$E2$ transition and Q_J^+ systematics of even mass palladium nuclei

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The yrast spectra with $J^{\pi}_{\max}=10^+$, $B(E2)$ transition probabilities, and Q_J^+ values are calculated for even-even palladium isotopes by carrying out variation-after-projection calculations in conjunction with the Hartree-Fock-Bogoliubov (HFB) ansatz employing a pairing-plus-quadrupole-quadrupole effective interaction operating in a reasonably large valence space outside the ^{76}Sr core. Our calculations describe the shape changes as a function of mass number for palladium isotopes and also reveal that both the HFB technique and the quadrupole-quadrupole-plus-pairing model of the two-body interaction are fairly reliable in this mass region.

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The low-lying yrast spectra in doubly even Pd isotopes have been the subject of a large number of recent experimental studies involving in-beam γ-ray spectroscopy. A thorough compilation of the literature on experimental studies has been given by Sakai [1]. A striking feature of the observed spectra (see Table I) is that the E_2^+ excitation energy decreases in its value as we go from ^{100}Pd to ^{102}Pd and then it remains almost constant in $^{102-106}$Pd. Thereafter, the E_2^+ excitation energy decreases faster in $^{106-112}$Pd nuclei. From the above observation it appears as if there is a slow shape transition taking place in palladium isotopes at ^{108}Pd. Palladium isotopes with $100 \leqslant A \leqslant 106$ seem to be soft vibrational nuclei and those with $A \geqslant 110$ seem to be quasirotational. ^{108}Pd is thought to be a transitional nucleus forming a zone between soft vibrators on one side and nearly deformed nuclei on the other side. This is also confirmed by a similar trend exhibited by the observed $B(E2;0^+ \rightarrow 2^+)$ and Q_2^+ values given in Table II.

In contrast to the large scale effort that has been made on the experimental side, only a few theoretical models [6–11] have been proposed to explain the character of the yrast spectra in these nuclei. Some time back, Stachel, Van Isacker, and Heyde [11] attempted a study of the experimental excitation energies and $E2$ transition probabilities of neutron-rich Pd isotopes in the framework of the interacting-boson model (IBA-I). It has been established by them that palladium isotopes follow the SU(5)→O(6) transition. The mechanism of this shape transition is not very clear from their calculation. In view of this it would be interesting to carry out a study of the isotopic mass chain of palladium isotopes in a microscopic framework.

In this paper, we carry out a microscopic study of the yrast states, $B(E2)$ transition probabilities, and Q_J^+ values in the nuclei $^{100-114}$Pd by employing the variation-after-projection (VAP) [12] formalism in conjunction with the Hartree-Fock-Bogoliubov (HFB) [13] ansatz for the axially symmetric intrinsic wave functions. In the present variational calculation of the yrast levels in the nuclei $^{100-114}$Pd, we have employed the usual pairing-plus-quadrupole-quadrupole effective interaction operating in a valence space spanned by the $3s_{1/2}$, $2p_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, $1g_{9/2}$, and $1h_{11/2}$ orbits for protons as well as neutrons. The nucleus ^{76}Sr has been considered as a core.

The details of interaction parameters, the projection of states of good angular momentum from axially symmetric HFB intrinsic states, the VAP method, and electric quadrupole transition matrix elements of yrast states are the same as the ones given in Ref. [14].

We now discuss the results of HFB and VAP calculations on Pd isotopes. It is well known from Grodzins' rule [15] that E_2^+ systematics bear an inverse correlation to the observed Q_2^+ systematics. Since Q_2^+ of a nucleus is directly related to its intrinsic quadrupole moment, the observed systematics of E_2^+ with A should produce a corresponding inverse systematics of the intrinsic quadrupole moment of Pd nuclei with increasing A. Based on the above logic, the calculated values of intrinsic quadrupole moments should increase in going from ^{100}Pd to ^{102}Pd and also for the set of nuclei $^{106-112}$Pd. The intrinsic quadrupole moment should remain nearly constant in the nuclei $^{102-106}$Pd. In Table I, the results of HFB calculations are presented. Note that the intrinsic quadrupole moments increase as we move from ^{100}Pd to ^{102}Pd and from ^{106}Pd to ^{112}Pd. For example, the $\langle Q_0^2 \rangle$ values for ^{100}Pd, ^{102}Pd, ^{106}Pd, and ^{112}Pd are 44.75, 51.86, 70.80, and 79.33 units, respectively. However, the intrinsic quadrupole moments for the set of nuclei $^{102-106}$Pd are not consistent with the E_2^+ systematics.

