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Δ decay in the nuclear medium

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Proton-nucleus collisions, where the beam proton gets excited to the Δ resonance and then decays to $p\pi^+$, either inside or outside the nuclear medium, are studied. Cross sections for various kinematics for the $(p,p'\pi^+)$ reaction between 500 MeV and 1 GeV beam energy are calculated to see the effects of the nuclear medium on the propagation and decay of the resonance. The cross sections studied include proton energy spectra in coincidence with the pion, four momentum transfer distributions, and the invariant $p\pi^+$ mass distributions. We find that the effect of the nuclear medium on these cross sections mainly reduces their magnitudes. Comparing these cross sections with those considering the decay of the Δ outside the nucleus only, we further find that at 500 MeV the two sets of cross sections have large differences, while by 1 GeV the differences between them become much smaller.

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I. INTRODUCTION

In the description of nuclear reactions at intermediate energy, in addition to nucleons and pions, Δ play an important role. Because of this, there has been great interest over the years in the study of the Δ-nucleus interaction [1]. Experimentally, since the Δ is a spin-isospin excitation of the nucleon, it is readily excited in charge-exchange reactions such as (p,n) and $(^3\mathrm{He},t)$. Measurements of inclusive spectra on ejectiles in these reactions show broad bumps with large cross sections around 300 MeV excitation in all targets ranging from ^{12}C to ^{208}Pb [2]. These excitations correspond to the excitation of a nucleon in the target to a Δ isobar. They are, therefore, capable of exploring Δ-nucleon-hole (ΔN^{-1}) excitations in nuclei and thereby the collective aspects of these modes.

Another class of intermediate energy reactions, such as (p,Δ^{++}), is those where the Δ appears as one of the final reaction products. In these reactions the measurements can be done directly on the Δ or its decay products. They are, therefore, well suited to investigate Δ dynamics in the continuum and the transition interaction $pp\rightarrow n\Delta^{++}$. The presence of a Δ in these reactions can be inferred either by measuring the recoiling nucleus, as in the pioneering experiment on the $^6\mathrm{Li}(p,\Delta^{++})\,^6\mathrm{He}$ reaction at Saturne [3], or the ejectile nucleus on proton targets, as in the experiments on $p(^3\mathrm{He},t)\Delta^{++}$ and $p(^{12}\mathrm{C},{}^{12}\mathrm{B})\Delta^{++}$ reactions carried out at Saturne and Dubna [2,4]. These kind of experiments, however, can only be performed on few nuclei as the ejectile nucleus is required to be stable or sufficiently long lived. Theoretical analyses of these reactions, which consider the Δ as a stable elementary particle, show that this reaction proceeds in one step and the measured cross sections can be adequately described in the framework of the distorted-wave Born approximation (DWBA) [5]. It has also been found that, due to the very large momentum transfer ($\sim$ beam momentum) involved in the excitation of the bound nucleon in the target to the Δ in the continuum in the final state, the "target excitation" in the (p,Δ^{++}) contributes little [6].

Yet another way to study the (p,Δ^{++}) reaction is to detect the Δ directly by measuring its decay products p and π^+. This procedure has the great advantage that the experiment can be performed on any target nucleus. Furthermore, by choosing a different energy of the outgoing Δ we can vary the energy transfer to the nucleus, and thereby explore its spin-isospin response at different excitations. Experiments on $(p,p'\pi^+)$ are now being done at TRIUMF with the availability of dual magnetic spectrometers [7]. For analyzing these reactions, it is necessary to develop a formalism which, unlike earlier work, incorporates the unstable nature of the Δ and includes the effect of the nucleus on its propagation and on the decay products p' and π^+. In this work we present such a formalism. Then using it, we calculate various cross sections which can be measured on the $(p,p'\pi^+)$ reaction. We study the effects on these reactions of the nuclear distortion of the Δ. We compare these cross sections with those calculated with the Δ decaying outside the nucleus only. This comparison determines the region of applicability of the latter model, where the calculations can be done with much ease and where the Δ-nucleus interaction can be explored without the complicating effects due to the pπ^+ interaction with the nucleus.

The reaction mechanism which we follow for the $(p,p'\pi^+)$ reaction is shown in Fig. 1. Accordingly, the incoming proton interacts with a nucleon at some point **r** and is converted into a Δ. The target nucleon, to provide one unit of spin and isospin to the beam proton, undergoes a spin and isospin flip. It also gets accelerated by a momentum

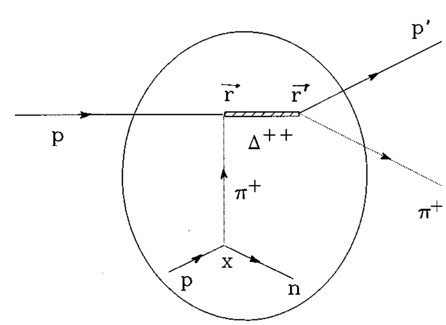

FIG. 1. Projectile excitation diagram for the $(p,p'\pi^+)$ reaction.

corresponding to the excitation energy ($\approx$300 MeV) of the Δ. The Δ then propagates from point $\mathbf{r}$ to $\mathbf{r}'$, where it decays into a proton and pion. The decay point $\mathbf{r}'$ may lie inside or outside the nucleus. During this propagation, the Δ interacts with the nuclear medium and suffers distortion. The incoming proton and the decay products p' and π^+ get distorted by the nuclear medium before reaching the point $\mathbf{r}$ and in propagating out of the point $\mathbf{r}'$, respectively. In our formalism, we include all these distortions through appropriate optical potentials. The formalism itself is written following Gottfried and Julius [8], who, among several other workers [9], have studied the effect of the nuclear medium on the decay of a ρ meson.

For the elementary process $pp\rightarrow n\Delta^{++}$ in the above mechanism we have used the one-pion-exchange potential. We have not included the contribution from the ρ exchange for this excitation. The reason for this omission is that attempts to include ρ exchange in the $pp\rightarrow n\Delta^{++}$ reaction have yielded very unsatisfactory results. A detailed study by Jain and Santra [10] and earlier work by Dmitriev *et al.* [11] both showed that the experimental data on the spin-averaged cross sections for this reaction are better reproduced by the one-pion-exchange interaction alone. The inclusion of rho exchange worsens this agreement. This means that either the strength of the $\rho N\Delta$ coupling, $f_{\rho N\Delta}$, is considerably weaker than what is usually assumed or some additional amplitude tends to cancel the contribution from the rho. A recent theoretical study on the microscopic analysis of the $\rho N\Delta$ vertex due to Haider and Lui [12], in fact, suggests that the value of $f_{\rho N\Delta}$ is much smaller.

In Sec. II we present the formalism in detail. Sections III and IV contain results, discussion, and the conclusions. Since the contribution of the Δ decay inside the nucleus depends upon the beam energy and the size of the nucleus, we have done calculations at 500 MeV and 1 GeV for ^{12}C and ^{208}Pb.

In general, our findings are that (i) around 1 GeV and for lighter target nuclei the calculated cross sections do not differ greatly from those which consider the decay of the Δ only outside the nucleus, and (ii) the Δ-nucleus interaction mainly reduces the magnitude of the cross sections.

