$1/N_c$ expansion of the quark condensate at finite temperature

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(Received 27 October 1995)

Previously the quark and meson properties in a many quark system at finite temperature have been studied within effective QCD approaches in the Hartree approximation. In the present paper we consider the influence of the mesonic correlations on the quark self-energy and on the quark propagator within a systematic $1/N_c$ expansion. Using a general separable ansatz for the nonlocal interaction, we derive a self-consistent equation for the $1/N_c$ correction to the quark propagator. For a separable model with cutoff form factor, we obtain a decrease of the condensate of the order of 20% at zero temperature. A lowering of the critical temperature for the onset of the chiral restoration transition due to the inclusion of mesonic correlations is obtained with results that seem to be closer to those from lattice calculations.

PACS number(s): 24.85.+p, 12.38.Aw, 12.40.Yx, 14.40.Aq

I. INTRODUCTION

QCD motivated effective theories are the most promising approaches to the low energy behavior of QCD and meson physics in terms of quark and gluon degrees of freedom and symmetries. Starting from chiral quark model Lagrangians a perturbative approach to the occurrence of a chiral condensate below a critical temperature T_c in a mean-field approximation is usually considered. Simultaneously, the pseudoscalar Goldstone boson—the pion—occurs. Perturbation theory can be formulated in $1/N_c$, where N_c is the number of colors [1]. The leading order is the Hartree approximation. (Results are reported in Refs. [2–5].) A more general approach, where a nonlocal instantaneous interaction is applied, has been presented in Refs. [6–11]. A still open but very important question is the influence of mesonic degrees of freedom, which are neglected in the Hartree approximation. These degrees of freedom are supposed to be dominant in the low-temperature limit. For the NJL model, an effective $1/N_c$ expansion which accounts for the mesonic fluctuations has been considered in [12]. However, the set of diagrams for the self-energy in next-to-leading order considered in this reference was not complete. This has been observed in Ref. [13] where the role of the scalar isovector mesons in the $1/N_c$ approximation at zero temperature was also discussed. It was shown that in the $1/N_c$ expansion the Schwinger-Dyson equation for the quark self-energy is different from the gap equation for the quark condensate and has to be solved separately. A complete collection of diagrams in $1/N_c$ was given in Ref. [14] and recently studied in the chiral limit $m_0=0$ at $T=0$ in Ref. [15]. At $T=0$, effects of the order of 10–20% have been obtained in these approaches, showing that mesonic fluctuations play an important role.

In this work, we consider the influence of mesonic correlations on the quark condensate at finite temperature. It is expected that such a calculation beyond the Hartree level of description will lead to corrections to the temperature behavior of the quark condensate since the medium allows for mesonic degrees of freedom. The relation of a generalized gap equation to the thermodynamical potential of a quark meson plasma has been considered in Ref. [16]. The present paper is a first step for a consistent description of a meson gas at finite temperature within a chiral quark model.

The paper is organized as follows. In Sec. II the nonlocal chiral quark model is briefly introduced, which is used in Sec. III to derive a generalized formula for the quark condensate in $\mathcal{O}(1/N_c)$ expansion. In Sec. IV we include dynamical fluctuations into the self-energy and treat the scalar and pseudoscalar contributions within the pole approximation. The numerical results for a calculation within the NJL model at finite temperature are discussed in Sec. V.

II. THE MODEL

Our starting point is the chiral-symmetric effective Lagrangian in the quark sector of the general form

$$\mathcal{L}=\overline{q}_1(p)(\gamma_\mu p^\mu - m_0)q_1(p)+\mathcal{L}_{\rm int}, \quad (1)$$

where the interaction term

$$\mathcal{L}_{\rm int}=-\frac{1}{2}\overline{q}_1(p_1)\Lambda^{\phi}_{12}q_2(p_2) \times K(p_1,p_2,p_{1'},p_{2'})\overline{q}_{2'}(p_{2'})\Lambda^{\phi}_{1'2'}q_{1'}(p_{1'}) \quad (2)$$

is given as a nonlocal generalization of the current-current type interaction. Here the matrices Λ^{ϕ}_{12} denote the decomposition into the color (c), flavor (f), and Dirac (D) channels. In this work we restrict ourselves to scalar and pseudoscalar channels. Therefore we choose $\Lambda^{\sigma}_{12}=[1_c\cdot 1_f\cdot 1_D]_{12}$ and $\Lambda^{\pi}_{12}=[1_c\cdot \boldsymbol{\tau}_f\cdot i\gamma_5]_{12}$.

