Effect of isospin mixing on superallowed Fermi β decay

H. Sagawa*
Center for Mathematical Sciences, the University of Aizu, Aizu-Wakamatsu, Fukushima 965, Japan

Nguyen Van Giai†
Division de Physique Théorique, Institut de Physique Nucléaire, Université de Paris-Sud, F-91406 Orsay CEDEX, France

Toshio Suzuki‡
Department of Physics, College of Humanities and Science, Nihon University, Sakurajosui 3-25-40, Setagaya-ku, Tokyo 156, Japan
(Received 27 November 1995)

We study the effect of isospin impurity on the superallowed Fermi β decay using the microscopic Hartree-Fock and random phase approximation (or Tamm-Dancoff approximation) model taking into account charge symmetry and independence breaking interactions. It is found that the isospin impurity of $N=Z$ nuclei gives enhancement of the sum rule of Fermi transition probabilities. On the other hand, the superallowed transitions between odd-odd $J=0$ nuclei and even-even $J=0$ nuclei are quenched because of the cancellation of the isospin impurity effects of mother and daughter nuclei. An implication of the calculated Fermi transition rate on the unitarity of the Cabibbo-Kobayashi-Maskawa mixing matrix is also discussed.

PACS number(s): 23.40.Bw, 12.15.Hh, 21.10.Hw, 21.60.Jz

I. INTRODUCTION

Nuclei near the proton drip line are particularly interesting in the study of isospin impurity because of favorable isospin geometrical factors [1]. The question of isospin impurity in nuclear ground states is especially important because of its direct bearing on the experimental determination of the vector coupling constant G_V of nucleon β decay. Indeed, superallowed Fermi β decays have been studied intensively for several decades in relation to this vector coupling constant under the conserved vector current (CVC) hypothesis. Combining the vector coupling constant of nuclear β decay with that of muon β decay, it is possible to determine the Cabibbo-Kobayashi-Maskawa (CKM) mixing amplitude between u and d quarks [2]. This amplitude, together with the mixing amplitudes of u and s quarks and u and b quarks, provides an opportunity to test experimentally the standard three-generation quark model for the electroweak interaction.

Two nuclear medium corrections have been studied for obtaining the *nucleus-independent ft* value [3,4]. The first one is the ''outer'' radiative correction and the second one is the effect of isospin nonconserving forces in nuclei. These two corrections have been studied intensively during the last two decades and found to be important to obtain the nucleus-independent ft value. However, there is still a substantial deviation from unitarity in the empirical CKM matrix elements. It has been claimed that the empirical data of Fermi decay after subtracting the nuclear structure effects give a somewhat smaller value than that required by the unitarity condition (three times more than the standard deviation). In this study, we will examine two new effects which have not been thoroughly discussed in the previous studies of isospin mixing effects in β decay, namely, the isospin impurity of the core, which is not considered in shell model calculations, and the effect of the charge symmetry breaking (CSB) and charge independence breaking (CIB) forces on the mean field potentials [5,6]. It will be interesting to see the predictions of the random phase approximation (RPA) model for this unitarity problem since there are these two essential differences between our model and the previous studies. In particular, we study the Fermi β decay in ^{10}C, ^{14}O, ^{26}Al, ^{34}Cl, ^{38}K, ^{42}Sc, and ^{54}Co nuclei for which the most accurate experimental ft values are available [7,8]. We also report results of the heavier nuclei ^{62}Ga, ^{66}As, and ^{74}Rb for further study of the CVC, hypothesis, and to clarify the differences between our model and previous calculations with other models. This paper is organized in the following way. The effect of the isospin impurity on the sum rule of Fermi decay is discussed in Sec. II. The Fermi transitions between ($J^\pi=0^+ \rightarrow J^\pi=0^+$) states are studied by using the Hartree-Fock (HF) + RPA model in Sec. III. Since the RPA model cannot be applied for ^{10}C and ^{14}O because of the negative final state energy, the Tamm-Dancoff approximation (TDA) is adopted for these nuclei, correcting for the spurious component of the isospin impurity. Section IV is devoted to a perturbation theory discussion of the isospin impurity effect on the Fermi decay in order to clarify the numerical results of Sec. III. The relation of the nuclear Fermi decays to the CKM mixing matrix is discussed in Sec. V. A summary is given in Sec. VI.

II. SUM RULE OF FERMI β DECAY

There is a model-independent sum rule on the difference $S_+ - S_- = Z - N$ between the Fermi-type T_+ and T_- transition strengths when all final states are summed up, N and Z being the neutron and proton number of the parent nucleus. Furthermore, the sum rule S_+ itself can be calculated model

*Electronic address: sagawa@u-aizu.ac.jp
†Electronic address: nguyen@ipncls.in2p3.fr
‡Electronic address: suzuki@chs.nihon-u.ac.jp

independently if the parent nucleus has pure isospin (T,T_z):

$$S_+ = T(T+1) - T_z(T_z+1). \tag{1}$$

For the nuclei in which we are now interested, the isospin is $T=1$ and $T_z=0$ or -1. When the ground state has isospin impurity, the sum rule S_+ may change proportionally to the mixing probabilities since the transition operators are the isospin raising and lowering ones. Most of the relevant nuclei for superallowed β decay are odd-odd ones with $J=0$ and $T_z=0$, for example, ^{26}Al, ^{34}Cl, ^{38}K, ^{42}Sc, and ^{54}Co. The dominant isospin component of these nuclei is $T=|T_z|+1$ because of the spin-isospin symmetry of the ground state wave function. On the other hand, two lighter nuclei ^{10}C and ^{14}O have $T_z=-1$ and the dominant isospin is $T=|T_z|=1$. The sum rule for the Fermi β decay of all these nuclei is expressed in terms of a deviation factor δ from the value for conserved isospin [6]. This factor δ can be further separated into valence and core particle contributions:

$$\begin{aligned} S_+ &= \langle \phi | T_- T_+ | \phi \rangle \equiv 2(1+\delta) \\ &= 2(1+\delta_{cc} - \delta_{j_p,c} - \delta_{j_n,c} - \delta_{j_p,j_n}), \end{aligned} \tag{2}$$

where the factor δ contains contributions from core-core, valence-proton–core, valence-neutron–core, and valence-valence particles. All four factors are defined to be positive. If the core-core contribution is ignored, the factor δ is negative and it gives a reduction of the sum rule. It is found that the core contribution is much larger than the other contributions from valence particles.

It has been pointed out [9] that the CSB and CIB forces are important to solve the Coulomb (Nolen-Schiffer) anomaly problem. The effects of isospin nonconserving interactions on the Fermi transitions, however, have been discussed only in shell model calculations [10], but not in HF calculations. We take them into account in the HF calculations by parametrizing them as short-range interactions in order to evaluate these δ factors. We use the following Skyrme-type parametrizations:

$$\begin{aligned} V_{\rm CSB} = \tfrac{1}{4}\{\tau_z(1)+\tau_z(2)\}\{s_0(1+y_0P_\sigma) + \tfrac{1}{2}s_1(1+y_1P_\sigma) \\ \times(k^2+k'^2) + s_2(1+y_2P_\sigma)\mathbf{k}'\cdot\mathbf{k}\} \end{aligned} \tag{3}$$

and

$$\begin{aligned} V_{\rm CIB} = \tfrac{1}{2}\tau_z(1)\tau_z(2)\{u_0(1+z_0P_\sigma) + \tfrac{1}{2}u_1(1+z_1P_\sigma)(k^2+k'^2) \\ + u_2(1+z_2P_\sigma)\mathbf{k}'\cdot\mathbf{k}\}, \end{aligned} \tag{4}$$

where P_σ is the spin-exchange operator. The parameters s_i and u_i are given in Ref. [6]. All exchange parameters are $y_i=z_i=-1$ because of the singlet-even character of the isospin breaking forces.

The calculated enhancement factors δ are shown in Fig. 1 and Tables I and II where the commonly used parameters sets SGII and SIII are adopted for the HF calculations. The HF equations are solved in coordinate space by using the Numerov algorithm. In the cases of ^{10}C and ^{14}O, the mean field calculations have a problem of spurious isospin mixing. These spurious components are subtracted by the following method. First, the spurious isospin mixing amplitude is evaluated by a HF calculation without Coulomb interaction.

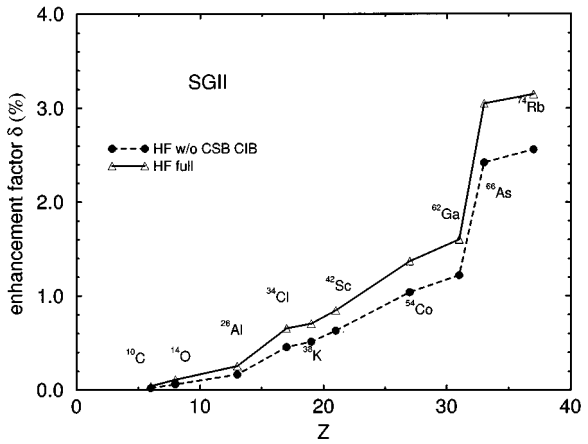

FIG. 1. Calculated enhancement factor δ of the Fermi β decay sum rule. The Skyrme interaction SGII is used as the HF interaction. The dashed line with black circles is obtained without the CSB and CIB interactions, while the solid line with triangles is calculated with those interactions.

TABLE I. Calculated values of the enhancement factor δ for the sum rule in Eq. (2) and the quenching factor δ_c for $0^+\rightarrow 0^+$ transitions in Eq. (5). E_x is the energy difference between the mother and the daughter nuclei. A Skyrme interaction SGII is used for the calculations with/without CSB and CIB interactions. For each case, the first line shows the results without CSB and CIB interactions, while the second line corresponds to those with CSB and CIB interactions. The values δ are calculated using the HF wave functions, while δ_c are obtained by self-consistent HF+RPA calculations except for ^{10}C and ^{14}O, for which only TDA results are available because of the negative energies of daughter nuclei. The HF+TDA results are marked by asterisks.

SGII	HF	RPA (TDA)	
A	δ(%)	E_x (MeV)	δ_c(%)
^{10}C→^{10}B	0.02	−2.32*	0.01*
	0.04	−2.61*	0.00*
^{14}O→^{14}N	0.06	−3.34*	0.21*
	0.11	−3.56*	0.29*
^{26}Al→^{26}Mg	0.16	4.93	0.18
	0.25	5.18	0.27
^{34}Cl→^{34}S	0.46	5.84	0.24
	0.65	6.04	0.33
^{38}K→^{38}Ar	0.51	6.43	0.24
	0.71	6.64	0.33
^{42}Sc→^{42}Ca	0.63	7.00	0.33
	0.85	7.25	0.44
^{54}Co→^{54}Fe	1.04	8.71	0.37
	1.37	9.00	0.49
^{62}Ga→^{62}Zn	1.22	9.68	1.15
	1.60	9.93	1.42
^{66}As→^{66}Ge	2.42	9.89	0.61
	3.05	10.11	0.78
^{74}Rb→^{74}Kr	2.56	10.83	0.59
	3.15	11.06	0.74

TABLE II. Calculated values of the enhancement factor δ for the sum rule in Eq. (2) and the quenching factor δ_c for $0^+ \rightarrow 0^+$ transitions in Eq. (5). Skyrme interactions SGII and SIII are used for the calculations together with CSB and CIB interactions. The quantities δ and δ_c are calculated as explained in Table I. The TDA results are marked by asterisks.

