U(6/20) supersymmetry in $^{115-119}$Sn isotopes

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(Received 24 January 1995)

In this paper we have studied the low energy structure of $^{115-119}$Sn isotopes within the vibrational limit for bosons under $U^{BF}(5)\otimes SU^{F}(4)$ dynamical symmetry of the graded Lie group U(6/20). Calculation of excitation energy spectra of positive parity levels and $B(E2)$ values of these isotopes has been performed using this dynamical symmetry. Comparison of experimental and calculated energy spectra and $B(E2)$ values of these nuclei suggests the existence of an approximate U(6/20) supersymmetry in these isotopes.

PACS number(s): 21.60.Fw, 21.10.Ky, 27.60.+j

In the last few years supersymmetry in the framework of an interacting boson-fermion model has been extensively used to describe both even-even and odd-A nuclei simultaneously under the same Bose-Fermi symmetry. In the vibrational limit for bosons, supersymmetry has been found to work well in the $1f$-$2p$ shell for both types of odd fermion, proton [1], and neutron [2]. The nuclei ^{76}Se and ^{75}As were studied [3] under different supersymmetry schemes in the single particle space $j=\frac{3}{2},\frac{5}{2}$ and $j=\frac{1}{2},\frac{3}{2}$, and $\frac{5}{2}$. Existence of supersymmetry in the Ru-Pd region was reported [4] with the limitation that the odd fermion be in the state $j=\frac{1}{2}$. No work has been reported in the U(5) limit for bosons, with the odd fermion in single particle space $j=\frac{1}{2},\frac{3}{2},\frac{5}{2}$, and $\frac{7}{2}$. In this paper we present the results of our calculations on the positive parity levels of a number of even and odd mass Sn isotopes using U(6/20) supersymmetry.

Even Sn nuclei are known to exhibit collective vibrational structure in the low energy excitation region. Ring and Schuck have described the lowest 2^+ state in ^{116,118,120}Sn as vibrational states [5]. The odd nucleus, ^{117}Sn, has been described in the framework of particle vibration coupling [6]. The $B(E2)$ values for the known strong transitions in even Sn nuclei are one order of magnitude greater than the single-particle estimates. This indicates that the even-even core is vibrational in nature. Therefore, the low energy structure of odd Sn nuclei is expected to be well described in terms of U(5), i.e., the vibrational limit for bosons, with the odd fermion in single particle space $j=\frac{1}{2},\frac{3}{2},\frac{5}{2}$, and $\frac{7}{2}$. The appropriate group structure of odd-mass Sn isotopes in which the neutrons occupy the $3s_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, and $1g_{7/2}$ levels is $U^{B}(6)\otimes U^{F}(20)$ where $U^{B}(6)$ is the usual boson group describing the collective excitations and $U^{F}(20)$ is the fermion group associated with the single-particle degrees of freedom. The generators of the fermion group are

$$(a_j^\dagger \tilde{a}_{j'})^\rho, \text{ with } j,j'=\tfrac{1}{2},\ \tfrac{3}{2},\ \tfrac{5}{2},\ \text{or } \tfrac{7}{2}. \quad (1)$$

Using the idea of pseudoangular momentum, the fermion angular momentum $j=\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ can be viewed as a pseudo-orbital angular momentum $\lambda=2$, coupled to a pseudospin, $s=3/2$. The group $U^{F}(20)$ therefore can be divided into a $U^{F}(5)$ and $SU^{F}(4)$ group corresponding to pseudo-orbital angular momentum and pseudospin respectively. The generators of the group $U^{F}(5)$ and $SU^{F}(4)$ are A^ρ and S^ρ respectively, and can be obtained from those of $U^{F}(20)$ by the transformation

$$A^\rho=\sum_{jj'} \hat{j}\hat{j}'(-1)^{j'+\rho+3/2}\begin{Bmatrix} 2 & j & 3/2 \\ j' & 2 & \rho \end{Bmatrix}(a_j^\dagger \tilde{a}_{j'})^\rho, \quad (2)$$

$$S^\rho=\sum_{jj'} \hat{j}\hat{j}'(-1)^{j+\rho+3/2}\begin{Bmatrix} 3/2 & 3/2 & \rho \\ j' & j & 2 \end{Bmatrix}(a_j^\dagger \tilde{a}_{j'})^\rho, \quad (3)$$

where $\hat{j}=(2j+1)^{1/2}$. We thus obtain the group reduction $U^{F}(20)\supset U^{F}(5)\otimes SU^{F}(4)$.