The gluonic degrees of freedom do not occur explicitly in this effective approach to the low-energy sector of QCD. They are assumed to form a condensate which is responsible for the nonperturbative character of the quark-quark interaction in this domain. We make the phenomenological ansatz

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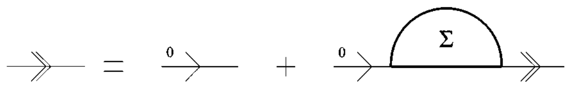

FIG. 1. The Dyson equation with the full self-energy.

of an instantaneous interaction kernel, which can be formulated in a covariant way [11]. We employ here a separable form for the nonlocal 4-point interaction of the form

$$K(p_1,p_2,p_1',p_2') = -\frac{K_0}{N_c} g\left(\frac{|\boldsymbol{p}_1+\boldsymbol{p}_2|}{2}\right) g\left(\frac{|\boldsymbol{p}_1'+\boldsymbol{p}_2'|}{2}\right) \times \delta_{p_1-p_2,p_1'-p_2'}. \tag{3}$$

The N_c dependence arises from the Fierz transformation of the quark current-current interaction in the color singlet channel considered here (see, e.g., [17]). For our numerical calculations in Sec. V we use $N_c=3$. The dependence of the form factor on the modulus of the three-momentum ($|\boldsymbol{p}|=\mathrm{p}$) has been discussed for different shapes, e.g., a Gaussian one ($g(\mathrm{p})=\exp[-(\mathrm{p}/\Lambda_{\mathrm{Gauss}})^2]$) or the well-known NJL-type interaction [$g(\mathrm{p})=\Theta(1-\mathrm{p}/\Lambda_{\mathrm{NJL}})$] (see [10]). Note that the potential does not depend on the energy and we therefore obtain the NJL model with a three-momentum cutoff. The spectral properties of the quark model defined by the Lagrangian (1) are obtained from the single-particle propagator

$$G_{12}(p_1p_2) = [G(p_1)1_c1_f]_{12}\delta_{p_1,p_2}, \tag{4}$$

which is a diagonal matrix in color, flavor, and momentum space. The matrix element $G(p)$ obeys the Dyson equation

$$G(p) = [G_0^{-1}(p) - \Sigma(p)]^{-1}, \tag{5}$$

where $G_0^{-1}(p)=\gamma_\mu p^\mu - m_0$ is the vacuum Green function (see Fig. 1). The self-energy $\Sigma(p)$ is defined by an analysis of all one-particle irreducible diagrams contributing to the propagator. Having the single-particle propagator at our disposal, the physical quantity of interest which is straightforwardly evaluated is the quark condensate. For our separable potential we introduce the nonlocal quark condensate as

$$\langle \overline{q}q\rangle = N_cN_f\sum_p g(\mathrm{p})\mathbf{Tr}[G(p)], \tag{6}$$

where **Tr** stands for the trace over the Dirac space only. The finite temperature investigations are performed using the Matsubara technique [18–20], where $p_0=i\omega_n$ with the fermionic Matsubara frequencies $\omega_n=(2n+1)\pi T$, and Σ_p is short for $T\Sigma_n\int d\boldsymbol{p}/(2\pi)^3$. In order to obtain estimates for the quark condensate one has to make approximations for the self-energy.

The first step towards a systematic investigation of the Dyson equation (5) is the self-consistent Hartree approximation (see Fig. 2),

$$\Sigma^H(\mathrm{p};G^H) = -K_0N_fg(\mathrm{p})\sum_k g(\mathrm{k})\mathbf{Tr}[G^H(k)], \tag{7}$$

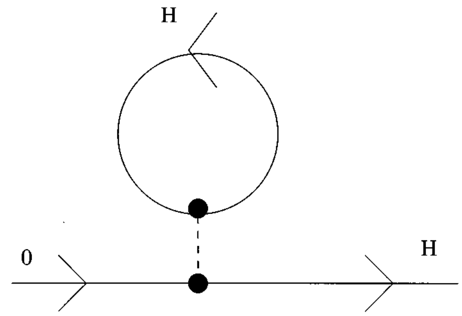

FIG. 2. The Dyson equation in self-consistent Hartree approximation.

which defines upon insertion in (5) the propagator in Hartree approximation

$$G^H(k) = (G_0^{-1}(k) - \Sigma^H[\mathrm{k};G^H])^{-1}. \tag{8}$$

The Hartree self-energy (7) is a Dirac scalar and appears as a mass term in the propagator,

$$m^H(\mathrm{p}) = m_0 - g(\mathrm{p})\frac{K_0}{N_c}\langle\overline{q}q\rangle^H, \tag{9}$$

with a momentum dependence due to the nonlocality of the interaction kernel (3). The quark condensate in Hartree approximation is [cf. Eqs. (6)–(8)]

$$\langle\overline{q}q\rangle^H = N_cN_f\sum_k g(\mathrm{k})\mathbf{Tr}[G^H(k)] = -2N_cN_f\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}g(\mathrm{k})\frac{m^H(\mathrm{k})}{E(\mathrm{k})}\{1-2f[E(\mathrm{k})]\} \tag{10}$$

(for details see Refs. [2–5,10]). In this approximation, the magnitude as well as the temperature dependence of the dynamical mass generation is determined from the condensate only. Note that the restoration of the chiral symmetry at temperatures above the critical one ($T_c\sim 200$ MeV, [10]) is governed by the Fermi distribution function of quarks in the medium, $f[E(\mathrm{k})]=\{\exp[(E(\mathrm{k}))/T]+1\}^{-1}$, where the quasiparticle dispersion relation

$$E(\mathrm{k}) = \sqrt{\mathrm{k}^2+[m^H(\mathrm{k})]^2} \tag{11}$$

contains the momentum-dependent Hartree mass (9).