	SGII			SIII		
A	δ(%)	E_x (MeV)	δ_c(%)	δ (%)	E_x (MeV)	δ_c (%)
$^{10}C \rightarrow {}^{10}B^*$	0.04	−2.61	0.00	0.03	−2.14	−0.01
$^{14}O \rightarrow {}^{14}N^*$	0.11	−3.56	0.29	0.09	−3.46	0.22
$^{26}Al \rightarrow {}^{26}Mg$	0.25	5.18	0.27	0.21	5.19	0.27
$^{34}Cl \rightarrow {}^{34}S$	0.65	6.04	0.33	0.58	6.00	0.34
$^{38}K \rightarrow {}^{38}Ar$	0.71	6.64	0.33	0.58	6.61	0.32
$^{42}Sc \rightarrow {}^{42}Ca$	0.85	7.25	0.44	0.68	7.25	0.43
$^{54}Co \rightarrow {}^{54}Fe$	1.37	9.00	0.49	1.07	8.95	0.46
$^{62}Ga \rightarrow {}^{62}Zn$	1.60	9.93	1.42	1.24	9.81	1.56
$^{66}As \rightarrow {}^{66}Ge$	3.05	10.11	0.78	2.70	10.05	0.78
$^{74}Rb \rightarrow {}^{74}Kr$	3.15	11.06	0.74	2.63	11.01	0.70

The spurious mixing amplitude is then subtracted from that of the full HF calculation. The calculated values of δ with SGII force are shown in Fig. 1 as a function of Z. The values are also listed in Table I. They are always positive due to the dominant term δ_{cc} which leads to an enhancement of the sum rule. The core contribution is one order of magnitude larger than the other contributions because of the large number of particles in the core. In general, the CSB and CIB forces increase the enhancement factors by 30–40 % as compared to the Coulomb interaction alone. The Z dependence of δ is found to be Z^2, on average, which is somewhat smaller than that of the isospin impurity calculated in the hydrodynamical model by Bohr and Mottelson [11]. We have used also the Skyrme interaction SIII and obtained somewhat smaller δ values to those obtained with the SGII force as is shown in Table II. This might be due to the lower incompressibility of SGII interaction compared with that of SIII.

III. RPA CALCULATIONS FOR FERMI TRANSITIONS

The enhancement of the sum rule is expected to have a significant effect on the analysis to extract the nucleus-independent ft value, especially in the large-Z nuclei. We have performed HF+RPA self-consistent calculations of $0^+ \rightarrow 0^+$ transitions for the above mentioned nuclei. The CSB and CIB interactions are taken into account only in the HF mean field since both interactions have no contributions to the p-h matrix elements except the exchange p-h matrix element of the CIB interaction. First, a spherical HF calculation is performed for the $(Z, N=Z+2)$ nucleus (or $N=Z-2$ in the case of ^{10}C and ^{14}O) using a filling approximation for the last partially occupied orbital. Note the difference from the calculations of the previous section where the HF approximation was done for the odd-odd parent nuclei. Next, we take the ground state of the even-even nucleus (the daughter nucleus of the β decay except for ^{10}C and ^{14}O) as the RPA vacuum and calculate the excited states with $J^\pi = 0^+$ in the neighboring odd-odd nucleus. The appropriate scheme is charge-exchange RPA [12], which insures that no spurious isospin symmetry breaking is present in the $N \neq Z$ RPA ground state. This point is particularly important for our purpose and therefore we have checked it numerically. When all isospin breaking interactions are switched off in the Hamiltonian, the calculated isospin impurity of the RPA ground state is indeed less than 10^{-4}. The lowest states in the RPA spectra are identified as the isobaric analogue state (IAS) of the β decay. We adopt the filling approximation for the HF state in which the particles occupy the single-particle orbitals from the bottom of the HF potential in order and the last orbit is partially occupied according to the mass number. The HF+RPA calculations are performed by using a harmonic oscillator basis. The model space adopted is $8\hbar\omega$ for ^{10}C and ^{14}O and $10\hbar\omega$ for other nuclei. We checked the convergence of the results by extending the model space up to $12\hbar\omega$ and found that the adopted model space gives a good convergence. For example, the HF method in the harmonic oscillator basis gives the same results as those of the coordinate space calculations in Sec. II up to four-digit accuracy as far as the single-particle energies are concerned.