In Table II, the calculated values of $E2$ transition prob-

TABLE I. The experimental values of excitation energy of E_2^+ state (ΔE) and intrinsic quadrupole moments of the HFB states in some doubly even Pd isotopes. Here $\langle Q_0^2 \rangle_\pi$ ($\langle Q_0^2 \rangle_\nu$) gives the contribution of the protons (neutrons) to the total intrinsic quadrupole moment. The quadrupole moments have been computed in units of b^2 where b ($=\sqrt{\hbar/m\omega}$) is the harmonic oscillator parameter.

Nucleus	E_2^+(MeV)	$\langle Q_0^2 \rangle_{\rm HFB}$	$\langle Q_0^2 \rangle_\pi$	$\langle Q_0^2 \rangle_\nu$
^{100}Pd	0.66	44.75	19.80	24.95
^{102}Pd	0.55	51.86	21.10	30.76
^{104}Pd	0.55	58.48	23.07	35.41
^{106}Pd	0.51	70.80	29.49	41.31
^{108}Pd	0.43	75.76	31.86	43.90
^{110}Pd	0.37	79.58	34.05	45.53
^{112}Pd	0.34	79.33	33.99	45.34
^{114}Pd	0.33	76.13	32.25	43.88

TABLE II. The reduced transition probabilities for $E2$ transitions as well as the static quadrupole moments for the yrast levels in the nuclei $^{100-114}$Pd. Here e_p (e_n) denotes the effective charge for protons (neutrons). The reduced matrix elements as well as the static moments have been expressed in a form that brings out their explicit dependence on the effective charges. The entries presented in the fifth and seventh columns correspond to the effective charges indicated in the first column. The $B(E2)$ values are in units of $10^{-48}\ e^2\ \mathrm{cm}^4$ and static quadrupole moments have been given in units of $10^{-24}\ e\ \mathrm{cm}^2$.