It is, however, questionable whether the Hartree approximation is appropriate for the description of the nonperturbative low-energy region of QCD where free quarks should be absent due to confinement. Since mesonic correlations are supposed to dominate the low-energy excitation spectrum of the quark matter system, one has to study their influence on the results obtained within the Hartree approximation. A systematic perturbation theory for strong interactions, however, is lacking. Instead, one resorts to an expansion of diagrams to orders $1/N_c$, which we will investigate in this work at finite temperatures.

III. $1/N_c$ EXPANSION

The self-energy in the self-consistent Hartree approximation appears of the order $\mathcal{O}[1]$ as the leading term in the $1/N_c$ expansion, as can be seen from Eq. (7). In order to improve this approximation, we will study next-to-leading order diagrams, i.e., $\mathcal{O}[1/N_c]$ contributions. Therefore we make the following Ansätze for the self-energy and for the quark propagator:

$$\Sigma(p)=\Sigma^H[\mathrm{p};G]+\frac{1}{N_c}\delta\Sigma[p;G]+\mathcal{O}[1/N_c^2], \tag{12}$$

$$G(p)=G^H(p)+\frac{1}{N_c}\delta G(p)+\mathcal{O}(1/N_c^2), \tag{13}$$

where the corrections to the self-energy depend in the general case on the full Green function $G(p)$ and on the 4-momentum p. The corrections to the self-energy $\delta\Sigma[p;G]$ are not yet specified and will be discussed in the following section. Using the $1/N_c$ approximation (12) for $\Sigma[p;G]$ in the Dyson equation (5), the $1/N_c$ expansion to the propagator is given as

$$\begin{aligned}G(p)&=(G_0^{-1}(p)-\Sigma[p;G])^{-1}\\&=\left(G_0^{-1}(p)-\Sigma^H[\mathrm{p};G^H]\right.\\&\quad\left.-\frac{1}{N_c}[\Sigma^H[\mathrm{p};\delta G]+\delta\Sigma[p;G^H]+\mathcal{O}(1/N_c)]\right)^{-1}.\end{aligned} \tag{14}$$

Expanding the $1/N_c$ contribution in the denominator and comparing with (13), we obtain a self-consistent equation for $\delta G(p)$ in the form

$$\begin{aligned}\delta G(p)&=G^H(p)\Sigma^H[\mathrm{p};\delta G]G^H(p)\\&\quad+G^H(p)\delta\Sigma[p;G^H]G^H(p).\end{aligned} \tag{15}$$

Note, that this consistent $1/N_c$ expansion for the quark propagator is a new result of this paper. In particular, the first term on the right-hand side (rhs) of (15) has not been considered in some of the previous approaches (see [12,21]). In order to get a closed expression, we use the fact that the functional dependence of the Hartree self-energy on the $1/N_c$ corrections to the quark propagator δG is known from Eq. (7). After insertion of $\Sigma^H[\mathrm{p};\delta G]$ on the rhs of Eq. (15), we obtain

$$\begin{aligned}\sum_p g(\mathrm{p})\mathbf{Tr}[\delta G(p)]&=-K_0N_f\sum_p g^2(\mathrm{p})\mathbf{Tr}[G^H(p)G^H(p)]\sum_k g(\mathrm{k})\mathbf{Tr}[\delta G(k)]+\sum_p g(\mathrm{p})\mathbf{Tr}[G^H(p)\delta\Sigma[p;G^H]G^H(p)]\\&=\frac{1}{1-J^\sigma(0)}\sum_p g(\mathrm{p})\mathbf{Tr}[G^H(p)\delta\Sigma[p;G^H]G^H(p)],\end{aligned} \tag{16}$$

where the scalar quark loop integral $J^\sigma(0)$ is defined in Appendix A. The $1/N_c$ expansion of the quark condensate corresponding to that of the propagator (13) and the definiton of the quark condensate (6) reads

$$\langle\overline{qq}\rangle=\langle\overline{qq}\rangle^H+\delta\langle\overline{qq}\rangle+\mathcal{O}[1/N_c^2]. \tag{17}$$

The $1/N_c$ correction to the condensate is obtained in closed form using the result (16)

$$\delta\langle\overline{qq}\rangle=Z\cdot N_f\sum_p g(\mathrm{p})\mathbf{Tr}[G^H(p)\delta\Sigma[p;G^H]G^H(p)] \tag{18}$$

with a prefactor $Z=1/[1-J^\sigma(0)]$ as derived in (16) coming from the $1/N_c$ contributions to the Hartree self-energy $\Sigma^H[\mathrm{p},\delta G]$ (see in Fig. 3). This prefactor leads to a considerable rescaling ($Z\sim4$) which has first been pointed out in Refs. [13,15] for the NJL model at zero temperature and is obtained here for the more general case of a nonlocal separable interaction at finite temperature.

