathrm{H}+{}^3\overrightarrow{\mathrm{He}}\to d^2\mathrm{H}+{}^4\mathrm{He}$ and $^3\vec{\mathrm{H}}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ scattering.

(II-3) The target is polarized along the $\hat{\vec{\kappa}}$ direction: As usual it is necessary to calculate the corresponding trace, namely,

$$\tfrac{1}{2}TrFF^\dagger\vec{\sigma}\cdot\hat{\vec{\kappa}}=\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^\star\cdot\hat{\vec{n}})+\vec{U}^\star\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Re}(f_1+f_2)f_4^\star+(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot+\vec{U}^\star\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Re}(f_1-f_2)f_3^\star$$
$$+i(\vec{U}\cdot\hat{\vec{\kappa}}\vec{U}^*\cdot\hat{\vec{n}}-\vec{U}^\star\cdot\hat{\vec{\kappa}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Im}(f_1+f_2)f_4^\star-i(\vec{U}\cdot\hat{\vec{m}}\vec{U}^*\cdot\hat{\vec{n}}-\vec{U}^\star\cdot\hat{\vec{m}}\vec{U}\cdot\hat{\vec{n}})\,\mathrm{Im}(f_1-f_2)f_3^\star. \tag{65}$$

This result allows us to obtain the following set of the standard predictions.

(a) The analyzing power, corresponding to the production of unpolarized deuterons, must be equal to zero

$$A^{(0,m)}=0, \tag{66}$$

for any values of the independent amplitudes f_1-f_4 (due to the P invariance).

(b) The vector deuteron polarization is characterized by two nonzero components (in the decay plane: due also to P invariance)

$$P^{(0,\kappa)}_m\frac{d\sigma}{d\Omega}\approx\tfrac{1}{2}\,\mathrm{Im}(f_1+f_2)f_4^\star,$$

$$P^{(0,\kappa)}_\kappa\frac{d\sigma}{d\Omega}\approx\tfrac{1}{2}\,\mathrm{Im}(f_1-f_2)f_3^\star. \tag{67}$$

(c) The components of the tensor polarization of the produced deuterons is characterized by the following expressions:

$$Q^{(0,\kappa)}_{\kappa n}\frac{d\sigma}{d\Omega}=Q^{(0,\kappa)}_{n\kappa}\frac{d\sigma}{d\Omega}\approx\tfrac{1}{2}\ \mathrm{Re}(f_1+f_2)f_4^{\star},$$

$$Q^{(0,\kappa)}_{mn}\frac{d\sigma}{d\Omega}=Q^{(0,\kappa)}_{nm}\frac{d\sigma}{d\Omega}\approx\tfrac{1}{2}\ \mathrm{Re}(f_1-f_2)f_3^{\star}. \qquad (68)$$

After this the analysis of the polarization phenomena in the $^3\vec{\mathrm{H}}+{}^3\mathrm{He}\to\vec{d}^2\mathrm{H}+{}^4\mathrm{He}$ and $^3\mathrm{H}+{}^3\overrightarrow{\mathrm{He}}\to\vec{d}^2\mathrm{H}+{}^4\mathrm{He}$ collisions with production at 90° of unpolarized and polarized deuterons becomes complete.

In the next step, we will consider polarization phenomena for the case of collisions of both polarized nuclei, $^3\vec{\mathrm{H}}+{}^3\overrightarrow{\mathrm{He}}\to d^2\mathrm{H}+{}^4\mathrm{He}$ (for $\theta=90°$).

