Thermostatic properties of semi-infinite polarized nuclear matter

S. Ramadan, M. Abd-Alla, and M. Y. M. Hassan
Physics Department, Faculty of Science, Cairo University, Giza, Egypt
(Received 2 March 1995)

The surface and curvature properties of semi-infinite polarized nuclear matter (SPNM) are calculated using an expansion for the Fermi integrals up to T^2. A density matrix expansion is obtained for a modified form of the Seyler-Blanchard interaction. New parameters that characterize the surface and curvature properties of SPNM are introduced. The level density parameter is extracted from the low temperature expansion of the free energy and compared with previous calculations. A reasonable agreement is obtained with the parameters calculated before.

PACS number(s): 21.65+f, 21.30.Fe

I. INTRODUCTION

One of the most exciting features of finite and semi-infinite quantum systems is their surfaces [1,2]. The surface region of a nucleus greatly influences its binding energy, level spectra, and reactions.

One of the widely used models in studying these finite and semi-infinite systems is the liquid drop model (LDM). In the liquid drop model, the binding energy of a nucleus is written as a sum of volume, surface, curvature, and Coulomb terms. The volume term of polarized nuclear matter (PNM) was studied before using Seyler-Blanchard [3] and effective Skyrme interactions [4]. Surface and curvature energies are important in studying heavy ion collisions and in astrophysical applications [5]. Surface and curvature energies are also instrumental in determining the size distribution of fragments which are emitted from the expanding collision for temperatures less than the critical temperature T, at which the liquid-gas phase transition occurs [6–8]. These are crucial in calculating fission barrier heights and shapes of saddle point configurations [9]. Also, in describing neutron stars [10] and supernovas [11–13] it is important to determine the sizes of nuclei, electron capture rates [14], and level densities [15,16], all of which are sensitive to surface and curvature energies. The thermodynamical properties of nuclei have been studied theoreticaly by several authors [17–25] using Hartree-Fock (HF) or extended Thomas-Fermi (ETF) [26–30] methods at finite temperature. In fully self-consistent HF and ETF calculations at $T>0$ [17–26,28], the T dependence of surface and curvature terms is taken into account.

The surface symmetry energy is slightly increased with excitation energy [31–34], and it is a relevant quantity in studying nuclear fission. That is why knowledge of its variation with temperature has to be considered.

The surface energy term of the LDM, the semi-infinite slab model, originally suggested by Bethe [35] and later developed by Swiatecki [36], was used to explore the sensitivity of the underlying nucleon-nucleon (N-N) interactions. The study of the surface properties of semi-infinite nuclear matter (SNM) is usually done by two methods. In the first method, the surface energy is calculated by extracting from the total energy of SNM a reference energy which represents the bulk contribution. The second method is based on performing a T^2 approximation for the free energy and then a leptodermous expansion in powers of A to account for nuclear geometry [2,31,32,37–39].

Dividing the energy of the system into volume and surface terms means that the curvature energy is either included by rescaling the surface energy term [40] or neglecting it [4,41]. It is straightforward to show [37,42] that the curvature energy may be simply extracted from the SNM surface energy. Chu *et al.* [43] extracted the curvature energy term from semiclassical calculations of finite nuclei. For realistic nuclei the contribution of the curvature energy to the total energy is not insignificant and there is no reason to ignore it [28]. A model calculation has been done [41,44] by extracting the curvature energy from the surface energy of SNM. Farinel [45] used the soluble model of SNM to derive a closed expression for the curvature energy as a function of the surface profile asymmetry. Brack *et al.* [28] performed very accurate semiclassical variational calculations for the curvature energy term. Several attempts to carry out HF calculations for the curvature energy using the leptodermous expansion resulted in unacceptably large values with the same interaction. There are large variations in the curvature energy and the curvature symmetry energy among the results for different interactions [42].

Studying the temperature effect on the bulk properties of SNM leads to a sensitive quantity, namely, the level density parameter, which is a good quantity for testing the theoretical calculations. (Applying the methods of statistical mechanics, Weisskopf [46] and Bethe [35,47] introduced the level density parameter.) These calculations have been restricted to relatively low excitations comapared to those encountered in heavy ion collisions [48]. The nuclear level density parameter extracted from experimental data leads to the value $a_{v0}=0.125A$ MeV^{-1} [9,16,49,50], while that calculated using Skyrme forces, including volume effects only, gives $a_{v0}=0.055A$ MeV^{-1} [17,31,39]. This difference reflects the importance of including surface, curvature, and symmetry corrections in calculating the nuclear level density parameter.

The aim of this work is to test the interaction used before [3] and to study the effect of symmetry excess parameters on the surface and curvature properties of SNM. We also estimate their temperature dependence by studying the effect of these parameters on the level density parameter.

In this model, we start with a two-body (extended Seyler-Blanchard [3]) interaction to calculate the energy of semi-

infinite polarized nuclear matter (SPNM); then, we expand the density matrix in the relative coordinates up to second order to account for the gradient term of the potential. We use the extended. Thomas-Fermi form of kinetic energy to write the total energy of the system. At low temperature we perform an expansion in the T^2 approximation of the thermodynamical quantities to get the free energy. Using the Woods-Saxon form of the density, we expand the free energy in powers of $A^{1/3}$, where analytical expressions for the surface and curvature properties as a function of the potential parameters can be obtained up to second order in the symmetry excess parameter and temperature. We use these formulas to analyze the effect of temperature and symmetry excess parameters on the surface and curvature properties of semi-infinite polarized nuclear matter. Our results are compared with the other calculations. New quantities, such as a_{vi}, a_{si}, and a_{ci}, are introduced and discussed. In Sec. II the model and calculations are given. Section III contains the results and discussion, while Sec. IV is devoted to conclusions.

II. MODEL AND CALCULATIONS

The direct two-body matrix element between a pair of nucleons in states i and j is given by

$$\langle ij|V|ij\rangle=\int \rho_i(\vec{r}_1)\rho_j(\vec{r}_2)V(r,s)d\vec{r}d\vec{s}, \tag{1}$$

where $\vec{r}=\frac{1}{2}(\vec{r}_1+\vec{r}_2)$ is the center of mass coordinates and $\vec{s}=(\vec{r}_1-\vec{r}_2)$ is the relative coordinate. Expanding ρ, in powers of s up to second order [51], we obtain

$$\rho_i(r_1)=\rho_i(r_1)+\tfrac{1}{2}\vec{s}\cdot\nabla\rho_i(r)+\tfrac{1}{8}\vec{s}\vec{s}:\nabla\nabla\rho_i(r). \tag{2}$$

Substituting into Eq. (1) and integrating by parts over r, we get $\langle ij|V|ij\rangle=\int\rho_i(r)\rho_j(r)V(r,s)d\vec{r}d\vec{s}$

$$-\frac{1}{6}\int\nabla\int\rho_i(r)\cdot\nabla\rho_j(r)V(r,s)d\vec{r}\,s^2\,d\vec{s}. \tag{3}$$

In our model we use the two-body interaction of Seyler-Blanchard [3,52,53], with separation s and relative momentum p, which is a Yukawa force plus a momentum-dependent term. This interaction is

$$V(r,s)=-C_{1,u}\frac{e^{-s/a}}{s/a}\left(1-\frac{p^2}{b^2}\right)=V(r)V(s), \tag{4}$$

where

$$V(r)=-C_{1,u}\left(1-\frac{p^2}{b^2}\right) \tag{5}$$

and

$$V(s)=e^{-s/a}/(s/a). \tag{6}$$

The potential energy (E_p) of polarized NM is given in Appendix A where the exchange term is neglected [3].