In the vibrational limit, the bosons have $U^{B}(5)$ dynamical symmetry. The relevant group chain and quantum numbers for the U(6/20) supergroup are given below:

$$\begin{array}{ll}
U(6/20) & N \\
\supset U^{B}(6)\otimes U^{F}(20) & N_B, N_F=1 \\
\supset U^{B}(6)\otimes U^{F}(5)\otimes SU^{F}(4) & 1,1 \\
\supset U^{B}(5)\otimes U^{F}(5)\otimes SU^{F}(4) & n_d \\
\supset U^{BF}(5)\otimes SU^{F}(4) & [n_1,n_2] \\
\supset O^{BF}(5)\otimes Sp^{F}(4) & (v_1,v_2),(t_1,t_2) \\
\supset Spin^{BF}(5) & \{\tau_1,\tau_2\} \\
\supset Spin^{BF}(3) & J.
\end{array} \quad (4)$$

Since $Spin^{BF}(5)$ is not fully reducible to $Spin^{BF}(3)$, an additional quantum number ν_Δ is required to completely classify the states.

The Hamiltonian for excitation spectra written in terms of linear and quadratic Casimir operators of the subgroups in the above group chain is

$$H=AC_1[U^{B}(5)]+BC_2[U^{B}(5)]+CC_2[U^{BF}(5)] +DC_2[O^{BF}(5)]+FC_2[Spin^{BF}(5)]+GC_2[Spin^{BF}(3)], \quad (5)$$

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where $C_n[G]$ is the nth order Casimir operator for the group G and A, B, C, D, F, and G are free parameters. The other Casimir operators of the subgroups in the group chain are not contributing to the excitation energy calculation and are neglected.

The eigenvalues obtained from the above Hamiltonian written in terms of quantum numbers are given by

$$E=An_d+Bn_d(n_d+4)+C[n_1(n_1+4)+n_2(n_2+2)] +D[v_1(v_1+3)+v_2(v_2+1)] +F[\tau_1(\tau_1+3)+\tau_2(\tau_2+1)]+GJ(J+1). \quad (6)$$

The wave function in terms of isoscalar factors and six-j coefficients in the above chain is written as

$$|N,n_d,[n_1,n_2],(v_1,v_2),\{\tau_1,\tau_2\},JM\rangle=\sum_{v,L,k}\left\langle \begin{matrix} n_d & 1 \\ v & 1 \end{matrix} \middle| \begin{matrix} [n_1,n_2] \\ (v_1,v_2) \end{matrix} \right\rangle \left\langle \begin{matrix} v & 1 \\ L & 2 \end{matrix} \middle| \begin{matrix} (v_1,v_2) \\ k \end{matrix} \right\rangle \left\langle \begin{matrix} (v_1,v_2) & (1/2,1/2) \\ k & 3/2 \end{matrix} \middle| \begin{matrix} \{\tau_1,\tau_2\} \\ J \end{matrix} \right\rangle \times \sum_j (-1)^{L+J+3/2}\hat{k}\hat{j}\begin{Bmatrix} L & 3/2 & k \\ 2 & J & j \end{Bmatrix}[|N,n_d,v,L\rangle a^\dagger_{fm}]^J_M. \quad (7)$$

The electromagnetic $E2$ transition operator written in terms of boson and fermion one-body terms is given by [7]

$$T^{E2}=e_b(s^\dagger\tilde{d}+d^\dagger\tilde{s})^2+e_b'(d^\dagger\tilde{d})^2+\sum_{jj'}t_{jj'}[a_j^\dagger\tilde{a}]^2. \quad (8)$$

We have taken a simpler form of the operator where the fermion part is written in terms of generators of fermion group $U^F(5)$ and $SU^F(4)$:

$$T^{E2}=e_b(s\dagger\tilde{d}+d^\dagger\tilde{s})^2+e_b'(d^\dagger\tilde{d})^2+e_f(A^2)+e_f'(S^2). \quad (9)$$

The first two terms denote the bosonic part and the other two the fermionic part. The selection rules for the bosonic part are $\Delta n_d=\pm 1$ and $\Delta n_d=0$ respectively and for the fermionic part, $\Delta N_F=0$. Analytical expressions for $B(E2)$ values can be obtained for exact symmetry.

