Equivalent photon approach to simultaneous excitation in heavy ion collision

Kai Hencken*
Institut für Theoretische Physik der Universität Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland
and Institute for Nuclear Theory, University of Washington, Physics/Astronomy Building, Box 351550, Seattle, Washington 98195†

Dirk Trautmann
Institut für Theoretische Physik der Universität Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland

Gerhard Baur
Institut für Kernphysik (Theorie), Forschungszentrum Jülich, 52425 Jülich, Germany

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We apply the equivalent photon approximation to calculate cross sections for the simultaneous excitation of two heavy ions in relativistic collisions. We study especially the excitation of two nuclei to a 1^- state and show that the equations are symmetric with respect to both ions. We also examine the limit in which the excitation energy of one of the nuclei goes to zero, which gives the elastic case. Finally a few remarks about the limits of this approach are made. [S0556-2813(96)06605-8]

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Recently [1] a formalism was presented that allows the calculation of the equivalent photon spectra corresponding to nuclear transitions of relativistic nuclei ($\gamma \gg 1$). In this way it became possible to study corrections such as the influence of nuclear excitations on two-photon processes. The example that was studied was one nucleus making a transition to the giant dipole resonance (GDR) state, while at the same time emitting an equivalent photon. It is the purpose of this Brief Report to study simultaneous projectile and target excitation using the equivalent photon method. Using this method, we will see that the results derived previously by Benesh and Friar [2] can be simplified very much and the physics behind this process is made more transparent.

Throughout this report we assume that we can always use the long wavelength approximation. This requires that $kR<1$, where k is the space part of the photon momentum and R the radius of the nucleus. From this we conclude that the excitation energy Δ and therefore also the energy of the photon ω has to be low, that is, $\omega \approx \Delta < 1/R$.

In the equivalent photon approximation (EPA) the cross section for the simultaneous excitation is expressed as

$$\sigma_{\Delta_1\Delta_2} = \int \frac{d\omega}{\omega} n_{\Delta_1}(\omega) \sigma_{\gamma\Delta_2}(\omega), \tag{1}$$

where $n_{\Delta_1}(\omega)$ is the equivalent photon number of the inelastic photon emission process, and $\sigma_{\gamma\Delta_2}$ is the cross section for the photoabsorption process for a photon of energy ω in the rest frame of the nucleus.

In [1] the equivalent photon number for the inelastic photon emission was derived neglecting the longitudinal components of the electromagnetic field. In the limit of high ion energy ($\gamma \gg 1$) the result is

$$n_\Delta(\omega) = \int \frac{-2\omega^2 C + q_\perp^2 M^2 \gamma^2 D}{(2\pi)^3 2M^2 \gamma^2 (-q^2)^2} d^2 q_\perp . \tag{2}$$

Here q is the four-momentum transfer and C and D are given generally by

$$C = -2\pi[|T^e|^2 + |T^m|^2], \tag{3}$$

$$D = \frac{(-q^2)^2}{(\Delta^2 - q^2)^2 M^2} 2\pi \left[2|M^C|^2 + \frac{\Delta^2 - q^2}{-q^2} (|T^e|^2 + |T^m|^2) \right], \tag{4}$$

where M^C, T^e, and T^m are the usual Coulomb, electric, and magnetic matrix elements and $-q^2$ is determined by the kinematics of this process as

$$-q^2 = \frac{\omega^2}{\gamma^2} + 2\frac{\omega\Delta}{\gamma} + \frac{\Delta^2}{\gamma^2} + q_\perp^2 = q_{\min}^2 + q_\perp^2 . \tag{5}$$

For high values of γ we can safely neglect the term proportional to C compared to D in Eq. (2). The result is

$$n_\Delta(\omega) = \int \frac{q_\perp^2}{(2\pi)^2 2(\Delta^2 - q^2)^2 M^2} \left[2|M^C|^2 + \frac{\Delta^2 - q^2}{-q^2} (|T^e|^2 + |T^m|^2) \right] d^2 q_\perp . \tag{6}$$

Using the long wavelength limit, we can write the matrix elements in terms of the $B(EJ)$ and $B(MJ)$ values (see, e.g., [3]). Assuming that a single value of J dominates, the formulas are

$$|M^C|^2 = 4M^2 \frac{(\Delta^2 - q^2)^J}{[(2J+1)!!]^2} \alpha B(EJ), \tag{7}$$

$$|T^e|^2 = 4M^2 \frac{\Delta^2}{\Delta^2 - q^2} \frac{J+1}{J} \frac{(\Delta^2 - q^2)^J}{[(2J+1)!!]^2} \alpha B(EJ), \tag{8}$$

* Electronic address: hencken@phys.washington.edu
† Present address.