IV. MESONIC CORRELATIONS

Within the chiral quark model as defined in Sec. II, the complete set of diagrams contributing in $\mathcal{O}[1/N_c]$ to the self-energy is given in Fig. 4. The double line corresponds to the random-phase approximation (RPA)-type partial resummation of the chain of bubble diagrams, where the quark-antiquark loop in Hartree approximation defines the polarization functions $J^\phi(p-k)$ in the scalar and pseudoscalar channel ($\phi=\sigma,\pi$) (see Appendix A).

The $\mathcal{O}[1/N_c]$ self-energy contribution is given by

$$\begin{aligned}\delta\Sigma[p;G^H]=K_0\sum_k g^2\left(\frac{|\boldsymbol{p}+\boldsymbol{k}|}{2}\right)\left[\frac{G^H(k)}{1-J^\sigma(p-k)}\right.\\\left.-(N_f^2-1)\frac{\gamma_5G^H(k)\gamma_5}{1-J^\pi(p-k)}\right].\end{aligned} \tag{19}$$

The denominators $1-J^\phi(p-k)$ occur due to the resummation and thus strong correlations can be described. Note that the $1/N_c$ self-energy is a dynamical quantity and has not yet been solved in its complexity. The most dramatic effect is the occurrence of collective excitations in the quark-antiquark channel when $\mathrm{Re}J^\phi(P=M_\phi)=1$ [and $\mathrm{Im}J^\phi(P=M_\phi)=0$] which correspond to mesonic bound

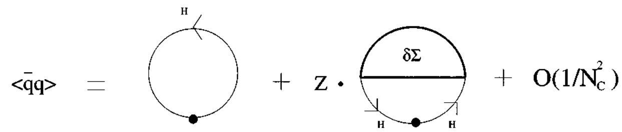

FIG. 3. $1/N_c$ expansion of the quark condensate.

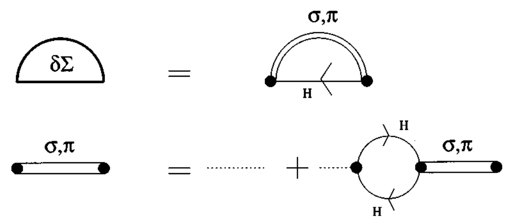

FIG. 4. $1/N_c$ approximation for the self-energy. The scalar and pseudoscalar correlations are described by a RPA-type partial summation of bubble diagrams.

states. In what follows we restrict ourselves to the consideration of bound states only and use an expansion of the polarization function at the mesonic poles (pole approximation), which leads to the introduction of meson propagators and meson-quark-antiquark form factors $g_{\phi q \overline{q}}$ (see Appendix A):

$$\frac{1}{1-J^{\phi}(P)} \approx \frac{1}{M_{\phi}^2-P^2}\frac{g^2_{\phi q\overline{q}}(M_{\phi})}{N_f K_0}. \tag{20}$$

The full treatment of the RPA approximation that contains bound and scattering states is possible for the separable interaction and will be regarded in an additional work.

Using the expression (19), (20) for the self-energy and the short notation with $\Gamma^{\sigma}=1_D$ and $\Gamma^{\pi}=i\gamma_5$, we obtain

$$\delta\langle\overline{q}q\rangle^{\phi}=\frac{g^2_{\phi q\overline{q}}}{1-J^{\sigma}(0)}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}g(\mathrm{p})\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}g^2\left(\frac{|\boldsymbol{p}+\boldsymbol{k}|}{2}\right)\times\int\frac{dp_0}{2\pi}\int\frac{dk_0}{2\pi}\frac{1}{M_{\phi}^2-(k-p)^2}\mathbf{Tr}[G^H(p)\Gamma^{\phi}\times G^H(k)\Gamma^{\phi}G^H(p)]. \tag{21}$$

After evaluation of the Dirac trace

$$\frac{1}{M_{\phi}^2-(k-p)^2}\mathbf{Tr}[G^H(p)\Gamma^{\phi}G^H(k)\Gamma^{\phi}G^H(p)]=-4\left[\frac{m(\mathrm{p})\pm m(\mathrm{k})}{[p^2-m^2(\mathrm{p})][k^2-m^2(\mathrm{k})][(k-p)^2-M_{\phi}^2]}+m(p)\times\left(\frac{1}{[p^2-m^2(\mathrm{p})]^2[(k-p)^2-M_{\phi}^2]}-\frac{M_{\phi}^2-[m(\mathrm{p})\pm m(\mathrm{k})]^2}{[\mathrm{p}^2-m^2(\mathrm{p})]^2[k^2-m^2(\mathrm{k})][(k-p)^2-M_{\phi}^2]}-\frac{1}{[p^2-m^2(\mathrm{p})]^2[k^2-m^2(\mathrm{k})]}\right)\right], \tag{22}$$