Energy values of $^{115-119}$Sn isotopes are calculated by using Eq. (6). The constant parameters in (6) are determined from fitting a number of energy levels in the even and odd isotopes of each supersymmetric representation. In the even Sn isotopes, $2p$-$2h$ type intruder states appear around 2 MeV. For example, the 0^+ state at 1.76 MeV, the 2^+ state at 2.11 MeV, and the 4^+ state at 2.53 MeV in ^{116}Sn isotope belong to the intruder band [8]. The IBM in the simplest form is incapable of explaining the structure of these states. In the neighboring Cd isotopes, these states have been described in the framework of O(5) symmetry [9]. Since we are concentrating on the states in U(5) symmetry limit in the even Sn isotopes, it is not possible for the present model to explain these intruder states. In the odd isotopes, we concentrate on the states that are obtained by coupling an odd quasifermion to the zero- and one-phonon states of even Sn isotopes. These states are given by the quantum number $n_d=0$ and 1. We expect that states up to about 2 MeV excitation energy can be so described. Even then, it is very difficult to find out the quantum numbers of the states unambiguously in view of the large-level density between 1 and 2 MeV. Therefore in the fitting procedure we have taken mostly those levels whose quantum numbers are confirmed through $B(E2)$ calculation. In the case of ^{117}Sn and ^{119}Sn isotopes, since the bosons and fermions are holelike, the supersym-

TABLE I. The values of the different parameters obtained from fitting the energy levels of different Sn isotopes.

	Parameters in keV					
	A	B	C	D	F	G
^{115,116}Sn	2348.2	−198.0	−24.4	54.9	−70.0	25.7
^{116,117}Sn	2221.0	−181.0	−20.3	−34.5	3.9	35.5
^{118,119}Sn	2460.9	−458.0	198.9	−141.2	32.9	65.6

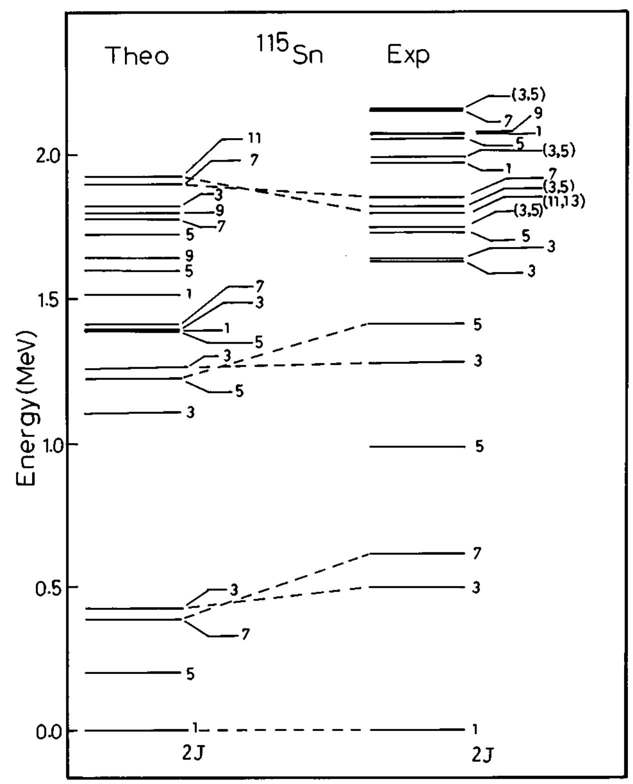

FIG. 1. Comparison of experimental and calculated low-energy positive-parity levels of ^{115}Sn. The levels are marked by twice their angular momentum values.