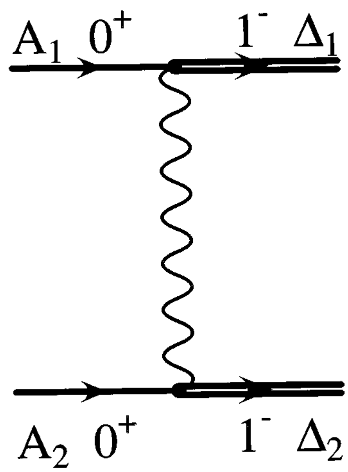

FIG. 1. Simultaneous excitation of two ions A_1 and A_2 to the GDR. The excitation energy of each GDR is denoted by Δ_1 and Δ_2.

$$|T^m|^2 = 4M^2 \frac{J+1}{J} \frac{(\Delta^2 - q^2)^J}{[(2J+1)!!]^2} \alpha B(MJ). \tag{9}$$

Similarly the cross section for the photoabsorption can be written as follows in the narrow resonance limit:

$$\sigma_\gamma(\omega) = \frac{\pi^2}{2\Delta M^2} (|T^e|^2 + |T^m|^2) \delta(\omega - \Delta). \tag{10}$$

For the photoabsorption of on-shell photons we have of course to set also $q^2 = 0$ for the process in Eqs. (8) and (9).

Let us examine the simultaneous excitation of the GDR of two nuclei A_1 and A_2. The corresponding Feynman diagram is shown in Fig. 1. We can view this process in two different ways, depending on which we consider the emitter and absorber of the equivalent photon. Let us assume that nucleus A_2 is at rest and is excited to the GDR by absorption of a photon with energy $\omega = \Delta_2$, where Δ_2 is the excitation energy of the GDR. This equivalent photon is emitted by the fast moving nucleus A_1, which is excited to its GDR due to this emission.

The excitation to the 1^- GDR is an $E1$ transition, proportional to $B(E1)$. Using this in Eq. (1), we get the cross section as

$$\sigma_{\Delta_1 \Delta_2} = \frac{4\alpha^2}{81} B(E1,\Delta_1) B(E1,\Delta_2) \int \frac{q_\perp^2 d^2 q_\perp}{(-q^2)}. \tag{11}$$

We see that the result is symmetric with respect to both ions.

When integrating over the transverse momenta, we have to keep in mind that the finite size of the nucleus limits the range of $q_\perp$. The integrand therefore has to be multiplied by the square of the form factor $F_i^2(q^2)$ of both ions. Using a Gaussian form factor for both ions $F_i(q^2) = \exp(-|q^2|/\lambda_i^2)$ as a model for the inelastic form factor, we can evaluate the equivalent photon number analytically as

$$n_\Delta(\omega) = \frac{\alpha B(E1)}{9\pi} \left[\frac{\lambda^2}{2} e^{-2q_{\min}^2/\lambda^2} - q_{\min}^2 E_1 \left(\frac{2q_{\min}^2}{\lambda^2} \right) \right], \tag{12}$$

where E_1 is the usual exponential integral [4], λ is given by $\lambda = (1/\lambda_1^2 + 1/\lambda_2^2)^{-1/2}$, and $q_{\min} = \Delta_1^2/\gamma^2 + 2\Delta_1 \omega/\gamma + \omega^2/\gamma^2$ as defined in Eq. (5).

An even simpler expression can be found by cutting off the integration over $q_\perp$ at $1/R$, where R is approximately the sum of the radii of the ions. Integrating and keeping only the leading term for large values of γ gives the equivalent photon number

$$n(\omega) = \frac{\alpha B(E1,\Delta_1)}{9\pi R^2}, \quad \text{for } \omega < \gamma/R, \tag{13}$$

and the cross section

$$\sigma_{\Delta_1 \Delta_2} = \frac{4\pi\alpha^2}{81 R^2} B(E1,\Delta_1) B(E1,\Delta_2). \tag{14}$$

We can see that the dependence of the cross section on Δ_1 and Δ_2 drops out. It is essentially only proportional to the product of the corresponding $B(E1)$ values. The limit of large γ in Eq. (12) has the form of Eq. (14) with R^2 given by $R^2 = 2/\lambda^2 = \frac{1}{3}(\langle r_A^2 \rangle + \langle r_B^2 \rangle)$. But please note that this result is only valid for a Gaussian form factor. As discussed in [1] (see, e.g., their Fig. [10]) $n(\omega)$ is much more sensitive to the detailed form of the form factor at large q^2 than in the elastic case due to the higher order of $q_\perp$ in the numerator of the integral. The cross section is also independent of γ unlike the elastic case where the cross section increases with $\ln(\gamma)$ for large γ. It can be shown easily that Eq. (14) is valid if $\Delta_i \ll \gamma/R$. As already shown earlier, it is also symmetric with respect to ions A_1 and A_2, so the role of both can be interchanged. Equation (14) has a similar form as found in [2], but in contrast to that work our result depends on R and therefore on the size of the nuclei.