For the kinetic energy, we used the extended TF formula. This formula was introduced to account for the density variation near the surface by including gradient-type corrections. The kinetic energy density in the ETF is given by

$$E_{\rm kin}=\frac{\hbar^2}{2m}\int\left[\alpha\rho_{\tau,s}^{5/3}+\beta\frac{(\nabla\rho_{\tau,s})^2}{\rho_{\tau,s}}+\gamma\nabla^2\rho_{\tau,s}\right]d\vec{r}, \tag{7}$$

where $\tau=$ neutron (n) or proton (p), and $s=$ spin up ($\uparrow$) or down ($\downarrow$),

$$\alpha=\tfrac{3}{5}(3\pi^2/2)^{2/3},$$

$$\beta=\frac{1}{36}\left[1-\frac{21\pi^2}{12\times8}\left(\frac{3\pi^2}{2}\right)^{4/3}T^2\rho^{4/3}+\frac{3\times31\pi^4}{12^2\times8}\left(\frac{3\pi^2}{2}\right)^{8/3}T^2\rho^{8/3}\right]. \tag{8}$$

At $T=0$, the values of β and γ are taken to be [43,54–56]

$$\beta=\tfrac{1}{36},\quad \gamma=\tfrac{1}{3}. \tag{9}$$

The nuclear density is calculated using the restricted variational method. We take for this purpose the Woods-Saxon form for the density. This choice has been shown to give reliable results for both the surface energy and the diffuseness parameters [2,41,42,57,58]. In our model, the parameters of the Woods-Saxon function, namely, the diffuseness parameter d and the half-value radius R, for SPNM are put in a general form, namely, $d_{\tau,s}$ and $R_{\tau,s}$, respectively. However, the diffuseness paramters for neutrons and protons are nearly equal [42] (exactly equal for small neutron excesses) as can be demonstrated by considering the two coupled Euler-Lagrange equations, and consequently $d_{\tau,s}=d$.

The Woods-Saxon form for SPNM becomes

$$\rho_{\tau,s}=\frac{\rho_{0\tau,s}}{1+\exp[r-R_{\tau,s}]/d}, \tag{10}$$

where $\rho_{0\tau,s}$ is the asymptotic density deep inside the system. The relation between $\rho_{0\tau,s}$ and the density ρ_0 of NM is given by

$$\begin{aligned}\rho_{0n\uparrow}&=\tfrac{1}{4}\rho_0(1+X+Y+Z),\\ \rho_{0n\downarrow}&=\tfrac{1}{4}\rho_0(1+X-Y-Z),\\ \rho_{0p\uparrow}&=\tfrac{1}{4}\rho_0(1-X+Y-Z),\\ \rho_{0p\downarrow}&=\tfrac{1}{4}\rho_0(1-X-Y+Z),\end{aligned} \tag{11}$$

where X, Y, and Z are the excess parameters defined by [3]

$$X=\frac{N\uparrow+N\downarrow-Z\uparrow-Z\downarrow}{A}=\text{neutron excess parameter},$$

$$Y=\frac{N\uparrow-N\downarrow+Z\uparrow-Z\downarrow}{A}$$

$$=\text{spin-up nucleon excess parameter}, \tag{12}$$

$$Z=\frac{N\uparrow-N\downarrow+Z\uparrow+Z\downarrow}{A}$$

$$=\text{spin-up neutron excess parameter},$$

where $N\uparrow$ or $N\downarrow$ is the number of neutrons with spin up or down and $Z\uparrow$ or $Z\downarrow$ is the number of protons with spin up or down.

The relation between the number of particles and their densities is given by

$$\int \rho_{n\uparrow}d\vec{r}=N\uparrow, \quad \int \rho_{n\downarrow}d\vec{r}=N\downarrow,$$
$$\int \rho_{p\uparrow}d\vec{r}=Z\uparrow, \quad \int \rho_{p\downarrow}d\vec{r}=Z\downarrow. \tag{13}$$

These integrations are solved analytically (see Appendix B) to give

$$\int \rho_{\tau,s}d\vec{r}=\frac{4\pi}{3}\rho_{0\tau,s}(R_{\tau,s}^3+\pi^2d^2R_{\tau,s}). \tag{14}$$

Solving for $R_{\tau,s}$ we get

$$R_{n\uparrow}=R_{n\downarrow}=R_{p\uparrow}=R_{p\downarrow}=R=(3A/4\pi\rho_0)^{1/3}+\frac{\pi^2d^2}{3}\left(4\frac{\pi\rho_0}{3A}\right)^{1/3}. \tag{15}$$

This means that in our calculations the half-value radius depends neither on the spin nor on the isotropic spin of the nucleons. This gives a zero neutron skin, which is the assumption of the liquid drop model and the compressible liquid drop model [40]. This has been shown, experimentally [59], to be a good assumption for ordinary nuclei. However, for neutron-rich neutron star matter nuclei, this may not be a good assumption.

With these simplifications we see that the total energy contains terms like $\int\rho^q d\vec{r}$ and $\int\rho^p\rho^q d\vec{r}$.

Following the method adopted by Srivastava [60], the integrals $\int\rho^q d\vec{r}$ can be approximated in the form

$$\int \rho_{\tau,s}^q d\vec{r}=\frac{4\pi}{3}\rho_{0\tau,s}[R^3-3A_1(q)R^2d+6A_2(q)Rd^2-6A_3(q)d^3], \tag{16}$$

where the coefficients $A_n(q)$ are given by

$$A_n(q)=\frac{1}{(n-1)!}\int_0^\infty[(1+e^{-x})^{-q}+(-1)^n(1+e^{-x})^{-q}]x^{n-1}dx. \tag{17}$$

Using the same technique, we found (see Appendix B) that

$$\int \rho_{\tau,d}^q\rho_{\tau,s}^p d\vec{r}=\frac{4\pi}{3}\rho_{0\tau,s}\rho_{0\tau',s'}[R^3-3A_1(p+q)R^2d+6A_2(p+q)Rd^2+6A_3(p+q)d^3]. \tag{18}$$

Applying these simplifications, we can calculate the total energy E and correspondingly the free energy F ($F=E-ST$, where S is the entropy). The expression for S in the case of polarized NM is taken [3], using the Fermi-liquid approximation of Landau, to be

$$S=\frac{1}{\rho}\sum_{\tau,s}\frac{1}{(2\pi)^3}\int d\vec{k}[\rho_{\tau,s}\ln(\rho_{\tau,s})+(1-\rho_{\tau,s})\ln(1-\rho_{\tau,s})], \tag{19}$$

which can be written in terms of Fermi integrals as

$$S=\frac{1}{\rho}\sum_{\tau,s}\rho_{\tau,s}\left[\frac{5}{3}\frac{j_{3/2}(\eta_{\tau,s})}{3j_{1/2}(\eta_{\tau,s})}-\eta_{\tau,s}\right]. \tag{20}$$

In the low temperature limit, the entropy per nucleon becomes

$$S=\frac{\Pi^2}{4}\sum_{\tau,s}\rho_{\tau,s}\left[\frac{2m}{h^2b_{\tau,s}k_f^2}\right]T+0(T^3). \tag{21}$$

This equation can be expanded to second order in X, Y, and Z [4] as

$$S=S_v+X^2S_x+Y^2S_y+Z^2S_z, \tag{22}$$

where

$$S_v=\pi^2mT/h^2b_{\tau,s}K_f^2 \tag{23}$$

and

$$S_x=S_y=S_z=-\pi^2mT/9h^2b_{\tau,s}K_f^2.$$

We can express the total free energy as a sum of volume, surface, and curvature terms. Each term is a function of the potential parameters, the density at saturation, and the surface diffuseness parameter. The resulting expression is written as (see Appendix A)

$$F=F_vA+F_sA^{2/3}+F_cA^{1/3}, \tag{24}$$

where

$$F_v=F_{v0}+F_{vx}X^2+F_{vy}Y^2+F_{vz}Z^2, \tag{25}$$

$$F_s=F_{s0}+F_{sx}X^2+F_{sy}Y^2+F_{sz}Z^2, \tag{26}$$

$$F_c=F_{c0}+F_{cx}X^2+F_{ey}Y^2+F_{cz}Z^2. \tag{27}$$

The surface diffuseness parameter d is obtained (as a function of the symmetry parameters and temperature) by minimization of the volume free energy with respect to d; thus, $\partial F/\partial d=0$. Since the volume free energy is independent of d and the curvature energy depends quadratically on d, we have

$$\frac{\partial F}{\partial d}=\frac{\partial F_s}{\partial d}=0.$$

If we write

$$F_s=P(\rho_0)d+N(\rho_0)/d,$$

TABLE I. Coefficients of the density expansion [Eq. (31)]. The units of these coefficients are fm^{-3}. (PW: present work.)

