Expansion, thermalization, and entropy production in high-energy nuclear collisions

H. Heiselberg
NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Xin-Nian Wang
Nuclear Science Division, MS 70A-3307, Lawrence Berkeley Laboratory,
University of California, Berkeley, California 94720

The thermalization process is studied in an expanding parton gas using the Boltzmann equation with two types of collision terms. In the relaxation time approximation we determine the criteria under which a time-dependent relaxation time leads to thermalization of the partons. We calculate the entropy production due to collisions for the general time-dependent relaxation time. In a perturbative QCD approach on the other hand, we can, given the initial conditions, estimate the effective relaxation time due to elastic collisions; this will be an upper limit only since radiative processes will also contribute to thermalization. We find that the parton gas does thermalize eventually but only after having undergone a phase of free streaming and gradual equilibration where considerable entropy is produced ("after burning"). The final entropy and thus particle density depends on the collision time as well as the initial conditions (a "memory effect"). Results for entropy production are presented based upon various model estimates of early parton production.

I. INTRODUCTION

Hard and semihard parton scatterings are expected to be the dominant processes in ultrarelativistic heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) and Large Hadron Collider (LHC) energies [1,2]. These hard or semihard processes happen in a very short time scale after which a dense parton gas will be formed. However, this parton gas is not immediately in thermal and chemical equilibrium. Secondary interactions among the produced partons may lead to equilibrium if the interactions are sufficiently strong. Exactly how this parton gas equilibrate and what are the time scales are under intense investigations [3–6] and that is also the focus of this paper.

It is commonly assumed in relativistic heavy ion collisions that the matter expands hydrodynamically shortly after the nuclei collide and that it is in thermal equilibrium locally in space and time. In the classic Bjorken model [7] hydrodynamic expansion is assumed after a time $\tau_0 \simeq 1$ fm/c. Yet, when initial parton formation times are estimated to be a fraction of a fm/c [3,5,8] the expansion might be much more rapid than the typical collision time of the partons produced in the collisions. For times shorter than the typical collision time of partons immediately after the initial collisions, the rapid expansion is closer to free streaming than hydrodynamic expansion. Only at times much larger than the characteristic collision time may the parton gas thermalize, equilibrate and expand hydrodynamically [9]. However, if the collision time increases with time the gas may never thermalize. The characteristic collision time is therefore a crucial parameter. Since it among other things depends on the density, which decreases in time, it may be time dependent. In fact, when the collision time is proportional to the expansion time, the system neither expands hydrodynamically nor as free streaming at large times but somewhere in between [10].

In this paper we study the thermalization process of a parton gas expanding in one dimension using the Boltzmann equation with two types of collision terms. In the relaxation time approximation we show generally for which time-dependent relaxation times the partons thermalize and estimate when they do so and how much entropy is produced in the collisions. Secondly, we describe the elastic parton scatterings by perturbative QCD in order to estimate the important collision time and its dependence on expansion time. Inelastic collisions will not be included but are expected to contribute to thermal and chemical equilibration thus leading to earlier relaxation. The effective "out of equilibrium" collision time differ from the standard transport relaxation time, $\tau_{\text{tr}} \simeq [\alpha_s^2 \ln(1/\alpha_s) T]^{-1}$, by a prefactor and a weak time dependence, which are calculated in the Appendix. It is, however, still the Debye screening and Landau damping that screen the singular forward parton scattering processes. Finally, we will give results for entropy production based upon various model estimates of the pre-equilibrium conditions.

II. FREE STREAMING VS HYDRODYNAMIC EXPANSION

In order to have a tractable approach we assume that the spatial variations are sufficiently small along the longitudinal (z) direction so that we can describe the parton gas by the Boltzmann equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v_p} \cdot \nabla_{\mathbf{r}} \right) f = \left(\frac{\partial f}{\partial t} \right)_{\text{coll}}. \tag{1}$$

We also assume along the lines of the Bjorken model [7] that the transverse dimension of the parton gas formed in the nuclear collisions is sufficiently large that the initial expansion is one dimensional. Furthermore, we assume the central

rapidity regime is boost invariant in rapidity, i.e., depending only on the invariant time $\tau = \sqrt{t^2 - z^2}$. The Boltzmann equation thus reduces to [9]

$$\left(\frac{\partial f(p_\perp, p_z, \tau)}{\partial \tau}\right)\Bigg|_{p_z \tau} = \left(\frac{\partial f(p_\perp, p_z, \tau)}{\partial \tau}\right)_{\text{coll}}. \qquad (2)$$

The collision term on the right-hand side of Eqs. (1,2) determines the equilibration and has been studied extensively within perturbative QCD [11,12,14–17] near equilibrium in a quark-gluon plasma. In a relaxation time approximation, the collision term is

$$\left(\frac{\partial f(p_\perp, p_z, \tau)}{\partial \tau}\right)\Bigg|_{p_z \tau} = -\frac{f - f_{\text{eq}}}{\theta}, \qquad (3)$$

where θ is the relaxation time, which determines the time scale for thermalization and f_{eq} is the equilibrium distribution function.

A. The hydrodynamic limit

When collisions are sufficient to thermalize the system, i.e., the corresponding relaxation times are short as compared to expansion times, $\theta \ll \tau$, hydrodynamics applies and the distribution function in the local comoving frame is the thermal one

$$f_{\text{eq}}(\mathbf{p}) = \left[\exp\left(\frac{E_p - \mu}{T}\right) \pm 1\right]^{-1}. \qquad (4)$$

If particle production is sufficiently rapid, chemical equilibrium can also be reached and $\mu \simeq 0$ for gluons. We assume that the net baryon density is relatively low in the midrapidity region so that the quark chemical potential vanishes as well. In the Bjorken flow model [7] the expansion is idealized in one dimension and assumed to be rapidity independent. Including viscous dissipation one finds generally in hydrodynamics [11–13]

$$\frac{d\epsilon}{d\tau} + \frac{\epsilon + P}{\tau} = \frac{\frac{4}{3}\eta + \xi}{\tau^2}, \qquad (5)$$

where ϵ is the energy density, P the pressure, η and ξ the shear and bulk viscosities, respectively. We assume that parton gas is a weakly interacting quark-gluon plasma (QGP) consisting of relativistic quarks and gluons. Consequently, the gas is described by an ideal equation of state with pressure $P = \epsilon/3$ where the energy density is $\epsilon = aT^4$; $a = 8\pi^2/15 (1 + (21/32)N_f)$ ($a = 15.6$ for $N_f = f$). A bag pressure can be included but will not change the entropy production that we are concerned with here.

In the relaxation time approximation the shear viscosity is $\eta = \theta \epsilon$. In a weakly interacting QGP the temperature is the only scale besides factors of the interaction strengths, e.g., the Debye wave number is $q_D \sim gT$. Thus the shear viscosity necessarily scales like

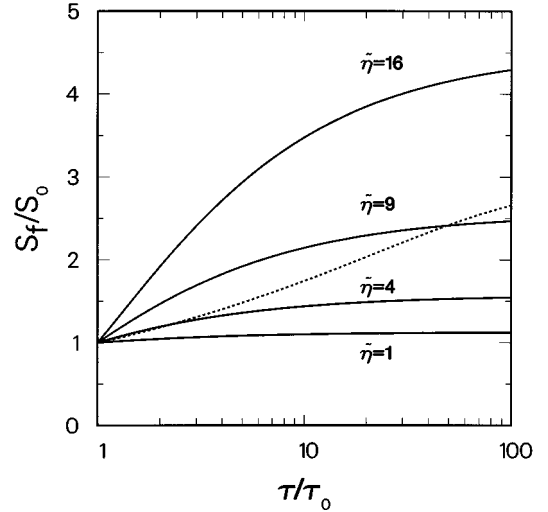$$\eta = \tilde{\eta} T^3, \qquad (6)$$

FIG. 1. Total entropy in one-dimensional viscous hydrodynamical expansion of Eq. (7) for various viscosities $\eta = \tilde{\eta} T^3$. For comparison, the upper limit on the entropy by solving the Boltzmann equation, Eq. (35), is shown with dashed curve for $\theta = \tau_\eta$ in the case $\tilde{\eta} = 16$. In both cases $T_0 = \hbar/\tau_0$ is assumed.

where $\tilde{\eta}$ is a dimensionless constant depending on the coupling constant g. The bulk viscosity ξ vanishes for an ideal relativistic gas [18].