we perform the Matsubara summation (see Appendix B) and obtain as the final result for the $1/N_c$ mesonic contributions (21) to the quark condensate

$$\begin{aligned}\delta\langle\overline{q}q\rangle^{\phi}=&\frac{g^2_{\phi q\overline{q}}}{1-J^{\sigma}(0)}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}g(\mathrm{p})\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}g^2\left(\frac{|\boldsymbol{p}+\boldsymbol{k}|}{2}\right)\left\{\frac{2m^H(\mathrm{p})}{E^2(\mathrm{p})}\left(\frac{1-2f[E(\mathrm{p})]}{2E(\mathrm{p})}-\frac{f[E(\mathrm{p})]\{1-f[E(\mathrm{p})]\}}{T}\right)\left(\frac{1-2f[E(\mathrm{k})]}{2E(\mathrm{k})}\right.\right.\\&\left.-\frac{1+2n[E_{\phi}(\boldsymbol{k}-\boldsymbol{p})]}{2E_{\phi}(\boldsymbol{k}-\boldsymbol{p})}\right)+\left(\left[\frac{\{1-f[E(\mathrm{p})]-f[E(\mathrm{k})]\}\{1+n[E(\mathrm{p})+E(\mathrm{k})]+n[E_{\phi}(\boldsymbol{k}-\boldsymbol{p})]\}}{E^3(\mathrm{p})E(\mathrm{k})E_{\phi}(\boldsymbol{k}-\boldsymbol{p})[E_{\phi}(\boldsymbol{k}-\boldsymbol{p})+E(\mathrm{p})+E(\mathrm{k})]}\right[E^2(\mathrm{p})[m^H(\mathrm{p})\pm m^H(\mathrm{k})]\\&+m^H(\mathrm{p})\{M_{\phi}^2-[m^H(\mathrm{p})\pm m^H(\mathrm{k})]^2\}\left(\frac{E_{\phi}(\boldsymbol{k}-\boldsymbol{p})+2E(\mathrm{p})+E(\mathrm{k})}{E_{\phi}(\boldsymbol{k}-\boldsymbol{p})+E(\mathrm{p})+E(\mathrm{k})}+\frac{E(\mathrm{p})f[E(\mathrm{p})]\{1-f[E(\mathrm{p})]\}}{T}\right)\left.\right]+[E(\mathrm{k})\to-E(\mathrm{k})]\left.\right]\\&\left.\left.+[E(\mathrm{p})\to-E(\mathrm{p})]\right)\right\},\end{aligned} \tag{23}$$

with the energies $E_{\phi}(\boldsymbol{k}-\boldsymbol{p})=\sqrt{(\boldsymbol{k}-\boldsymbol{p})^2+M_{\phi}^2}$ and the bosonic distribution function $n(E)=[\exp(E/T)-1]^{-1}$. The upper sign holds for the scalar, the lower one for the pseudoscalar meson, respectively. The $\mathcal{O}[1/N_c]$ contribution (23) consists of two parts, and the numerical analysis shows that the contribution due to mesonic correlations is dominated by the first one, i.e.,

$$\delta\langle\overline{q}q\rangle^{\phi}\approx\frac{g^2_{\phi q\overline{q}}}{1-J^{\sigma}(0)}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}g(\mathrm{p})\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}g^2\left(\frac{|\boldsymbol{p}+\boldsymbol{k}|}{2}\right)\frac{m^H(\mathrm{p})\{1-2f[E(\mathrm{p})]\}}{E^3(\mathrm{p})}\left(\frac{1-2f[E(\mathrm{k})]}{2E(\mathrm{k})}-\frac{1+2n[E_{\phi}(\boldsymbol{k}-\boldsymbol{p})]}{2E_{\phi}(\boldsymbol{k}-\boldsymbol{p})}\right), \tag{24}$$

which has a simpler structure than (23).

In order to compare our results with previous works we will now discuss the $T=0$ case. At zero temperature the Fermi and Bose distribution functions vanish. In the $T=0$ limit of Eqs. (10) and (23) we obtain

$$\delta\langle\overline{q}q\rangle^{\phi}=\frac{g^2_{\phi\overline{q}q}}{1-J^{\sigma}(0)}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}g(\mathrm{p})\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}g^2\left(\frac{|\boldsymbol{p}+\boldsymbol{k}|}{2}\right)\left[\frac{m^H(\mathrm{p})}{2E^3(\mathrm{p})}\cdot\left(\frac{1}{E(\mathrm{k})}-\frac{1}{E_{\phi}(\boldsymbol{k}-\boldsymbol{p})}\right)\right.$$
$$\left.+\frac{\{M^2_{\phi}-[m^H(\mathrm{p})\pm m^H(\mathrm{k})]^2\}[E_{\phi}(\boldsymbol{k}-\boldsymbol{p})+2E(\mathrm{p})+E(\mathrm{k})]m^H(\mathrm{p})}{E^3(\mathrm{p})E(\mathrm{k})E_{\phi}(\boldsymbol{k}-\boldsymbol{p})[E_{\phi}(\boldsymbol{k}-\boldsymbol{p})+E(\mathrm{p})+E(\mathrm{k})]^2}+\frac{[m^H(\mathrm{p})\pm m^H(\mathrm{k})]}{E(\mathrm{p})E(\mathrm{k})E_{\phi}(\boldsymbol{k}-\boldsymbol{p})[E_{\phi}(\boldsymbol{k}-\boldsymbol{p})+E(\mathrm{p})+E(\mathrm{k})]}\right]. \tag{25}$$

In the following section we present the numerical evaluation and discussion of the above $1/N_c$ corrections to the quark condensate.

