# Feeding of hole states by proton decay of Gamow-Teller and isobaric analog state resonances

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Partial decay widths corresponding to the proton decay from the Gamow-Teller and isobaric analog state resonances in $^{208}$Bi to hole states in $^{207}$Pb are evaluated. The giant resonances are described by using the random-phase approximation within the Berggren representation. A good agreement with available experimental data is obtained.

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The study of giant resonances in nuclei is attracting much attention both experimentally and theoretically [1,2]. The importance of these studies is that they may provide information about the continuum part of nuclear spectra as well as an understanding of the nuclear dynamics in the continuum. However, there is not much experimental evidence regarding partial decay widths from giant resonances—even the theoretical results are scarce. The reason for this is that the processes of formation and decay of giant resonances are time dependent and a proper treatment of such processes is a very difficult task. Experimentally, it is not easy to subtract the interesting signals from the background. Moreover, if the resonance is not isolated what one measures is not related to the resonance itself but rather to the result of the contribution of all the overlapping resonances, including their interference.

The relativistic random-phase approximation (RRPA) [3] is a natural extension of the random-phase approximation (RPA) which allows for the treatment of particle-hole states in the continuum. The eigenvalues of the RRPA can be complex due to the admixture of bound and resonant states in the unperturbed particle-hole basis (Berggren's representation). The imaginary part of a complex eigenvalue is associated with the escape width of the state [4]. The formalism has been presented in detail previously and several applications of it have been given, particularly for the description of charge-conserving multipole excitations in $^{208}$Pb [7]. As has been discussed in [4] the residues of the $S$ matrix at a complex energy $\omega_n$ are the partial decay widths of the resonance $n$ if the resonance is isolated (nonoverlapping). For such a resonance there exists a direct correspondence between the total escape width and the imaginary part of the energy, i.e., $\Gamma_n = -2\,\mathrm{Im}(\omega_n)$. This result has been confirmed for the case of the decay of the giant monopole resonance in $^{208}$Pb [7]. In the present case we shall show that similar results are also valid for charge-exchange modes, i.e., the isobaric analog (IAS) and Gamow-Teller giant resonances (GTGR's) in $^{208}$Bi.

Following the notation of [4,5] and assuming that the resonances are isolated one can write for the $S$ matrix the Breit-Wigner form

$$S_{cc'}(\mathscr{E}) = e^{i(\delta_c+\delta_{c'})}\left(\delta_{cc'} - i\sum_n \frac{\Gamma^{1/2}_{n,c}\Gamma^{1/2}_{n,c'}}{\mathscr{E}-\omega_n}\right), \qquad (1)$$

where $\Gamma^{1/2}_{n,c}$ is the partial decay amplitude from the $n$th resonance to the final channel state $(c)$. These channels are represented by the emitted particle and by the residual nuclear state. For the case of the decay of the IAS and GTGR the emitted particle is a proton and the final nuclear states are one-neutron-hole states. The partial decay amplitude can be calculated by expanding the outgoing wave of the emitted particle in partial waves and using the spectral representation of the continuum RPA Green function. Using a $\delta$ force as residual interaction and after performing the integration on the direction of the emitted particle one gets

$$\Gamma_{n,c} = \frac{2(2j_h+1)}{(2J+1)}\sum_{lj}\gamma(lj,h,n)^2, \qquad (2)$$

with

$$\gamma(lj,h,n) = \sqrt{2\pi}V_0\sum_i X(i,n)G(lj,h,i) \times\int dr r^2 u_{lj}(r)R_h(r)R_{p_i}(r)R_{n_i}(r), \qquad (3)$$

where $(lj)$ denotes the partial wave of the outgoing particle, $V_0$ is the strength of the interaction, the index $i$ labels the particle-hole configuration $\{p_i,n_i\}$ of the giant resonance with energy $\omega_n$ and total angular momentum $J$, $X(i,n)$ is the corresponding RRPA amplitude, and $h$ denotes the final one hole state. The factor $G(lj,h,i)$ is a geometrical factor given by

$$G(lj,h,i) = \sum_{LS} Z(lj,l_hj_h,LS;J)Z(l_pj_p,l_nj_n,LS,J)S_\sigma, \qquad (4)$$

where $Z(l_1j_1,l_2j_2,LS;J)$ is the recoupling coefficient:

 

TABLE I. Experimental and theoretical partial decay widths in units of keV, for the decay of the GTGR in $^{208}$Bi to neutron-hole states in $^{208}$Pb.

| Hole state | Experiment [6] | Theory |
|---|---|---|
| $p_{1/2}$ | 48.9 ± 9.3 | $29-i21$ |
| $f_{5/2}$ | incl. in $p_{1/2}$ | $46-i24$ |
| $p_{3/2}$ | 84.9 ± 13.1 | $47-i33$ |
| $i_{13/2}$ | 6.9 ± 7.7 | $0-i0$ |
| $f_{7/2}$ | 13.1 ± 6.2 | $3-i1$ |
| Total | 153.8 ± 57.2 | $125-i79$ |

TABLE II. Experimental and theoretical partial decay width in units of keV, for the decay of the IAS in $^{208}$Bi to neutron-hole states in $^{208}$Pb.

| Hole state | Experiment [6] | Theory |
|---|---|---|
| $p_{1/2}$ | 51.3 ± 5.6 | $36-i7$ |
| $f_{5/2}$ | incl. in $p_{3/2}$ | $42-i6$ |
| $p_{3/2}$ | 79.4 ± 9.4 | $36-i8$ |
| $i_{13/2}$ | | $2-i0$ |
| $f_{7/2}$ | 3.5 ± 1.6 | $2-i0$ |
| Total | 134.2 ± 35.6 | $118-i21$ |

$$Z(l_1j_1,l_2j_2,LS;J)=\sqrt{\frac{(2l_1+1)(2l_2+1)}{4\pi(2L+1)}}\langle l_10l_20|L0\rangle \times\langle(l_1\tfrac{1}{2})j_1(l_2\tfrac{1}{2})j_2;J|(l_1l_2)L(\tfrac{1}{2}\tfrac{1}{2})S;J\rangle, \quad (5)$$

and $S_\sigma=\langle S||(\sigma\sigma)||S\rangle$ for the GTGR and $S_\sigma=1$ for the IAS.

The radial wave functions for the initial proton and neutron states and for the final neutron hole are indicated by $R_p(r)$, $R_n(r)$, and $R_h(r)$, respectively, while the radial wave of the scattered particle is given by $u_{lj}(r)$. All these functions are normalized and in this representation the partial widths $\Gamma_{n,c}$ are given in units of energy.

Since the partial decay width $\Gamma_{n,c}$ is proportional to the probability that the resonance decays to the channel $c$ one expects that $u_n=\Sigma_c\Gamma_{n,c}/\Gamma_n=1$. The physical meaning of the quantity $u_n$ will be discussed in the applications below.

The scattering wave has been calculated by using a Coulomb corrected Woods-Saxon potential with a spin-orbit term; the particle-hole residual interaction is the spin-isospin dependent delta force. The strength of the residual interaction has been adjusted to reproduce the observed energy of the GTGR in $^{208}$Bi ($\omega_{\rm GTGR}$ = 15.6 MeV) and the observed position of the IAS ($\omega_{\rm IAS}$ = 15.2 MeV) [6]. Thus, for the GTGR it is $V_0$=320 MeV fm$^3$ while for the IAS it is $V_0$=260 MeV fm$^3$.

The particle(proton)-hole(neutron) basis has been constructed by using as a representation the eigenvalues of the Woods-Saxon potential of [3]. All single-particle states up to an energy of 120 MeV and narrower than 5 MeV have been included. The resulting number of single-particle states is 244 for neutrons and 185 for protons. The largest dimension allowed for the RRPA basis was 500.

The first significant quantity extracted from our RRPA calculation is the imaginary part of the complex energies. We have obtained Im($\omega_{\rm IAS}$)$=-78$ keV and Im($\omega_{\rm GTGR}$)$=-62$ keV, i.e., $\Gamma_{\rm IAS}$=156 keV and $\Gamma_{\rm GTGR}$=124 keV. This agrees well with the corresponding experimental data of Tables I and II.