Solving Eq. (5), the entropy density $s = (4/3)\epsilon/T$ is obtained. The total entropy, S, is proportional to τs and is thus [11,12]

$$S = S_0 \left[1 + \frac{\tilde{\eta}}{2a\tau_0 T_0}\left(1 - \left(\frac{\tau_0}{\tau}\right)^{2/3}\right)\right]^3, \qquad (7)$$

where S_0, τ_0, and T_0 are the initial entropy, time, and temperature. As seen in Fig. 1 the entropy production increases rapidly with viscosity $\tilde{\eta}$. If the decoupling (freeze-out) or hadronization time is sufficiently late the entropy approaches the asymptotic value $S = S_0(1 + \tilde{\eta}/2a\tau_0 T_0)^3$.

To get an idea of the magnitude of $\tilde{\eta}$ we refer to recent calculation within perturbative QCD [14,16]. For weak couplings, $\alpha_s \lesssim 0.1$, the quark and gluon viscosities depend on the coupling constant as

$$\tilde{\eta} \simeq [\alpha_s^2 \ln(1/\alpha_s)]^{-1}, \qquad (8)$$

where the logarithm arises from the sensitivity to screening of long range (small momentum transfer) interactions. The quark viscosity is approximately four times larger than the gluon because quarks interact more weakly. For strongly interacting plasmas, $\alpha_s \gtrsim 0.1$, the Debye and dynamical screening must be replaced by some effective cutoff due to correlations in the plasma, which leads to $\tilde{\eta} \propto \alpha_s^{-2}$. Within the range $\alpha_s \sim 0.1$–0.5, the quark (gluon) viscosity decrease from $\tilde{\eta} \sim 50$ (15) to $\tilde{\eta} \sim 1.5$ (0.8). If the initial density of quarks and gluons, n_i, is less than that in chemical equilibrium, n_{eq}, then the viscosities are larger by a factor $\sim n_{\text{eq}}/n_i$.
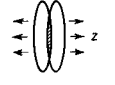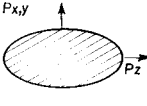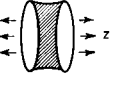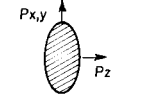
The corresponding viscous relaxation time
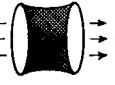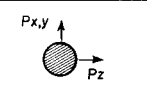
TIME PERIOD	EXPANSION MODE	COORDINATE SPACE	MOMENTUM SPACE
$0 \lesssim \tau \lesssim \tau_0$	FORMATION		
$\tau_0 \lesssim \tau \lesssim \theta$	FREE STREAMING		
$\theta \ll \tau$	HYDRO- DYNAMIC		

FIG. 2. Qualitative picture of the expanding system following a nuclear collision. The formation time from $0-\tau_0$, free streaming from $\tau_0 - \theta$, and hydrodynamical flow for $\tau \gg \theta$.

$$\tau_\eta = 5 \frac{\eta}{\varepsilon + P} = \frac{15}{4a} \tilde{\eta} T^{-1}, \tag{9}$$

is, however, larger than the expansion time and so hydrodynamics does not apply at early times as pointed out by Danielewicz and Gyulassy [11]. The entropy production will be much lower and will be calculated in the following sections by solving the Boltzmann equation in the relaxation time approximation. For comparison the upper limit on the entropy production in Eq. (35) from solving the Boltzmann equation with a finite collision time, $\theta = \tau_\eta$, for the case $\tilde{\eta} = 16$ corresponding to $\theta \approx 4\tau_0$ is also shown in Fig. 1 as a dashed line. Note that the entropy production continues long after θ and τ_η.

B. Free streaming

In the opposite extreme to the hydrodynamic limit examined above, collisions are absent and partons stream freely. According to Eq. (2) the distribution function evolves as [9]

$$f(\mathbf{p}) = f_0(\mathbf{p}_\perp, p_z \tau / \tau_0), \tag{10}$$

where f_0 is the initial distribution function at time τ_0.

As will be described in the following section the parton gas will stream freely until collisions thermalize the system around a time θ. The free streaming alters the distribution in phase space drastically. Initially the partons have large longitudinal momenta due to the high relative energy of the incoming nucleons in the nucleus-nucleus collisions and the relatively small transverse momentum transfer in hadronic collisions. However, in the one-dimensional free streaming expansion of the system at later times only those partons with similar longitudinal velocity will travel together locally in space and time. Thus the phase space separates the longitudinal momenta and the distribution function changes from a wide to a narrow one in p_z locally in space and time (see Fig. 2). Collisions will then attempt to thermalize the system towards an isotropic distribution.

When the longitudinal expansion has extended the system to a size similar to the transverse size, which happens at a time of order the nuclear transverse dimension $\tau \sim R$, three

dimensional expansion takes over. Hereafter the densities will decrease rapidly reducing collisions drastically and a free streaming scenario is again likely. Around the same time, however, the system may break up, freeze-out, and fragment.

III. THERMALIZATON IN THE RELAXATION TIME APPROXIMATION

Baym solved the Boltzmann equation in the relaxation time approximation Eq. (3) with a relaxation time θ *independent* of expansion time [9]. He found that the parton gas started out free streaming and gradually thermalized to hydrodynamical flow on a time scale given by θ. The distribution function at any point in the local rest frame changes from being highly anisotropic, with only small longitudinal momenta, to being isotropic for $\tau \gg \theta$. A similar calculation by Gavin [10], however, with a collision time scaling *linearly* with time, $\theta = \alpha t$, gave a qualitatively different result. In this case the gas ends up somewhere between free streaming and thermal equilibrium depending on the values of α. For small α the collision time is short and the final state is close to Bjorken flow but for large α the state is closer to free streaming. The time dependence and magnitude of the relaxation time is thus essential for describing the degree of thermalization and its time scale.

We will in the following solve the Boltzmann equation within the relaxation time approximation for a more general time dependence of the collision time proportional to the expansion time τ to some power,

$$\theta \propto \tau^p. \tag{11}$$

This covers the constant relaxation time of Baym [9] ($p=0$), the linear one of Gavin [10] $\tau_{\text{coll}} = 1/n\sigma \sim \tau$ (i.e., $p=1$) as well as the near equilibrium transport relaxation time $\tau_{\text{tr}} \sim T^{-1} \sim n^{-1/3} \sim \tau^{1/3}$ (i.e., $p=1/3$). We shall study under which circumstances the parton gas thermalizes, estimate the relaxation times and predict the degree and time scale for equilibration.

