Dynamical analysis of the evolution of nuclear density modes

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We study the evolution of density waves in an expanding nuclear system taking into account nonlinearity and finiteness effects. The main differences with the standard spinodal decomposition are that the transition from the stable to the nonstable region is gradual, and that it takes place at densities higher than those associated to the spinodal line.

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In recent years emerged the concept that, after a heavy-ion collision at intermediate energies, the highly excited resulting nucleus expands and finally decays through fragment emission. In order to describe the instabilities that precede the decay of the nuclear system into fragments, Pethick and Ravenhall [1] considered the linearized hydrodynamical equations for infinite nuclear matter that lead to an anomalous dispersion relation for the density waves inside this system. Using this dispersion relation, the density-temperature phase diagram separates into stability and instability zones. This is the well-known spinodal decomposition for hot nuclear matter. Therefore, in this approach, when an initially homogeneous hot nucleus expands beyond a threshold value, some compression modes become unstable. According to these authors, such instabilities are responsible for the fragmentation of the nuclear system.

The picture mentioned above is based on a linear analysis of the growth of the amplitudes of the density modes and on the hypothesis of infinite nuclear matter. In a previous work dealing with this problem [2], we tried to remove these constraints. We performed a nonlinear analysis of the growth of these instabilities on a finite nucleus with spherical symmetry. Those calculations showed that the higher-order terms tend to stabilize the density modes that are unstable in the linear analysis. The average density of the system was, however, considered to be fixed in time. Therefore, such an analysis is valid only for cases in which the growth of the instabilities is much faster than the expansion of the nuclear system. In a later work [3], we studied the growth of the density modes in an expanding nucleus and showed that their growth and stability depend strongly on the initial conditions of the expansion.

Here we continue the previous work, performing a systematic study of the behavior of the system for a diversity of initial conditions. We show that the inclusion of the expansion dynamics leads to a division of the phase diagram into stable and unstable regions that of very different from the one found in the linear and quasistatic study of Ref. [1].

In order to describe the dynamical evolution of the nuclear system as it expands, we use the Euler and continuity equations for an irrotational ideal fluid, modeling the system as a spherical liquid drop at a uniform temperature. The pressure is given by the equation of state [3]

$$P = P(\rho) - \frac{\beta\rho^2}{\bar{\rho}}\nabla^2\rho, \tag{1}$$

where $P(\rho)$ is the pressure associated with the homogeneous nucleus, ρ is the density, $\bar{\rho}$ the average density, and $\beta=80$ MeV fm^5 the van der Waals parameter [4,5].

The nuclear density may be written as a superposition of radial compression modes [6],

$$\rho = \bar{\rho}\left(1 + \sum_n b_n j_0(k_n r)\right), \tag{2}$$

where $j_0(k_n r)$ are the zeroth-order spherical Bessel functions that satisfy the boundary condition $j_0(k_n R)=0$, in which R stands for the radius of the expanded nucleus, i.e., $k_n = n\pi/R$. In what follows we restrict our discussion to the $n=1$ mode, since it is the first to become unstable as the system expands. Using this model and assuming that the appearance of compression waves has little effect on the time evolution of the radius of the system, in the previous papers [2,3] we derived the following approximate equations for the first mode amplitude and the nuclear radius:

$$\ddot{R}(t) = \frac{20\pi}{3mA}P(R)R^2, \tag{3}$$

$$\frac{R\ddot{R}}{2} - \frac{R^2}{\pi^2}\ddot{b}_1\left[1+\frac{b_1}{2}\right] - \frac{2R\dot{R}}{\pi^2}\dot{b}_1\left[1+\frac{b_1}{2}\right] - \frac{R^2\dot{b}_1^2}{2\pi^2}\left[1+\frac{1}{\pi}\right] = \left[\frac{\partial P}{\partial\rho} + \beta\bar{\rho}k_1^2\right]\frac{b_1}{m} - \left[\frac{\partial P}{\partial\rho} - 3\beta\bar{\rho}k_1^2\right]\frac{b_1^2}{2m}, \tag{4}$$

where $R(t)$ is the radius of an isentropically expanding uniform nucleus [4], P is the pressure, expressed as a function of $R(t)$, A is the nuclear mass number, $\bar{\rho}=A/(4/3)\,\pi R(t)^3$ the space-averaged density, and b_1 the amplitude of the $n=1$ compression mode. We should keep in mind that the assumption of an isentropic expansion, generally employed in hydrodynamical treatments of the evolution of a hot nuclear system, may influence the appearance and growth of the density modes we purport to study. A recent comparison between