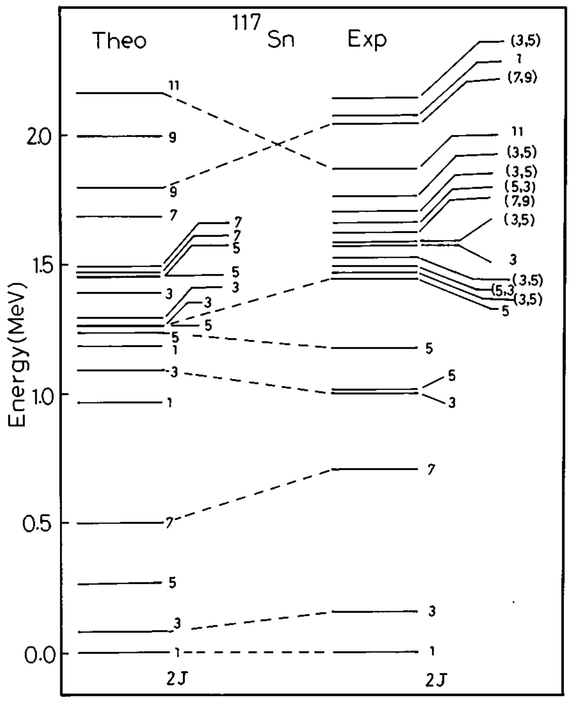

FIG. 2. Comparison of experimental and calculated low-energy positive-parity levels of ^{117}Sn. The levels are marked by twice their angular momentum values.

metric partners of ^{117}Sn and ^{119}Sn are ^{116}Sn and ^{118}Sn, respectively. The value of the different parameters obtained from the fitting procedure are given in Table I.

Theoretical and experimental energy levels with positive parity for odd *A* Sn isotopes are compared in Figs. 1, 2, and

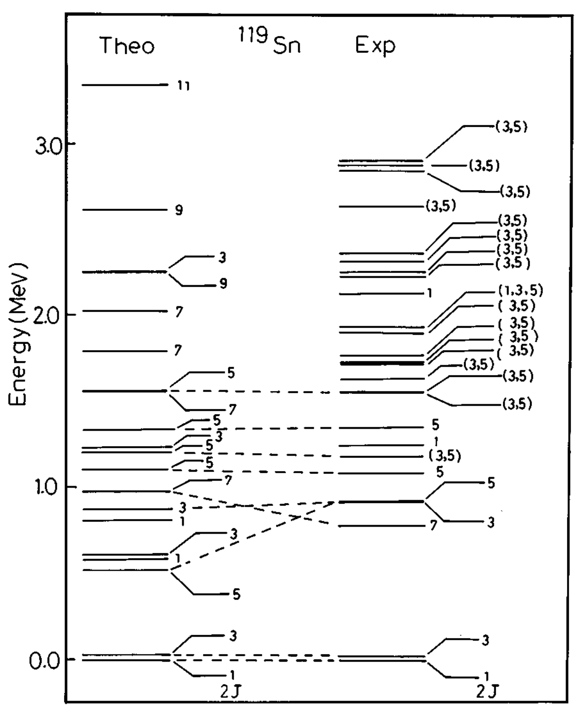

FIG. 3. Comparison of experimental and calculated low-energy positive-parity levels of ^{119}Sn. The levels are marked by twice their angular momentum values.

TABLE II. The absolute values of different parameters used in $B(E2)$ calculation for Sn isotopes.

	Parameters in efm^2		
	$\lvert e_b\rvert$	$\lvert e_f\rvert$	$\lvert e'_f\rvert$
^{115,116}Sn	7.5	18.2	2.7
^{116,117}Sn	6.5	22.1	7.4
^{118,119}Sn	8.3	17.3	27.0

3. The experimental and theoretical levels which are used in the fitting procedure are connected by dashed lines in the figures. Due to the uncertainties in assignment of spin and absence of the experimental $B(E2)$ values for transitions from higher energy levels in these nuclei, assignment of quantum numbers becomes very difficult. Experimental energy values for these isotopes can be obtained from Refs.