The solution to the Boltzmann equation (3) can be written in terms of an integral equation

$$f = f_0(p_\perp, p_z \tau / \tau_0) e^{-x}$$
$$+ \int_0^x dx' e^{x'-x} f_{\text{eq}}[\sqrt{p_\perp^2 + (p_z \tau / \tau')^2}, T', \mu'], \tag{12}$$

where the time dependence of the temperature T and chemical potential μ in $f_{\text{eq}}(p, T, \mu)$ are determined by demanding the energy density and number density (assuming no particle production) be the same for f_{eq} and f at any time, i.e.,

$$\epsilon(T, \mu) = \int d\Gamma_{\mathbf{p}} E_p f(\mathbf{p}) \equiv \int d\Gamma_{\mathbf{p}} E_p f_{\text{eq}}(p, T, \mu), \tag{13}$$

$$n(T, \mu) = \int d\Gamma_{\mathbf{p}} f(\mathbf{p}) \equiv \int d\Gamma_{\mathbf{p}} f_{\text{eq}}(p, T, \mu), \tag{14}$$

where $d\Gamma_{\mathbf{p}} = d^3 \mathbf{p}/(2\pi)^3$. The function $x(\tau)$ is related to τ by
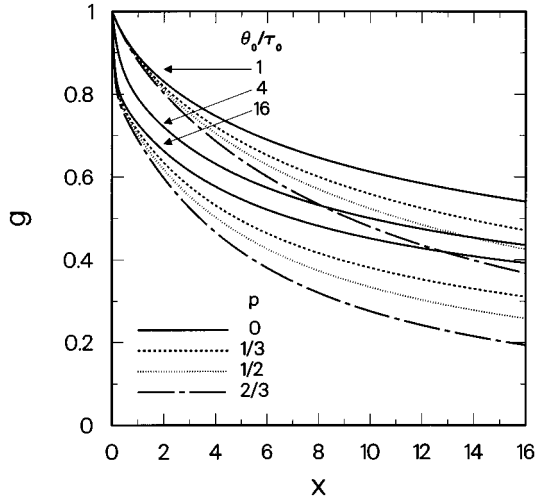
FIG. 3. The function $g[\tau(x)]$ found by solving Eq. (16) numerically. Its dependence on the relaxation time $\theta = \theta_0(\tau/\tau_0)^p$ is shown for various θ_0 and p.

$$x(\tau) = \int_{\tau_0}^{\tau} d\tau'/\theta = \frac{1}{1-p}\left[\frac{\tau}{\theta(\tau)} - \frac{\tau_0}{\theta_0}\right], \quad p \neq 1, \quad (15)$$

where $\theta_0 = \theta(\tau_0)$. The marginal case $p=1$ was studied in [10].

The evolution of the parton gas can be conveniently studied by taking moments of Eq. (12) with respect to particle energies. Summing over particle momenta gives a simple integral equation for the particle density which has the simple solution $n(\tau) = n_0\tau_0/\tau$ which also follows directly from Eq. (3). Multiplying by particle energy and summing over momentum we obtain

$$e^x g(\tau) = h(\tau_0/\tau) + \int_0^x dx' e^{x'} g(\tau')h(\tau'/\tau), \quad (16)$$

where

$$g(\tau) = \frac{\tau}{\tau_0}\frac{\epsilon(\tau)}{\epsilon(\tau_0)}, \quad (17)$$

and

$$h(r) = \int_0^1 d\cos(v)\sqrt{1+\cos^2(v)(r^2-1)}$$

$$= \frac{1}{2}\left(r + \frac{\sin^{-1}\sqrt{1-r^2}}{\sqrt{1-r^2}}\right). \quad (18)$$

Here v is the polar angle of the particle momenta with respect to the z axis. The function $h(r)$ is a monotonically increasing function between $h(0) = \pi/4$ and $h(1) = 1$. The function $g(x)$ is calculated numerically and is shown in Fig. 3 for various θ_0 and $p<1$.

Performing a partial integration on Eq. (16) gives

$$\int_0^x dx' e^{x'}\frac{d}{dx'}[g(\tau')h(\tau'/\tau)] = 0. \quad (19)$$

Note that by assuming an initially spherical symmetric distribution we differ slightly from Baym [9]. He assumes the distribution to be peaked in transverse directions initially. The term $h(\tau_0/\tau)$ in Eq. (16) is then replaced by unity and the right-hand side of Eq. (19) changes to $1 - \pi/4$. The difference in initial conditions will not affect any of our subsequent arguments concerning the p dependence of the thermalization.

When $p<1$, $x(\tau)$ increases with increasing τ as seen from Eq. (15). The exponential factor $e^{x'}$ thus weights large x' and Eq. (19) implies for large x or τ

$$\frac{d}{d\tau'}[g(\tau')h(\tau'/\tau)]\Big|_{\tau'=\tau} = 0, \quad (20)$$

since $d\tau = \theta dx$. The slope of $h(r)$ near $r=1$ is 1/3 and so we conclude that $g(\tau) \propto \tau^{-1/3}$ for large τ. Thus $\epsilon(\tau) \propto \tau^{-4/3}$ which is the one-dimensional hydrodynamical limit.

When $p>1$, $x(\tau)$ is negative and decreases with increasing τ to a minimal but finite value $x_{\min} = -\tau_0/\theta_0/(1-p)$. By differentiating Eq. (16) we find that $g'(\tau)$ is always negative and therefore $g(\tau)$ decreases monotonically. Yet, by definition, g is positive and must therefore have an asymptotic value larger than or equal to zero. From the integral equation (16) we see that this value at $\tau=\infty$ or $x=x_{\max}$ is nonvanishing and thus $g(\infty)>0$. Consequently, $\epsilon(\tau) = g(\infty)\tau_0/\tau$ at large times which is the one-dimensional free streaming limit.

We conclude that thermalization will be reached when $p<1$ and the parton gas will expand hydrodynamically at large times whereas when $p>1$ it will continue to stream freely. The marginal case $p=1$ was studied by Gavin [10] who found that the parton gas ends up in a state between hydrodynamic expansion and free streaming depending on the size of the prefactor α where $\theta = \alpha\tau$. When α is small the collision time is always relatively short and the gas equilibrates near the hydrodynamic limit. When α is large the collision time is always longer than the expansion time and the gas continues to stream freely. One should, however, keep in mind the finite decoupling, freeze-out, or hadronization time. When it is shorter than the collision time, which will be the case for large θ_0 or p close to unity or larger, the parton gas does not thermalize.

IV. A PERTURBATIVE QCD ESTIMATE OF THE RELAXATION TIME

As we have just seen, the magnitude of the collision time as well as its dependence on expansion time is crucial for the equilibration. We shall therefore study the collision term and calculate it within perturbative QCD where we know the scattering matrix elements between quarks and gluons. We shall here only consider elastic scattering processes even though inelastic (radiative) processes have been found to be very important in the cases of initial parton production [3,5], energy loss of a fast parton going through a QGP [19], and later parton chemical equilibration [4,6].

In general, radiative processes are important for a system far from equilibrium. During the very initial stage of heavy-ion collisions, the processes leading to minijet (parton) production have relatively large momentum transfers Q^2. For

such hard processes, the radiative "corrections" in powers of $\alpha_s \ln(Q^2/Q_0^2)$ can be very large [20], where Q_0^2 is some scale below which perturbative QCD is no longer applicable. They lead to both initial and final-state radiation of additional partons and contribute significantly to the initial parton production. Immediately after the initial scatterings when the system is still far away from thermal equilibrium, radiative processes might also be important as in initial multiple scattering processes. Radiative processes in multiple collisions and the Landau-Migdal-Pomeranchuk effect has recently been discussed within perturbative QCD by a number of people. According to recent work by Baier *et al.* [21], the radiative energy loss of a fast parton with energy E penetrating a QGP is $dE/dx \sim -\alpha_s \sqrt{Eq_D^2/\lambda_g} \ln(E/q_D^2\lambda_g)$, where λ_g is the gluon mean free path. They included also rescattering of the radiated gluons which was ignored by Gyulassy and Wang [19] and thus obtained a different energy dependence. Weldon also showed [22] that inclusion of absorption processes as well as production and absorption of virtual particles reduces the energy loss further. Interestingly, if one takes (as generally assumed) $\lambda_g \sim \alpha_s T$ and ignores the weak logarithmic dependences, the energy loss is proportional to α_s^2 in both these cases as well as for energy loss due to elastic scatterings only [23]. Another situation in which radiative processes are important is the chemical equilibration of a kinetically thermalized system. In this case, the leading order contribution to the chemical equilibration is $2 \rightarrow 3$, with the elastic scatterings maintaining the thermal equilibrium [4].