If the polarization of the final deuteron is not detected, the dependence of the differential cross section on the polarization vectors $\vec{P}_1(^3\mathrm{H})$ and $\vec{P}_2(^3\mathrm{He})$ can be written in the following form:

$$\frac{d\sigma}{d\Omega}(\vec{P}_1,\vec{P}_2)=\frac{d\sigma^0}{d\Omega}[1+A^{(m,m)}\hat{\vec{m}}\cdot\vec{P}_1\hat{\vec{m}}\cdot\vec{P}_2+A^{(n,n)}\hat{\vec{n}}\cdot\vec{P}_1\hat{\vec{n}}\cdot\vec{P}_2+A^{(\kappa,\kappa)}\hat{\vec{\kappa}}\cdot\vec{P}_1\hat{\vec{\kappa}}\cdot\vec{P}_2+A^{(m,\kappa)}\hat{\vec{m}}\cdot\vec{P}_1\hat{\vec{\kappa}}\cdot\vec{P}_2+A^{(\kappa,m)}\hat{\vec{\kappa}}\cdot\vec{P}_1\hat{\vec{m}}\cdot\vec{P}_2], \qquad (69)$$

where $A^{(m,m)}\ \cdots\ A^{(\kappa,m)}$ are the corresponding analyzing powers for the $^3\vec{\mathrm{H}}+{}^3\overrightarrow{\mathrm{He}}\to d\ ^2\mathrm{H}+{}^4\mathrm{He}$ reaction. As can be seen, it is the most general expression for the $(d\sigma/d\Omega)(\vec{P}_1,\vec{P}_2)$ which is valid for any two-particle collisions (with vector polarization of both colliding particles). The structure of Eq. (69) is valid not only for the collision of two particles with spin 1/2, but for any values of nonzero spin (in the special case of vector polarization if the spin is greater than 1/2). Equation (69) generalizes the spin dependence of the total cross section for any $\vec{a}+\vec{b}$ collisions (a and b are some hadrons or nuclei). Instead of the traditional two-spin correlations [40] for the total cross section, we have five independent spin correlations for the differential cross section which is fixed by the P invariance only.

Equation (69) is valid for any d production angle, except $\theta=0$ or $\theta=\pi$. In this case the spin structure of the differential cross sections is defined by two-spin correlations only (as in the case of the total cross section) as

$$\frac{d\sigma}{d\Omega}(\vec{P}_1,\vec{P}_2)_{\theta=0,\pi}=\frac{d\sigma^0}{d\Omega}(1+A_1\vec{P}_1\cdot\vec{P}_2+A_2\vec{P}_1\cdot\hat{\vec{\kappa}}\vec{P}_2\cdot\hat{\vec{\kappa}}). \qquad (70)$$

Asymmetries in Eq. (69), $\theta=90°$, are given with the following expressions in terms of the amplitudes f_1-f_4:

$$A^{(m,m)}\frac{d\sigma^0}{d\Omega}\approx\tfrac{1}{2}(|f_3|^2-|f_4|^2)+2\ \mathrm{Re}f_1f_2^*, \qquad (71)$$

$$A^{(n,n)}\frac{d\sigma^0}{d\Omega}\approx|f_1|^2+|f_2|^2)-\tfrac{1}{2}(|f_3|^2+|f_4|^2),$$

$$A^{(\kappa,\kappa)}\frac{d\sigma^0}{d\Omega}\approx-\tfrac{1}{2}(|f_3|^2-|f_4|^2)+2\ \mathrm{Re}f_1f_2^*,$$

$$A^{(m,\kappa)}\frac{d\sigma^0}{d\Omega}=-A^{(\kappa,m)}\frac{d\sigma^0}{d\Omega}\approx2\ \mathrm{Im}f_1f_2^*.$$

The vector and tensor polarizations of the deuteron are characterized in terms of the set of 41 SF's $t_i(s,0)$. Using the amplitude parametrization, Eq. (44), we can obtain the following expression for these SF's in terms of the transverse amplitudes f_i, $i=1-4$:

$$t_1=\tfrac{1}{2}|f_3|^2, \qquad (72)$$

$$t_2=2\ \mathrm{Re}f_1f_2^*,$$

$$t_3=-\tfrac{1}{2}|f_4|^2,$$

$$t_4=t_5=0,$$

$$t_6=-\tfrac{1}{2}|f_3|^2,$$

$$t_7=2\ \mathrm{Re}f_1f_2^{\star},$$

$$t_8=\tfrac{1}{2}|f_4|^2,$$

$$t_9=t_{10}=0,$$

$$t_{11}=-\tfrac{1}{2}|f_3|^2,$$

$$t_{12}=|f_1|^2+|f_2|^2,$$

$$t_{13}=-\tfrac{1}{2}|f_4|^2,$$

$$t_{14}=t_{15}=t_{16}=t_{18}=0,$$

$$t_{17}=2\ \mathrm{Im}f_1f_2^*,$$

$$t_{19}=2\ \mathrm{Im}f_3f_4^*,$$

$$t_{20}=-\tfrac{1}{2}\ \mathrm{Re}f_3f_4^*,$$

$$t_{21}=t_{23}=0,$$

$$t_{22}=-2\ \mathrm{Im}f_1f_2^{\star},$$

$$t_{24}=-\tfrac{1}{2}\ \mathrm{Im}f_3f_4^{\star},$$

$$t_{25}=-\tfrac{1}{2}\ \mathrm{Re}f_3f_4^{\star},$$

$$t_{26}=\ \mathrm{Im}(f_1-f_2)f_3^{\star},\quad t_{27}=\ \mathrm{Im}(f_1+f_2)f_4^{\star},$$

$$t_{28}=-\ \mathrm{Re}(f_1-f_2)f_3^{\star},\quad t_{29}=-\ \mathrm{Re}(f_1+f_2)f_4^{\star},$$

$$t_{30}=\ \mathrm{Im}(f_1-f_2)f_3^{\star},\quad t_{31}=-\ \mathrm{Im}(f_1+f_2)f_4^{\star},$$

$$t_{32}=-\ \mathrm{Re}(f_1-f_2)f_3^\star,\quad t_{33}=\ \mathrm{Im}(f_1+f_2)f_4^\star,$$

$$t_{34}=\ \mathrm{Re}(f_1+f_2)f_3^\star,\quad t_{35}=\ \mathrm{Re}(f_1-f_2)f_4^\star,$$

$$t_{36}=\ \mathrm{Im}(f_1+f_2)f_3^\star,\quad t_{37}=\ \mathrm{Im}(f_1-f_2)f_4^\star,$$

$$t_{38}=-\ \mathrm{Re}(f_1+f_2)f_3^\star,\quad t_{39}=\ \mathrm{Re}(f_1-f_2)f_4^\star,$$

$$t_{40}=-\ \mathrm{Im}(f_1+f_2)f_3^\star,\quad t_{41}=\ \mathrm{Im}(f_1-f_2)f_4^\star.$$

We can see from these expressions that there are many different observables for the polarized particles in the initial and final states in the $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction. Measurements allow us to determine not only $|f_i(s,0)|^2$, $i=1-4$, but the relative phases of the four independent amplitudes through their interference contribution, ($\mathrm{Re} f_i f_j^\star$ and $\mathrm{Im} f_i f_j^\star$). Therefore there are many ways for obtaining full experimental knowledge of the spin structure of the amplitude for the $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction. The first step of such experiments must be the determination of the moduli of the four independent amplitudes as a function of the single variable s. The simplest polarization effects that arise in $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ as defined by $|f_i|^2$ only are as follows:

$$\frac{d\sigma^0}{d\Omega},\quad A^{(n,0)}=A^{(0,n)},\quad A^{(n,n)}\frac{d\sigma^0}{d\Omega},$$

$$A^{(m,m)}\frac{d\sigma^0}{d\Omega}-A^{(\kappa,\kappa)}\frac{d\sigma^0}{d\Omega},\quad Q^{(0,0)}_{m,m},\quad Q^{(0,0)}_{\kappa,\kappa}. \tag{73}$$