TABLE III. Comparison of experimental and calculated $B(E2)$ values for Sn isotopes.

	$J_i\rightarrow J_f$	$B(E2)$ Exp.	in e^2fm^4 Theo.
^{115}Sn	$\frac{3}{2}_1\rightarrow\frac{1}{2}_1$	69.8	71.7
	$\frac{7}{2}_1\rightarrow\frac{3}{2}_1$	4.3	14.0
	$\frac{5}{2}_1\rightarrow\frac{7}{2}_1$	106.3	33.3
	$\frac{5}{2}_1\rightarrow\frac{3}{2}_1$	10.0	8.2
	$\frac{5}{2}_1\rightarrow\frac{1}{2}_1$	79.4	91.2
	$\frac{5}{2}_2\rightarrow\frac{5}{2}_1$	150.2	59.2
	$\frac{5}{2}_2\rightarrow\frac{3}{2}_1$	12.9	0.0
	$\frac{5}{2}_2\rightarrow\frac{1}{2}_1$	252.5	233.7
^{116}Sn	$2_1\rightarrow 0_1$	389.8	445.2
	$4_1\rightarrow 2_1$	638.9	779.1
^{117}Sn	$\frac{3}{2}_1\rightarrow\frac{1}{2}_1$	3.1	86.5
	$\frac{7}{2}_1\rightarrow\frac{3}{2}_1$	11.1	20.6
	$\frac{5}{2}_1\rightarrow\frac{3}{2}_1$	44.8	50.7
	$\frac{5}{2}_1\rightarrow\frac{1}{2}_1$	203.9	149.2
	$\frac{3}{2}_2\rightarrow\frac{3}{2}_1$	5.8	0.0
	$\frac{3}{2}_2\rightarrow\frac{1}{2}_1$	271.9	104.7
	$\frac{5}{2}_2\rightarrow\frac{3}{2}_1$	$>$200.5	209.5
	$\frac{5}{2}_3\rightarrow\frac{3}{2}_1$	5.8	0.0
	$\frac{5}{2}_3\rightarrow\frac{1}{2}_1$	119.9	179.6
^{116}Sn	$2_1\rightarrow 0_1$	389.8	342.2
	$4_1\rightarrow 2_1$	638.9	598.8
^{118}Sn	$\frac{3}{2}_1\rightarrow\frac{1}{2}_1$	$<$23.3	5.0
	$\frac{7}{2}_1\rightarrow\frac{3}{2}1$	11.5	12.6
	$\frac{5}{2}_1\rightarrow\frac{3}{2}_1$	591.4	611.9
	$\frac{5}{2}_1\rightarrow\frac{1}{2}_1$	180.6	167.0
	$\frac{5}{2}_3\rightarrow\frac{3}{2}_1$	27.8	288.6
	$\frac{5}{2}_3\rightarrow\frac{1}{2}_1$	94.0	41.2
	$\frac{3}{2}_2\rightarrow\frac{1}{2}_1$	382.8	144.3
^{118}Sn	$2_1\rightarrow 0_1$	432.7	481.1
	$4_1\rightarrow 2_1$	584.1	824.7

[10–14]. In all the nuclei, the lowest $\frac{5}{2}{}_1^+$ state is poorly reproduced. This points to a possible departure from exact symmetry. The symmetry demands that the single-particle levels $3s_{1/2}$, $2d_{5/2}$, and $1g_{7/2}$ follow a $J(J+1)$ rule. However the poor reproduction of $\frac{5}{2}{}_1^+$ level indicates that the symmetry is broken. In the γ-soft limit of U(6/20) supersymmetry, Jolie *et al.* have introduced a perturbation δE_j to accommodate this type of symmetry breaking [15].