II. FORMALISM

The differential cross section for the $A(p,p'\pi^+)B$ reaction is given by

$$d\sigma=[\mathrm{PS}]\langle|T_{fi}|^2\rangle, \tag{1}$$

where [PS] is the factor associated with phase space and the beam current,

$$[\mathrm{PS}]=\frac{1}{j(2\pi)^5}\frac{m_\Delta m^2}{E_\pi E_p E_{p'}}\delta^4(P_i-P_f)d\mathbf{k}_{p'}d\mathbf{k}_\pi d\mathbf{K}_B. \tag{2}$$

Here j is the beam current and m is the mass of the proton. P_x denotes the four-momentum.

In the center-of-mass system, j is given by

$$j=\frac{p_c\sqrt{s}}{E_pE_A}, \tag{3}$$

where $\sqrt{s}$ is the available energy in the center-of-mass system and p_c is the c.m. momentum in the initial state.

T_{fi} in Eq. (1) is the transition amplitude. The angular brackets around its square denote the appropriate sum and average over the final and initial spins, respectively.

A. Transition amplitude T_{fi}

For the diagram in Fig. 1, the transition amplitude can be written as

$$T_{fi}=\int d\mathbf{r}d\mathbf{r}'\chi_{p'}(\mathbf{r}')^{-*}\chi_\pi^{-*}(\mathbf{r}')\Gamma_{\Delta N\pi}G_\Delta(\mathbf{r}',\mathbf{r})\psi_\Delta(\mathbf{r}), \tag{4}$$

where $\Gamma_{\Delta N\pi}$ is the decay operator for the Δ decaying into $p\pi^+$. In momentum space and in a nonrelativistic static approximation, it is given by

$$\Gamma_{\Delta N\pi}=\frac{f_\pi^*}{m_\pi}\mathbf{S}^\dagger\cdot\boldsymbol{\kappa}_\pi\mathbf{T}^\dagger\cdot\boldsymbol{\phi}_\pi, \tag{5}$$

where $\mathbf{S}$ and $\mathbf{T}$ are the spin and isospin transition operators, respectively, for $\frac{1}{2}\rightarrow\frac{3}{2}$. κ is the pion momentum in the Δ rest frame. Because the final pion is on shell (if we neglect the effect of distortions on it), the above form for $\Gamma_{\Delta N\pi}$ does not contain the usual form factor F^* .

$G_\Delta(\mathbf{r}',\mathbf{r})$ is the Δ propagator. It satisfies the inhomogeneous wave equation

$$[\nabla^2+E_p^2-m_\Delta^2+i\Gamma_\Delta m_\Delta-\Pi_\Delta(\mathbf{r})]G_\Delta(\mathbf{r}',\mathbf{r})=\delta(\mathbf{r}'-\mathbf{r}), \tag{6}$$

where m_Δ (=1232 MeV) and Γ_Δ are the resonance parameters associated with the free Δ. The width of the free Δ, Γ_Δ, depends upon the invariant mass according to

$$\Gamma(\mu)=\Gamma_0\left[\frac{k(\mu^2,m_\pi^2)}{k(m_\Delta^2,m_\pi^2)}\right]^3\frac{k^2(m_\Delta^2,m_\pi^2)+\gamma^2}{k^2(\mu^2,m_\pi^2)+\gamma^2}, \tag{7}$$

with Γ_0=120 MeV and γ=200 MeV. μ is the invariant mass given by

$$\mu^2=(E_{p'}+E_\pi)^2-(\mathbf{k}_{p'}+\mathbf{k}_\pi)^2. \tag{8}$$

In Eq. (7), k, for an on-shell pion, is given by

$$k(\mu^2,m_\pi^2)=[(\mu^2+m^2-m_\pi^2)^2/4\mu^2-m^2]^{1/2}. \tag{9}$$

This relation reflects the restrictions on the available phase space for the decay of a delta of mass μ into an on-shell pion of mass m_π (=140 MeV).

Π_Δ in the Green's function, Eq. (6), describes the collisions of the Δ with the medium. In the mean field approximation, it is related to the Δ optical potential V_Δ through

$$\Pi_\Delta=2E_\Delta V_\Delta. \tag{10}$$

One of the important channels which contribute significantly to this potential is $\Delta N\rightarrow NN$.

$\psi_\Delta(\mathbf{r})$ in Eq. (4) is the amplitude for the production of the Δ at a point $\mathbf{r}$. It is given by

$$\psi_\Delta(\mathbf{r})=\int d\boldsymbol{\xi}\psi^*_\beta(\xi)\Gamma_{\pi NN}\psi_\alpha(\boldsymbol{\xi})G_\pi(\mathbf{r},\mathbf{x})\Gamma_{\pi N\Delta}\chi^+_p(\mathbf{r}). \quad (11)$$

Here $\Gamma_{\pi N\Delta}$ is the operator for the excitation of the beam proton to the Δ, and $\Gamma_{\pi NN}$ is the interaction operator at the πNN vertex in the nucleus. Like $\Gamma_{\Delta N\pi}$, their forms are

$$\Gamma_{\pi N\Delta}=i\frac{f^*_\pi F^*(t)}{m_\pi}\mathbf{S}\cdot\mathbf{q}\mathbf{T}\cdot\boldsymbol{\phi}_\pi \quad (12)$$

and

$$\Gamma_{\pi NN}=i\frac{f_\pi F}{m_\pi}\sigma\cdot\mathbf{q}\tau\cdot\boldsymbol{\phi}_\pi, \quad (13)$$

with f^*_π and f_π the coupling constants for the $\pi N\Delta$ and πNN vertices. Their values are 2.156 and 1.008, respectively. F^* and F are the form factors associated with these vertices. They incorporate the off-shell extrapolation of the pion-nucleon coupling vertex. t is the four momentum transfer (squared). However, for the "projectile excitation" diagram, if we ignore the nuclear recoil, it is the same as the three-momentum $\mathbf{q}$ squared.

χ_p in Eq. (4) is the distorted wave for the beam proton. In this paper we will consider beam energies above 500 MeV. We use the eikonal approximation for this and the other continuum particles (viz., Δ, p', and π^+). This implies that the main effect of the nuclear medium on these waves is absorptive, and the dominant momentum components in them are their asymptotic momenta. Because of this, and also because the interaction $V_{NN\to N\Delta}$ in the region of t of the present studies is known to depend weakly on the momentum transfer q, [5] the evaluation of $\psi_\Delta(\mathbf{r})$ simplifies. In the case of closed shell nuclei we find

$$\psi_\Delta(\mathbf{r})\approx\frac{\langle\Gamma_{\pi NN}(\mathbf{q})\rangle}{(m^2_\pi-t)}\Gamma_{\pi N\Delta}\rho_{\beta\alpha}(\mathbf{r})\chi^+_p(\mathbf{r}). \quad (14)$$

Here the angular brackets around $\Gamma_{\pi NN}$ represent its expectation value over the nuclear spin-isospin wave functions. $\rho_{\beta\alpha}(\mathbf{r})$ is the spatial part of the nuclear transition density. The momentum transfer $\mathbf{q}$ is given in terms of the asymptotic momenta of the continuum particles as

$$\mathbf{q}=\mathbf{k}_p-\mathbf{k}_\Delta(=\mathbf{k}_{p'}+\mathbf{k}_\pi). \quad (15)$$