ρ_{0n}	ρ_x	ρ_y	ρ_z	ρ_t	ρ^1	Ref.
0.145	1.176	1.214	1.115	0.002	−0.0002	PW3
0.144	0.09	1.62	0.84	0.012		[4]
0.225	0.49					[62]
0.185	0.45					[56]

then

$$d=[N(\rho_0)/P(\rho_0)]^{1/2}, \tag{28}$$

where

$$N(\rho_0)=N_0+N_xX^2+N_yY^2+N_zZ^2,$$

$$P(\rho_0)=P_0+P_xX^2+P_yY^2+P_zZ^2,$$

where the functions N_0, N_i, ρ_0, and ρ_i are given in Appendix C.

The density in the above equations is the equilibrium density for zero temperature symmetric NM. The equilibrium density, for zero temperature NM with a neutron excess, was deduced by Weiss and Cameron [61], Bethe [62], and Dworzecka [63]. Hassan *et al.* [4,64] generalized the equilibrium density for nonzero temperature and polarized NM. In SPNM, to get the equilibrium density as a function of the symmetry excess parameters and temperature, we minimize the free energy with respect to ρ_0. Since A is finitely large in the case of SPNM, we have [65]

$$\frac{\partial F}{\partial \rho_0}=\frac{\partial F_v}{\partial \rho_0}=0. \tag{29}$$

Following the strategy adopted by Hassan *et al.* [4], we get ρ_0 up to second order in X, Y, Z, and T as

$$\rho_0=\rho_{0n}\left[1-\frac{9\rho_{0n}}{K}\frac{\partial F_x}{\partial \rho_{0n}}\bigg|\rho_{0n}X^2+\frac{\partial F_y}{\partial \rho_{0n}}\bigg|\rho_{0n}Y^2+\frac{\partial F_z}{\partial \rho_{0n}}\bigg|\rho_{0n}Z^2+\frac{\pi^2mT}{3\rho^{5/3}}\left(\frac{2}{3\pi^2}\right)^{2/3}\right] \tag{30}$$

and we get

$$\rho_0=\rho_{0n}[1-\rho_\tau T^2-(\rho_x-\rho^1T^2)X^2-(\rho_y-\rho^1T^2)Y^2-(\rho_z-\rho^1T^2)Z^2]. \tag{31}$$

To calculate the surface and curvature properties at the equilibrum density ρ_{0n}, we have to rewrite the functions $N(\rho_0)$ and $P(\rho_0)$ in terms of ρ_{0n} (see Appendix C).

The surface and curvature properties as well as the diffuseness parameter have been calculated at the equilibrium density of symmetric NM. The results up to second order in X, Y, Z, and T are of the form

$$d=d_0+\alpha_0T^2+(d_x+\alpha_xT^2)X^2+(d_y+\alpha_yT^2)Y^2+(d_z+\alpha_zT^2)Z^2, \tag{32}$$

$$F=F_{s0}+a_{s0}T^2+(F_{sx}+a_{sx}T^2)X^2+(F_{sy}+a_{sy}T^2)Y^2+(F_{sz}+a_{sz}T^2)Z^2, \tag{33}$$

$$F_c=F_{c0}+a_{c0}T^2+(F_{cx}+a_{cx}T^2)X^2+(F_{cy}+a_{cy}T^2)Y^2+(F_{cz}+a_{cz}T^2)Z^2. \tag{34}$$

Analytic expressions are obtained for all the parameters appearing in Eqs. (32)–(34) (see Appendix C).

It is straightfoward to deduce the level density parameter from the dependence of the total free energy as well as the total energy on T^2 through the low temperature expansion [22,39] and using the TF or ETF formula (for kinetic energy). This gives voume (a_v), surface (a_s), and curvature (a_c) level density parameters. The results are

$$a_i=a_{i0}+a_{ix}X^2+a_{iy}Y^2+a_{iz}Z^2, \quad i=v,s,c, \tag{35}$$

where a_{v0} is obtained as

TABLE II. Coefficients of the diffuseness parameter expansion equation (32). The units of d_i are fm and the units of α_i are $\mathrm{fm\,MeV}^{-2}$. (PW: present work.)

Force \ Parameter	d_0	d_x	d_y	d_z	$\alpha_0\times10^3$	$\alpha_x\times10^3$	$\alpha_y\times10^3$	$\alpha_z\times10^3$
PW3	0.431	−0.01	−0.01	−0.01	0.0025	0.0043	0.0024	0.0074
PW2	0.431	0.36	0.37	0.35	0.0025	0.023	0.022	0.025
PW1	0.431	0.36	0.37	0.35	0.0038	0.040	0.40	0.041
SII	0.421				0.0035			
SIII	0.398 [a]				0.0033 [a]			
SIV	0.445 [a]				0.0037 [a]			
SV	0.467 [a]				0.0038 [a]			
SVI	0.394 [a]				0.003 [a]			
SKT	0.487 [a]				0.0043 [a]			
SEI	0.469 [a]				0.0041 [a]			

[a]Reference [32].

TABLE III. Coefficients of the surface free energy expansion [Eq. (33)]. The units of F_{si} are MeV and the units of a_{si} are MeV^{-1}.

Force \ Parameter	$-F_{s0}$	$-F_{sx}$	$-F_{sy}$	$-F_{sz}$	a_{s0}	a_{sx}	a_{sy}	a_{sz}
PW3	215	−31.0	−30.0	−32.0	−0.145	0.320	0.259	0.419
PW2	21.5	−27.0	−26.2	−28.2	−0.145	1.018	0.979	1.08
PW1	21.5	−27.0	−26.2	−28.2	−0.191	1.616	1.6091	1.627
SII	19.65 [a]	52.0 [b]			−0.189 [a]			
	20.08 [c]				−0.199 [c]	−0.9174 [c]		
	20.17 [d]	60.0 [d]						
SIII	18.9 [a]	29.0 [b]			0.183 [a]			
	19.29 [c]	47.9 [c]			−0.194 [c]	0.0171 [c]		
	18.89 [e]	33.2 [e]						
	20.37 [f]				−0.233 [g]			
	18.79 [d]	35.00 [d]						
	18.04 [h]	88.11 [h]						
SIV	20.28 [a]	55.00 [b]			−0.193 [a]			
	18.75 [c]	29.6 [c]			−0.185 [c]	0.0161 [c]		
	20.12 [d]	64.00 [d]						
SV	21.15 [a]	74.00 [b]			−0.195 [a]			
	21.21 [d]	97.00 [d]						
SVI	18.31 [a]	22.00 [b]			−0.176 [a]			
	18.75 [c]	23.20 [c]			−0.185 [c]			
	18.13 [d]	25.00 [d]			−0.218 [g]			
SKM	18.72 [a]	48.2 [c]			0.22 [c]	0.0181 [c]		
	18.83 [e]	58.9 [e]						
	19.16 [f]				0.228 [g]			
	17.34 [d]	57.00 [d]						
SKM*	19.37 [c]	47.9 [c]			−0.253 [c]	0.0188 [c]		
	19.06 [e]	58.0 [e]						
	17.96 [d]	55.00 [d]						
	17.22 [h]	60.37 [h]						
SKT	19.54 [a]	53.00 [b]						
SI	17.96 [e]	38.9 [e]						
SKa	19.57 [d]	74.00 [d]						
	18.52 [h]	71.29 [h]						
SKM(ITF)	61.61 [h]	58.64 [h]			−0.139 [g]			

[a]Reference [32].
[b]Reference [65].
[c]Reference [31].
[d]Reference [34].
[e]Reference [42].
[f]Reference [66].
[g]Reference [26].
[h]Reference [30].

$$a_{v0}=\frac{\pi^2 m}{3\hbar^2\rho_0^{2/3}} \tag{36}$$

and

$$a_{vx}=a_{vy}=a_{vz}=-\tfrac{1}{9}a_{v0}. \tag{37}$$

III. RESULTS AND DISCUSSION

In previous work [3] we have studied the bulk properties of polarized nuclear matter (PNM), focusing our attention on the equation of state. Here we restrict ourselves to the study of the surface and curvature properties of SPNM using the extended form of the Seyler-Blanchard interaction introduced before. Our extended potential has a set of fitting parameters, and we took into account only the direct term.