In view of the uncertain status of the radiative rate in QGP, we will not here attempt to give an accurate calculation of the radiative corrections to the relaxation time in system out of equilibrium. For an order-of-magnitude estimate, we neglect these higher order contributions. What we will do is to estimate the effective thermalization time due to *elastic collisions*. This time will then only serve as an upper limit if inelastic collisions add significantly to the equilibration at later times. As we shall see, this upper limit will still allow us to make predictions concerning the eventual thermalization during the expansion stage.

The collision integral for scattering particles elastically from initial states 1 and 2 to final states 3 and 4 is

$$\left(\frac{\partial f_1}{\partial t}\right)_{\text{coll}} = -(2\pi)^4 \nu_2 \int d\Gamma_{\mathbf{p}_2} d\Gamma_{\mathbf{p}_3} d\Gamma_{\mathbf{p}_4} |M_{12 \rightarrow 34}|^2$$

$$\times [f_1 f_2 (1 \pm f_3)(1 \pm f_4) - f_3 f_4 (1 \pm f_1)(1 \pm f_2)]$$

$$\times \delta^4(p_1 + p_2 - p_3 - p_4), \quad (21)$$

where p_i are the parton four-momentum. We assume they are massless, i.e., $E_i = |\mathbf{p}_i|$. The $(1 \pm f_i)$ factors correspond physically to the Pauli blocking of final states, in the case of fermions, and to (induced or) stimulated emission, in the case of bosons. ν_2 is the statistical factor, 16 for gluons, and $12N_f$ for quarks and antiquarks. $|M_{12 \rightarrow 34}|^2$ $= |\mathcal{M}_{12 \rightarrow 34}|^2/(16E_1 E_2 E_3 E_4)$ is the matrix element squared summed over final states and averaged over initial states. For scattering of gluons

$$|\mathcal{M}_{12 \rightarrow 34}^{(gg)}|^2 = \frac{9}{2} g^4 \left(3 - \frac{us}{t^2} - \frac{st}{u^2} - \frac{ut}{s^2}\right); \quad (22)$$

quark-gluon and quark-quark interactions are just 4/9 and $(4/9)^2$ times weaker, respectively, near forward scattering ($t \simeq 0$). In a $t = \omega^2 - q^2$ channel, a singularity occurs for small momentum $\mathbf{q}$ and energy ω transfers. In a medium the t^{-2} singularity is screened as given by Dyson's equation in which a gluon self-energy $\Pi_{L,T}$ is added to the propagator

$$t^{-1} \rightarrow \omega^2 - q^2 - \Pi_{L,T}. \quad (23)$$

As was shown in [14–16] that Debye screening and dynamical screening due to Landau damping effectively screen the longitudinal and transverse interactions off in most transport problems at a length scale of order the Debye screening length q_D^{-1}. For small momentum transfer, $q \ll E_1, E_2$, one can split the matrix element into longitudinal and transverse parts [16],

$$|M_{gg}|^2 = \frac{9}{8} g^4 \left[\frac{1}{q^2 + \Pi_L} - \frac{(1 - \omega^2/q^2)\cos\phi}{q^2 - \omega^2 + \Pi_T}\right]^2, \quad (24)$$

where $\cos\phi = (\mathbf{v}_1 \times \hat{\mathbf{q}}) \cdot (\mathbf{v}_1 \times \hat{\mathbf{q}})$. The gluon self-energies, Π_L and Π_T are given in the long wavelength limit ($q \ll T$) by [24]

$$\Pi_L(q, \omega) = q_D^2 \left[1 - \frac{\omega}{2q} \ln\left(\frac{q + \omega}{q - \omega}\right)\right], \quad (25)$$

$$\Pi_T(q, \omega) = q_D^2 \left[\frac{\omega^2}{2q^2} + \frac{\omega}{4q}\left(1 - \frac{\omega^2}{q^2}\right) \ln\left(\frac{q + \omega}{q - \omega}\right)\right]. \quad (26)$$

The Debye screening wave number in thermal QCD is $q_D^2 = g^2(1 + N_f/6)T^2$ where N_f is the number of quark flavors.

The Boltzmann equation with the full collision term, Eq. (21), has been solved for quark-gluon plasmas near equilibrium and a number of transport coefficients have been calculated to leading orders in the coupling constant [14,15,25]. For viscous and thermal relaxation as well as momentum stopping, the "transport relaxation time" is generally

$$\tau_{\text{tr}} \simeq \left[\alpha_s^2 \ln\left(\frac{1}{\alpha_s}\right) \lambda T\right]^{-1}, \quad (27)$$

where λ (the "fugacity") is the ratio of the actual density to the one in chemical equilibrium at temperature T. This relaxation time may be used at later times when the parton gas is near equilibrium. In Bjorken flow the temperature scales like $T \propto \tau^{-1/3}$, i.e., $\tau_{\text{tr}} \propto \tau^{1/3}$. Since this power is less than unity, the parton gas should thus equilibrate according to the analysis in the previous section.

In nuclear collisions the parton gas may be far from equilibrium when first produced and the expansion may also drive it out of equilibrium as in free streaming. Solving the Boltzmann equation thus becomes a very difficult nonlinear problem that requires major computational efforts which is being undertaken in a number of parton cascade models as, e.g., in [3]. We take another approach in this paper. As mentioned above, hydrodynamics does not apply at early times because of long viscous relaxation times and the parton gas is expected to expand as free streaming initially. With this initial ansatz in Eq. (10) for the distribution function we can calculate the change in the distribution function at early

times from the Boltzmann equation with the full collision term of Eq. (21). More specifically, from the entropy density

$$s(\tau) = -\sum_{\mathbf{p}} [f \ln f \mp (1 \pm f)\ln(1 \pm f)], \qquad (28)$$

we have calculated the entropy production

$$\left(\frac{\partial s}{\partial t}\right)_{\mathrm{coll}} = \sum_{\mathbf{p}} \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}} \ln\left(\frac{f}{1 \pm f}\right), \qquad (29)$$

in the free streaming phase by approximating f by f_0 [see Eq. (A1) in the Appendix]. The initial entropy production can be estimated analytically with the full collision term to leading logarithmic order in the coupling constants and de-

tails of this calculation is given in the Appendix. Ignoring quarks the final result is [see (A12)]

$$\left(\frac{\partial s}{\partial t}\right)_{\mathrm{coll}} = \nu_g^2 \frac{9}{8\pi^4} \alpha_s^2 T_0^4 \lambda_{0,g}^2 \ln\left[\frac{9\tau}{\pi\lambda_{0,g}\alpha_s\tau_0}\right] \ln\left(\frac{2\tau}{\tau_0}\right), \quad (30)$$

where $\lambda_{0,g} = \exp(\mu_{0,g}/T_0)$ is the ratio of the initial density to that in chemical equilibrium.

We can match this entropy production to that obtained in the relaxation time approximation thus determining an effective and momentum averaged relaxation time θ. The relaxation time and entropy production during this early period, $\tau_0 \ll \tau \ll \theta$, are different from later times $\tau \gtrsim \theta$ when collisions change the free streaming distribution functions. The entropy production for the initial free streaming in the relaxation time approximation is from Eq. (29)

$$\left(\frac{\partial s}{\partial t}\right)_{\mathrm{coll}}^{\mathrm{relax}} = -\int d\Gamma_{\mathbf{p}} \left(\frac{f_0 - f_{\mathrm{eq}}}{\theta}\right) \frac{\sqrt{p_\perp^2 + (p_z\tau/\tau_0)^2} - \mu_0}{T_0}$$

$$= \frac{\epsilon(T_0)}{\theta T_0} \left\{ \frac{1}{4}\left[1 + \frac{\tau_0^2}{\tau^2}\frac{\ln(1 + \sqrt{1 - \tau_0^2/\tau^2})}{\sqrt{1 - \tau_0^2/\tau^2}}\right]\left[\frac{\tau_0}{\tau} + \frac{\sin^{-1}(\sqrt{1 - \tau_0^2/\tau^2})}{\sqrt{1 - \tau_0^2/\tau^2}}\right] - \frac{\tau_0}{\tau} \right\}$$

$$\simeq \frac{\pi}{8}\frac{\epsilon(T_0)/T_0}{\theta}, \qquad \tau \gg \tau_0. \tag{31}$$