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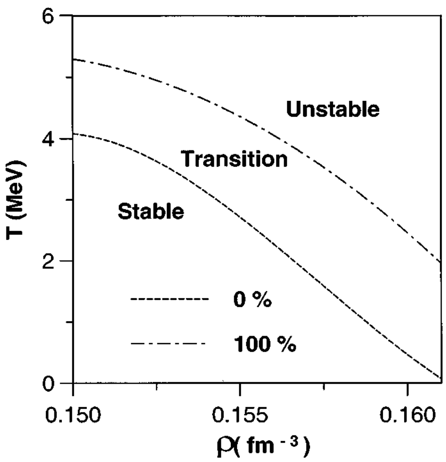

FIG. 1. Fragmentation probability as a function of the initial values of the temperature and density of the nuclear system.

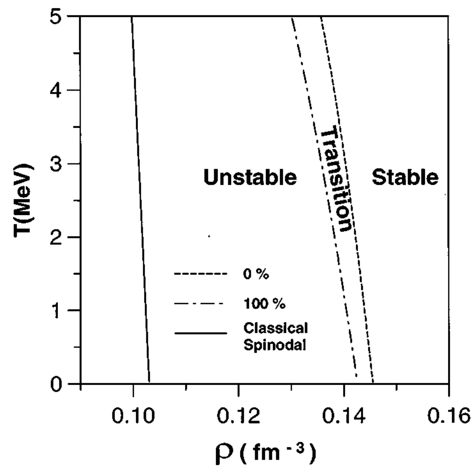

FIG. 2. Similar to Fig. 2, but for the conditions at the lowest densities reached during the dynamical expansion of the system. See text for further details.

molecular dynamics and Vlasov descriptions of the evolution has shown that, for some initial conditions, the average density has an oscillatory behavior in the Vlasov approach and a steadily decreasing one when studied through classical molecular dynamics [7]. It appears, from this study, that the expansion is isentropic until particles begin to be emitted. Since this effect is not included in our treatment, which is similar in spirit to the Vlasov description, we have limited the scope of our calculations to the low-temperature region, where, presumably, emission effects are not so important.

In order to fix the initial conditions for the evolution of the system, we assumed that it was prepared in thermodynamical equilibrium with an initial radius R_0 and an initial temperature T_0. The energy associated with the $n=1$ compression mode is assumed to be kT_0, in accordance with the equipartition theorem. Thus, the initial conditions $b_1, \dot{b}_1$ must satisfy the relationship

$$\frac{\pi^2\bar{\rho}}{k_1^3}\left[A_1 b_1^2+\frac{m}{k_1^2}\dot{b}_1^2\right]=kT_0, \tag{5}$$

where

$$A_1=\frac{\partial P}{\partial \rho}+\beta\bar{\rho}k_1^2, \tag{6}$$

kT_0 is the energy associated with the $n=1$ compression mode, and $k_1=\pi/R$. In Ref. [3] we showed that, for a given R_0, T_0, some initial conditions $b_1, \dot{b}_1$ lead to an indefinite growth of the $n=1$ mode, while for others the trajectories oscillate around the stable solution.

We now perform a systematic study, considering a large number of initial values of b_1 and $\dot{b}_1$, related in accordance with Eq. (5), for given initial values of the temperature and density. We determine how many trajectories of each set of initial conditions have amplitudes b_1 that grow indefinitely. We may assume, as in [1,3], that such large amplitudes correspond to the fragmentation of the nuclear system.

The results of these calculations, showing, as a function of the initial temperature and density, the fragmentation probability, are presented in Fig. 1. We notice that the ρ-T plane is divided into two large regions, corresponding to stability and instability with respect to the growth of the $n=1$ density mode, and a thin transition region where the fragmentation probability passes smoothly from 0 to 1. The reason why a high initial density or temperature leads to the instability of the system is that for them the system makes a deep excursion into the low-density regions where the breakup is more probable. This is illustrated in Fig. 2, where we give the fragmentation probability in a form similar to that of Fig. 1, but according to the minimum density reached in the dynamical evolution of the system, and its corresponding temperature. We note that, according to Eq. (3), this minimum is independent of the initial values of b_1 and $\dot{b}_1$. Thus there is a one-to-one correspondence between the initial conditions, depicted in Fig. 1, and the values of the density and temperature presented in Fig. 2.