The quenching factor δ_c for the superallowed transition is defined as

$$|\langle J^\pi = 0^+ T = 1 : \text{daughter} | T_+ | J^\pi = 0^+ T = 1 : \text{mother} \rangle|^2 \equiv 2(1 - \delta_c). \quad (5)$$

The results with SGII are shown in Fig. 2 with and without the CSB and CIB interactions. The calculated values are also given in Tables I and II including those with SIII. Contrary to the results of the sum rule in Fig. 1, the $(0^+ \rightarrow 0^+)$ transitions are quenched substantially except for ^{10}C. This is due to the fact that the isospin mixing of the mother state enhances the sum rule, but that of the daughter state cancels this enhancement in the transition. While there is no enhancement due to the couplings to the isovector giant monopole states, the coupling to neighboring $J^\pi = 0^+$ states decreases the decay strength. This is the same mechanism as found in the previous shell model calculations [3,4]. The CSB and CIB interactions give 20–30 % larger quenching factors in all nuclei as shown in Fig. 2. The quenching factors δ_c calculated with SIII are given in Table II. It is interesting to notice that the factors δ_c are almost the same as those of SGII, although there is substantial difference in the results of the factors δ.

In Fig. 3 we compare our full results with the HF + shell model and the Woods-Saxon (WS) + shell model calculations. In general, our results are surprisingly close to those of the previous studies except for a few cases, although the models are quite different from each other. The values δ_c of our calculations are smoothly increasing as a function of Z, while the two other models show rather strong variation at the middle of the shells. A large increase of δ_c in ^{62}Ga is induced by the last occupied $2p_{1/2}$ orbit for the RPA calculations, since single-particle wave functions with more nodes induce a worse overlap between proton and neutron states.

IV. PERTURBATIVE DISCUSSION OF ISOSPIN MIXING EFFECTS

Let us discuss the relation between the enhancement of the sum rule (2) and the quenching of the Fermi transitions.

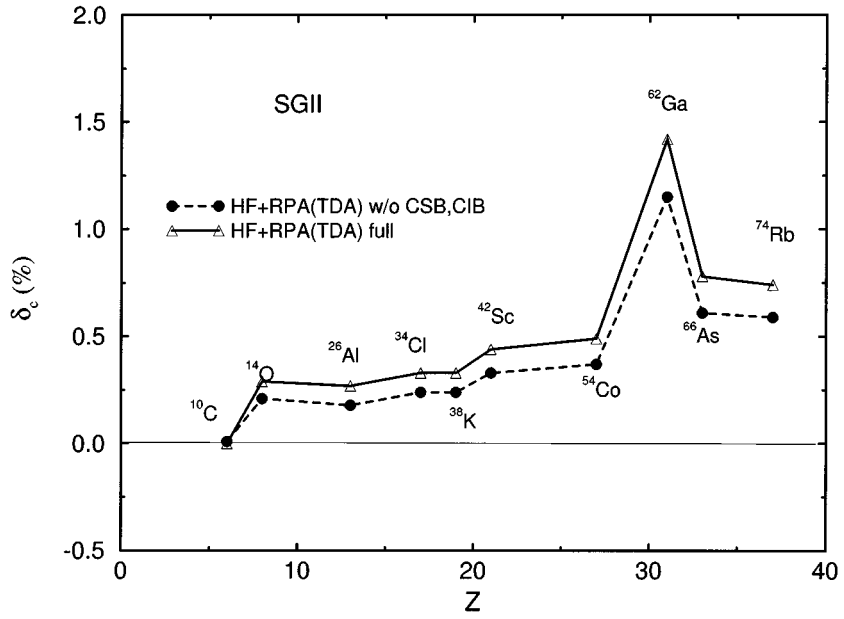

FIG. 2. Calculated quenching factor δ_c of the superallowed Fermi β decay of seven $J=0$ odd-odd nuclei. the Skyrme interaction SGII is used in the HF and RPA (or TDA) calculations. The dashed line with black circles is obtained without the CSB and CIB interactions, while the solid line with triangles is calculated with those interactions.

We will adopt a perturbative method to study the effect of isospin mixing on the Fermi transition between $J^\pi=0^+$ and $J^\pi=0^+$ states. The total Hamiltonian can be divided into two parts,

$$H=H_0+V_{Tz}^{T=1}, \tag{6}$$

where H_0 is the isospin invariant Hamiltonian and $V^{T=1}$ is the isospin nonconserving part due to the Coulomb, CSB, and CIB interactions. The parent state is an eigenstate of the full Hamiltonian,

$$H|\pi\rangle=E_\pi|\pi\rangle, \tag{7}$$

while the isospin projected state is an eigenstate of H_0:

$$H_0|TT_z\rangle=E_\pi^0|TT_z\rangle. \tag{8}$$

Let us consider the parent state to be a $J^\pi=0^+$ state of an odd-odd nucleus so that the dominant isospin is $T=1$ with $T_z=0$. The coupling to the isovector monopole giant resonance is taken into account in the calculation since the monopole state induces the isospin mixing as a first-order perturbation, while other multipoles give higher-order effects than the first order. The predominant role of the isovector monopole state in Coulomb mixing effects has been extensively studied in the past [13]. The parent state is now expressed in first-order perturbation as