Substituting ψ_Δ from Eq. (14) in Eq. (4), we get

$$T_{fi}=\frac{\langle\Gamma_{pn\pi^+}\rangle}{q^2+m^2_\pi}\langle\Gamma_{pp'\pi^+}\rangle F_{fi}(\mathbf{k}_{p'},\mathbf{k}_\pi;\mathbf{k}_p), \quad (16)$$

where

$$F_{fi}=\int d\mathbf{r}d\mathbf{r}'\chi^{-}_{p'}{}^*(\mathbf{r}')\chi^{-}_{\pi}{}^*(\mathbf{r}')G_\Delta(\mathbf{r}',\mathbf{r})\rho_{\beta\alpha}(\mathbf{r})\chi^+_p(\mathbf{r}) \quad (17)$$

and

$$\langle\Gamma_{pp'\pi^+}\rangle=\left[\frac{f^*_\pi}{m_\pi}\right]^2F^*(t)\langle\sigma_{p'}|\mathbf{S}^\dagger\cdot\boldsymbol{\kappa}_\pi\mathbf{T}^\dagger\cdot\phi_\pi\mathbf{S}\cdot\mathbf{q}\mathbf{T}\cdot\phi_\pi|\sigma_p\rangle. \quad (18)$$

$\langle|T_{fi}|^2\rangle$ is given by

$$\langle|T_{fi}|^2\rangle=\frac{|\langle\Gamma_{pp'\pi^+}\rangle|^2|\langle\Gamma_{pn\pi^+}\rangle|^2}{(m^2_\pi-t)^2}|F_{fi}(\mathbf{k}_{p'},\mathbf{k}_\pi;\mathbf{k}_p)|^2, \quad (19)$$

where $|\langle\Gamma_{pp'\pi^+}\rangle|^2$ is given by

$$|\langle\Gamma_{pp'\pi^+}\rangle|^2=\frac{1}{2}\Sigma_{\sigma_p\sigma_{p'}}|\Gamma_{pp'\pi^+}|^2$$
$$=\frac{1}{9}\left[\frac{f^*}{m_\pi}\right]^4F^{*2}(t)[4|\kappa_\pi\cdot\mathbf{q}|^2+|\mathbf{q}\times\kappa_\pi|^2]. \quad (20)$$

In the last evaluation we have used the identity

$$\mathbf{S}^\dagger\cdot\mathbf{q}\mathbf{S}\cdot\boldsymbol{\kappa}_\pi=\frac{2}{3}\kappa_\pi\cdot\mathbf{q}-\frac{i}{3}\boldsymbol{\sigma}\cdot(\mathbf{q}\times\kappa_\pi). \quad (21)$$

B. Evaluation of $F_{fi}(\mathbf{k}_{p'},\mathbf{k}_\pi;\mathbf{k}_p)$

To evaluate

$$F_{fi}=\int d\mathbf{r}d\mathbf{r}'\chi^{-}_{p'}{}^*(\mathbf{r}')\chi^{-}_{\pi}{}^*(\mathbf{r}')G_\Delta(\mathbf{r}',\mathbf{r})\rho_{\beta\alpha}(\mathbf{r})\chi^+_p(\mathbf{r}), \quad (22)$$

we define, for convenience, a function

$$G_\Delta(\mathbf{r};\mathbf{k}_\Delta,\mu)=\int d\mathbf{r}'\chi^{-}_{p'}{}^*(\mathbf{r}')\chi^{-}_{\pi}{}^*(\mathbf{r}')G_\Delta(\mathbf{r}',\mathbf{r}). \quad (23)$$

This function physically gives the probability amplitude for finding a proton and a pion in the detector with the total momentum

$$\mathbf{k}_\Delta=\mathbf{k}_{p'}+\mathbf{k}_\pi \quad (24)$$

and the invariant mass μ if the Δ is produced at a point $\mathbf{r}$ in the nucleus. The wave equation for this new function can be obtained from Eq. (6) by multiplying it on both sides by $\chi^{-}_{p'}{}^*\chi^{-}_{\pi}{}^*$ and integrating over $\mathbf{r}'$. This gives

$$[\nabla^2+E^2_p-m^2_\Delta+i\Gamma_\Delta m_\Delta-\Pi_\Delta(\mathbf{r})]G_\Delta(\mathbf{r};\mathbf{k}_\Delta,\mu)$$
$$=\chi^{-}_{p'}{}^*(\mathbf{r})\chi^{-}_{\pi}{}^*(\mathbf{r}). \quad (25)$$

Equation (22) for F_{fi} then becomes

$$F_{fi}=\int d\mathbf{r}G_\Delta(\mathbf{r};\mathbf{k}_\Delta,\mu)\rho_{\beta\alpha}(\mathbf{r})\chi^+_p(\mathbf{r}). \quad (26)$$

To proceed further, as mentioned earlier, we treat all the continuum waves in the eikonal approximation. We write

$$\chi_p(\mathbf{r})=e^{i\mathbf{k}_p\cdot\mathbf{r}}D_{\mathbf{k}_p}(\mathbf{r}) \quad (27)$$

and

$$G_\Delta(\mathbf{r};\mathbf{k}_\Delta,\mu)=e^{-i\mathbf{k}_\Delta\cdot\mathbf{r}}\Phi(\mathbf{r};\mathbf{k}_\Delta,\mu), \quad (28)$$

where the distortion functions D and Φ are slowly varying functions of z, the coordinate taken along the beam momentum $\mathbf{k}_p$. We have evaluated these distortion functions along $\mathbf{k}_p$,

$$D_{\mathbf{k}_p}(\mathbf{r})=\exp\left[\frac{-i}{\hbar v_p}\int_{-\infty}^{z}dz' V_p(\mathbf{b},z')\right], \tag{29}$$

where V_p is the optical potential for the beam proton. The function Φ, whose dependence on $\mathbf{k}_\Delta$ and μ will henceforth be suppressed for brevity, is obtained from Eq. (25). With the eikonal approximation this simplifies to

$$\left[G_0^{-1}(\mu)-\Pi_\Delta-2ik_\Delta\frac{\partial}{\partial z}\right]\Phi(b,z)=D_{p'}(b,z)D_\pi(b,z), \tag{30}$$

where

$$G_0(\mu)=[\mu^2-m_\Delta^2+i\Gamma_\Delta m_\Delta]^{-1} \tag{31}$$

is the propagator for the free Δ. The solution of Eq. (30) gives

$$\Phi(\mathbf{b},z)=\frac{1}{2ik_\Delta}\int_z^\infty \Phi_\Delta(\mathbf{b};z,z')f(\mathbf{b},z')dz', \tag{32}$$

where

$$\Phi_\Delta(\mathbf{b};z,z')=\exp\left[\frac{i}{2k_\Delta}(\mu^2-m_\Delta^2+i\Gamma_\Delta m_\Delta)(z'-z)\right]\exp\left[\frac{-i}{v_\Delta}\int_z^{z'}V_\Delta(\mathbf{b},z'')dz''\right] \tag{33}$$

and

$$f(\mathbf{b},z)=\exp\left[-i\int_{z'}^{\infty}\left(\frac{V_{p'}(\mathbf{b},z'')}{v_{p'}}+\frac{V_\pi(\mathbf{b},z'')}{v_\pi}\right)dz''\right]. \tag{34}$$

Here, as we see, Φ_Δ describes the propagation of the Δ from z to its decay point z'. $f(\mathbf{b},z')$ describes the same for the decay products p' and π^+ of the Δ from z' to the detectors. Equation (30) for $\Phi(\mathbf{b},z)$ thus gives the probability amplitude for detecting $p'\pi^+$ in the detector when the Δ is distorted by the medium from its production point z to its decay point z' and the decay products p$'$ and π^+ are distorted from z' to the detectors.