As in Eq. (30) we ignore quarks; $\epsilon(T_0) \approx \nu_g(\pi^2/30)T_0^4\lambda_{0,g}$ is the initial gluon energy density. By equating the entropy production of Eq. (31) for $\tau \gg \tau_0$ to that from the perturbative QCD collision term Eq. (30) we find

$$\frac{1}{\theta} \simeq 1.43\alpha_s^2 T_0\lambda_{0,g}\ln\left[\frac{3\pi\tau}{2\lambda_{0,g}\alpha_s\tau_0}\right]\ln\left(\frac{2\tau}{\tau_0}\right). \qquad (32)$$

Note that it is the initial temperature T_0 that enters here and not $T(t)$ as in Eq. (27). The collision time depends only logarithmically on the expansion time. As explained in the Appendix, the relaxation time is only weakly time dependent in a free-streaming parton gas because the phase space for small momentum scattering opens up quadratically with time thus effectively compensating the decrease in parton densities. On the other hand, if large momentum transfers are imposed to each parton scattering, the entropy production rate will decrease quadratically with time, leading to a much stronger time dependence of the relaxation time. The long range interactions (small momentum transfers) are therefore very important in an expanding parton gas. The two logarithms in Eq. (32) arise from integrals over momentum and energy transfers respectively. The "fugacity" factor λ_0 arises from the correspondingly smaller density of scatterers.

The relaxation time θ depends sensitively on the initial conditions through T_0, λ_0, τ_0, and $\alpha_s(T)$. A reliable estimate of these initial parameters will require additional non-

trivial calculations treating the early hard collisions. In the following we list the results from a number of such model calculations.

From the HIJING model calculation [5,26,27] it was found that at $\tau_0 = 0.7$ fm/c the produced partons in central Au+Au collision at RHIC energy can reach a local isotropy in momentum distribution temporarily with effective temperature $T_0 = 0.57$ GeV; in addition $\lambda_{0,g} = 0.09$ and $\lambda_{0,q} = 0.02$. At LHC energies $\tau_0 = 0.5$ fm/c, $T_0 = 0.83$ GeV, $\lambda_{0,g} = 0.14$, and $\lambda_{0,q} = 0.03$. Eskola et al. [28] found similar results for the temperature $T_0 = 1(1.5)$ GeV at the RHIC (LHC) energy in their minijet plasma calculation. However, they used a smaller initial time, $\tau_0 = 0.1$ fm/c, and find $\lambda_{0,g} \sim 1$ and $\lambda_{0,q} \sim 0$. If one were to allow this minijet plasma to stream freely shortly after τ_0 the result would be consistent with HIJING estimates at the later time $\tau_0 \approx 0.7$ fm/c. The newest set of parton distribution functions in the calculation of Eskola et al. also increase the initial parton density, especially at LHC. In the parton cascade model [3] the initial parton density is found to be larger due to a different treatment of soft parton interactions. These numbers for τ_0 and T_0 are surprisingly similar to those found by Shuryak [8] in a different analysis. By estimating the particle rapidity distributions dN/dy in relativistic nuclear collisions he obtains a particle density in the Bjorken scenario $(dN/dy)/(\pi R^2\tau)$ which at a time $\tau = \tau_{\mathrm{coll}} \sim (\alpha_s T)^{-1}$ is assumed to be the same as the equilibrium one $\sim T^3$. Hereby the initial values for T and τ are found. One should bear in mind that all these

estimates are based on perturbative QCD inspired models. There are many uncertainties (see, e.g., [29]) due to our limited knowledge of strong interactions.

If we ignore the slow logarithmic time dependence and assume $T_0 \simeq 1/\tau_0$ and $\lambda_{0,q} \sim 0$, as motivated by the above mentioned models, we find

$$\theta_0 \simeq \alpha_s^{-2} \lambda_{0,g}^{-1} \tau_0. \tag{33}$$

With $\alpha_s \simeq 0.3$ we find a rather long collision time even if $\lambda_{0,g} = 1$.

V. ENTROPY PRODUCTION

With the weak time dependence of the relaxation time, we can now estimate the entropy production during the early thermalization. We still assume the relaxation time has a power dependence on time as in Eq. (11) with small p. The scaling behavior of the function $g(\tau)$ with $\theta_0 = \theta(\tau = \tau_0)$ can be obtained when $\theta_0 \gg \tau_0$. In that case the integral equation (16) for $g[\tau(x)]$ depends almost solely on x, since $x'/x \simeq (\tau'/\tau)^{1-p}$. Thus for given p, $g[\tau(x)]$ is a generic function of x. Its behavior at large x is

$$g[\tau(x)] \simeq g_p x^{-1/3(1-p)} = g_p \left[(1-p) \frac{\theta_0}{\tau_0} \right]^{1/3(1-p)} \left(\frac{\tau}{\tau_0} \right)^{-1/3}, \tag{34}$$

where g_p is some p-dependent constant of order unity. In the case $p=0$ Baym [9] found $g_p = 1.22$ but due to the different initial conditions (isotropic versus peaked in the transverse directions) this value differs from our case by a factor $\pi/4$ when $\theta_0 \gg \tau_0$.

Until the parton gas reaches equilibrium its entropy at time τ is always less than the equilibrium entropy at temperature $T(\tau)$, i.e., $s \leq s_{eq} = (4/3)\epsilon/T = s_0 g(\tau) T_0 \tau_0 /(T\tau)$. Thus the total entropy is

$$S \leq S_0 g(\tau) \frac{T_0}{T} = S_0 \left(\frac{\tau}{\tau_0} \right)^{1/4} g(\tau)^{3/4}. \tag{35}$$

If the parton gas equilibrates the equal sign holds at large times and the final entropy is from Eqs. (34,35)

$$\frac{S_f}{S_0} = s_p (\theta_0/\tau_0)^{1/4(1-p)}, \tag{36}$$

where s_p is a p-dependent number. In Fig. 4, the final entropy is plotted for various values of p and θ_0. The formula Eq. (36) is a good approximation with coefficient $s_p \sim 1$. For comparison the final entropy calculated by Baym

$$\frac{S_f}{S_0} = 1.16(\theta_0/\tau_0)^{1/4}, \tag{37}$$

with a constant relaxation time ($\theta = \theta_0$, $p=0$) is also shown in Fig. 4, being multiplied by a factor $(\pi/4)^{3/4}$ because of the different initial conditions.

It is interesting to compare the entropy production to the case when the parton gas is streaming freely until a time
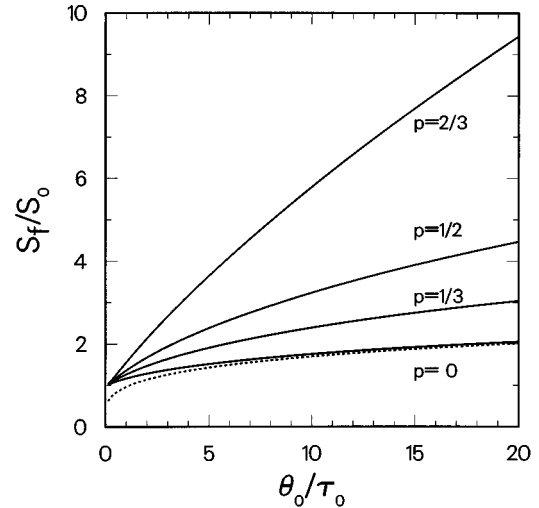

FIG. 4. Entropy production in the relaxation time approximation as function of the relaxation time $\theta = \theta_0(\tau/\tau_0)^p$ for various powers p. The entropy found by Baym [9] (see text) is shown by dashed curve.