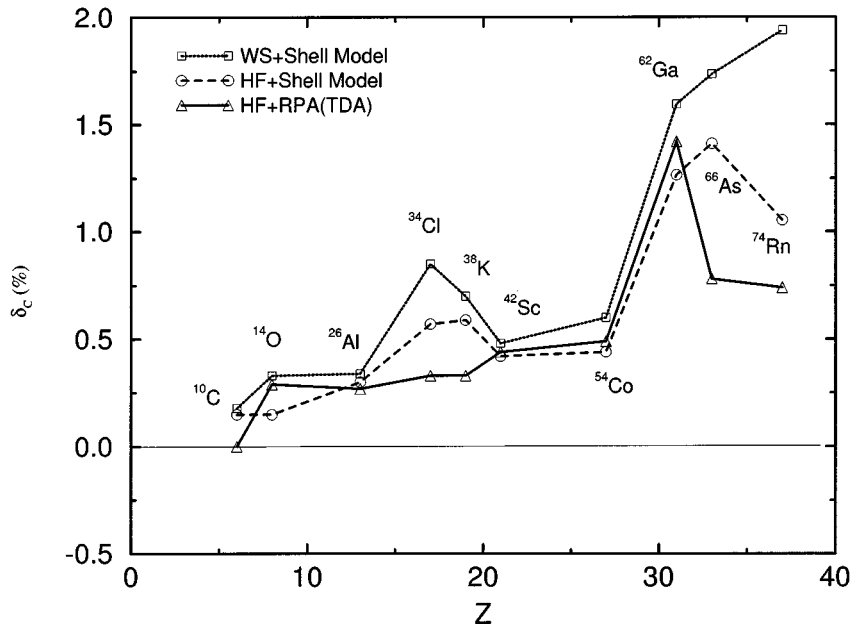

FIG. 3. Calculated quenching factor δ_c of the superallowed Fermi β decay of seven $J=0$ odd-odd nuclei. The solid line with triangles is calculated using the HF and RPA (or TDA) with the Skyrme interaction SGII plus the CSB and CIB interactions. The results of HF (Woods-Saxon) + shell model denoted by the dashed (dotted) line with circles (squares) are taken from Ref. [10].

$$|\pi\rangle=\{|T,T-1\rangle+\varepsilon_{-1}|M;T-1,T-1\rangle+\varepsilon_0|M;T,T-1\rangle + \varepsilon_{+1}|M;T+1,T-1\rangle\}\frac{1}{\sqrt{N}}, \tag{9}$$

where the perturbed state coupled to the isovector monopole state is defined as

$$|M;T+i,T_z\rangle=|\{|T,T-1\rangle\times(ph)^{T=1}\}^{T+i}_{T_z}\rangle, \tag{10}$$

and the normalization N is

$$N=1+\sum_{i=-1}^{1}\varepsilon_i^2. \tag{11}$$

The coefficients ε_i of the perturbed states $|M;T+i,T_z\rangle$ are calculated to be

$$\varepsilon_i=\frac{\langle M;T+i,T-1|V_0^1|T,T-1\rangle}{\Delta E_M^i} =\frac{\langle TT-110|T+iT-1\rangle}{\Delta E_M^i\sqrt{3}}\langle(ph)^{T=1}||V^1||0\rangle, \tag{12}$$

where ΔE_M^i is the energy difference between the parent state and the $T+i$ component of the isovector monopole state in the daughter nucleus. One can define an ''ideal'' IAS state from the parent by operating with the isospin raising operator T_+:

$$|A\rangle=\frac{T_+|\pi\rangle}{\sqrt{S}} =\frac{\{|T,T\rangle+\varepsilon_0|M;TT\rangle\}\sqrt{2T}+\varepsilon_1|T+1,T\rangle\sqrt{2(2T+1)}}{\sqrt{S}\sqrt{N}}, \tag{13}$$

where the factor S in the denominator is identical to the sum rule of Fermi transitions and is expressed as

$$S=\langle\pi|T_-T_+|\pi\rangle=2T\left(1+\varepsilon_0^2+\frac{2T+1}{T}\varepsilon_1^2\right)\frac{1}{N} \approx 2T\left(1-\varepsilon_{-1}^2+\frac{T+1}{T}\varepsilon_1^2\right). \tag{14}$$

The terms of higher order than ε^2 are neglected in the above formula. The factor δ of the sum rule (1) is now given by the coefficients ε_i,

$$\delta=-\varepsilon_{-1}^2+\frac{T+1}{T}\varepsilon_1^2. \tag{15}$$

We express the isospin dependence of the energy denominator ΔE_M^i by using an isovector potential,

$$\Delta E_M^i=\hbar\omega+\frac{V_1}{A}\mathbf{t}\cdot\mathbf{T}_c, \tag{16}$$

where $\hbar\omega$ and $\mathbf{t}(=1)$ are the excitation energy and isospin of the isovector monopole state, respectively, $\mathbf{T}_c$ is the isospin of the core, and V_1 is the symmetry potential coefficient. The amplitudes ε_i can be related to each other by calculating the isospin Clebsch-Gordan coefficients:

$$\varepsilon_0=\frac{T-1}{2T}\sqrt{2T+1}\frac{\hbar\omega+V_1T/A}{\hbar\omega-V_1/A}\varepsilon_1, \tag{17}$$

$$\varepsilon_{-1}=-\frac{1}{2T}\sqrt{(T+1)(2T-1)}\frac{\hbar\omega+V_1T/A}{\hbar\omega-V_1(T+1)/A}\varepsilon_1.$$

The relations (17) give for the factor δ a positive value:

$$\delta\simeq\left[\frac{(2T+1)(T+1)}{4T^2}-\frac{(2T-1)(T+1)}{T}\frac{V_1}{A\hbar\omega}\right]\varepsilon_1^2. \tag{18}$$

Equation (18) is valid up to order $O(\varepsilon_1^2V_1/A\hbar\omega)$ since $V_1/A\hbar\omega$ will be around 0.1 for $A=50$. Certainly, the sign of δ agrees with that obtained by the microscopic evaluations in Fig. 1. One can show that the Fermi transition rate between the parent and the ''ideal'' IAS state exhausts the sum rule value S,

$$|\langle A|T_+|\pi\rangle|^2=\frac{1}{S}|\langle\pi|T_-T_+|\pi\rangle|^2=S. \tag{19}$$