The explicit forms of the quantities in Eq. (73) are given by the following expressions:

$$\frac{d\sigma^0}{d\Omega}\approx|f_1|^2+|f_2|^2+|f_3|^2+|f_4|^2,$$

$$A^{(n,0)}\frac{d\sigma^0}{d\Omega}\approx|f_1|^2-|f_2|^2,$$

$$A^{(n,n)}\frac{d\sigma^0}{d\Omega}\approx|f_1|^2+|f_2|^2-\tfrac{1}{2}(|f_3|^2+|f_4|^2), \tag{74}$$

$$A^{(m,m)}\frac{d\sigma^0}{d\Omega}-A^{(\kappa,\kappa)}\frac{d\sigma^0}{d\Omega}\approx|f_3|^2-|f_4|^2.$$

Therefore we can reconstruct all $|f_i(s,0)|^2$ without measuring the polarization of the produced deuterons. This reconstruction is unique.

The next step is the determination of the $\mathrm{Re}(f_i f_j^\star)$ and $\mathrm{Im}(f_i f_j^\star)$ combination of independent amplitudes.

VII. CONCLUSION

We analyzed all possible polarization effects in the $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction. We use a standard phenomenological description of the spin structure of the amplitude, which is based on general symmetry properties of the fundamental interaction only. Taking into account the P invariance of the strong interaction, the amplitude of the $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction was parametrized in terms of six independent complex amplitudes. The nuclei $^3\mathrm{H}$ and $^3\mathrm{He}$ are described by two-component spinors and the deuteron by an axial vector. The structure of the interacting nuclei and the mechanism of the reaction $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ define a definite form of these six scalar amplitudes. A general analysis of the polarization effects in this reaction can be performed in terms of these amplitudes.

Despite the evident equivalence of different possible parametrizations [14,41,42] of the spin structure of the total amplitude for $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$, the parametrization which is known as transverse amplitudes [36–39] is more appropriate for this particular reaction. The main advantage of such a parametrization is simplification of the expressions for the polarization observables.

The most trivial set of polarization observables of $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ reaction, which are dependent on the squares of moduli of transverse amplitudes only and do not contain any interference contributions, is found. An important point to be noted is that the differential cross section (with all unpolarized particles in the initial and final states) has this property. The analyzing powers in $^3\vec{\mathrm{H}}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ and $^3\mathrm{H}+{}^3\overrightarrow{\mathrm{He}}\to d^2\mathrm{H}+{}^4\mathrm{He}$ due to the polarization of the one colliding nuclei (with vector polarization orthogonal to the reaction plane) are also defined by the moduli of the scalar amplitudes. The normal to the reaction plane, which is produced by the three-momenta of the initial and final particles, can be chosen as the axis of quantization for the spins of the interacting particles. This choice of quantization axis allows, in some sense, to compensate the nonequivalence of the initial and final particles. The same procedure is applied in the method of helicity amplitudes.

It is evident that the method of scalar amplitudes, which employs the two-component spinors and Pauli matrixes $\vec{\sigma}$, simplifies essentially the analysis of the polarization effects.

The reaction $^3\mathrm{He}+{}^3\mathrm{H}\to d^2\mathrm{H}+{}^4\mathrm{He}$ proceeds via the strong interaction and then the properties of the internal symmetry, which is represented by the isospin invariance, are correlated to the polarization characteristics of the process. Indeed, the isospin invariance produces a definite behavior of the scalar amplitudes relative to a $\cos\theta\to-\cos\theta$ change, which develops in the relations between polarization observables. Similar relations are valid because nuclei $^3\mathrm{H}$ and $^3\mathrm{He}$ in the initial state belong to the same isospin multiplet but differing by only isospin projections. Specifically, the isospin invariance simplifies the spin structure of the total amplitude of $^3\mathrm{H}+{}^3\mathrm{He}\to d^2\mathrm{H}+{}^4\mathrm{He}$ at $\theta=90°$, leaving (under such kinematical conditions) only four (from six) independent amplitudes.

All the above-mentioned consequences of isospin invariance are not exactly correct. Electromagnetic effects and the mass difference of u and d quarks invoke violations of the above-mentioned symmetry. Therefore, this violation can produce real physical effects in the different polarization observables.