With the RRPA wave functions obtained for the IAS and the GTGR we have calculated the contribution of each state to the associated sum rules [8] and found that these two states indeed exhaust most of the strength. Moreover, the sum rules are almost purely real, showing that the resonances are isolated [4]. It guarantees that the partial decay widths, calculated in this fashion, are physically meaningful. When the resonance is not an isolated one the quantity to be compared with the experiment is the branching ratio between the decay and the excitation cross sections [4,10]. When the resonance is an isolated state, the partial decay width, as defined in Eq. (2), is a quantity which can be directly compared with the experimental results [4].

Once the structure of the giant resonances is defined by the RRPA procedure we are in position to evaluate the other important ingredient entering Eq. (2), i.e., the scattering wave of the outgoing proton. Different from the case of the giant monopole resonance, where the outgoing particle is a neutron, the decay of the IAS and GTGR states proceeds through the emission of a proton. Therefore one has to calculate the Coulomb scattering wave functions well below the Coulomb barrier, where the regular Coulomb function $F_l$ is practically zero and the irregular solutions $G_l$, $H_l^+$, and $H_l^-$ coincide within the machine precision. Therefore we were not able to calculate the scattered proton wave function accurately at energies less than 100 keV. The Coulomb wave functions were calculated by the code COULCC [9].

We have calculated the partial decay width for the decay of the IAS and GTGR in $^{208}$Bi leading to neutron-hole states in $^{207}$Pb. The results are listed in Tables I and II. Considering the difficulties associated with these calculations, one can say that the agreement between theory and experiment is rather good. One notices that for the GTGR the sum of the partial decay widths practically coincides with twice the absolute value of the imaginary part of the GTGR energy, i.e., $u\approx1$. This is another indication of the consistency of the theory: The GTGR resonance is isolated. Yet, the imaginary parts of the partial decay widths in Table I are rather large, i.e., $|{\rm Im}(\Gamma)/{\rm Re}(\Gamma)|$= 0.63. We have looked into this feature in detail and found that the imaginary parts of the partial decay widths are sensitive to variations of the interaction strength. For instance, for $V_0$=280 MeV fm$^3$ one obtains $\omega_{\rm GTGR}$=14.83 MeV and $|{\rm Im}(\Gamma)/{\rm Re}(\Gamma)|$= 0.30, still maintaining the consistency condition $u\approx1$. We have traced down this sensitivity and found that it is due to the Coulomb waves, which feel the difference in energy between the two cases (i.e., 0.77 MeV) very strongly.

For the case of the IAS in Table II the imaginary parts of the partial decay widths are (in absolute value) small, but $u$ is not unity. Again in this case we found that by changing the strength of the interaction within reasonable limits the consistency criterion improves. For instance, for $V_0$=235 MeV fm$^3$ it is $u$=1 while $\omega_{\rm IAS}$=(13.80 $-i$ 0.076) MeV. This variation of the energy causes, as before, strong changes in the Coulomb wave.

One feature to be noticed in Tables I and II is that the agreement between theory and experiment is good although no effects induced by two-particle–two-hole excitations have been introduced in our formalism. As mentioned above, this is expected since the resonance is isolated. However, in Refs. [11,12] the contribution of the particle-hole excitations to the partial decay widths was found to be larger, and the agreement with experiment is not as good as here. This may be due to precision problems related to the calculation of the Coulomb scattering wave, but it may also be due to the use in Refs. [11,12] of a Hartree-Fock mean field, which is nonlocal, instead of the local Woods-Saxon potential used here.

In conclusion, in this paper we have calculated partial decay widths corresponding to the decay of the Gamow-Teller and IAS giant resonances in $^{208}$Bi. We found that these resonances, contrary to the case of the isoscalar monopole giant resonance in $^{208}$Pb [7], are isolated, and therefore the concept of partial decay width is meaningful. This is supported by the good agreement between our calculated quantities and the available experimental data.

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