We should notice that the total Hamiltonian H does not commute with T_+ so that the state $|A\rangle$ is not an eigenstate of the full Hamiltonian. We construct a ''physical'' analog state by the same perturbative method as for the parent state in Eq. (9),

$$|``A''\rangle=\{|T,T\rangle+\tilde{\varepsilon}_0|M;T,T\rangle+\tilde{\varepsilon}_{+1}|M;T+1,T\rangle\}\frac{1}{\sqrt{N'}}, \tag{20}$$

where the coefficients $\tilde{\varepsilon}_i$ of the perturbed states $|M;T+i,T_z\rangle$ are given by

$$\tilde{\varepsilon}_i=\frac{\langle M;T+i,T|V_0^1|T,T\rangle}{\Delta\tilde{E}^{\,i}_{\,M}} =\frac{\langle TT10|T+iT\rangle}{\Delta\tilde{E}^{\,i}_{\,M}\sqrt{3}}\langle(ph)^{T=1}||V^1||0\rangle, \tag{21}$$

and the normalization N' is

$$N'=1+\sum_{i=0}^{1}\tilde{\varepsilon}_i^2. \tag{22}$$

It is reasonable to take the energy denominator $\Delta\tilde{E}^{\,i}_{\,M}$ in Eq. (21) to be the same as that of the parent state ΔE_M^i since the variation of $\hbar\omega$ and of the isovector potential is very small between mother and daughter nuclei,

$$\Delta\tilde{E}^{\,i}_{\,M}\simeq\Delta E_M^i. \tag{23}$$

The coefficients $\tilde{\varepsilon}_i$ are then related to ε_1 as

$$\tilde{\varepsilon}_0=\frac{1}{2}\sqrt{2T+1}\,\frac{\hbar\omega+V_1T/A}{\hbar\omega-V_1/A}\varepsilon_1,$$
$$\tilde{\varepsilon}_1=\frac{1}{2}\left(\frac{2T+1}{T}\right)^{1/2}\varepsilon_1. \tag{24}$$

It is a straightforward calculation to show that the normalization N' is the same as N by using these coefficients within order $O((V_1/A\hbar\omega)\varepsilon_1^2)$,

$$N\simeq N'\simeq 1+\frac{(2T+1)(T+1)}{4T}\left[1+\frac{2V_1T}{A\hbar\omega}\right]\varepsilon_1^2. \tag{25}$$

The transition probability between the states $|\pi\rangle$ and $|\text{``A''}\rangle$ is also calculated as

$$\begin{aligned}|\langle\text{``A''}|T_+|\pi\rangle|^2&=\frac{1}{NN'}\{\sqrt{2T}(1+\varepsilon_0\tilde{\varepsilon}_0)+\sqrt{2(2T+1)}\varepsilon_1\tilde{\varepsilon}_1\}^2\\&\simeq\frac{2T}{[1+(T+1)(2T+1)(1+2V_1T/A\hbar\omega)\varepsilon_1^2/4T]^2}\\&\quad\times\left\{1+\frac{(T-1)(2T+1)}{4T}\left(1+2\frac{V_1}{A}(T+1)\right)\varepsilon_1^2+\frac{2T+1}{2T}\varepsilon_1^2\right\}^2\\&\simeq 2T\left\{1-\frac{(T+1)(2T+1)}{2T}\frac{V_1}{A\hbar\omega}\varepsilon_1^2\right\}^2.\end{aligned} \tag{26}$$

Thus the quenching factor δ_c is shown to be positive and proportional to δ, but the magnitude is smaller because of the factor $V_1/A\hbar\omega$. This result is due to a strong cancellation of the isospin impurity effect in the parent state with that of the daughter state as far as the superallowed Fermi transition is concerned.

V. CVC HYPOTHESIS AND CABIBBO-KOBAYASHI-MASKAWA MIXING MATRIX

Precise measurements of superallowed β decay between nuclei with $(J^\pi=0^+,\ T=1)$ provide the most stringent probe of the electroweak interaction and have been the subject of intensive study for several decades. Since the axial current does not contribute to transitions in the lowest order, the experimental ft value is directly related to the vector coupling constant G_V:

$$ft=\frac{K}{G_V^2|M_F|^2}, \tag{27}$$

where the constant $K/(\hbar c)^6$ and the matrix element M_F are defined as

$$\begin{aligned}K/(\hbar c)^6&=2\pi^3\ln(2)\hbar/(m_ec^2)^5=(8120.271\pm0.012)\\&\quad\times10^{-10}\text{GeV}^{-4}\text{ sec},\end{aligned} \tag{28}$$

$$\begin{aligned}|M_F|^2&=|\langle J^\pi=0^+T=1\text{: daughter}|T_+|J^\pi=0^+T=1\text{:mother}\rangle|^2\\&=2(1-\delta_c).\end{aligned} \tag{29}$$

Up to now, nine ft values of $0^+\to0^+$ transitions have been reported experimentally with enough accuracy of less than 0.2% error to test the CVC hypothesis, from the lightest, ^{10}C, to the heaviest, ^{54}Co. The constancy of these values is the key issue of the prediction of the CVC hypothesis. On top of the CVC problem, the CKM mixing matrix element between u and d quarks (v_{ud}) can be determined by comparing the decay rates for muon and nuclear Fermi β decay. A test of the unitarity of the matrix, made possible by the empirical value v_{ud}, is an important measure of accuracy for the three-generation standard model.