Nucleus	Transition			$B(E2;J_i^+\rightarrow J_f^+)$		$Q(J_f^+)$	
(e_p,e_n)	$(J_i^+\rightarrow J_f^+)$	$[B(E2;J_i^+\rightarrow J_f^+)]^{1/2}$	$[Q(J_f^+)]$	Theory	Expt.	Theory	Expt.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
^{100}Pd	$0^+\rightarrow2^+$	$0.45e_p+0.55e_n$	$-0.28e_p-0.35e_n$				
	$2^+\rightarrow4^+$	$0.53e_p+0.67e_n$	$-0.36e_p-0.45e_n$				
	$4^+\rightarrow6^+$	$0.55e_p+0.70e_n$	$-0.40e_p-0.50e_n$				
	$6^+\rightarrow8^+$	$0.58e_p+0.75e_n$	$-0.42e_p-0.54e_n$				
	$8^+\rightarrow10^+$	$0.58e_p+0.76e_n$	$-0.43e_p-0.56e_n$				
^{102}Pd							
(1.18,0.18)	$0^+\rightarrow2^+$	$0.47e_p+0.68e_n$	$-0.30e_p-0.43e_n$	0.46	0.46 ± 0.03[a]	-0.43	-0.20 ± 0.2[b]
(1.0,0.0)	$2^+\rightarrow4^+$	$0.56e_p+0.81e_n$	$-0.38e_p-0.55e_n$	0.31	0.26 ± 0.01[c]	-0.38	
	$4^+\rightarrow6^+$	$0.58e_p+0.84e_n$	$-0.41e_p-0.60e_n$				
	$6^+\rightarrow8^+$	$0.59e_p+0.85e_n$	$-0.43e_p-0.64e_n$				
	$8^+\rightarrow10^+$	$0.59e_p+0.84e_n$	$-0.43e_p-0.65e_n$				
^{104}Pd							
(1.17,0.17)	$0^+\rightarrow2^+$	$0.50e_p+0.78e_n$	$-0.32e_p-0.50e_n$	0.51	0.535 ± 0.035[a]	-0.46	-0.25 ± 0.12[d]
(1.0,0.0)	$2^+\rightarrow4^+$	$0.60e_p+0.93e_n$	$-0.41e_p-0.63e_n$	0.36	0.273 ± 0.016[c]	-0.41	
	$4^+\rightarrow6^+$	$0.63e_p+0.98e_n$	$-0.45e_p-0.69e_n$				
	$6^+\rightarrow8^+$	$0.59e_n+0.88e_n$	$-0.50e_p-0.74e_n$				
	$8^+\rightarrow10^+$	$0.75e_p+1.06e_n$	$-0.57e_p-0.77e_n$				
^{106}Pd							
(1.11,0.11)	$0^+\rightarrow2^+$	$0.64e_p+0.90e_n$	$-0.41e_p-0.57e_n$	0.65	0.656 ± 0.035[a]	-0.51	-0.52 ± 0.12[d]
(1.0,0.0)	$2^+\rightarrow4^+$	$0.76e_p+1.08e_n$	$-0.52e_p-0.73e_n$	0.57	0.396 ± 0.054[c]	-0.52	
	$4^+\rightarrow6^+$	$0.66e_p+0.85e_n$	$-0.64e_p-0.82e_n$				
	$6^+\rightarrow8^+$	$0.95e_p+1.22e_n$	$-0.71e_p-0.88e_n$				
	$8^+\rightarrow10^+$	$0.96e_p+1.23e_n$	$-0.73e_p-0.91e_n$				
^{108}Pd							
(1.11,0.11)	$0^+\rightarrow2^+$	$0.69e_p+0.95e_n$	$-0.44e_p-0.61e_n$	0.75	0.76 ± 0.04[a]	-0.55	
(1.0,0.0)	$2^+\rightarrow4^+$	$0.83e_p+1.14e_n$	$-0.56e_p-0.77e_n$	0.68	0.504 ± 0.072[c]	-0.56	
	$4^+\rightarrow6^+$	$0.74e_p+0.97e_n$	$-0.67e_p-0.86e_n$				
	$6^+\rightarrow8^+$	$0.97e_p+1.26e_n$	$-0.72e_p-0.91e_n$				
	$8^+\rightarrow10^+$	$0.97e_p+1.27e_n$	$-0.74e_p-0.94e_n$				
^{110}Pd							
(1.1,0.1)	$0^+\rightarrow2^+$	$0.76e_p+1.01e_n$	$-0.47e_p-0.63e_n$	0.87	0.870 ± 0.04[a]	-0.58	-0.82 ± 0.18[d]
(1.0,0.0)	$2^+\rightarrow4^+$	$0.91e_p+1.21e_n$	$-0.60e_p-0.80e_n$	0.82	0.558 ± 0.072[c]	-0.60	
	$4^+\rightarrow6^+$	$0.95e_p+1.27e_n$	$-0.67e_p-0.88e_n$				
	$6^+\rightarrow8^+$	$0.97e_p+1.30e_n$	$-0.70e_p-0.92e_n$				
	$8^+\rightarrow10^+$	$0.97e_p+1.31e_n$	$-0.72e_p-0.94e_n$				
^{112}Pd							
(1.02,0.02)	$0^+\rightarrow2^+$	$0.76e_p+1.01e_n$	$-0.48e_p-0.65e_n$	0.63	0.630 ± 0.01[a]	-0.50	
	$2^+\rightarrow4^+$	$0.90e_p+1.21e_n$	$-0.61e_p-0.82e_n$				
	$4^+\rightarrow6^+$	$0.95e_p+1.27e_n$	$-0.68e_p-0.90e_n$				
	$6^+\rightarrow8^+$	$0.96e_p+1.29e_n$	$-0.71e_p-0.94e_n$				
	$8^+\rightarrow10^+$	$0.97e_p+1.31e_n$	$-0.74e_p-0.97e_n$				
^{114}Pd							
(1.0,0.0)	$0^+\rightarrow2^+$	$0.71e_p+0.98e_n$	$-0.46e_p-0.63e_n$	0.50	0.340 ± 0.01[a]	-0.46	
	$2^+\rightarrow4^+$	$0.85e_p+1.17e_n$	$-0.58e_p-0.80e_n$				
	$4^+\rightarrow6^+$	$0.89e_p+1.23e_n$	$-0.64e_p-0.87e_n$				
	$6^+\rightarrow8^+$	$0.91e_p+1.25e_n$	$-0.68e_p-0.92e_n$				
	$8^+\rightarrow10^+$	$0.92e_p+1.26e_n$	$-0.70e_p-0.94e_n$				

[a]From [4].
[b]From [2].
[c]From [5].
[d]From [3].