In the Goldhaber-Teller model [5,6] or the sum-rule model [7] the $B(E1)$ is given by

$$B(E1, 0^+ \to 1^-, \Delta) = \frac{9}{2 m_N \Delta} \frac{NZ}{A}, \tag{15}$$

and therefore depends on the excitation energies. This dependence on Δ_1 and Δ_2 remains of course in the final formula.

In Fig. 2 we compare the results of the different equations. For the exact calculation we have used the Goldhaber-Teller model for matrix element and have used a Gaussian form factor, with λ chosen to reproduce the usual $\langle r^2 \rangle$ of the nucleus. No further approximations are made. This is compared with the results of the equivalent photon approximation of Eqs. (12) and (14). We see that for not too high energies we get good agreement between all three. Also shown are the results as given by Eq. (11) of [2]. They agree reasonably well with our results. Please note that our results depend on the detailed form of the form factor at large q^2; the good agreement between our three results therefore depends also on the use of the same form factor in all of them.

Of course the mutual excitation, as shown in Fig. 1, is not the only process leading to two excited nuclei in the final state. There are also other ones, like the one shown in Fig. 3, whose contribution can become more important for higher Z_1 and Z_2 [8].

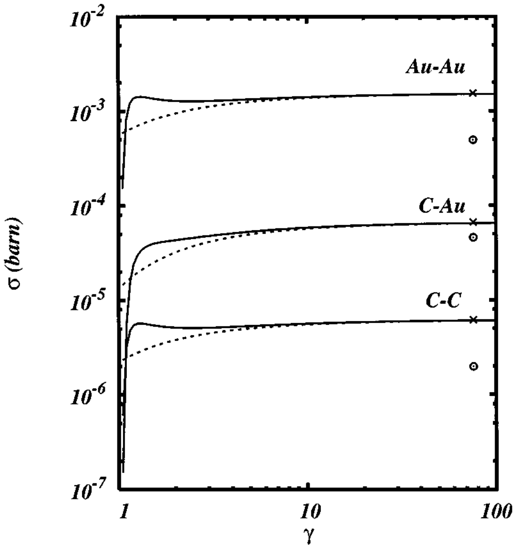

FIG. 2. Comparison of the different equations for the simultaneous excitation cross section as a function of the Lorentz factor γ. Shown are the results for C-C, C-Au, and Au-Au collisions as examples. The solid line shows the result of the exact calculation by using the Goldhaber-Teller model and using a Gaussian form factor, the dashed line the one for the equivalent photon approximation calculation using also the Gaussian form factor. The cross is the approximation for large values of γ using the cutoff form factor. Also shown are the results of Eq. (11) of [2] as circles.

Let us look also at other multipolarities. Magnetic $M1$ transitions can be treated in a similar way. As an example we look at the case of a $1/2^+ \to 1/2^+$ excitation, as shown in Fig. 4. As initial and final states have the same quantum numbers, this case can also be used to study the limit of elastic interaction, i.e., $\Delta_1 \to 0$. We expect to get then the equivalent photon spectrum due to the static magnetic moment of the nucleus. The equivalent photon spectrum for the static magnetic moment was already studied in [9,10]. Here we are looking at the excitation of a $1/2^+ \to 1/2^+$ transition in the other nucleus as well. The ion that emits the photon, has an equivalent photon number

$$n_{\Delta_1}(\omega) = \frac{\alpha}{9\pi}\frac{B(M1)}{R^2}, \quad \text{for } \omega < \gamma/R, \tag{16}$$

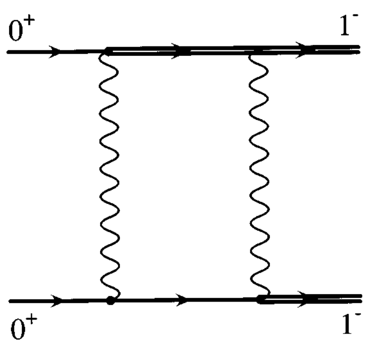

FIG. 3. Higher order process leading also to the excitation of both ions to the GDR by the exchange of two photons. The contribution of this process can be dominant over the one of Fig. 1 for large values of Z_1 and Z_2