For these purposes, nucleus-dependent corrections should be subtracted from the experimental ft values. The first one is the radiative correction to the statistical rate function f, denoted conventionally δ_R. There are also nucleus-independent radiative corrections Δ_R^V. The factor δ_R is called the ''outer'' radiative correction and Δ_R^V is the ''inner'' radiative correction including axial-vector interference terms. The second correction is the nuclear structure factor due to the isospin impurity as is discussed in Secs. III and IV. Including all these corrections, the nucleus-independent Ft value is defined as

$$Ft=ft(1+\delta_R+\Delta_R^V)(1-\delta_c), \tag{30}$$

and it is this nucleus-independent Ft which must be used on the left-hand side of Eq. (27). Then the matrix element v_{ud} is given by

$$|v_{ud}|^2=\frac{\pi^2\ln2}{Ft}\frac{\hbar^7}{G_F^2m_e^5c^4}=\frac{2984.38(6)}{Ft}, \tag{31}$$

where the Fermi coupling G_F is obtained from muon β decay.

In Table III, we list the experimental ft values, the nucleus-dependent radiative correction δ_R, the nuclear structure factor δ_c, and the Ft values. The inner radiative correction Δ_R^V is taken from Ref. [8],

$$\Delta_R^V=(2.46\pm0.09)\%, \tag{32}$$

TABLE III. Empirical ft values of superallowed Fermi transitions and nucleus-dependent corrections δ_R and δ_c. The nucleus-independent radiative corrections $\Delta_R^V=(2.46\pm0.09)\%$ are included in the Ft values in Eq. (30). The error of Δ_R^V, however, is not included in those of the Ft's. The data for ft, δ_R, and Δ_R^V are taken from Ref. [8].

		SGII		
A	ft (sec)	δ_R (%)	δ_c (%)	Ft (sec)
^{10}C	3040.1 (50)	1.30 (4)	0.00	3154.4 (53)
^{14}O	3038.1 (18)	1.26 (5)	0.29	3142.0 (24)
^{26}Al	3035.8 (17)	1.45 (2)	0.27	3146.0 (19)
^{34}Cl	3048.4 (19)	1.33 (3)	0.33	3153.5 (22)
^{38}K	3047.9 (26)	1.33 (4)	0.33	3153.0 (30)
^{42}SC	3045.1 (14)	1.47 (5)	0.44	3150.8 (21)
^{54}Co	3045.8 (11)	1.39 (7)	0.49	3147.6 (25)
			av.	3148.8 (9)

and added to obtain the Ft values. The statistical factor f adopted is slightly different from the values in Ref. [5], but this does not make any significant change for the final results.

The average value $\overline{Ft}$ of the seven data in Table III is

$$\overline{Ft}=3148.8\pm3.0 \text{ sec.} \tag{33}$$

Here, the error of Δ_R^V is also taken into account. The standard deviation σ is 3.0, which could be small enough to justify the CVC hypothesis. There is no sign of a Z dependence of the Ft values in Fig. 4, a dependence which was claimed in Ref. [5] to solve the deviation of the unitarity condition of the CKM matrix. In Ref. [8], the calculated $\overline{Ft}$ of the seven data is

$$\overline{Ft}=3148.3\pm3.0 \text{ sec,} \tag{34}$$

while Ormand and Brown [10] give

$$\overline{Ft}=3150.0\pm3.1 \text{ sec.} \tag{35}$$

The average value of our model is surprisingly close to previous calculations in which the models are essentially different in two respects, namely, the configuration space and the effect of CSB and CIB interactions on the mean field. The matrix element v_{ud} is now calculated from Eq. (31) to be

$$v_{ud}=0.9735(5). \tag{36}$$

The other two matrix elements v_{us} and v_{ub} are determined from independent experimental information on weak decays [14],

$$v_{us}=0.2205(18), \qquad v_{ub}=0.0032(9). \tag{37}$$

Finally, the sum of all three matrix elements squared becomes

$$|v_{ud}|^2+|v_{us}|^2+|v_{ub}|^2=0.9963(18), \tag{38}$$

which deviates two times more than the standard deviation $\sigma=0.0018$ from the unitarity condition. This deviation is certainly more than the ambiguity of the nuclear structure effect since our RPA model prediction differs at only the 0.05% level from that of the shell model on average and corresponds to less than one standard deviation. We calculated also the δ_c and Ft values by using another Skyrme interaction, SIII. The average $\overline{Ft}$ value obtained with SIII is 3149.3 ± 3.0 sec, which is essentially identical to that of SGII in Table III. It should be noticed that the final result in Eq. (33) relies on many small effects of the radiative corrections and the nuclear structure. The final conclusion could be easily changed by one unknown effect since the values dis-