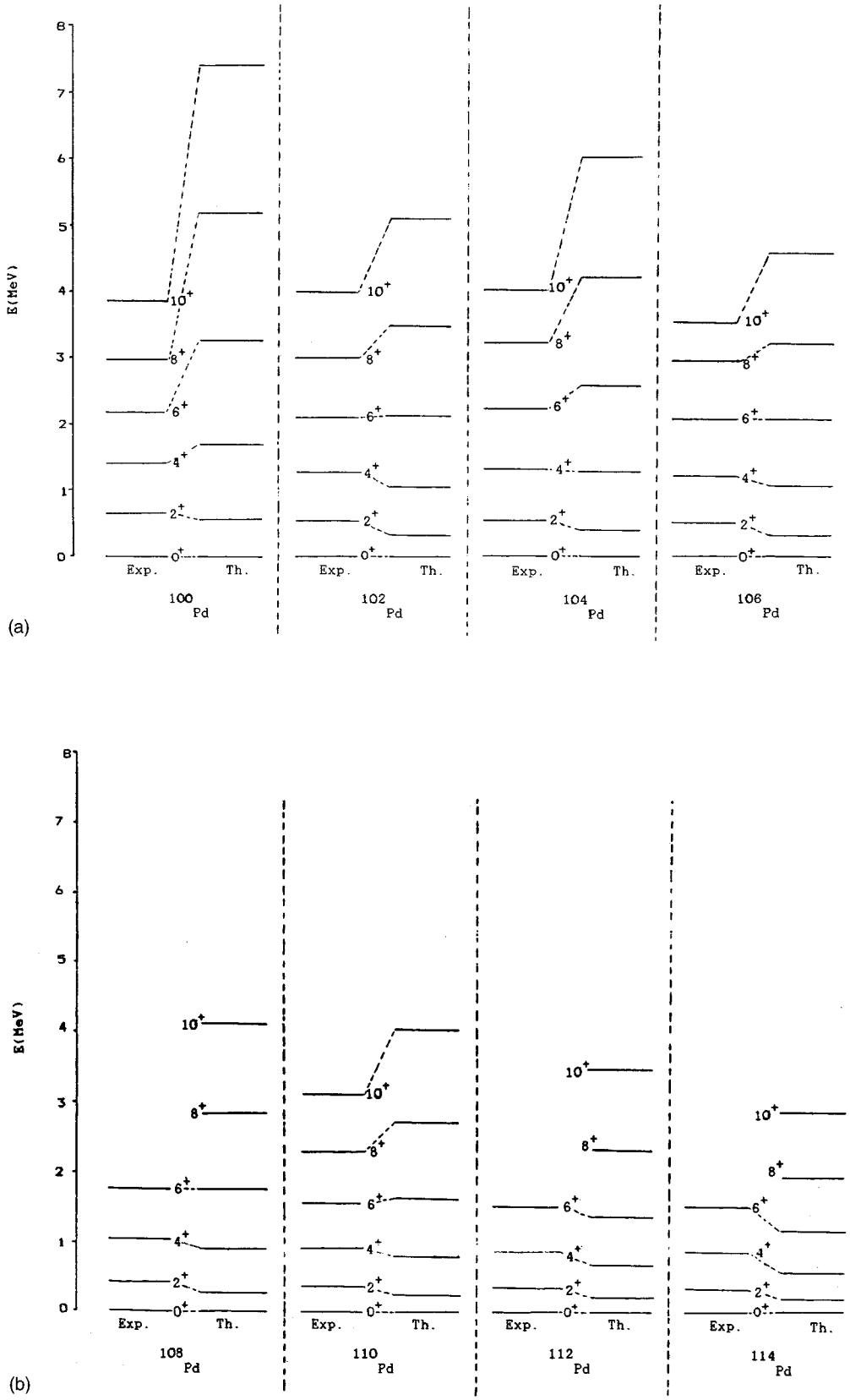

FIG. 1. (a) Comparison of the observed (expt.) as well as the calculated yrast spectra in the nuclei $^{100-106}$Pd. (b) Comparison of the observed (expt.) as well as the calculated yrast spectra in the nuclei $^{108-114}$Pd.