V. NUMERICAL RESULTS AND DISCUSSION

In Sec. II we have introduced a general nonlocal interaction kernel in separable form. In order to compare the numerical results with previous approaches within the NJL model, we will restrict ourselves in this paper to the discussion of a cutoff form factor:

$$g\left(\frac{|\boldsymbol{p}+\boldsymbol{k}|}{2}\right)=\Theta\left(1-\frac{|\boldsymbol{p}+\boldsymbol{k}|}{2\Lambda_{\mathrm{NJL}}}\right). \tag{26}$$

The chiral quark model with soft form factors (e.g., a Gaussian one) has been discussed in Refs. [9,10].

After fixing the parameters of the model as described in Appendix A, we obtain for the quark condensate in the Hartree approximation $\langle\overline{u}u\rangle^H=-(250\ \mathrm{MeV})^3$ and for the quark mass $m^H=300$ MeV, in agreement with the well-known data of the literature [2–5].

In the next step, discussed in Sec. IV, we have included dynamical self-energy contributions due to mesonic correlations. Compared with the Hartree term (10) where we have to solve a one-loop integral, the next-to-leading order contributions are two-loop integrals that after summation over both Matsubara frequencies (k_0,p_0) reduce to three-dimensional integrals over the variables k,p,z, where $z=\cos\theta$, if θ denotes the angle between the momenta $\boldsymbol{k}$ and $\boldsymbol{p}$.

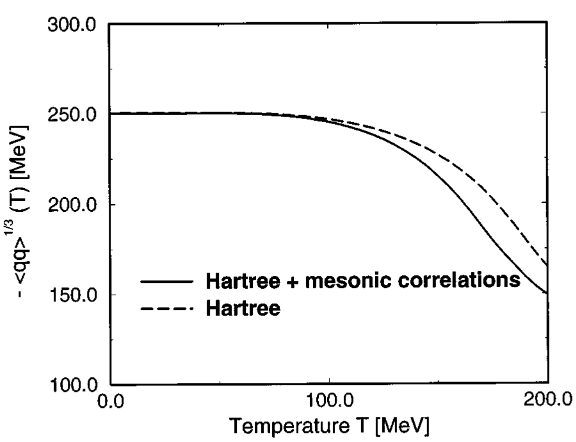

FIG. 5. The quark condensate as a function of the temperature in self-consistent Hartree approximation (dotted line) and with inclusion of mesonic correlations (solid line).

At first we want to discuss the $T=0$ limit. An open question which occurs in the conventional NJL model is the choice of the cutoff for the second momentum integral in Eqs. (23) and (25) over k. A very crude approximation presented in Ref. [21] is the omission of the second integral by assuming that $\boldsymbol{k}=0$. In Refs. [13,15] the additional cutoff $\overline{\Lambda}$ was discussed. Reference [13] assumes that $\overline{\Lambda}=\Lambda_{\mathrm{NJL}}$ and in Ref. [15] upper and lower limits are determined from a calculation of f_{π} in $\mathcal{O}[1/N_c]$. In the formulation we have chosen, such a problem does not exist since the integrals are regularized in the separable approach by the proper treatment of the form factors. The parameter Λ_{NJL} in the cutoff form factor (26) regularizes the integral over p. The upper limit of the k integration is given by $\overline{\Lambda}=-\mathrm{p}z+\sqrt{4\Lambda^2_{\mathrm{NJL}}-\mathrm{p}^2(1-z^2)}$ and runs between $\Lambda_{\mathrm{NJL}}<\overline{\Lambda}<3\Lambda_{\mathrm{NJL}}$. Note that in solving (23) one has to check the integral limits for each term separately due to different combinations of form factors partly hidden in the momentum-dependent quark mass (9). Thus we have removed the ambiguity in regularizing the second momentum integration which occurred in the previous approaches to the $1/N_c$ expansion in the NJL model.

The result for such a calculation in the $T=0$ limit is that for fixed model parameters the absolute value of the condensate is decreased by 20% compared to the Hartree approximation. For comparison, a decrease of the quark mass due to the mesonic correlations in $1/N_c$ at $T=0$ has been obtained in Ref. [13]. This result can be understood qualitatively since the Hartree contribution (10) and the $1/N_c$ mesonic contribution (23) to the quark condensate have opposite signs, the latter one being smaller in magnitude.