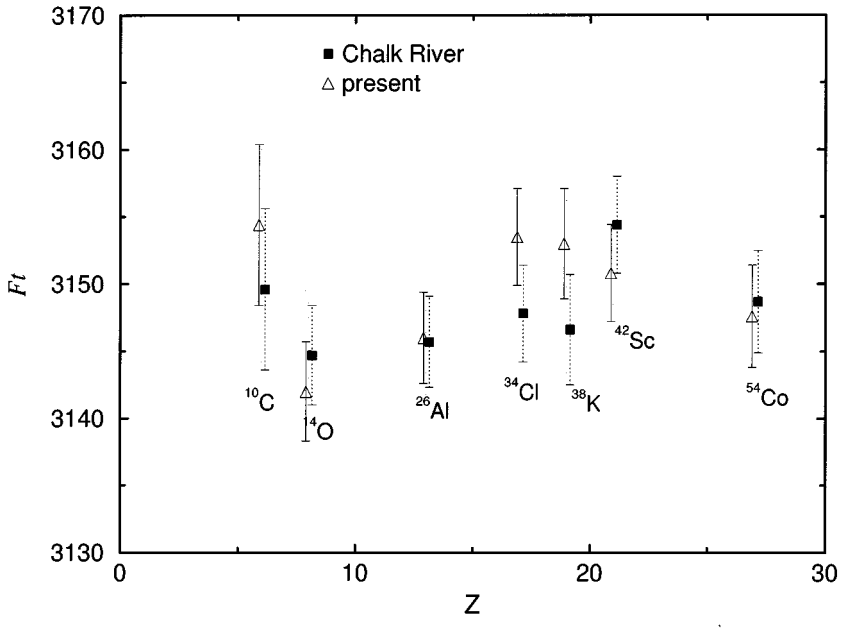

FIG. 4. Nucleus-independent Ft values in Eq. (30) of superallowed Fermi transitions. The present results are plotted by triangles, while those of Ref. [8] are given by black squares.

cussed are always at the 1% level, which would certainly be discarded in most nuclear physics studies. For example, an empirical approach has been proposed by Wilkinson [5] to fit the Ft values by a smooth Z-dependent function. Extrapolation of this function to $Z=0$ provides the value which should be used to extract the v_{ud} matrix element. This approach gives an Ft value which satisfies the unitarity condition within the statistical errors, although the physical reason for this Z dependence is not at all clear as either a nuclear structure or a radiative correction effect.

VI. SUMMARY

We have studied the effect of isospin impurity on the superallowed Fermi β decay using the HF and RPA (or TDA) model. The Skyrme force SGII is adopted for both HF and RPA calculations. The CSB and CIB interactions are also taken into account in the HF calculations for the first time. The superallowed Fermi transition probabilities of the light nuclei ^{10}C, ^{14}O, and ^{26}Al are shown to be quenched by less than 0.3%, while those of sd-shell and pf-shell nuclei are quenched by up to 1%. These calculated values are close to those obtained in the literature with HF and shell model calculations. It is interesting to notice that the shell model configuration space is $1\hbar\omega$ while, in our RPA calculations, we took into account up to $12\hbar\omega$ configurations in the harmonic oscillator basis. Another difference is the pairing, which is properly taken into account in the shell model but not in the HF+RPA calculations. We can notice in Fig. 3 that the quenching factor δ_c of the shell model is somewhat larger than those of our results in nuclei at the middle of the shells because of strong correlations in open-shell nuclei.

We estimate the so-called nucleus-independent Ft value taking into account both the nuclear structure and the radiative effects. The average Ft value is obtained as $\overline{Ft}=3148.8(30)$ sec, which is very close to the averaged value of those reported by the Towner and Hardy [8] and by Ormand and Brown [10] recently. It is shown that the Ft values are rather Z independent and show no sign of the quadratic Z dependence which is suggested by Ref. [5]. Our result shows that the average Ft value is still larger than the value required to satisfy strictly the unitarity condition of the Cabibbo-Kobayashi-Maskawa matrix within the 0.1% level, although two new effects, the core polarization effects and the CSB and CIB interactions, are taken into account for the first time in our study.

ACKNOWLEDGMENTS

We would like to thank N. Auerbach for enlightening discussions and correspondence related to Sec. IV. We acknowledge also B. A. Brown, T. Shinozuka, I. Towner, and D. H. Wilkinson for fruitful discussions.

[1] I. Hamamoto and H. Sagawa, Phys. Rev. C **48**, R960 (1993).
[2] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Threor. Phys. **49**, 652 (1973).
[3] W. E. Ormand and B. A. Brown, Phys. Rev. Lett. **62**, 866 (1989).
[4] I. S. Towner, J. C. Hardy, and M. Harvey, Nucl. Phys. **A284**, 269 (1977); J. C. Hardy, I. S. Towner, V. T. Koslowsky, E. Hagberg, and H. Schmeing, *ibid.* **A509**, 429 (1990).
[5] D. H. Wilkinson, Nucl. Instrum. Methods Phys. Res. Sect. A **335**, 201 (1993); in Proceedings of 5th WEIN conference, 1995, Osaka.
[6] H. Sagawa, Nguyen Van Giai, and T. Suzuki, Phys. Lett. B **353**, 7 (1995).
[7] G. Savard *et al.*, Phys. Rev. Lett. **74**, 1521 (1995).
[8] I. S. Towner and J. C. Hardy, in *Symmetries and Fundamental Interactions*, edited by E. M. Henley and W. C. Haxton (World Scientific, Singapore, 1995), p. 183.
[9] T. Suzuki, H. Sagawa, and Nguyen Van Giai, Phys. Rev. C **47**, R1360 (1993).
[10] W. E. Ormand and B. A. Brown, Phys. Rev. C **52**, 2455 (1995).
[11] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I, p. 171.
[12] A. M. Lane and J. Martorell, Ann. Phys. (N.Y.) **129**, 273 (1980).
[13] N. Auerbach, Phys. Rep. **98**, 273 (1983).
[14] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).