The Δ dynamics implicit in $\Phi(\mathbf{b},z)$ becomes more transparent if we consider a nucleus with distorting potentials having a sharp surface. For a radius R of this surface, we can split the "distorted" Δ propagator Φ into two parts, i.e.,

$$\Phi(\mathbf{b},z)=\Phi_{\text{in}}(\mathbf{b},z)+\Phi_{\text{out}}(\mathbf{b},z), \tag{35}$$

where

$$\Phi_{\text{in}}(\mathbf{b},z)=\frac{1}{2ik_\Delta}\int_z^{\sqrt{(R^2-\mathbf{b}^2)}}dz'\Phi_\Delta(\mathbf{b};z,z')f(\mathbf{b},z') \tag{36}$$

and

$$\begin{aligned}\Phi_{\text{out}}(\mathbf{b},z)&=\frac{1}{2ik_\Delta}\int_{\sqrt{(R^2-\mathbf{b}^2)}}^{\infty}dz'\Phi_\Delta(\mathbf{b};z,z')f(\mathbf{b},z')\\&=\frac{1}{2ik_\Delta}\int_{\sqrt{(R^2-\mathbf{b}^2)}}^{\infty}dz'\Phi_\Delta(\mathbf{b};z,z')\end{aligned} \tag{37}$$

These two functions, as we see, describe, respectively, the contributions to the cross section from the decay of the Δ inside and outside the nuclear medium. The relative contribution of these two regions, as seen from Eq. (33) for $\Phi_\Delta(\mathbf{b};z,z')$, is determined by the intrinsic decay length

$$\lambda_{\text{in}}=\frac{k_\Delta}{m_\Delta\Gamma_\Delta}=\tau v_\Delta \tag{38}$$

of the Δ. Because of this factor, the Δ, in traveling a distance, $L(\mathbf{b},z)[=\sqrt{(R^2-\mathbf{b}^2)}-z]$ from its production point z to the nuclear surface, gets attenuated by a factor $\exp[-L(\mathbf{b},z)/\tau v_\Delta]$. This attenuation, as is obvious, decreases with an increase in the delta (hence the beam) momentum and its lifetime. Consequently, the ratio $\Phi_{\text{in}}/\Phi_{\text{out}}\to 0$ as the beam momentum $k_p\to\infty$ and/or $\tau\to\infty$.

In case there is no distortion by the nuclear medium, Φ can be shown to be independent of $(\mathbf{b}$,z) and reduces to the free Δ propagator, i.e.,

$$\Phi(\mathbf{b},z)=[\mu^2-m_\Delta^2+i\Gamma_\Delta m_\Delta]^{-1}\equiv G_0. \tag{39}$$

The effect of the nuclear field on the Δ and its decay products, as is obvious from here, results in the modification of this mass distribution and, through it, the modification of other experimental observables associated with p' and π^+.

For no distortions, Eq. (17) for the transition amplitude integral F_{fi} simplifies to

$$F_{fi}=\rho_{\beta\alpha}(\mathbf{q})G_0, \tag{40}$$

where

$$\rho_{\beta\alpha}(\mathbf{q})=\int d\mathbf{r}e^{i\mathbf{q}\cdot\mathbf{r}}\rho_{\beta\alpha}(\mathbf{r}) \tag{41}$$

is the nuclear transition density in momentum space. This expression demonstrates that, apart from G_0 (or its nuclear modified version), F_{fi} and the cross sections are determined by the nuclear transition density.

III. RESULTS AND DISCUSSION

The results presented here are motivated by two aims: (i) to see how these results compare with those in a model which considers the decay of the Δ only outside the nucleus and (ii) to see the extent to which the Δ distortions affect the cross sections. Since the outcome of both these investigations depends on the speed of the Δ in the nucleus and the distance it travels through the nucleus, we present calculations for 500 MeV and 1 GeV beam energies and for ^{12}C and ^{208}Pb target nuclei. The specific cross sections which we calculate are the four-momentum transfer distributions, invariant mass spectra of $p\pi^+$, and the proton energy spectra in coincident p and π^+ measurements.

A. Input quantities

The calculations require the following quantities as input: (i) the pion-nucleon coupling constants and form factors at the vertices, (ii) the resonance parameters of the Δ, (iii) the nuclear transition density $\rho_{\beta\alpha}$, and (iv) the optical potentials for protons, the pion, and the Δ.

The pion coupling and the Δ resonance parameters have already been given in Sec. II. These parameters reproduce the scattering data on the $p\pi^+ \rightarrow p\pi^+$ [13] reaction. For the form factors F and F^*, we have used the monopole form

$$F(t)=\frac{\Lambda^2-m_\pi^2}{\Lambda^2-t}, \tag{42}$$

with $\Lambda = 1.2$ GeV/c for both vertices.

For nuclear transition densities we have used forms which reproduce electron scattering data. Since in the present paper our emphasis is not on the investigation of nuclear structure effects on the $(p,p'\pi^+)$ cross sections, this choice of transition densities should be adequate. For ^{12}C we take

$$\rho_{\beta\alpha}(r)=\rho_0[1+a(r/b)^2]\exp(-r^2/b^2), \tag{43}$$

and for ^{208}Pb we choose

$$\rho_{\beta\alpha}(r)=\frac{\rho_0}{1+\exp[(r-c)/d]}, \tag{44}$$

where ρ_0 is fixed by the appropriate normalization of the density. Other parameters are taken from the electron scattering analyses compilations by Jackson and Barrett and De Jager [14]. They are

$$a=1.247, \quad b=1.649 \text{ fm},$$

$$c=6.54 \text{ fm}, \quad d=0.5 \text{ fm}.$$

For optical potentials, since we expect their effect to be mostly absorptive at intermediate energies [15], we use only their imaginary parts. For pions we fix them using the method of Ericson and Hüfner [16], which is valid around the Δ resonance. In this method, the strength of the optical potential is obtained from the refractive index n of the pion in the nuclear medium, where

$$n(E)=\frac{K(E)}{k(E)} \tag{45}$$

and

$$W(E)=-\frac{k^2}{E}n_I(E). \tag{46}$$

Here n_I is the imaginary part of the refractive index and K is the pion wave number in the nuclear medium. The latter is obtained by solving the dispersion relation

$$K^2=k^2+4\pi\rho f_{\pi N}(K,E), \tag{47}$$

where $f_{\pi N}$ is the π-N scattering amplitude in the nuclear medium in the forward direction. Considering that the pion scattering is dominated by p-wave scattering, the expression for π^+ scattering from a nucleus with N neutrons and Z protons is

$$f_{\pi^+N}=\frac{1}{A}(Nf_{\pi^+n}+Zf_{\pi^+p})\approx\left[\frac{N+3Z}{3A}\right]f_{33}. \tag{48}$$

A Breit-Wigner resonant form for the amplitude f_{33} gives

$$n_I(E)=\frac{X\Gamma/4}{(E-E_R+\frac{3}{4}X)^2+\frac{1}{4}\Gamma^2}, \tag{49}$$

with

$$X=4\pi n_0 c \tag{50}$$

and

$$c=-\left[\frac{N+3Z}{3A}\right]\frac{58(\text{MeV})a^3}{1+(ka)^2}. \tag{51}$$

Here n_0 is the nuclear density, and $a = 1.24$ fm.