For the kinetic energy density, we used the ETF formula which contains gradient and Laplacian terms to account for variation of the density near the surface. The Laplacian term is equal to zero as a result of neglecting the neutron skin thickness. We considered $\beta=1/36$ [43,54–56], and we found

TABLE IV. Same as Table III but for the curvature free energy expansion [Eq. (34)].

Force \ Parameter	F_{c0}	$-F_{cy}$	$-F_{cx}$	$-F_{cz}$	a_{c0}	a_{cx}	a_{cy}	a_{cz}
PW3	9.9	20.0	19.3	21.0	0.429	0.229	0.223	0.329
PW2	9.9	17.0	16.0	18.0	0.429	0.358	0.356	0.431
PW1	9.9	17.0	16.0	18.0	0.422	0.440	0.445	0.431
SI	6.49 [a]	27 [a]						
SII	11.6 [b]							
SIII	7.22 [a]	26 [a]						
	11.29 [c]				0.103 [d]			
	10.00 [b]							
	8.66 [e]	49.32 [e]						
SIV	12.10 [b]							
SV	13.8 [b]				0.039 [d]			
SVI	9.30 [b]				0.166 [d]			
SKM	9.99 [a]	51 [a]						
	15.35 [c]				0.175 [d]			
	12.9 [b]							
	12.31 [e]	37.27 [e]						
SKM*	10.63 [a]	52 [a]						
	14.50 [b]							
	14.13 [e]	40.7 [e]						
	10.24 [f]							
SKa	13.6 [b]							
	13.88 [e]	47.28 [e]						
SKM(ITF)					0.267 [d]			

[a]Reference [42].
[b]Reference [34].
[c]Reference [66].
[d]Reference [26].
[e]Reference [30].
[f]Reference [58].

that a large value of β gives rise to a large value of the surface energy [43,54–56].

The results below are carried out for different forms of ρ_0, namely,

$$\rho_0=\rho_{0n}, \tag{38}$$

$$\rho_0=\rho_{0n}(1-\rho_T T^2), \tag{39}$$

and the full expansion form of ρ_0 given by Eq. (31). Equations (38), (39), and (31) are referred to in the text as (PW1), (PW2), and (PW3), respectively.

Table I gives the coefficients of the density expansion [Eq. (31)] compared with previous calculations. We notice that there is fair agreement between the values of the coefficients ρ_y and ρ_z with those of Hassan *et al.* [4], but there is a difference in the values of the coefficient ρ_x. It has been shown [61] that these coefficients are strongly correlated with the values of E_x, E_y, and E_Z. In our case, we have $E_x=33.4$ MeV, which differs from that used by Hassan *et al.* [4] ($E_x=28$ MeV), while the values of E_y and E_Z are nearly the same. This is reflected in the agreement of ρ_y and ρ_Z in both calculations.

The half-value radius R can be easily deduced from the relation $R=(3A/4\pi\rho_0)^{1/3}$, keeping only the $A^{1/3}$ term in the expansion of R as a first approximation [9]. Using Eq. (31) we get

$$R=(3A/4\pi\rho_{0n})^{1/3}[1+\tfrac{1}{3}\rho_t T^2+\tfrac{1}{3}(\rho_x-\rho^l T^2)X^2$$

$$+\frac{1}{3}(\rho_y-\rho^l T^2)Y^2+\tfrac{1}{3}(\rho_z-\rho^l T^2)Z^2]. \tag{40}$$

From this equation, the half-value radius R increases with increasing X, Y, Z, and T. The coefficient $\rho^l T^2$ has a small effect in the range of temperature considered (9–12 MeV).

Table II gives the coefficients of the expansion of the diffuseness parameter d, Eq. (32). The value of d_0 is slightly less than the empirical value ($d_0=0.5$ fm [51]). This result can be modified by using the Fermi distribution for the density [31,42]. The parameter α_0 has different values corresponding to the application of different forms of ρ_0. It is also sensitive to the two-body interaction used [32].

To the best of our knowledge, the values d_x, d_y, d_z, α_x, α_y, and α_z have not been reported before. The effect of d_i ($i=x,y,z$) is to decrease the diffuseness parameter in the case of PW3 [9]. An opposite effect is obtained for the two cases PW1 and PW2. This is in agreement with the other calculations [62,66]. The effect of α_i is to decrease (increase) the diffuseness parameter in case of PW2 (PW1 and PW3).

The parameters of the surface energy, Eq. (33), are listed in Table III. The values of F_{s0} are in reasonable agreement with the previous calculations [67,68]. The value of a_{s0} is very sensitive to the form of the density used (PW1, PW2, and PW3), and it varies largely with the type of the force used. We notice from Table III that the values found in the literature for the surface symmetry energy F_{sx} range between -22 and 97 MeV [29,30,50] and our value of F_{sx} lies in this range. The surface free energy for the symmetric unpolarized system ($X=Y=Z=0$) vanishes at temperatures between 10.6 and 12.1 MeV for PW1 and PW2. This temperature range is critical, because in this temperature range, the symmetry free energies have very large negative values and the diffuseness parameter increases to very large positive values. This critical temperature may be understood from the fact that with the increasing of temperature, the system undergoes a phase transition from the liquid to the gas phase. This phase transition starts at a critical temperature $T_{c1}=9$ MeV [3] and continues until a layer of the gas phase covers the surface of the system at the temperature $T_c=12$ MeV at which the surface energy vanishes [3]. A similar critical temperature was obtained by Stocker and Burzlaff [26,49] for symmetric unpolarized NM. It was also found in the HF calculations of Bonche *et al.* [18,19,22] that there exists a limiting temperature ($T_c=8-10$ MeV) beyond which the nucleus becomes unstable. At this temperature, no solution for the HF equations can be obtained. The same kind of instability was found by Bonche and Levitt [19] and Krappe *et al.* [69], using a semiclassical approximation for hot nuclei ($T_c=8$ MeV). The parameters F_{sy} and F_{SZ} have a similar effect on F_S as F_{SX} has; namely, F_S decreases with the increasing of X, Y, and Z. The parameters a_{si} differ for different forms of ρ_0. The value of a_{sx} is larger than that obtained numerically in Ref. [31].

The coefficients of the expansion of the curvature free energy [Eq. (34)] are listed in Table IV. F_{c0} agrees with the known theoretical value of 10 MeV [70]. If we take into account the compression of the bulk [71], one obtains a smaller value of $F_{c0}=5.64$ MeV. This value is in agreement with the findings of the recent analysis of nuclear masses and fission barrier heights [69,72]. The curvature symmetry energy F_{cx} agrees with the values extracted from the thermodynamic potential of Kolehmainen *et al.* [42]. More calculations are still necessary to determine the precise value of F_{cx}.

The surface and curvature parameters a_{si} and a_{ci}, discussed before, are essentially those which appear in the level density parameter formula [Eq. (35)]. The volume level density parameters [Eq. (36)] are given in Table V for PW3. The results for PW1 and PW2 are the same as those of PW3 for a_{v0}, but differ slightly for a_{vi}. The value of a_{v0} is in agreement with the value [8,61,73–76] $a=(1/8.8)A$ MeV^{-1}. The values of a_{vi} are directly related to a_{v0}. The symmetry effects have been found to be negligible. In order to have a closer assessment of the level density parameter a ($a=a_vA+a_sA^{2/3}+a_cA^{1/3}$), it is depicted in Fig. 1 together with previous results as well the experimental data [70]. We notice that the agreement between our results and the experimental values is reasonable. It has been stressed before [39,77] that the value of the level density parameter depends on the value of the effective mass m^*, and we obtain better results when the value of the effective mass m^* is near to that of the bare mass m. In our case, $m^*/m=1$.

TABLE V. Volume level density parameters [Eqs. (35) and (36)] in units of MeV^{-1}.

Force	a_{v0}	a_{vx}	a_{vy}	a_{vz}
PW3	0.075	-0.008	-0.008	-0.008
SII	0.041 [a]	-0.0035 [b]		
SIII	0.054 [b,c]	-0.0061 [c]		
SIV	0.068 [b]	-0.0082 [b]		
SV	0.026 [c]			
SVI	0.068 [c]			
SKM	0.053 [b]	-0.005 [b]		

[a]Reference [32].
[b]Reference [31].
[c]Reference [26].