τ_{sudden} when the gas suddenly collides violently and immediately thermalizes. Conserving energy per volume (but not particle density) we find

$$\frac{S_f}{S_0} = \left(\frac{\pi}{4} \right)^{3/4} (\tau_{sudden}/\tau_0)^{1/4}, \tag{38}$$

which is only slightly lower than Eq. (37) if $\theta_0 \simeq \tau_{sudden}$. The exact way by which the parton gas equilibrates is therefore not so important; it is the collision time that determines the entropy production.

From Eqs. (37,36) it is evident that the final entropy production increases slowly with the collision time. Yet it takes much longer time and at a given time ($\tau \lesssim \theta$) the entropy production rate [cf. Eq. (31)] is inversely proportional to θ. If the parton gas decouples, freeze-out, or hadronize at time, τ_d, and entropy is no longer produced, then the total entropy production will decrease as θ increases above τ_d.

With the approximate collision time of Eq. (33) and assuming $T_0 \simeq 1/\tau_0$ the final entropy is obtained from Eq. (36) or (37)

$$\frac{S_f}{S_0} \simeq \alpha_s^{-1/2} \lambda_{0,g}^{-1/4}. \tag{39}$$

For $\alpha_s \sim 0.3$ this gives an increase in entropy by a factor of 2–3 when varying λ_0 from unity down to 0.1. Should the relaxation time decrease by, for example, a factor of 2 due to additional radiative processes, the entropy production decrease by a factor $\sim 2^{-1/4} \simeq 0.84$.

VI. SUMMARY

We have studied a one-dimensionally expanding parton gas created in the wake of nuclear collisions. Within the Boltzmann equation in the relaxation time approximation we find that the rapid expansion is closer to free streaming than hydrodynamic expansion for times shorter than typical collision times of partons, $\tau_0 \lesssim \tau \lesssim \theta$. Only at times much larger

than the characteristic collision times, $\tau \gg \theta$ may the parton gas thermalize and expand hydrodynamically. However, if the collision time increases with time the gas may never thermalize. Parametrizing the collision time as $\theta = \theta_0 (\tau/\tau_0)^p$ the condition for equilibration and hydrodynamical expansion is $p < 1$. We calculate how much entropy is produced in the collisions. These different expansion modes and the corresponding parton phase space distributions at different stages of evolution in high-energy heavy ion collisions are illustrated in Fig. 2.

We have calculated the elastic parton scatterings within perturbative QCD and point out that inelastic processes such as gluon emission may well add to the thermalization rate by an amount of the same order. The effective elastic relaxation time is thus only an upper limit but that is still sufficient to determine the eventual thermalization during expansion. The elastic relaxation time depends sensitively on the initial conditions present in the very early phase of the collisions and we have taken these from other models.

We find that the parton gas does equilibrate eventually with these collision times but only after having undergone a phase of free streaming and gradual thermalization where considerable entropy is produced ("after burning"). The final entropy and thus particle density depends on the collision time as well as the initial conditions (a "memory effect"). For various models predicting the preequilibrium scenarios the entropy production is significant. The total entropy and particle production is estimated to be doubled or tripled with respect to the initial value.

These estimates do not include particle production which by itself adds to the entropy production. On the other hand particle production will also increase the density and thus shorten the effective collision time which leads to a decrease in entropy production according to Eq. (36).

Most analyses assume a constant density in space but large density fluctuations may well be present in the initial parton plasma. This will increase the average entropy production for both elastic and inelastic scatterings since these are proportional to the initial densities squared as well as the final densities through the stimulated emission factors (for bosons) or Pauli blocking factors (for fermions). High density regions ("hot spots") will equilibrate thermally and chemically faster than low density regions. At the same time, however, the free streaming will tend to reduce density fluctuations.

It was emphasized in the Appendix that the very singular small momentum transfers provides strong scattering and the opening up of phase space compensates for the decreasing densities. If a larger momentum transfer cutoff of the order of particle momenta ($\sim$ temperature) is applied then the collision time will increase quadratically with expansion time and the parton gas will never thermalize. Also, when the longitudinal extension of the system exceeds the transverse

size, which is in the order of the nuclear radius, the expansion proceeds in three dimensions. The densities will then decrease cubically with expansion time and collision might never catch up with the expansion.

APPENDIX: THE RELAXATION TIME

In this Appendix we give a detailed derivation of the entropy production by elastic parton collisions in a system near the free streaming case at times $\tau_0 \ll \tau \ll \theta$.

The initial distribution of partons has been estimated in several models [3,26]. Typically, one finds that the partons are formed within the first $\tau_0 \simeq 0.2$–0.7 fm/c after the nuclear collision and that rapid longitudinal expansion takes place. The local momentum distribution of partons at the formation time is not isotropic but forward/backward peaked, i.e., $\langle |p_z| \rangle \gg \langle p_\perp \rangle$. However, due to streaming the local distribution changes rapidly since $\langle |p_z| \rangle$ scales as τ_0/τ [see Eq. (10)] and at later times the particles have $\langle |p_z| \rangle \ll \langle p_\perp \rangle$. At the crossover time τ_0 the parton gas is isotropic in momentum space and seems to be in approximate thermal equilibrium with temperature T_0 even though it is streaming freely in space and time. From Eq. (10) we obtain the free streaming distribution function

$$f_0(\mathbf{p}, \tau) = \left[\exp\left(\frac{\sqrt{p_\perp^2 + (p_z \tau/\tau_0)^2} - \mu_0}{T_0} \right) \pm 1 \right]^{-1}, \quad (A1)$$

where μ_0 is the chemical potential determined by the density. Since the parton gas may not have reached chemical equilibrium yet, μ_0 may not vanish. In fact most models predict that particle densities are rather low initially [3,27,28] and that $-\mu_0 \simeq (1-2) T_0$. The temperature increases with collision energy and typically $T_0 \simeq 0.5$–2 GeV. We shall use Eq. (10) with Eq. (A1) for the free streaming initially with parameters τ_0, T_0, and μ_0. The entropy production due to collisions is negligible at times around τ_0 because the parton gas is near thermal equilibrium and so we shall ignore collisions earlier than τ_0. On the other hand, continuing particle production will produce entropy but we shall not consider that contribution here.

With the free streaming distribution function, we obtain from the Boltzmann equation by changing variables from $p_{z,i} \rightarrow p_{z,i} \tau_0/\tau$, $i = 1,2$, and $q_z \rightarrow q_z \tau_0/\tau$

$$\left(\frac{\partial s}{\partial t} \right)_{\text{coll}} = -2\pi \nu_1 \nu_2 \frac{\tau_0^3}{\tau^3} \int d\Gamma_{\mathbf{p}_1} d\Gamma_{\mathbf{p}_2} d\Gamma_{\mathbf{q}} \left| M_{12 \rightarrow 34}\left(\mathbf{q}_\perp, q_z \frac{\tau_0}{\tau} \right) \right|^2 f_0(E_1) f_0(E_2) [1 \pm f_0(E_3)][1 \pm f_0(E_4)]$$

$$\times \frac{E_1 - \mu_0}{T_0} \left[1 - \exp\left(\frac{E_1 + E_2 - E_3 - E_4}{T_0} \right) \right] \delta(\tilde{E}_1 + \tilde{E}_2 - \tilde{E}_3 - \tilde{E}_4), \quad (A2)$$

where $\tilde{E}_3 = \sqrt{(\mathbf{p}_{1\perp} + \mathbf{q}_\perp)^2 + (p_{1,z} + q_z)^2 \tau_0^2/\tau^2}$ and $\tilde{E}_1 = \sqrt{p_{1\perp}^2 + (p_z \tau_0/\tau)^2}$. The same expressions are valid for $i=2,4$ when the sign of $\mathbf{q}$ is changed.