The dispersion relation given in Eq. (47) is equivalent to an optical potential approach, if the latter is defined through the folding of the π-N t matrix with the nuclear density. Tandy *et al.* [17] have shown that such optical potentials contain nucleon knockout as the primary reactive content. Therefore, the main contribution to $n_I(E)$ in Eq. (49) is the nucleon knockout (π^+,π^+N) channel. However, in addition to this, the pion flux can also be lost through real absorption of the pion in the nuclear medium. The dominant channel which contributes to such an absorption is $\Delta N \rightarrow NN$. We have approximated this contribution to the pion optical potential by

$$W_{\text{abs}}=g\Gamma_s/2, \tag{52}$$

where Γ_s is the spreading width. We take $\Gamma_s = 70$ MeV from the Δ-hole model of pion absorption [18]. The factor g is added to account for the fact that the $(p,p'\pi^+)$ reaction is a peripheral reaction. This factor represents the fraction of the central nuclear density in the region where this reaction takes

TABLE I. Optical potentials for pion and Δ.

T_Δ (MeV)	$-W_\Delta$ (MeV)	T_π (MeV)	$-W_\pi$ (MeV)
<100	45	40	25
120	32	100	67
200	35	150	88
350	45	200	90
450	48	250	57
650	51	300	38
800	52	350	33
-	-	400	28
-	-	600	26

place. We have chosen $g=0.7$. The resulting pion optical potentials are listed in Table I.

For protons, the distorting potentials beyond 300 MeV are obtained using the high energy ansatz

$$W=\frac{1}{2}v\sigma_T^{pN}n_0, \tag{53}$$

where σ_T^{pN} is the total proton-nucleon cross section at the proton speed v in their c.m. The values for the total cross section are taken from the experimental data on proton-nucleon scattering [19]. The radial shapes $\rho(r)/\rho(0)$ of the potentials are taken to be the same as those given in Eqs. (43), (44), with $n_0=0.17$ fm $^{-3}$. Below 300 MeV the potentials are taken from those available in the literature from the analyses of elastic scattering data of protons on ^{12}C at various energies [20]. The radial shape for these potentials is the two-parameter Woods-Saxon form. The optical potentials for ^{208}Pb are taken of the same form except that the radius parameter for it is enhanced in proportion to $A^{1/3}$.

On delta optical potentials, not much information exists. For $T_\Delta \leqslant 100$ MeV we make recourse to the Δ-hole model of Hirata *et al.* [18] and take $W_\Delta=-45$ MeV. For higher energies, as for protons, we make the high energy ansatz as given in Eq. (54). Here, however, for obtaining $\sigma_T^{\Delta N}$ first we write it as a sum of the elastic and the reactive parts, $\sigma_T^{\Delta N}=\sigma_{\rm el}^{\Delta N}+\sigma_r^{\Delta N}$. Then assuming that the spin-averaged elastic dynamics of the proton and Δ are not very different, we assume $\sigma_{\rm el}^{\Delta N}\approx\sigma_{\rm el}^{NN}$. For the reactive part, since up to about 1.5 GeV the main reactive channel in ΔN scattering is $\Delta N \rightarrow NN$, using the reciprocity theorem we write

$$\sigma_r^{\Delta N}\approx\sigma^{\Delta N\rightarrow NN}=\frac{1}{2}\frac{k_{NN}^2}{2k_{\Delta N}^2}\sigma(pp\rightarrow n\Delta^{++}), \tag{54}$$

where k_{NN} is the proton c.m. momentum corresponding to the same energy as is available in the ΔN c.m. An extra factor 1/2 is introduced to account for the identity of two particles in the final state. Using the experimentally known cross sections for the $pp\rightarrow n\Delta^{++}$ reaction [21], the resulting Δ optical potentials at representative energies are also listed in Table I.

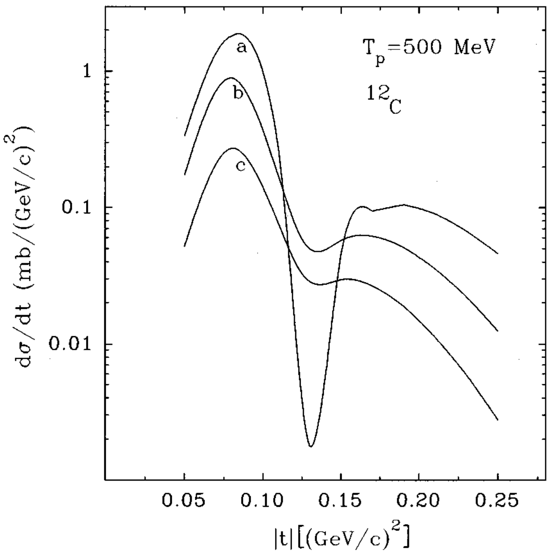

FIG. 2. Four-momentum transfer distribution at 500 MeV beam energy for ^{12}C. Curve a, no distortion for any continuum wave (PW results). Curve b includes distortions for the beam and Δ. Curve c includes distortions for beam, Δ, p', and π^+.

B. Calculated cross-sections

In Figs. 2 and 3 we show the four-momentum transfer distributions for ^{12}C at 500 MeV and 1 GeV beam energies. Theoretically, the cross sections are calculated by integrating Eq. (59) over the Δ mass between 1120 and 1300 MeV. Experimentally, these cross sections can be obtained either by measuring the four-momentum of the recoil nucleus or the four-momenta of the decay products p' and π^+ of the Δ. The results in Figs. 2 and 3 have three curves. Curve a corresponds to a situation where we assume no nuclear distortion of continuum particles. These are the plane wave (PW) results. Curve b includes the distortion of the beam proton and the propagating Δ. This curve, therefore, corresponds to a situation when the Δ is assumed to decay

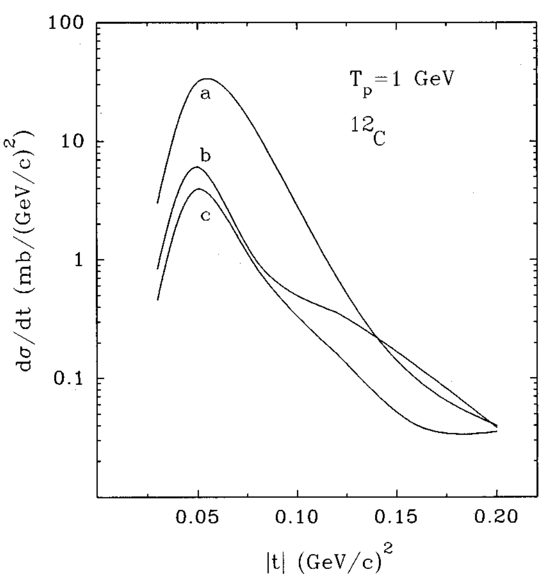

FIG. 3. Four-momentum transfer distribution at 1 GeV beam energy for ^{12}C. The description of various curves is the same as in Fig. 2.

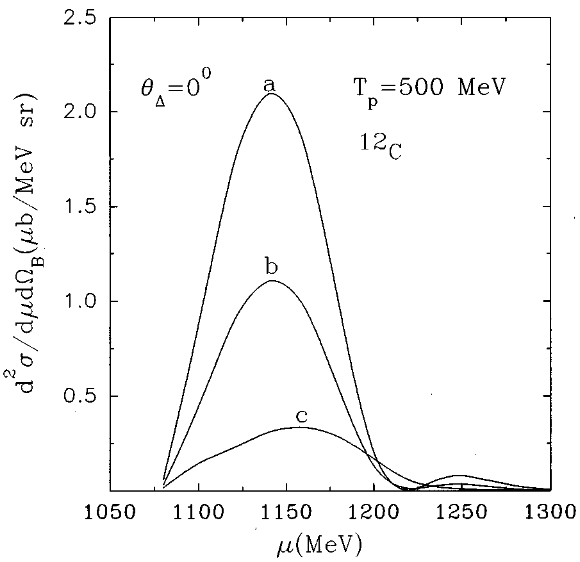

FIG. 4. Invariant mass distribution at 500 MeV beam energy for ^{12}C. $\theta_\Delta=0°$. The description of various curves is the same as in Fig. 2.