IV. CONCLUSION

The agreement between the calculated level density parameter and the experimental values reflects the fact that our formulas for the temperature dependence of the surface and curvature properties are reasonable. If the level density parameter a comes out to be close to experiment [39], this is a pure accident due to cancellation of two different errors.

(i) The HF and ETF methods cannot give the experimental level densities. They are known to be systematically too low by 20–30% due to correlations. Only at large temperatures do these [78,79] correlations become less important.

(ii) The present approximation of using $T=0$, ETF functionals, and adding the lowest T^2 correlation—the so-called low temperature approximation—has been shown to fail for the level density parameter a [28], giving an overestimation of about 30%.

Several new parameters were introduced in this work such as a_{vi}, a_{si}, and a_{ci}. We believe that their values need to be tested for different interactions. This will be done in the near future.

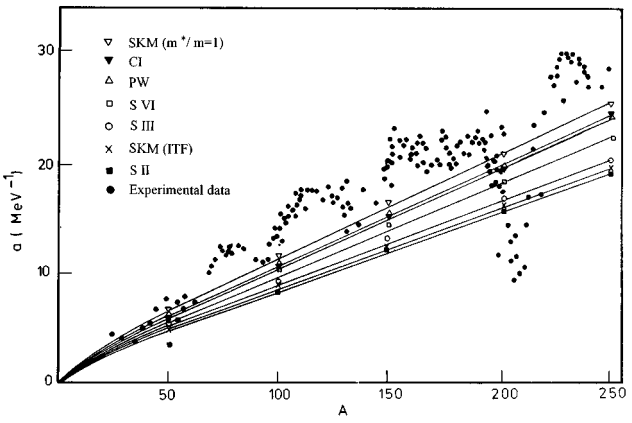

FIG. 1. Level density parameter a as a function of A. Ref. [26] = SKM ($m^*/m=1$), Ref. [74] = CI, present work = PW, Ref. [39] = SVI, SIII, SKM (ITF), SII, and experimental data = Ref. [70].

APPENDIX A

The potential energy (E_p) of polarized NM (the exchange term is neglected [3]) is

$$
\begin{aligned}
E_p = & -2\pi a^3 C_{11}\int(\rho_{n\uparrow}^2+\rho_{n\downarrow}^2+\rho_{p\uparrow}^2+\rho_{p\downarrow}^2)d\vec{r}+\frac{12\pi a^3 C_{11}}{5b^2}\left(\frac{3\pi^2}{2}\right)^{2/3}\int(\rho_{n\uparrow}^{8/3}+\rho_{n\downarrow}^{8/3}+\rho_{p\uparrow}^{8/3}+\rho_{p\downarrow}^{8/3})d\vec{r} \\
& -\frac{12\pi a^5 C_{11}}{5b^2}\left(\frac{3\pi^2}{2}\right)^{2/3}\int(\nabla\rho_{n\uparrow})^2\rho_{n\uparrow}^{2/3}+(\nabla\rho_{n\uparrow})^2\rho_{n\uparrow}^{2/3}+(\nabla\rho_{n\uparrow})^2\rho_{n\uparrow}^{2/3}+(\nabla\rho_{n\uparrow})^2\rho_{n\uparrow}^{2/3}d\vec{r} \\
& +2\pi a^5 C_{11}\int(\nabla\rho_{n\uparrow})^2+(\nabla\rho_{n\uparrow})^2+(\nabla\rho_{n\uparrow})^2+(\nabla\rho_{n\uparrow})^2+(\nabla\rho_{n\uparrow})^2 d\vec{r}-4\pi a^3 C_{1u}\int(\rho_{n\uparrow}\rho_{n\downarrow}+\rho_{p\uparrow}\rho_{p\downarrow})d\vec{r} \\
& +\frac{12\pi a^3 C_{1u}}{5b^2}\left(\frac{3\pi^2}{2}\right)^{2/3}\int(\rho_{n\uparrow}^{5/3}\rho_{n\downarrow}+\rho_{n\downarrow}^{5/3}\rho_{n\uparrow}+\rho_{p\uparrow}^{5/3}\rho_{p\downarrow}+\rho_{p\downarrow}^{5/3}\rho_{p\uparrow})d\vec{r}+4\pi a^5 C_{1u}\int(\nabla\rho_{n\uparrow}\nabla\rho_{n\downarrow}+\nabla\rho_{p\uparrow}\nabla\rho_{p\downarrow})d\vec{r} \\
& -\frac{12\pi a^2 C_{1u}}{5b^2}\left(\frac{3\pi^2}{2}\right)^{2/3}\int(\nabla\rho_{n\uparrow}\nabla\rho_{n\downarrow}\nabla\rho_{n\uparrow}^{2/3}+\nabla\rho_{n\uparrow}\nabla\rho_{n\downarrow}\nabla\rho_{n\uparrow}^{2/3}+\nabla\rho_{p\uparrow}\nabla\rho_{p\downarrow}\nabla\rho_{p\uparrow}^{2/3}+\nabla\rho_{p\uparrow}\nabla\rho_{p\downarrow}\nabla\rho_{p\downarrow}^{2/3})d\vec{r} \\
& +\frac{12\pi a^3 C_{1u}}{5b^2}\left(\frac{3\pi^2}{2}\right)^{2/3}\int(\rho_{p\uparrow}\rho_{n\uparrow}^{5/3}+\rho_{p\uparrow}^{5/3}\rho_{n\uparrow}+\rho_{n\downarrow}^{5/3}\rho_{p\downarrow}+\rho_{p\downarrow}^{5/3}\rho_{n\downarrow})d\vec{r}+4\pi a^3 C_{1u}\int(\nabla\rho_{n\uparrow}\nabla\rho_{p\uparrow}+\nabla\rho_{n\downarrow}\nabla\rho_{p\downarrow})d\vec{r} \\
& +4\pi a^5 C_{u1}\int(\nabla\rho_{n\uparrow}\nabla\rho_{p\uparrow}+\nabla\rho_{n\downarrow}\nabla\rho_{p\downarrow})d\vec{r}-\frac{12\pi a^5 C_{u1}}{5b^2}\left(\frac{3\pi^2}{2}\right)^{2/3}\int(\nabla\rho_{n\uparrow}\nabla\rho_{p\uparrow}\rho_{n\uparrow}^{2/3}+\nabla\rho_{n\uparrow}\nabla\rho_{p\uparrow}\rho_{p\uparrow}^{2/3} \\
& +\nabla\rho_{n\downarrow}\nabla\rho_{p\downarrow}\rho_{n\downarrow}^{2/3}+\nabla\rho_{n\downarrow}\nabla\rho_{p\downarrow}\rho_{p\downarrow}^{2/3})d\vec{r}-4\pi a^5 C_{uu}\int(\rho_{n\downarrow}\rho_{p\uparrow}+\rho_{n\uparrow}\rho_{p\downarrow})d\vec{r} \\
& +\frac{12\pi a^5 C_{uu}}{5b^2}\left(\frac{3\pi^2}{2}\right)^{2/3}\int(\rho_{n\downarrow}^{5/3}\rho_{p\uparrow}+\rho_{n\downarrow}\rho_{p\uparrow}^{5/3}+\rho_{n\downarrow}^{5/3}\rho_{p\downarrow}+\rho_{n\uparrow}\rho_{p\downarrow}^{5/3})d\vec{r}+4\pi a^5 C_{uu}\int(\nabla\rho_{n\downarrow}\nabla\rho_{p\uparrow}+\nabla\rho_{n\uparrow}\nabla\rho_{p\downarrow})d\vec{r} \\
& -\frac{12\pi a^5 C_{uu}}{5b^2}\left(\frac{3\pi_2}{2}\right)^{2/3}\int(\nabla\rho_{n\uparrow}\nabla\rho_{p\downarrow}\rho_{n\uparrow}^{2/3}+\nabla\rho_{n\uparrow}\nabla\rho_{p\downarrow}\rho_{p\downarrow}^{2/3}+\nabla\rho_{n\downarrow}\nabla\rho_{p\uparrow}\rho_{n\downarrow}^{2/3}+\nabla\rho_{n\downarrow}\nabla\rho_{p\uparrow}\rho_{p\uparrow}^{2/3})d\vec{r}. \qquad \text{(A1)}
\end{aligned}
$$