Energy values alone are not sufficient to confirm the existence of supersymmetry because they depend only on the group chain and are independent of the wave function of the system. It is therefore necessary to calculate other observables like transition probabilities which depend on the wave function of the system. Here we have calculated $B(E2)$ values of some of the lower-level transitions in $^{115-119}$Sn isotopes. $B(E2)$ values are calculated by evaluating the matrix element of the T^{E2} operator in (9) between the basis states given by Eq. (7). The necessary isoscalar factors can be obtained from [7]. In this calculation the same parameters are used for the supersymmetric partners. The different parameters used in this calculation are given in Table II. The absolute signs of e_b and e_f or e_f' cannot be predicted from the $B(E2)$ calculation though the relative sign of e_f and e_f' can be predicted. From the available experimental quadrupole moments in ^{115}Sn and ^{119}Sn, we conclude that e_f and e_f' are positive in the first isotope and negative in the second. This change of sign may be linked to the fact that the fermion is particlelike in ^{115}Sn to the holelike in ^{119}Sn. Calculated and experimental values of $B(E2)$ are compared in Table III. Comparison of theoretical and experimental $B(E2)$ values for the $\Delta n_d=0$ transitions for the bosonic part is not possible due to a lack of experimental data. The experimental values are obtained from Refs. [10–14]. It is seen that calculated $B(E2)$ values for most of the transitions are very close to the corresponding experimental values. However, some observed γ transitions, e.g., $\frac{5}{2}{}_2^+\rightarrow\frac{3}{2}{}_1^+$ transition in ^{115}Sn and $\frac{3}{2}{}_2^+\rightarrow\frac{3}{2}{}_1^+$ transition in ^{117}Sn, are forbidden in this model. These transitions may be due to a breaking of exact symmetry, but it is worthwhile to note that all these transitions found experimentally are very weak. On the other hand some γ transitions, e.g., $\frac{3}{2}{}_1^+\rightarrow\frac{1}{2}{}_1^+$ in ^{117}Sn and $\frac{5}{2}{}_3^+\rightarrow\frac{3}{2}{}_1^+$ transition in ^{119}Sn, are strong in our model though they are found experimentally to be weak. One of the reasons for this may be due to a breaking of symmetry. However, there is another reason that the $\frac{3}{2}{}_1^+\rightarrow\frac{1}{2}{}_1^+$ transition in ^{117}Sn may not be reproduced well. For simplicity of calculation, we have written the fermion part of the $B(E2)$ operator in terms of group generators of the fermionic group $\mathrm{U}^F(5)$ and $\mathrm{SU}^F(4)$. This is usually too restrictive and one may use more general operators to get better agreement with the experimental data. However, the number of known transition probabilities in this mass region is not large enough to establish whether this, rather than breakage of symmetry, is the reason for poor agreement with our calculation. The γ transition $\frac{3}{2}{}_2^+\rightarrow\frac{3}{2}{}_1^+$ in ^{119}Sn is found experimentally to be of pure $M1$ character. This is successfully explained by this model. The γ transition $\frac{5}{2}{}_3^+\rightarrow\frac{5}{2}{}_1^+$ in ^{119}Sn is also found experimentally to be of pure $M1$ character but our model gives a small $B(E2)$ value. On the whole the agreement is fairly good.

In this paper we have studied $^{115-119}$Sn isotopes using U(6/20) supersymmetry under $\mathrm{U}^{BF}(5)\otimes\mathrm{SU}^F(4)$ Bose-Fermi symmetry. Energy levels of odd *A*, and $B(E2)$ values of both odd *A* and even-even nuclei have been calculated and compared with experimental observations. From the above calculation it is seen that only a qualitative description of nuclear properties in this region is obtained if one adheres to the exact symmetry limit. Nevertheless, as pointed out by Kota and Van Isacker [16], dynamical symmetries of the interacting boson-fermion model can be used as a starting point in the analysis of an odd-mass nucleus and thus provide different bases or coupling schemes in which a boson-fermion Hamiltonian can be considered, which includes more complicated symmetry breaking terms. The practical advantage of dynamical symmetry is that solutions are obtained by simple procedures that are quite close to the exact ones.

The authors acknowledge R. Bhattacharya for stimulating discussions and making helpful comments. One of the authors (A.K.S.) wishes to thank the University Grants Commission, New Delhi, for financial support.

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