In this latter diagram, the region to the right of the dashed line is the one associated with stability against the unbound growth of the $n=1$ density mode. This is because it corresponds to the least excited systems, which make short excursions into the lower-density regions and, as a consequence, tend not to fragment. On the other hand, for the opposite argument, the instability zone has moved to the low-density region to the left of the dot-dashed line. The transition region between these two zones is placed much to the right of the classical spinodal line, which is also depicted (solid curve) in this diagram. Figure 3 shows the dependence of the fragmentation probability on the minimum density reached by the system in its evolution for a cut through the transition region at a typical temperature.

We may consider the diagram presented in Fig. 2 as the equivalent of the standard phase diagrams of nuclear matter showing regions of stability and instability, such as the one presented in Ref. [1]. However, important differences should be pointed out. First of all, it includes information on the dynamical evolution that, eventually, leads the system to its fragmentation. This dynamical evolution, starting from well-defined initial conditions and following the hydrodynamical trajectory, leads to results very different from the linear analysis of Ref. [1] and even from the nonlinear, albeit static, of Ref. [2]. For this reason we call it a "dynamical instability diagram" (DID).

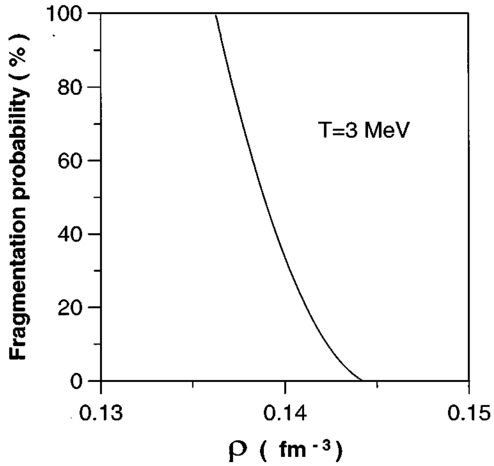

FIG. 3. Cut at $T=3$MeV through the transition region of Fig. 2.

The most important differences, however, are in the regions of instability. In the DID they lie well outside the standard spinodal region. The growth of the instabilities, leading to the breakup of the system, is thus strongly influenced by the noninfinitesimal values of the amplitude of the density mode. This is also related to the finiteness of the system in the present calculation, as opposed to standard phase diagrams which assume infinite nuclear matter. Furthermore, due to the finite time available for the growth of such modes, which is fixed by the dynamics of the nuclear expansion, not all trajectories starting with given conditions of density and temperature have similar breakup behavior. This leads to a fuzziness in the separation between the stable and unstable regions of nuclear matter in the DID diagram, very different from the abrupt transitions in the standard spinodal decomposition.

In conclusion, we have developed a new instability diagram, based on the dynamics of the evolution of the density modes, which is more appropriate than the spinodal decomposition to study the tendency to fragmentation. In particular, we have shown that the details of the dynamics of the expansion influence strongly the growth of the compression modes. The transition from stable to unstable behavior takes place over a region of the nuclear matter phase diagram, instead of over a sharp line, like in the standard spinodal decomposition, and is placed entirely outside this zone.

We should remark that although the study of the transition region was made for only the $n=1$ mode, its generalization to other radial modes is trivial. In this we have followed the spirit of Ref. [1], where the evolution of each compression mode was analyzed separately. However, in the dynamical evolution of the system coupling among different density modes and the feedback of their growth on the hydrodynamical expansion could play an important role. These effects will be discussed in a future work [8], based on a treatment of the expansion that lets the density evolve without specific reference to the possible radial modes [9].

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[1] C.J. Pethick and D.G. Ravenhall, Nucl. Phys. **A471**, 19c (1987).

[2] R. Donangelo, A. Romanelli, and A.C. Sicardi Schifino, Phys. Lett. B **263**, 342 (1991).

[3] R. Donangelo, A. Romanelli, H. Schulz, and A.C. Sicardi Schifino, Phys. Rev. C **49**, 3182 (1994).

[4] K. Sneppen. Ph.D. thesis, University of Copenhagen, 1989.

[5] H.A. Bethe, Phys. Rev. **167**, 879 (1968); J.A. Lopez and G. Lübeck, Phys. Lett. B **219**, 215 (1988).

[6] A. Bohr and B.R. Mottelson, *Nuclear Structure* (Benjamin, Reading, MA, 1975), Vol. II, p. 666.

[7] M. Belkacem, V. Latora, and A. Bonasera, Phys. Lett. B **326**, 21 (1994); Phys. Rev. C **52**, 271 (1995).

[8] C.E. Aguiar, R. Donangelo, H.D. Marta, A. Romanelli, and A.C. Sicardi Schifino (unpublished).

[9] C.E. Aguiar, R. Donangelo, C.O. Dorso, R.S. Gomes, and N. Gonçalves, Phys. Rev. C **46**, 1069 (1992).