The total free energy is $F=F_v+F_sA^{2/3}+F_cA^{1/3}$,

$$
F_{v0}=\frac{3\pi a^3}{5b^2}C\rho_0^{5/3}\left(\frac{3\pi^2}{2}\right)^{2/3}-\frac{3h^2}{10m}\rho_0^{2/3}\left(\frac{3\pi^2}{2}\right)^{2/3}-\frac{\pi^2 mT^2}{2h^2\rho_0^{2/3}}\left(\frac{2}{3\pi^2}\right)^{2/3}, \qquad \text{(A2)}
$$

$$
F_{vi}=\frac{2\pi a^3}{3b^2}C_i\rho_0^{5/3}-\frac{\pi a^3}{2}C_i\rho_0+\frac{h^2}{6m}\rho_0^{2/3}\left(\frac{3\pi^2}{2}\right)^{2/3}-\frac{\pi^2 mT^2}{18h^2\rho_0^{2/3}}\left(\frac{2}{3\pi^2}\right)^{2/3}, \qquad \text{(A3)}
$$

$$
\begin{aligned}
F_{s0}= & 2\pi^2 a^3\rho_0^{4/3}dC\left(\frac{3}{4\pi}\right)^{2/3}-\frac{12\pi^2 a^3 dC}{5b^2}\rho_0^2 A_1\left(\frac{8}{3}\right)\left(\frac{9}{8}\right)^{2/3}-\frac{2\pi^2 a^5}{d}C\left(\frac{3}{4\pi}\right)^{2/3}[A_1(2)+A_1(4)-2A_1(3)]\rho_0^{5/3} \\
& +\frac{12\pi^2 a^5 C}{b}\rho_0^2\left(\frac{9\pi}{8}\right)^{2/3}\left[A_1\left(\frac{8}{3}\right)+A_1\left(\frac{14}{3}\right)-2A_1\left(\frac{11}{3}\right)\right]-\frac{6h^2 d}{5m}\rho_0\left(\frac{9\pi}{8}\right)^{2/3}A_1\left(\frac{5}{3}\right)-\frac{2h^2\pi\beta}{md}\rho_0^{1/3}\left(\frac{3}{4\pi}\right)^{2/3}[A_1(3)-(2)] \\
& +\frac{2mT^2 d}{h^2\rho_0^{1/3}}A_1\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^{2/3}, \qquad \text{(A4)}
\end{aligned}
$$

$$
\begin{aligned}
F_{si}= & 2\pi^2 a^3 d\rho_0^{4/3}\left(\frac{3}{4\pi}\right)^{2/3}C_i-\frac{2\pi^2 a^5}{d}C_i\rho_0^{4/3}\left(\frac{3}{4\pi}\right)^{2/3}[A_1(2)+A_1(4)-2A_1(3)]-\frac{8\pi^2 a^5 d}{3b^2}C_i\rho_0^2\left(\frac{9\pi}{8}\right)^{2/3}A_1\left(\frac{8}{3}\right) \\
& -\frac{8\pi^2 a^5}{3b^2 d}C_i\rho_0^2\left(\frac{9\pi}{8}\right)^{2/3}\left[A_1\left(\frac{8}{3}\right)+a_1\left(\frac{14}{3}\right)-2A_1\left(\frac{11}{3}\right)\right]-\frac{2h^2\pi d}{3m}\rho_0\left(\frac{9\pi}{8}\right)^{2/3}A_1\left(\frac{5}{3}\right)-\frac{2\pi mT^2 d}{9h^2\rho^{1/3}}A_1\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^{2/3}, \qquad \text{(A5)}
\end{aligned}
$$

$$F_{c0}=\frac{4\pi^3a^3}{5b^2}\rho_0^{7/3}\left(\frac{1}{2}\right)^{1/3}\left[6A_2\left(\frac{8}{3}\right)d^2-\pi^3Ad^{-3}-6a^2A_2\left(\frac{8}{3}\right)+A_2\left(\frac{14}{3}\right)-2A_2\left(\frac{11}{3}\right)\right]C$$
$$-\frac{2\pi^2h^2d^2}{5m}\rho_0^{4/3}\left(\frac{1}{2}\right)^{1/3}\left[\pi^2-6A_2\left(\frac{5}{3}\right)\right]+\frac{2\pi h^2}{m}\beta\rho_0^{2/3}\left(\frac{3}{4\pi}\right)^{1/3}+\frac{2\pi^2mT^2d^2}{3h^2}\left(\frac{1}{3\pi^2}\right)^{1/3}\left[\pi^2-6A_2\left(\frac{1}{3}\right)\right], \quad \text{(A6)}$$

$$F_{ci}=\frac{8\pi^3a^3}{5b^2}\rho_0^{7/3}\left[6d^2A_2\left(\frac{8}{3}\right)-\pi^2d^2-6a^2A_2\left(\frac{8}{3}\right)+2A_2\left(\frac{14}{3}\right)-2A_2\left(\frac{11}{3}\right)\right]C_i$$
$$-\frac{2\pi^2h^2d^2}{3h^2}\rho_0^{4/3}\left(\frac{1}{2}\right)^{1/3}\left[\pi^2-6A_2\left(\frac{1}{3}\right)\right]-\frac{2\pi^2m^2d^2}{27h^2}\left(\frac{1}{3\pi^2}\right)^{1/3}\left[\pi^2-6A_2\left(\frac{1}{3}\right)\right], \quad \text{(A7)}$$

where

$$C=C_{11}+C_{1u}+C_{u1}+C_{uu},$$
$$C_x=2C_{11}+2C_{1u}-C_{1u}-C_{uu},$$
$$C_{x'}=C_{11}+C_{1u}-C_{u1}-C_{uu},$$
$$C_y=2C_{11}-C_{1u}+2C_{u1}-C_{uu},$$
$$C_{y'}=C_{11}-C_{1u}+C_{ul}-C_{uu},$$
$$C_z=2C_{11}-C_{1u}-C_{u1}+2C_{uu},$$
$$C_{z'}=C_{11}-C_{u1}-C_{u1}+C_{uu}, \quad \text{(A8)}$$

and i runs over x, y, and z.

APPENDIX B: CALCULATION OF INTEGRALS INVOLVING TWO MULTIPLIED WOODS-SAXON FUNCTIONS TO A POWER

The Woods-Saxon form of the density is given by

$$\rho(r)/\rho_0=\{1+\exp[(r-R)/d]\}^{-1}. \quad \text{(B1)}$$

In our model calculations, we need to calculate integrals of the form

$$I(p,q)=\int_0^\infty \rho^p(r)\rho^q(r)d\vec{r},$$

where

$$\rho^p(r)=\rho_0^p/\{1+\exp[(r+R_1)/d_1]\}^p$$

and

$$\rho^q(r)=\rho_0^q/\{1+\exp[r-R_2]/d_2\}q.$$

In our calculations we set

$$R_1=R_2=R, \quad d_1=d_2=d.$$

Equation (B1) can be evaluated by noting that

$$I=4\pi\int_0^R\rho^q(r)\rho^p(r)r^2dr+4\pi\int_R^\infty\rho^q(r)\rho^p(r)r^2dr$$
$$=I_1+I_2.$$

For I_1, $r<R$, and thus we define

$$-x=(r-R)/d$$

and we get

$$I_1=4\pi d\rho_0^p\rho_0^q\int_0^{R/D}(1+e^{-x})^{-q}(1+e^{-x})^{-p}(R-dx)^2dx$$
$$=4\pi\rho_q\rho_0^p\int_0^{R/D}(1+e^{-x})^{-(p+q)}(R-dx)^2dx$$

using

$$(1+e^{-x})^{-(p+q)}$$
$$=1+\sum_{m=1}^\infty(-1)^m\times\frac{(p+q)(p+q+1)\cdots(p+q+m-1)e^{-mx}}{m!}$$
$$=1+\sum_{m=1}^\infty A_m(p+q)e^{-mx},$$

$$I_1=4\pi d\rho_0^p\rho_0^q\int_0^{R/d}\left(1+\sum_{m=1}^\infty A_m(p+q)e^{-mx}\right)(R-dx)^2dx$$
$$=4\pi d\rho_0^p\rho_0^q\left\{\frac{R^3}{3d}+\sum_{m=1}^\infty A_m(p+q)e^{R/d}\left(\frac{R^2}{m}-\frac{2Rd}{m^2}+\frac{2d^2}{m^3}-\frac{2d^2}{m^3}e^{-mR/d}\right)\right\}. \quad \text{(B2)}$$