abilities between the states E_J and E_{J+2} are presented. The calculated values are expressed in parametric form in terms of the proton (e_p) and neutron (e_n) effective charges and have been obtained through a rigorous projection calculation. These have been chosen such that $e_p=1+e_{\rm eff}$ and $e_n=e_{\rm eff}$, so that only one parameter is introduced in the calculation. The $B(E2:J_i^+\rightarrow J_f^+)$ values have been calculated in units of 10^{-48} e^2 cm^4. The results indicate that by choosing $e_{\rm eff}\leqslant 0.18$, a good agreement with the observed values for $B(E2;0^+\rightarrow 2^+)$ transition probabilities is obtained for the entire isotope chain of palladium nuclei (i.e., $^{100-114}$Pd). For example, in ^{102}Pd if $e_{\rm eff}=0.18$, the calculated value of $B(E2;0^+\rightarrow 2^+)=0.46$ units and the experimental value is (0.46 ± 0.03) units. Similarly, for ^{110}Pd if $e_{\rm eff}=0.10$ then the calculated and observed values of the transition probability are 0.87 units and (0.87 ± 0.04) units, respectively. The use of effective charges [14] is generally invoked in nuclear structure calculations to mimic the contribution made by the core towards the electromagnetic properties due to its polarization as the nucleons are put in the valence space.

In Table II, the calculated values of Q_J^+ are also presented for the entire isotopic mass chain of palladium nuclei. These values are expressed in parametric form in terms of proton (e_p) and neutron (e_n) effective charges. Our results indicate that agreement with observed values of Q_2^+ for ^{106,110}Pd is obtained provided we choose $e_{\rm eff}=0.1$ and 0.3, respectively. In the case of ^{102,104}Pd the agreement is obtained at $e_{\rm eff}=0.0$. In view of the quality of agreement for Q_2^+ and also $B(E2;0^+\rightarrow 2^+)$ values, we are constrained to assume that the HFB wave function gives a reasonably good description of the palladium nuclei.

In Figs. 1(a) and 1(b), the low-lying yrast spectra of a chain of palladium isotopes are displayed. The low-lying yrast spectra are found to reproduce the experimental levels with $J^\pi\leqslant 6^+$ with a reasonably acceptable discrepancy, with the exception of ^{100}Pd. Further, the slow shape transition as we go from ^{106}Pd to ^{110}Pd via ^{108}Pd is also reproduced. At this stage it is important to mention the significance of the energy ratios such as $E(4_1^+)/E(2_1^+)$ which are used as a measure of the collectivity of nuclei and reflect the shape and structure of the nucleus. It was recently established by Zhang, Casten, and Zamfir [16] that $E(4_1^+)/E(2_1^+)$ depends on the overall shape and structure of the potential energy surface. Since our calculated spectra for low-lying states are in satisfactory agreement with experiments we are, therefore, reproducing the observed slow shape transition. It is true that agreement for higher spins is bad. This is due to the fact that there is no free parameter in the energy calculation for the full isotopic mass chain $^{100-114}$Pd. This level of agreement can be considered to be satisfactory because of a number of considerations. First, the calculation of yrast spectra is a complex many-body calculation involving a minimum of 24 valence particles for ^{100}Pd and a maximum of 38 valence particles for ^{114}Pd. The fact that the calculation reproduces the order of magnitude of the low-lying energy states in the entire set of palladium isotopes is remarkable. Secondly, the calculations are carried out for the entire set of palladium isotopes with a single set of input parameters. Thirdly, we have not used any parameter to simulate the contribution of the ^{76}Sr core towards the moment of inertia. As was pointed out by Faessler [17] as well as by Praharaj [18] the inclusion of $K\neq 0$ bands may provide improvements in the calculated yrast spectra.

Summarizing, the observed systematics of yrast spectra, $B(E2)$ transition probabilities, and Q_J^+ values are found to be reproduced within a reasonable accuracy. It turns out that the VAP technique and the quadrupole-quadrupole-plus-pairing model of interaction are fairly reliable in this mass region and hence should be used for studying the properties of other isotopic mass chains in this mass region.

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