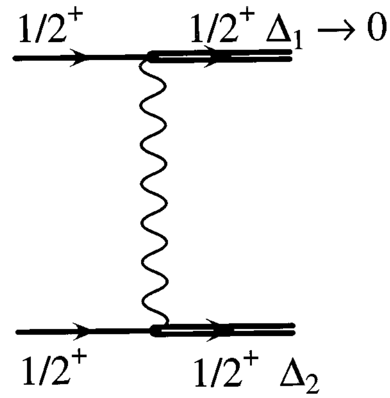

FIG. 4. Simultaneous excitation of two ions from a $1/2^+$ ground state to an excited $1/2^+$ excited state. In the limiting case of $\Delta_1 \to 0$ we recover the usual formula of the excitation due to the field of a magnetic dipole.

where we have used the simple cutoff form factor and kept only the leading term in the limit of $\gamma \to \infty$. The cross section for the simultaneous excitation is then given by

$$\sigma_{\Delta_1\Delta_2} = \frac{4\pi\alpha^2}{81R^2} B(M1,\Delta_1) B(M1,\Delta_2), \tag{17}$$

which again is proportional to the product of the two $B(M1)$ values and identical in form to Eq. (14). The known result for a static magnetic moment is obtained by replacing the $B(M1)$ value in the limit of $\Delta_1 \to 0$ by the magnetic moment μ according to

$$B(M1) = \frac{9}{4\pi}\mu^2. \tag{18}$$

On the other hand if we exchange the roles of both ions, we have the situation where the equivalent photon spectrum of the $1/2^+ \to 1/2^+$ transition in the fast moving nucleus A_2 (with $M1$ excitation energy Δ_2) is absorbed by the static magnetic moment of nucleus A_1 (with $\omega = \Delta_1 \to 0$). Although the integral over the equivalent photon spectrum taken by itself diverges for $\omega \to 0$, the product with the corresponding photoabsorption cross section (which vanishes for $\omega \to 0$) leads to a constant limit.

It seems interesting to study this limiting case for other possible transitions too, for example, for the transition $0^+ \to 0^+$. This is a special case as the photon emission is dominated by the $|M^C|^2$ matrix element with $J=0$ and no corresponding $|T^e|^2$ exists. Therefore the photoexcitation cross section vanishes for $J=0$ and we do not expect to be able to exchange the role of both ions.

In principle the equivalent photon approach can also be extended to other, higher multipolarities, for example, quadrupole excitations. We may also study transitions with different multipolarities for the two ions. We have only looked briefly at these processes, because the cross sections will be quite small. For the $0^+ \to 2^+$ transition, we expect the cross section to be proportional to $B(E2)/R^4$. These processes therefore become less and less important, as the higher order processes, as shown in Fig. 3, dominate the observed cross sections, see also [11].

We now discuss some of the limitations of this approach that one has to be aware of. One has to keep in mind that, in contrast to the elastic case, the inelastic equivalent photon approach is more and more dominated by the large values of $q_\perp$ when going to higher multipolarities. Therefore the results are much more sensitive to the detailed form of the form factors of both ions than in the elastic case. The equivalent photon spectrum is therefore no longer a property of the emitting nucleus alone, but of the absorbing nucleus as well. This should become more and more important as the multipolarity is increased.

In the case of the photon-photon collisions, as studied in [1], these limitations are less important, because we have two distinct transverse momentum scales; one scale is the limit of the q^2 of the emitted photon. This is given by the inverse of the nuclear radius R^{-1}. The other one is the range of q^2, where the virtual photon can be replaced by a real photon. This scale is normally given by the invariant mass of the produced system, which is much larger than the nuclear scale. The only exceptions are the e^+-e^- production and to a smaller extent the μ^+-μ^- production.

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[1] K. Hencken, D. Trautmann, and G. Baur, Z. Phys. C **68**, 473 (1995).
[2] C. J. Benesh and J. L. Friar, Phys. Rev. C **50**, 3167 (1994).
[3] A. de Shalit and H. Feshbach, *Theoretical Nuclear Physics Volume I: Nuclear Structure* (Wiley, New York, 1974), Chap. VIII.
[4] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1965), Eq. 5.1.1.
[5] M. Goldhaber and E. Teller, Phys. Rev. **74**, 1046 (1948).
[6] T. deForest and J. D. Walecka, Adv. Phys. **15**, 1 (1966).
[7] C.A. Bertulani and G. Baur, Phys. Rep. **163**, 299 (1988).
[8] G. Baur, in *Proceedings of the Rio de Janeiro International Workshop on relativistic aspects of nuclear physics, 1989*, edited by T. Kodama, K. C. Chung, S. J. B. Duarte, and M. C. Nemes (World Scientific, Singapore, 1990).
[9] V. M. Budnev, I. F. Ginzburg, G. V. Meledin, and V. G. Serbo, Phys. Rep. **15**, 181 (1975).
[10] N. Baron, Bericht des Forschungszentrum Jülich, Institut für Kernphysik Report No. Jul-2846, 1993 (unpublished).
[11] C. A. Bertulani, Phys. Lett. B**319**, 421 (1993).