For I_2, $r>R$, and therefore we define

$$y=(r-R)/d,$$

$$I_2=4\pi d\rho_0^p\rho_0^q\int_0^\infty(1+e^y)^{-(p+q)}(R+yd)^2dy=4\pi d\rho_0^p\rho_0^q\int_0^\infty e^{-(p+q)y}(1+e^{-y})^{-(p+q)}(R+yd)^2dy$$

$$=4\pi d\rho_0^p\rho_0^q\int_0^\infty e^{-(p+q)y}\left\{1+\sum_{m=1}^{\infty}A_m(p+q)e^{-my}\right\}(R+yd)^2dy$$

$$=4\pi d\rho_0^p\rho_0^q\left\{\int_0^\infty e^{-(p+q)y}(R+yd)^2dy+\sum_{m=1}^{\infty}A_m(p+q)\int_0^\infty e^{-(p+q+m)y}(R+yd)^2dy\right\}. \tag{B3}$$

Thus

$$I=4\pi\rho_0^p\rho_0^q\left\{\frac{R^3}{3}-2d^3\frac{R^2}{2d^2}A_1(p+q)+\frac{R}{d}A_2(p+q)-A_3(p+q)\right.$$
$$\left.+\sum_{m=1}^{\infty}(-1)^{m-1}\frac{(p+q)(p+q+1)\cdots(p+q+m-1)}{m!}\frac{e^{-mR/d}}{m^3}\right\}, \tag{B4}$$

with

$$A_n(p+q)=\frac{1}{(n-1)!}\int_0^\infty\{1-(1+e^{-x})^{-(p+q)}+(-1)^n(1+e^x)^{-(p+q)}\}x^{n-1}dx.$$

If $R>d$, the second term of I [in Eq. (B4)] can be neglected. Thus we can write, for I,

$$I(p,q)=\frac{4\pi}{3}\rho_0^p\rho_0^q\{R^3-3R^2dA_1(p+q)+6Rd^2A_2(p+q)-6d^3A_3(p+q)\}. \tag{B5}$$

APPENDIX C

The functions for $N(\rho_0)$ and $P(\rho_0)$ are given by

$$N(\rho_0)=N_0+N_xX^2+N_yY^2+N_zZ^2 \tag{C1}$$

and

$$P(\rho_0)=P_0+P_xX^2+P_yY^2+P_zZ^2, \tag{C2}$$

where

$$N_0=\frac{12\pi^2a^5}{5b^2}C\rho_0^2\left(\frac{9\pi}{8}\right)^{2/3}\left[A_1\left(\frac{8}{3}\right)+A_1\left(\frac{14}{3}\right)-2A_1\left(\frac{11}{3}\right)\right]-2\pi^2a^5C\rho_0^{4/3}\left(\frac{3}{4\pi}\right)^{2/3}[A_1(2)+A_1(4)-2A_1(3)]$$
$$-2h^2\pi\beta\rho_0^{1/3}\left(\frac{3}{4\pi}\right)^{2/3}[A_1(3)-2],$$

$$N_i=\frac{8\pi^2a^5}{3b^2}C_i\rho_0^2\left(\frac{9\pi}{8}\right)^{2/3}\left[A_1\left(\frac{8}{3}\right)+A_1\left(\frac{14}{3}\right)-2A_1\left(\frac{11}{3}\right)\right]-2\pi^2a^5\rho_0^{4/3}\left(\frac{3}{4\pi}\right)^{2/3}C_i[A_1(2)+A_1(4)-2A_1(3)],$$

$$P_0=2\pi^2a^3C\rho_0^{4/3}\left(\frac{3}{4\pi}\right)^{2/3}-\frac{12\pi^2a^3}{5b^2}C\rho_0^2\left(\frac{9\pi}{8}\right)^{2/3}A_1\left(\frac{8}{3}\right)-\frac{6h^2}{5m}\rho_0\left(\frac{9\pi}{8}\right)^{2/3}+\frac{2\pi mT^2}{h^2\rho^{1/3}}\left(\frac{1}{2}\right)^{2/3}A_1\left(\frac{1}{3}\right),$$

and

$$P_i=2\pi^2a^3C_i\rho_0^{4/3}\left(\frac{3}{4\pi}\right)^{2/3}-\frac{8\pi^2a^3}{3b^2}C_i\rho_0^2\left(\frac{9\pi}{8}\right)^{2/3}-\frac{2h^2\pi\rho_0}{3m}\left(\frac{9\pi}{8}\right)^{2/3}A_1(5/3)+\frac{2\pi mT^2}{h^2\rho_0^{1/3}}\left(\frac{1}{2}\right)^{2/3}A_1\left(\frac{1}{3}\right).$$

These functions are written at the equilibrium density. Using the density expansion [Eq. (31)] we get

$$N(\rho_{0n}) = N^0(\rho_{0n} + N^1(\rho_{0n})T^2 \tag{C3}$$

and

$$P(\rho_{0n}) = P^0(\rho_{0n}) + P^1(\rho_{0n})T^2, \tag{C4}$$

where the functions $N(\rho_{0n})$ and $P(\rho_{0n})$ are expanded in X, Y, and Z up to second order. The result is

$$F^{0,1} = F_0^{0,1} + F_x^{0,1}X^2 + F_y^{0,1}Y^2 + F_z^{0,1}Z^2, \tag{C5}$$

where

$$N_0^0 = \frac{12\pi^2 a^5}{5b^2} C\rho_{0n}^2 \left(\frac{9\pi}{8}\right)^{2/3} \left[A_1\left(\frac{8}{3}\right) + A_1\left(\frac{14}{3}\right) - 2A_1\left(\frac{11}{3}\right)\right] - 2\pi^2 a^5 C\rho_{0n}^{3/4} \left(\frac{3}{4\pi}\right)^{2/3} [A_1(2) + A_1(4) - 2A_1(3)]$$
$$- \frac{2\pi^2\beta}{m} \rho_{0n}^{1/3} \left(\frac{3}{4\pi}\right)^{2/3} [A(3) - 2], \tag{C6}$$

$$N_1^0 = -\frac{24\pi^2 a^5}{5b^2} C\rho_i \rho_{0n}^2 \left(\frac{9\pi}{8}\right)^{2/3} \left[A_1\left(\frac{8}{3}\right) + A_1\left(\frac{14}{3}\right) - 2A_1\left(\frac{11}{3}\right)\right] + \frac{8\pi^2 a^5}{3b^2} C\rho_i \rho_{0n}^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} [A_1(2) + A_1(4) - 2A_1(3)]$$
$$+ \frac{2h^2\pi\beta}{3m} \rho_i \rho_{0n}^{1/3} \left(\frac{3}{4\pi}\right)^{2/3} [A_1(3) - 2] + \frac{8\pi^2 a^5}{3b^2} C_i \rho_{0n}^2 \left[A_1\left(\frac{8}{3}\right) + A_1\left(\frac{14}{3}\right) - 2A_1\left(\frac{11}{3}\right)\right]$$
$$- 2\pi^2 a^5 \rho_{0n}^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} C_i [A_1(2) + A_1(4) - 2A_1(3)], \tag{C7}$$

$$N_0^1 = -\frac{24\pi^2 a^5}{5b^2} C\rho_{0n}^2 \rho_t \left(\frac{9\pi}{8}\right)^{2/3} \left[A_1\left(\frac{8}{3}\right) + A_1\left(\frac{14}{3}\right) - 2A_1\left(\frac{11}{3}\right)\right] + \frac{8\pi^2 a^5}{3b^2} C\rho_{0n}^{4/3} \rho_t \left(\frac{3}{4\pi}\right)^{2/3} [A_1(2) + A_1(4) - 2A_1(3)]$$
$$+ \frac{2h^2\pi\beta}{3m} \rho_t \rho_{0n}^{1/3} \left(\frac{3}{4\pi}\right)^{2/3} [A_1(3) - 2], \tag{C8}$$