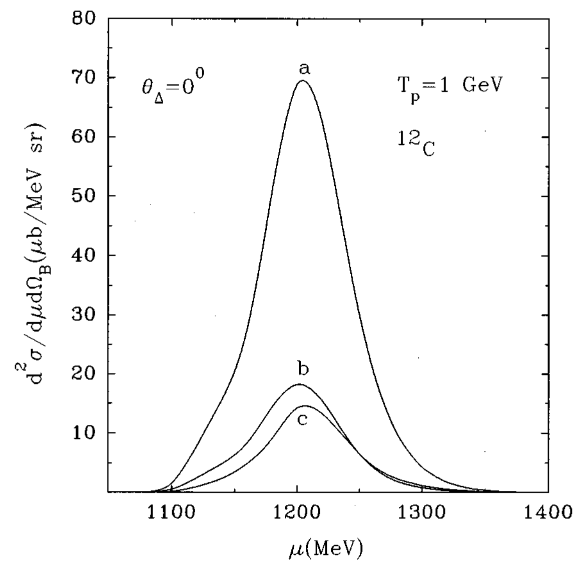

FIG. 5. Invariant mass distribution at 1 GeV beam energy for ^{12}C. $\theta_\Delta=0°$. The description of various curves is the same as in Fig. 2.

always outside the nucleus. Curve c includes, in addition, the nuclear distortion of p' and π^+. From these curves it is evident that $d\sigma/dt$, at both energies, can get modified in a major way by nuclear distortions. However, at 1 GeV the main modifications in the PW curves arise due to proton and delta distortions; at 500 MeV distortion of the outgoing proton and pion is equally important. We also notice that at higher energy the main effect of the distortions is the reduction in the magnitude of the cross sections. At lower energy distortions also fill in the minima in the $d\sigma/dt$ distributions. Quantitatively, the peak cross section at 1 GeV gets reduced by a factor of about 6 due to the beam proton and Δ distortions and by an additional factor of 3/2 due to p' and π^+ distortions. At 500 MeV the corresponding factors are about 2 and 3, respectively.

In Figs. 4 and 5 we display the calculated invariant mass (μ) distributions for the decay products p' and π^+ of the Δ. The results again are given for 500 MeV and 1 GeV beam energies and the Δ going in the forward direction. For an isolated Δ these distributions should peak around 1230 MeV. In our results, all the distributions are shifted towards a lower mass. At 1 GeV the peak appears around 1200 MeV, while at 500 MeV it appears around 1150 MeV. Since this shift appears in the PW results also, it is caused primarily by the nuclear transition density $\rho_{\beta\alpha}$. The distortion of the continuum particles controls the magnitude of the cross sections. The quantitative reductions in the peak cross sections at both energies are similar to those seen earlier in the four-momentum transfer distributions. This again means that at 1 GeV the additional effect of the p' and π^+ distortion is not much, while at 500 MeV it is of the same magnitude as that due to the beam and Δ distortions.

Since the inclusive measurements normally tend to have a large background, it is sometimes preferable to make exclusive measurements, though the cross sections in these measurements are smaller. In Figs. 6 and 7 we show the calculated proton energy spectra for the coincident measurements of the decay products p' and π^+. As an example, the results are given for the two particles being produced at 10° on either side of the beam direction. In these results we again find that at 1 GeV the dominant effect arises due to the distortion of the beam and delta. The distortion of p' and π^+ has a small effect. At 500 MeV, on the other hand, the distortions of all the continuum particles have equal effect.

To see the dependence of the above results on nuclear size we have calculated distributions for a ^{208}Pb target. Since for this nucleus we find that the calculated cross sections are very small at 500 MeV beam energy, only 1 GeV results are shown. In Figs. 8 and 9 we display the calculated invariant

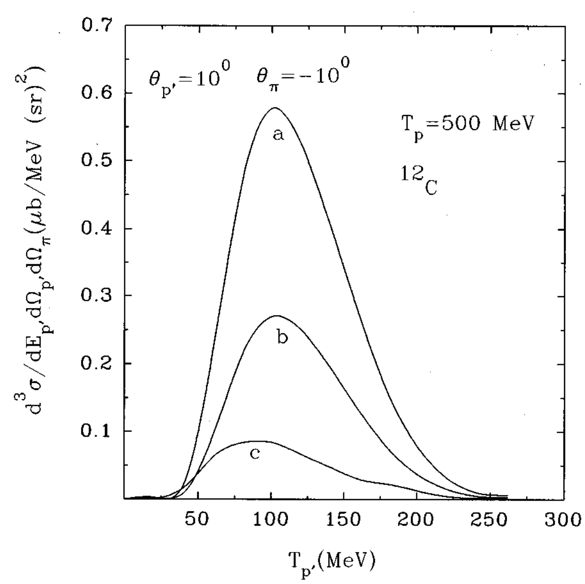

FIG. 6. The outgoing proton energy distribution in the $(p,p'\pi^+)$ reaction for a coplanar geometry, $\theta_{p'}=10°$ and $\theta_\pi=-10°$. The beam energy is 500 MeV and the target is ^{12}C. The description of various curves is the same as in Fig. 2.

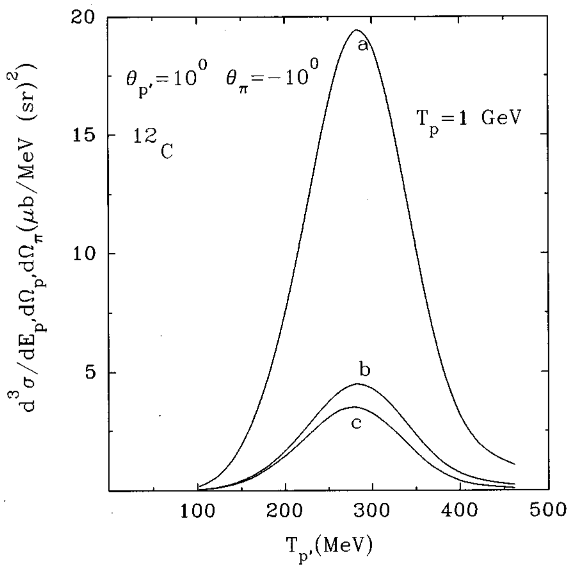

FIG. 7. The outgoing proton energy distribution in the $(p,p'\pi^+)$ reaction for a coplanar geometry, $\theta_{p'}=10°$ and $\theta_\pi=-10°$. The beam energy is 1 GeV and the target is ^{12}C. The description of various curves is the same as in Fig. 2.