Let us consider the finite temperature case. In order to compare the temperature behavior of the quark condensate in both models (Hartree approximation and Hartree approximation with mesonic correlations) we have to fix the parameters such that the same values for the observables at $T=0$ are obtained (see Appendix A). The numerical evaluation of the final result for the quark condensate is shown in Fig. 5. Paying attention to the shape of the chiral phase transition, we observe that the inclusion of $1/N_c$ mesonic correlations shifts the chiral symmetry restoration to lower temperatures when compared with the simple Hartree approximation. This finding is mainly due to the smaller $q\overline{q}$ coupling constant K_0 for the model with mesonic correlations.

VI. CONCLUSIONS

In conclusion we have obtained the following new results: (i) a consistent $1/N_c$ expansion for the self-energy as well as for the propagator and a closed formula for the quark condensate up to the order $1/N_c$, (ii) a finite temperature result for the $1/N_c$ quark condensate within the Matsubara formalism, and (iii) a consistent regularization of the two-loop diagrams.

The numerical evaluation for a NJL-type model shows that compared with the Hartree approximation mesonic correlations lead to a decrease of the absolute value of the quark condensate at $T=0$. After having compensated for this effect of quantum fluctuations at $T=0$ by readjusting the model parameters (Λ, m_0, K_0), the account for thermally excited mesonic correlations shifts the onset of the chiral-symmetry restoration to lower temperatures in comparison to the Hartree approximation. This behavior seems to be closer to the recent results of lattice calculations where the condensate remains unchanged with temperature up to the chiral transition which occurs at $T\approx 150$ MeV for $N_f=2$ [22].

The inclusion of quark-antiquark correlations is of principal interest because the treatment of the medium in free quasiparticle approximation seems not to be appropriate at low-temperatures. In contrast, in this region the mesonic degrees of freedom are expected to be relevant. This is supported by the fact that in the low temperature limit ($T\lesssim 50$ MeV) other thermodynamic properties of a quark-meson system (e.g., the pressure) are dominated by mesonic contributions [16]. The presented $1/N_c$ expansion should be considered as a first step in including mesonic correlations. However, at temperatures where the chiral phase transition occurs, higher orders of the $1/N_c$ expansion may become important.

Within the present approach, the treatment of the two-particle correlations was given in the usual pole approximation (20) for the $q\bar{q}$ T matrix. A next step in the evaluation of quark-antiquark correlations is the inclusion of the contribution of scattering states, which will be considered in a forthcoming paper. In this way the account of the corrections due to two-particle correlations will be completed on the basis of the approach presented here.

ACKNOWLEDGMENTS

Our thanks go to the Deutsche Forschungsgemeinschaft, which gave a research Grant No. Ro 905/7-2 to one of us (Yu.K.). G.R. and S.S. are grateful for the hospitality during a stay in Dubna. M.K.V. acknowledges the support provided by INTAS Grant No. W 94-2915 and RFFI Grant No. W 93-02-14411 as well as the hospitality extended to him during a stay in Rostock.

APPENDIX A: TABLE OF INTEGRALS AND PARAMETER FIXING

The polarization operators introduced in Eq. (19) of the main text are defined as

$$J^{\phi}(P)=-K_0 N_f \sum_q g^2(\mathrm{q})\mathbf{Tr}[\Gamma^{\phi}G^H(q+P/2)\times\Gamma^{\phi}G^H(q-P/2)]. \tag{A1}$$

The temperature-dependent meson masses are obtained from the solution of the Bethe-Salpeter equation

$$1-J^{\phi}[P_0=M_{\phi}(T),\boldsymbol{P}=0]=0, \tag{A2}$$

where the polarization operators $J^{\phi}(P_0)$ after evaluation of the Dirac trace and angular integration are given by

$$J^{\pi}(P_0)=\frac{2K_0N_f}{\pi^2}\int d\mathrm{q}\mathrm{q}^2 g^2(\mathrm{q})\frac{E(\mathrm{q})}{E^2(\mathrm{q})-(P_0/2)^2}\times\{1-2f[E(\mathrm{q})]\}, \tag{A3}$$

$$J^{\sigma}(P_0)=\frac{2K_0N_f}{\pi^2}\int d\mathrm{q}\mathrm{q}^2 \frac{g^2(\mathrm{q})}{E(\mathrm{q})}\frac{\mathrm{q}^2}{E^2(\mathrm{q})-(P_0/2)^2}\times\{1-2f[E(\mathrm{q})]\}. \tag{A4}$$

The quark meson coupling constants introduced in Eq. (20) are evaluated in the rest frame of the $q\bar{q}$ pair $(\boldsymbol{P}=0)$, where

$$g^{-2}_{\phi q\bar{q}}(M_{\phi})=\frac{1}{2K_0N_f}\frac{d}{dP^2}J^{\phi}(P)|_{P^2=M^2_{\phi}} \approx \frac{1}{2K_0N_f}\frac{d}{2P_0dP_0}J^{\phi}(P_0,0)|_{P_0=M_{\phi}}. \tag{A5}$$