At this point we want to emphasize the importance of small momentum transfer processes, $q \sim q_D \sim gT$ as compared to large momentum transfer ones, $q \sim T$. For the latter the exponential in Eq. (A2) can be ignored because $E_4 + E_3 - E_2 - E_1 \sim T$. One then finds that the integrations over the energy conservation δ function removes a factor τ_0/τ leaving an entropy production rate decreasing quadratically in time. This just reflects that the particle densities of the two scatterers decrease as τ_0/τ. The small momentum transfers have, however, a very singular scattering matrix element and, as we will now show, the phase space opens up quadratically with time for small q—effectively *compensating* the decreasing densities of scatterers.

Expanding around small q to second order, the term in the square bracket representing the difference between scattering in and out is

$$1 - \exp\left(\frac{E_1 + E_2 - E_3 - E_4}{T_0}\right) \simeq \frac{(\mathbf{v}_1 - \mathbf{v}_2) \cdot \mathbf{q}}{T_0} - \frac{[(\mathbf{v}_1 - \mathbf{v}_2) \cdot \mathbf{q}]^2}{2T_0^2} + \frac{q^2 - (\mathbf{v}_1 \cdot \mathbf{q})^2}{2E_1 T_0} + \frac{q^2 - (\mathbf{v}_2 \cdot \mathbf{q})^2}{2E_2 T_0}. \tag{A3}$$

Only when $\tau = \tau_0$ is $\tilde{E} = E$ and energy conservation requires that Eq. (A3) vanishes. Due to symmetry the first term vanishes when integrated over $\mathbf{p}_1$ and $\mathbf{p}_2$ and the leading term is second order in q^2.

For small q we can also replace E_3 by E_1 and E_4 by E_2 in the distribution functions. Thus we have

$$\left(\frac{\partial s}{\partial t}\right)_{\text{coll}} = -2\pi\nu_1\nu_2 \frac{\tau_0^3}{\tau^3} \int d\Gamma_{\mathbf{p}_2} f_0(E_2)[1 \pm f_0(E_2)] \int d\Gamma_{\mathbf{p}_1} \frac{E_1 - \mu_0}{T_0} f_0(E_1)[1 \pm f_0(E_1)] \int d\Gamma_{\mathbf{q}} \left| M_{12\to 34}\left(\mathbf{q}_\perp, q_z \frac{\tau_0}{\tau}\right)\right|^2$$

$$\times \int d\omega\, \delta(\omega - \tilde{E}_1 + \tilde{E}_3)\, \delta(\omega + \tilde{E}_2 - \tilde{E}_4)\left\{\frac{q^2 - (\mathbf{v}_1 \cdot \mathbf{q})^2}{2E_1 T_0} + \frac{q^2 - (\mathbf{v}_2 \cdot \mathbf{q})^2}{2E_2 T_0} - \frac{[(\mathbf{v}_1 - \mathbf{v}_2) \cdot \mathbf{q}]^2}{2T_0^2}\right\}. \tag{A4}$$

Here we have introduced an auxiliary integral over energy transfer ω.

We can use up these δ functions by performing the angular integrals $d\Omega_i$, $i=1,2$. For example,

$$I_1 \equiv \int d\Omega_1 \delta(\omega - \tilde{E}_1 + \tilde{E}_3) = \int_0^{2\pi} d\phi_1 \int_0^\pi \sin\theta_1 d\theta_1$$

$$\times \delta\left[\omega - \frac{\cos\phi_1 \sin\theta_1 q_\perp + \cos\theta_1 q_z \tau_0^2/\tau^2}{\sqrt{1 - \cos^2\theta_1(1 - \tau_0^2/\tau^2)}}\right], \tag{A5}$$

where θ_1 and ϕ_1 are the polar and azimuthal angles of $\mathbf{p}_1$ in a coordinate system with z axis along the collision beam direction. For $\tau = \tau_0$ the prefactor in Eq. (A3) vanishes due to energy conservation. For $\tau \gg \tau_0$ we can ignore the $(q_z \tau_0/\tau)^2$ term in Eq. (A5), which then yields

$$I_1 \simeq \frac{2}{q_\perp} \int_{-1}^1 dx \int_{-1}^1 \frac{dy}{\sqrt{1 - y^2}} \delta\left(\frac{\omega}{q_\perp} - \frac{y\sqrt{1 - x^2}}{\sqrt{1 - bx^2}}\right), \tag{A6}$$

where $b = 1 - \tau_0^2/\tau^2$ and $x = \cos\theta_1$ and $y = \cos\phi_1$. Changing variables to $\sin\chi = x\sqrt{(1 - \omega^2/q_\perp^2)/(1 - b\omega^2/q_\perp^2)}$, this integral gives

$$I_1 = \frac{4}{q_\perp \sqrt{1 - b\omega^2/q_\perp^2}} \int_0^{\pi/2} d\chi \sqrt{1 - x^2}$$

$$\simeq \frac{4}{q_\perp \sqrt{1 - b\omega^2/q_\perp^2}}, \quad \text{for } \tau_0/\tau \ll 1. \tag{A7}$$

The angular integrals of $d\Omega_2$ yields the same factor as Eq. (A7) and the integral over energy transfers in Eq. (A4) thus gives at large times

$$\int d\omega\, I_1 I_2 = \int d\omega\, d\Omega_1\, d\Omega_2\, \delta(\omega - \tilde{E}_1 + \tilde{E}_3)\, \delta(\omega + \tilde{E}_2$$

$$- \tilde{E}_4) \simeq 16 \int_{-q_\perp}^{q_\perp} \frac{d\omega}{q_\perp^2 - b\omega^2} \simeq \frac{32}{q_\perp} \ln\left(\frac{2\tau}{\tau_0}\right). \tag{A8}$$

An angular dependence arising from Eq. (A3) should also be included when performing integration in Eq. (A5). For example, an extra factor of $\cos^2\theta_1$ leads to an additional factor of $1/3$ in Eq. (A8). In addition the matrix element depends on energy transfer through the transverse part of the self-energy in Eq. (24). After evaluating the integrals over momentum transfers this dependence is, however, only logarithmic in ω [16] and can be ignored.

Let us first consider the entropy production due to gluon-gluon scatterings and include quarks later. We assume for convenience that $\mu_0 \lesssim -T_0$ which allows us to use Boltzmann distribution functions, $f_0(E) = \exp[(\mu_0 - E)/T_0] = \lambda_0 \exp(-E/T)$. The momentum integrals of $\mathbf{p}_1$ and $\mathbf{p}_2$ in Eq. (A4) are straightforward, which leave one remaining integral over momentum transfer,

$$\left(\frac{\partial s}{\partial t}\right)_{\text{coll}} = \frac{3\nu_g^2}{2\pi^6} \alpha_s^2 \frac{\tau_0^3}{\tau^3} \ln\left(\frac{2\tau}{\tau_0}\right) T_0^4 \lambda_{0,g}^2 \frac{3}{2}$$

$$\times \int d^3q \frac{q^2}{q_\perp [q_\perp^2 + (q_z \tau_0/\tau)^2 + q_D^2]^2}, \tag{A9}$$

where, we have for simplicity approximated Π_L by q_D^2 in the matrix element of Eq. (22). Furthermore, we have replaced the transverse part of interaction by the longitudinal one times a factor $\langle\cos^2\phi\rangle=1/2$. Both these approximations are exact to leading logarithmic order in the coupling constant for the calculation of a number of transport coefficient [16]. Note that the limit for the integral over q_z is $\pm q_{max}\tau/\tau_0$ and the maximum momentum transfer, q_{max}, is determined by the distribution functions which cut off large momentum transfers. It has been estimated in [16] to be $q_{max}\sim 3T_0$. The integral over momentum transfers in (48) then gives $(\tau/\tau_0)^3\pi^2\ln(2q_{max}/q_D)$ when $\tau/\tau_0\ll q_{max}/q_D$. It includes the usual logarithm (see, e.g., [16] for details) as well as a factor $(\tau/\tau_0)^3$ from the integral over q_z. The entropy production in QCD to leading order in the coupling coupling constant is thus

$$\left(\frac{\partial s}{\partial t}\right)_{\text{coll}}=\frac{9}{8\pi^4}T_0^4\alpha_s^2\left[\nu_g^2\lambda_{0,g}^2+\frac{4}{9}\nu_g\nu_q\lambda_{0,g}\lambda_{0,q}\right.$$
$$\left.+\left(\frac{4}{9}\right)^2\nu_q^2\lambda_{0,q}^2\right]\ln\left[\frac{4q_{max}^2}{q_D^2}\right]\ln\left(\frac{2\tau}{\tau_0}\right). \quad (A10)$$

Here, we have included contributions from quarks and antiquarks to the entropy production; $\nu_g=16$, $\nu_q=12N_f$, $\lambda_q=e^{\mu_q/T}$ and we assume $\lambda_q=\lambda_{\bar{q}}$. The quark-gluon and quark-quark forward scattering interactions are smaller than the gluon-gluon ones by a factor $(4/9)$ and $(4/9)^2$, respectively. According to the models in Refs. [3,5,4] fewer quark and antiquark than gluons are produced in relativistic nuclear collisions, i.e., $\lambda_{0,q\bar{q}}\ll\lambda_{0,g}$.