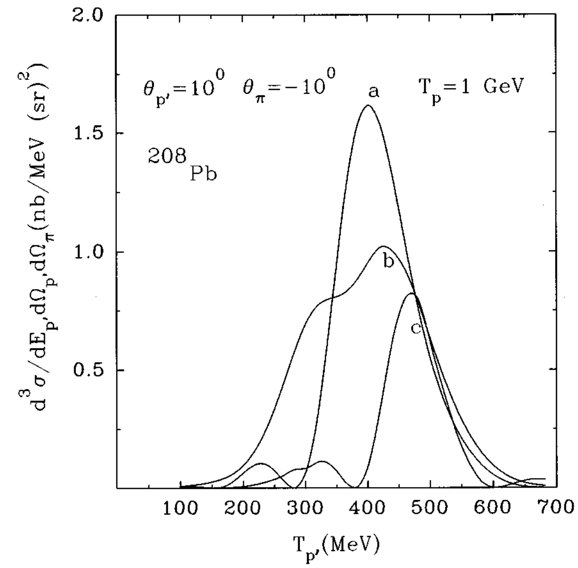

FIG. 9. The outgoing proton energy distribution in the $(p,p'\pi^+)$ reaction for a co-planar geometry, $\theta_{p'}=10°$ and $\theta_\pi=-10°$. The beam energy is 1 GeV and the target is ^{208}Pb. The description of various curves is the same as in Fig. 2, except that the cross sections in curves *a* and *b* are shown after dividing them by factors of 100 and 10, respectively.

mass distribution and the outgoing proton energy spectrum for the coincident p$'$ and π^+ measurements. Unlike the ^{12}C results, here we find that, even at 1 GeV, the effect of p' and π^+ distortion is as important as that of the beam proton and Δ. Furthermore, the distortions here not only reduce the magnitude of the cross sections, they also change their shapes. The plane wave invariant mass distribution, which previously had three peaks, has only one peak when all the distortions are included.

Finally, in Figs. 10 and 11 we present some results to isolate the effect on the measured cross sections due to the Δ-nucleus interaction alone, and also to see the effect of the real part of its potential on the cross sections. As an illustration we show $d\sigma/d\mu$ for ^{12}C at 500 MeV and 1 GeV. The results are shown for three situations. Curves *a* include no delta distortion, *b* include only the imaginary part W_Δ, of V_Δ, and *c* includes both real and imaginary parts of V_Δ. The beam, p', and π^+ distortions are included in all the curves. However, unlike earlier curves, these distortions now include

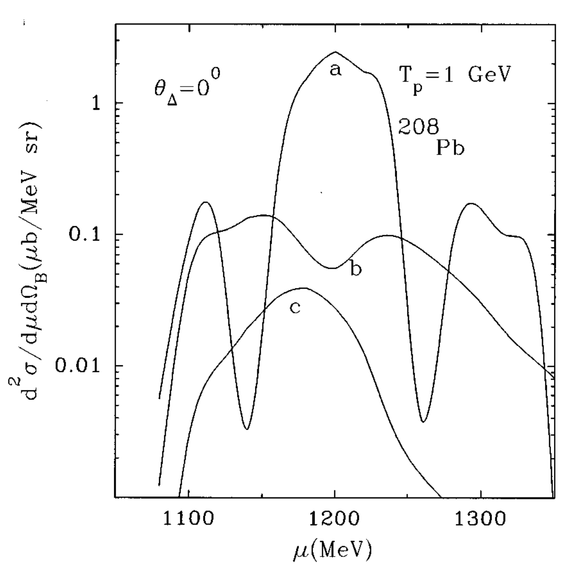

FIG. 8. Invariant mass distribution at 1 GeV beam energy for ^{208}Pb. $\theta_\Delta=0°$. The description of various curves is the same as in Fig. 2.

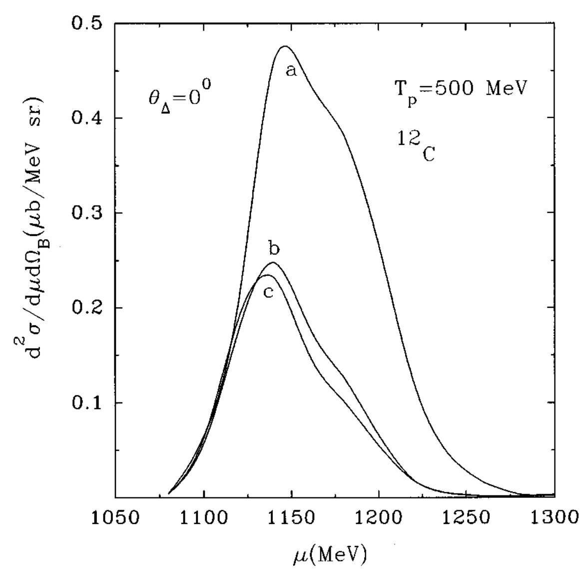

FIG. 10. Sensitivity of the invariant mass distribution at 500 MeV beam energy to the distortion of the Δ in the nuclear medium. The target nucleus is ^{12}C and $\theta_\Delta=0°$. Curve *a*, no Δ distortion. Curve *b*, Δ distortion with only the imaginary part W_Δ of its potential. Curve *c*, Δ distortion with the imaginary as well as real part of its optical potential. All curves include the distortion (including the real part of their potentials) of beam, p', and π^+.

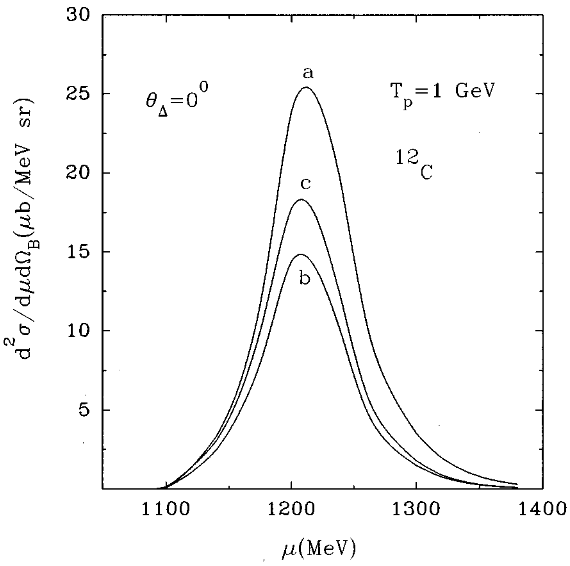

FIG. 11. Sensitivity of the invariant mass distribution at 1 GeV beam energy to the distortion of the Δ in the nuclear medium. The target nucleus is ^{12}C and $\theta_\Delta = 0°$. The description of different curves is the same as in Fig. 10.

the effect of their real potentials (U) also. From the comparison of various curves in these figures, we find that the effect of the Δ-nuclear collisions does get reflected in the final results to a significant extent. At 1 GeV, the term W_Δ suppresses the peak cross section by about 30%, while at 500 MeV this suppression factor is 50%. The effect of U_Δ on the cross sections, however, is not much. At 1 GeV its effect on the magnitude of the cross sections is within 10%, while at 500 MeV it is insignificant. Inclusion of U_Δ also does not lead to any perceptible shift in the peak position of the invariant mass distributions.

The real parts of the optical potentials for protons and deltas in the above figures are fixed using the same procedure as given earlier for their imaginary parts. For pions, like the imaginary part, they are obtained through the real part of its refractive index, i.e.,

$$\frac{U_\pi}{E} = 1 - n_r^2, \tag{55}$$

where

$$n_r(E) = 1 - \frac{\frac{1}{2}X(E - E_R + \frac{3}{4}X)}{(E - E_R + \frac{3}{4}X)^2 + \frac{1}{4}\Gamma^2}. \tag{56}$$

IV. CONCLUSIONS

In conclusion, for the $(p,p'\pi^+)$ reaction, proceeding through a Δ excitation in the intermediate state, (i) around 1 GeV beam energy and in lighter target nuclei (like ^{12}C), the cross sections obtained in a model, which considers the Δ decay only outside the nuclear medium, are not significantly different from those obtained in a model which also includes the Δ decay inside the nuclear medium. At lower energies (∼500 MeV) and/or for heavier nuclei (like ^{208}Pb) the situation is different. The calculated cross sections in two models can differ significantly in magnitude as well as shape.