Using Eqs. (A3) and (A4), they are given by the integrals

$$g^{-2}_{\pi q\bar{q}}(M_{\pi})=\frac{1}{2\pi^2}\int d\mathrm{q}\mathrm{q}^2 g^2(\mathrm{q})\frac{E(\mathrm{q})}{(E^2(\mathrm{q})-M^2_{\pi}/4)^2}\times(1-2f[E(\mathrm{q})]), \tag{A6}$$

$$g^{-2}_{\sigma q\bar{q}}(M_{\sigma})=\frac{1}{2\pi^2}\int d\mathrm{q}\mathrm{q}^2 g^2(\mathrm{q})\frac{\mathrm{q}^2}{E(\mathrm{q})}\frac{1}{[E^2(\mathrm{q})-M^2_{\sigma}/4]^2}\times(1-2f[E(\mathrm{q})]).$$

The pion decay constant which we use for the parameter fixing at zero temperature is calculated by [10]

$$f_{\pi}=\frac{\sqrt{N_c}g_{\pi q\bar{q}}}{2\pi^2}\int d\mathrm{q}\mathrm{q}^2 g(\mathrm{q})\frac{m^H(\mathrm{q})}{E(\mathrm{q})[E^2(\mathrm{q})-M^2_{\pi}/4]}. \tag{A7}$$

The model contains three parameters: the coupling constant K_0, the current quark mass m_0, and the range of the form factor of the potential. We fix these three parameters to reproduce the pion mass ($M_{\pi}=140$ MeV) [Eqs. (A2) and (A3)], the pion decay constant ($f_{\pi}=93$ MeV) [Eqs. (A6) and (A7)], and the quark condensate $(-\langle q\bar{q}\rangle^{1/3}=250$ MeV) at zero temperature. The resulting parameter sets are given in Table I for the Hartree approximation with and without the $1/N_c$ contribution from mesonic correlations, respectively. The parameters for the Hartree approximation are similar to those of the standard NJL model (see Refs. [3,4,12,13,15,21]).

APPENDIX B: MATSUBARA SUMMATION

The following formulas summarize the results of the one- and two-loop Matsubara sums performed in this paper:

TABLE I. Sets of parameters for the Hartree approximation with and without the inclusion of mesonic correlations for a fixing scheme described in the text.

Approximations	Λ (MeV)	m_0 (MeV)	K_0 (GeV^{-2})
Hartree	660	5.35	9.45
Hartree+mesonic correlations	765	4.4	6.49

$$\int \frac{dk_0}{2\pi}\frac{1}{k_0^2-E^2(\mathrm{k})} = -\frac{1-2f[E(\mathrm{k})]}{2E(\mathrm{k})}, \tag{B1}$$

$$\int \frac{dp_0}{2\pi}\frac{1}{[p_0^2-E^2(\mathrm{p})]^2} = \frac{1}{4E^2(\mathrm{p})}\Bigg(\frac{1-2f[E(\mathrm{p})]}{2E(\mathrm{p})} - \frac{f[E(\mathrm{p})]\{1-f[E(\mathrm{p})]\}}{T} + [E(\mathrm{p})\rightarrow -E(\mathrm{p})]\Bigg), \tag{B2}$$

$$\int \frac{dk_0}{2\pi}\frac{1}{(k_0-p_0)^2-E_\phi^2(\boldsymbol{k}-\boldsymbol{p})} = -\frac{1+2n[E_\phi(\boldsymbol{k}-\boldsymbol{p})]}{2E_\phi(\boldsymbol{k}-\boldsymbol{p})}, \tag{B3}$$

$$\int \frac{dk_0}{2\pi}\frac{1}{[(k_0-p_0)^2-E_\phi^2(\boldsymbol{k}-\boldsymbol{p})][k_0^2-E^2(\mathrm{k})]} = \Bigg(\frac{f[E(\mathrm{k})]+n[E_\phi(\boldsymbol{k}-\boldsymbol{p})]}{4[p_0+E_\phi(\boldsymbol{k}-\boldsymbol{p})-E(\mathrm{k})]E(\mathrm{k})E_\phi(\boldsymbol{k}-\boldsymbol{p})} + [E(\mathrm{k})\rightarrow -E(\mathrm{k})]\Bigg) + [E_\phi(\boldsymbol{k}-\boldsymbol{p})\rightarrow -E_\phi(\boldsymbol{k}-\boldsymbol{p})], \tag{B4}$$

$$\int \frac{dp_0}{2\pi}\frac{1}{[p_0^2-E^2(\mathrm{p})][p_0+E_\phi(\boldsymbol{k}-\boldsymbol{p})-E(\mathrm{k})]} = \frac{f[E(\mathrm{p})]-f[E(\mathrm{k})]}{2E(\mathrm{p})[E_\phi(\boldsymbol{k}-\boldsymbol{p})-E(\mathrm{k})+E(\mathrm{p})]} + [E(\mathrm{p})\rightarrow -E(\mathrm{p})]. \tag{B5}$$

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