The Debye screening mass in a quark-gluon gas in thermal and chemical equilibrium with no net baryon density (i.e., $\mu_g=\mu_q=\mu_{\bar{q}}=0$) is $q_D^2=4\pi(1+N_f/6)\alpha_sT^2$. Out of chemical equilibrium when $\mu_q=\mu_{\bar{q}}$ and $\mu_{g,q}\ll-T$ the Bose and Fermi distribution functions can be replaced by Maxwell-Boltzmann distribution functions and we find

$$q_D^2=\frac{24}{\pi}\left(\lambda_g+\frac{N_f}{3}\lambda_q\right)\alpha_sT^2. \quad (A11)$$

The quark and gluon densities and thus fugacities decrease with time due to the one-dimensional expansion such that $\lambda=\lambda_0\tau_0/\tau$.

Inserting these expressions for q_{max} and q_D in Eq. (A10) we find the entropy production rate

$$\left(\frac{\partial s}{\partial t}\right)_{\text{coll}}=\frac{9}{8\pi^4}\alpha_s^2T_0^4\left[\nu_g^2\lambda_{0,g}^2+\frac{4}{9}\nu_g\nu_q\lambda_{0,g}\lambda_{0,q}\right.$$
$$\left.+\left(\frac{4}{9}\right)^2\nu_q^2\lambda_{0,q}^2\right]\ln\left[\frac{3\pi\tau}{2\left(\lambda_{0,g}+\frac{N_f}{3}\lambda_{0,q}\right)\alpha_s\tau_0}\right]\ln\left(\frac{2\tau}{\tau_0}\right),$$
$$(A12)$$

to leading order in a free streaming gluon gas. The result is approximately valid for $\tau\gtrsim 2\tau_0$ and as long as the free streaming assumption is valid, i.e., $\tau\lesssim\theta$.

We emphasize the important result that τ^3 factor is cancelled by the integral over momentum transfers. Only a slow logarithmic dependence on time remains. The physical explanation for this cancellation is the following. From Eq. (A2) a factor $(\tau_0/\tau)^2$ appears from the substitutions of integration variables $p_{z,i}$, $i=1$, 2. This represents the fact that the densities of each of the colliding partons drop like τ_0/τ. If we keep the original momentum transfer variables [i.e., do not replace $q_z\to q_z\tau_0/\tau$ in Eq. (A2)] the factor in the square bracket, Eq. (A3), leads to a factor $(q\tau/\tau_0)^2$. This is because the phase space for small momentum scattering *opens up quadratically with time* and it *balances the decrease in parton densities*. On the other hand, for large momentum transfers the exponential in the square bracket, Eq. (A3), simply vanish leaving a factor of unity which leads to a much reduced entropy production rate decreasing quadratically with time. The long range interactions (small momentum transfers) are therefore very important in expanding plasmas and sensitive to screening or the cutoff as is applied in some models [3].

[1] J. P. Blaizot and A.H. Mueller, Nucl. Phys. **B289**, 847 (1987).

[2] K. Kajantie, P. V. Landshoff, and J. Lindfors, Phys. Rev. Lett. **59**, 2517 (1987); K. J. Eskola, K. Kajantie, and J. Lindfors, Nucl. Phys. **B323**, 37 (1989); K. J. Eskola, K. Kajantie, and J. Lindfors, Phys. Lett. B **214**, 613 (1989).

[3] K. Geiger and B. Müller, Nucl. Phys. **B369**, 600 (1992); K. Geiger, Phys. Rev. D **47**, 133 (1993).

[4] T. S. Biro, E. van Doorn, B. Müller, M. H. Thoma, and X.-N. Wang, Phys. Rev. **C 48**, 1275 (1993).

[5] K. J. Eskola and X.-N. Wang, Phys. Rev. D **48**, 1275 (1993).

[6] L. Xiong and E. Shuryak, Phys. Rev. C **49**, 2207 (1994).

[7] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).

[8] E. Shuryak, Phys. Rev. Lett. **68**, 3270 (1992).

[9] G. Baym, Phys. Lett. **138B**, 18 (1984).

[10] S. Gavin, Nucl. Phys. **B351**, 561 (1991).

[11] P. Danielewicz and M. Gyulassy, Phys. Rev. D **31**, 53 (1985).

[12] A. Hosoya and K. Kajantie, Nucl. Phys. **B250**, 666 (1985).

[13] S. Gavin, Nucl. Phys. **A435**, 826 (1985).

[14] G. Baym, H. Monien, C. J. Pethick, and D. G. Ravenhall, Phys. Rev. Lett. **64**, 1867 (1990); Nucl. Phys. **A525**, 415c (1991).

[15] G. Baym, H. Heiselberg, C. J. Pethick, and J. Popp, Nucl. Phys. **A544**, 569c (1992); H. Heiselberg, in *Proceedings of Pre-equilibrium Parton Dynamics in Heavy Ion Collisions*, Berkeley, 1993, edited by X.-N. Wang, Report No. LBL-34831 (unpublished), p. 125.

[16] H. Heiselberg, Phys. Rev. D **49**, 4739 (1994).

[17] M. H. Thoma, Phys. Rev. D **49**, 459 (1994).

[18] For relativistic particles the bulk viscosity vanishes to order α_s^2 in the coupling constant (see Refs. [12,13,15]). Higher order contributions arise from the trace anomaly [R. Pisarski (private communications)].

[19] M. Gyulassy and X.-N. Wang, Nucl. Phys. **B420**, 583 (1994);
X.-N. Wang, M. Gyulassy, and M. Plümer, Phys. Rev. D **51**,
3436 (1995).

[20] R. D. Field, *Application of Perturbative QCD*, Frontiers in
Physics Vol. 77 (Addison-Wesley, Reading, MA, 1989).

[21] R. Baier, Yu. L. Dokshitzer, S. Peigné, and D. Schiff, Phys.
Lett. B **345**, 277 (1995).

[22] A. Weldon, Nucl. Phys. **A566**, 581c (1994).

[23] M. H. Thoma and M. Gyulassy, Nucl. Phys. **B351**, 491 (1991).

[24] A. Weldon, Phys. Rev. D **26**, 1394 (1982).

[25] M. Gyulassy and A. Selikhov, Phys. Lett. B **316**, 316 (1993);
H. Heiselberg, Phys. Rev. Lett. **72**, 3013 (1994).

[26] X.-N. Wang and M. Gyulassy, Phys. Rev. D **44**, 3501 (1991).

[27] P. Lévai, B. Müller, and X.-N. Wang, Report No. LBL-36594,
1995, hep-ph/9412352.

[28] K. J. Eskola, K. Kajantie, and P. V. Ruuskanen, Nucl. Phys.
B323, 37 (1989); Report No. HU-TFT-94-6 (1994).

[29] X.-N. Wang, Nucl. Phys. **A590**, 47c (1995).