(ii) The distortion of the Δ by the nuclear medium yields a suppression of the magnitude of the measured cross sections. The peak position of the invariant mass and other distributions remain uneffected by it.

(iii) The peak positions in the invariant mass and other distributions are determined by the range of momentum transfer involved and the nuclear transition density. Compared to an isolated delta, the peak positions in the μ distributions get shifted towards lower mass. At 1 GeV this shift is small ($\mu_{\mathrm{peak}} - 1232 \approx 30$ MeV), while at 500 MeV it is large ($\mu_{\mathrm{peak}} - 1232 \approx 80$ MeV).

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APPENDIX

The phase space factor [PS] in Eq. (2) can be calculated for different kinematic settings. In the c.m. system, for the energy distribution of the outgoing protons, it is

$$[\mathrm{PS}] = \frac{1}{(2\pi)^5}\frac{m_\Delta m^2 E_A E_B}{\sqrt{s}}\frac{k_{p'}k_\pi^3}{p_c}\frac{1}{k_\pi^2(\sqrt{s} - E_{p'}) + E_\pi(\mathbf{k}_{p'}\cdot\mathbf{k}_\pi)}dE_{p'}d\Omega_{p'}d\Omega_\pi \tag{A1}$$

For the mass distribution of the Δ it is

$$[\mathrm{PS}] = \frac{\mu}{(2\pi)^5}\frac{m_\Delta m^2 E_A E_B}{s}\frac{K_B k_\pi^3}{p_c}\frac{1}{k_\pi^2(\sqrt{s} - E_B) + E_\pi(\mathbf{K}_B\cdot\mathbf{k}_\pi)}d\mu d\Omega_B d\Omega_\pi, \tag{A2}$$

and for the four momentum transfer distribution it is given by

$$[\mathrm{PS}] = \frac{\mu}{2(2\pi)^5}\frac{m_\Delta m^2 E_A E_B}{s}\frac{k_\pi^3}{p_c^2}\frac{1}{k_\pi^2(\sqrt{s} - E_B) + E_\pi(\mathbf{K}_B\cdot\mathbf{k}_\pi)}d\mu dt d\phi_B d\Omega_\pi. \tag{A3}$$

[1] See articles in *Delta Excitation in Nuclei*, edited by H. Toki *et al.* (World Scientific, Singapore, 1994); A. B. Migdal, E. E. Sapirstein, M. A. Troitsky, and D. N. Voskresensky, Phys. Rep. **192**, 179 (1990); G. E. Brown, *ibid.* **163**, 167 (1988); B. K. Jain and A. B. Santra, *ibid.* **230**, 1 (1993); A. B. Migdal, Rev. Mod. Phys. **50**, 107 (1978); C. Gaarde, Annu. Rev. Nucl. Sci. **41**, 187 (1991); T. Udagawa, S.-W. Hong, and F. Osterfeld, Phys. Lett. B **245**, 1 (1990); P. Oltmanns, F. Osterfeld, and T. Udagawa, *ibid.* **299**, 194 (1993); T. Udagawa, P. Oltmanns, F. Osterfeld, and S. W. Hong, Phys. Rev. C **49**, 3162 (1994); C. Guet, M. Soyeur, J. Bowlin, and G. E. Brown, Nucl. Phys. **A494**, 558 (1989); P. Fernandez de Cordoba, E. Oset, and M. J. Vicente-Vacas, Nucl. Phys. **A592**, 472 (1995); H. J. Morsch *et al.*, Phys. Rev. Lett. **69**, 1336 (1992); T. Hennino *et al.*, Phys. Lett. B **283**, 42 (1992); **303**, 236 (1993); J. Chiba *et al.*, Phys. Rev. Lett. **67**, 1982 (1991).

[2] B. E. Bonner, Phys. Rev. C **18**, 1418 (1978); C. Ellegaard *et al.*, Phys. Rev. Lett. **50**, 1745 (1983); C. Ellegaard *et al.*, Phys. Lett. B **154**, 110 (1985); D. Contardo *et al.*, *ibid.* **168**, 331 (1986); I. Bergqvist *et al.*, Nucl. Phys. **A469**, 648 (1987).

[3] T. Hennino *et al.*, Phys. Rev. Lett. **48**, 997 (1982).

[4] V. G. Ableev *et al.*, Pis'ma Zh. Éksp. Teor. Fiz. **40**, 35 (1984) [JETP Lett. **40**, 763 (1984)].

[5] B. K. Jain, Phys. Rev. Lett. **50**, 815 (1983); Phys. Rev. C **29**, 1396 (1984).

[6] B. K. Jain, J. T. Londergan, and G. E. Walker Phys. Rev. C **37**, 1564 (1988).

[7] S. Yen *et al.*, TRIUMF-Research Proposal No. 636, 1994.

[8] K. Gottfried and D. I. Julius, Phys. Rev. D **1**, 140 (1970).

[9] G. Chapline, Phys. Rev. D **1**, 949 (1970); P. Osland and D. Treleani, Nucl. Phys. **B107**, 493 (1976).

[10] B. K. Jain and A. B. Santra, Phys. Lett. B **244**, 5 (1990); Nucl. Phys. **A519**, 697 (1990).

[11] V. F. Dmitriev, O. Sushkov, and C. Gaarde, Nucl. Phys. **A459**, 503 (1986).

[12] Q. Haider and L. C. Liu, Phys. Lett. B **335**, 253 (1994).

[13] D. V. Bugg, A. A. Carter, and J. R. Carter, Phys. Lett. **44B**, 278 (1973); O. Dumbrajs *et al.*, Nucl. Phys. **B216**, 277 (1983); V. Flamino, W. G. Moorhead, D. R. O. Morrison, and N. Rivoire, CERN Report No. CERN-HERA 83-01, 1983.

[14] D. F. Jackson and R. C. Barrett, *Nuclear Sizes and Structure* (Clarendon, Oxford, 1977); C. W. De Jager, H. De Vries, and C. De Vries, At. Data Nucl. Data Tables **14**, 479 (1974).

[15] N. G. Kelkar and B. K. Jain, Phys. Rev. C **46**, 845 (1992).

[16] T. E. O. Ericson and J. Hüfner, Phys. Lett. **33B**, 601 (1970).

[17] P. C. Tandy, E. F. Redish, and D. Bolle, Phys. Rev. Lett. **35**, 921 (1975); Phys. Rev. C **16**, 1924 (1977).

[18] M. Hirata, F. Lenz, and K. Yazaki, Ann. Phys. (N.Y.) **108**, 116 (1977).

[19] D. V. Bugg *et al.*, Phys. Rev. **146**, 900 (1966); S. Barshay, C. B. Dover, and J. P. Vary, Phys. Rev. C **11**, 360 (1975).

[20] I. Abdul Jalil and D. F. Jackson, J. Phys. **5**, 1699 (1979); R. M. Haybron and H. McManus, Phys. Rev. B **136**, 1730 (1964); P. G. Roos and N. S. Wall, Phys. Rev. **140**, B1237 (1965); K. Seth, Nucl. Phys. **A138**, 61 (1969).

[21] F. Shimizu *et al.*, Nucl. Phys. **A386**, 571 (1982); **A389**, 445 (1982).