In this connection, it is necessary to stress that the more recent view of the theory of the strong interaction, which is based on QCD, changed essentially transposes the problem of isospin invariance violation [43]. Indeed, the previous so called electromagnetic conception [29,44–48] of this violation now seems less interesting. Moreover, such a model is incomplete because, for example, the electromagnetic contributions to the splitting of the masses of hadrons belonging to the same isospin multiplet incorrectly predicted the sign of these mass differences. The main effects here are due to the mass difference of u and d quarks.

Of course, the effects of the u and d quark mass difference are manifested in different physical quantities. The simplest are the mass splittings of baryons and mesons belonging to single isospin multiplets. But these simple characteristics demand for interpretation nonsimple theoretical models (especially in the case of baryons). The same is correct for the analysis of the effects of isospin invariance violation in different hadronic (and nuclei) processes. Such analysis touches the most delicate problems of QCD.

The problem of isospin invariance violation is transformed now in the interesting problem of fundamental particle physics, which unifies almost all low energy QCD phenomenology. Therefore the search for isospin invariance violations in different polarizations effects for different processes, such as $^3H+{}^3He\to d^2H+{}^4He$, forms an additional source of information.

In order to activate interest in this problem, it is necessary to have the most adequate formalism for describing the polarization effects. In our opinion, the technique of transverse amplitudes is the most suitable formalism. From a kinematical point of view, the most preferable must be $\theta=90°$, i.e., ‘‘transverse’’ kinematics.

Therefore, at first step, we parametrized the spin structure of the amplitude of the process $^3H+{}^3He\to d^2H+{}^4He$ in terms of six independent scalar amplitudes and established the symmetry properties of these amplitudes relative to the change $\cos\theta\to-\cos\theta$ which follows, from the isospin invariance.

Using these amplitudes, we systematically calculated all polarization effects in $^3H+{}^3He\to d^2H+{}^4He$: one-, two-, and three-spin correlations. We characterized the polarization properties of nuclei ^3H and ^3He in terms of the vector polarization, the deuteron polarization properties in terms of the vector and tensor polarizations. For all these polarization observables, we obtained the general expressions in terms of the scalar amplitudes and found all the relations between these observables which follow from the isospin invariance.

In such analysis, we do not use any concrete model for calculation of the reaction amplitudes. We assumed that the fact of the violation of these general relations between the polarization observables must be the unique signal of isospin invariance violation.

In any case, such a signal must be interesting because it demonstrates the connection between the polarization effects, from one side, and the isospin invariance violation effects, from another side. Of course, the quantitative interpretation of these effects will involve a very complicated dynamical analysis using all about the achievements of modern QCD, not only of the problem of isospin invariance violation, but the problems of the nuclear structure and nuclear dynamics.

APPENDIX

Following [36–38,49], the amplitudes for the process $^3He+{}^3H\to d^2H+{}^4He$ are denoted by $D(c,a;b)$ where $a={}^3H$, $b={}^3He$, $c=d$. The spin projection indices of ^3He and ^3H nuclei will be denoted by $+$ or $-$, and that of the deuteron by $=$, 0, and $-$. Then in a transversity formalism the following relations between amplitudes $D(c,a;b)$, and amplitudes f_i are valid:

$$D(+,+;+)=\sqrt{2}(if_3+f_4),$$

$$D(+,-;-)=\sqrt{2}(if_5+f_6),$$

$$D(+,+;-)=0,$$

$$D(-,-;+)=0,$$

$$D(0,+;+)=0,$$

$$D(0,-;-)=0,$$

$$D(0,+;-)=-2f_1,$$

$$D(0,-;+)=2f_2,$$

$$D(-,+;+)=\sqrt{2}(-if_3+f_4),$$

$$D(-,-;-)=\sqrt{2}(-if_5+f_6),$$

$$D(-,+;-)=0,$$

$$D(+,-;+)=0.$$

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