$$N_i^1 = -\frac{16\pi^2 a^2}{3b^2} C_i \rho_t \rho_{0n}^2 \left(\frac{9\pi}{8}\right)^{2/3} \left[A_1\left(\frac{8}{3}\right) + A_1\left(\frac{14}{3}\right) - 2A_1\left(\frac{11}{3}\right)\right] + \frac{8\pi^2 a^5}{3b^2} \rho_t \rho_{0n}^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} [A_1(2) + A_1(4) - 2A_1(3)], \tag{C9}$$

$$P_0^0 = 2\pi^2 a^3 C\rho_{0n}^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} - \frac{12\pi^2 a^3}{5b^2} C\rho_{0n}^2 \left(\frac{9\pi}{8}\right)^{2/3} A_1\left(\frac{8}{3}\right) - \frac{6\pi h^2}{5m} \rho_{0n} \left(\frac{9\pi}{8}\right)^{2/3} A_1\left(\frac{5}{3}\right), \tag{C10}$$

$$P_i^0 = -\frac{8\pi^2 a^3 C}{3} \rho_i \rho_{0n}^{4/3} \left(\frac{3}{4\pi}\right)^{2/3} - \frac{8\pi^2 a^3}{3b^2} \rho_{0n}^2 C_i A_1\left(\frac{8}{3}\right) \left(\frac{9\pi}{8}\right)^{2/3} + \frac{24\pi^2 a^3 C}{5b^2} \rho_i \rho_{0n}^2 \left(\frac{9\pi}{8}\right)^{2/3} A_1\left(\frac{8}{3}\right) + 2\pi^2 a^3 \rho_{0n}^{4/3} C_i \left(\frac{3}{4\pi}\right)^{2/3}$$
$$+ \frac{6\pi\hbar^2}{5m} \rho_i \rho_{0n} \left(\frac{9\pi}{8}\right)^{2/3} A_1\left(\frac{5}{3}\right), \tag{C11}$$

$$P_0^1 = -\frac{8\pi^2 a^3}{3} C\rho_t \rho_{0n}^{4/3} \left(\frac{3}{4\pi}\right)^{3/2} + \frac{24\pi^2 a^3}{5b^2} C\rho_t \rho_{0n}^2 \left(\frac{9\pi}{8}\right)^{2/3} A_1\left(\frac{8}{3}\right) + \frac{6\pi h^2}{5m} \rho_t \rho_{0n} \left(\frac{9\pi}{8}\right)^{2/3} A_1\left(\frac{5}{3}\right) + \frac{2\pi m}{\hbar^2 \rho^{1/3}} \left(\frac{1}{2}\right)^{2/3} A_1\left(\frac{1}{3}\right), \tag{C12}$$

and

$$P_i^1 = -\frac{8\pi^2 a^3}{3} C_i \rho_t \rho_{0n}^{3/4} + \frac{16\pi^2 a^3}{5b^2} C_i \rho_t \rho_{0n}^2 \left(\frac{9\pi}{8}\right)^{2/3} A_1\left(\frac{8}{3}\right) - \frac{2m}{\hbar^2 \rho_{0n}^{1/3}} \left(\frac{1}{2}\right)^{2/3} A_1\left(\frac{1}{3}\right) \left(\frac{1+3\rho_i}{9}\right) + \frac{2\pi\hbar^2}{3m} \rho_t \rho_{0n} \left(\frac{9\pi}{8}\right)^{2/3} A_1\left(\frac{5}{3}\right). \tag{C13}$$

APPENDIX D

We shall use Eqs. (C3) and (C4) to express the surface energy and the diffuseness parameter in terms of ρ_{0n}. The result up to second order in X, Y, Z, and T is written in the form

$$d = d_0 + \alpha_0 T^2 + (d_x + \alpha_x T^2)X^2 + (d_y + \alpha_y T^2)Y^2 + (d_z + \alpha_z T^2)Z^2, \tag{D1}$$

$$F=F_{s0}+a_{s0}T^2+(F_{sx}+a_{sx}T^2)X^2+(F_{xy}+a_{sy}T^2)Y^2+(F_{sz}+a_{sz}T^2)Z^2, \tag{D2}$$

where

$$d_0=(N_0^0/P_0^0)^{1/2}, \tag{D3}$$

$$\alpha_0=\tfrac{1}{2}d_0[(N_0^1/N_0^0)-(P_0^1/P_0^0)], \tag{D4}$$

$$\begin{aligned}\alpha_i=\tfrac{1}{2}d_0\{&\tfrac{1}{2}[(N_i^0/N_0^0)-(P_i^0/P_0^0)][(N_0^1/N_0^0)-(P_0^1/P_0^0)]+(N_0^1/N_0^0)[(N_i^1/N_0^1)-(N_i^0/N_0^0)]\\&-(P_0^1/P_0^0)[(P_i^1/P_0^1)-(P_i^1/P_0^0)]\},\end{aligned} \tag{D5}$$

$$d_i=\tfrac{1}{2}d_0[(N_i^0/N_0^0)-(P_i^0/P_0^0)], \tag{D6}$$

$$F_{s0}=2(N_0^0P_0^0)^{1/2}, \tag{D7}$$

$$a_{s0}=2(\alpha_0P_0^0+d_0P_0^1)^{1/2}, \tag{D8}$$

$$a_{si}=2(\alpha_0P_i^0+d_0P_i^1+\alpha_iP_0^0)^{1/2}, \tag{D9}$$

and

$$F_{si}=2(d_iP_0^0+d_0P_i^0). \tag{D10}$$

In order to obtain a similar formula for the curvature energy, we expand d^2 up to second order in X, Y, Z, and T. The result is

$$F_c=F_{c0}+a_{c0}T^2+(F_{cx}+a_{cx}T^2)X^2+(F_{cy}+a_{ay}T^2)Y^2+(F_{cz}+a_{cz}T^2)Z^2, \tag{D11}$$

where

$$\begin{aligned}F_{c0}=&\frac{6\pi^3a^3}{5b^2}C\rho_{0n}^{7/3}\left(\frac{1}{2}\right)^{1/3}\left[6A_2\left(\frac{8}{3}\right)-\pi^2\right]\left(\frac{N_0^0}{P_0^0}\right)+\frac{3\pi^2h^2}{5m}\rho_{0n}^{4/3}\left[6A_2\left(\frac{5}{3}\right)-\pi^2\right]\left(\frac{N_0^0}{P_0^0}\right)+\frac{2\pi^2h^2\beta}{m}\rho^{2/3}\left(\frac{3}{4\pi}\right)^{1/3}\\&-\frac{36\pi^3a^3}{5b^2}C\rho_{0n}^{7/3}\left(\frac{1}{2}\right)^{1/3}\left[A_2\left(\frac{8}{3}\right)+A_2\left(\frac{14}{3}\right)-2A_2\left(\frac{11}{3}\right)\right],\end{aligned} \tag{D12}$$

$$\begin{aligned}a_{c0}=&\frac{6\pi^3a^3}{5b^2}C\rho_{0n}^{7/3}\left(\frac{1}{2}\right)^{1/3}\left[6A_2\left(\frac{8}{3}\right)-\pi^2\right]\left(d_1-\frac{7\rho_tN_0^0}{3P_0^0}\right)+\frac{3\pi^2h^2}{5m}\rho_{0n}^{4/3}\left[6A_2\left(\frac{5}{3}\right)-\pi^2\right]\left(d_1-\frac{4\rho_tN_0^0}{3P_0^0}\right)\\&-\frac{4\pi h^2\beta}{3m}\rho_t\rho_{0n}^{2/3}\left(\frac{3}{4\pi}\right)^{1/3}+\frac{2\pi^2m}{3h^2}\left(\frac{1}{3\pi^2}\right)^{1/3}\left(\frac{N_0^0}{P_0^0}\right)+\frac{84\pi^3a^5}{5b^2}C\rho_t\rho_{0n}^{7/3}\left(\frac{1}{2}\right)^{1/3}\left[A_2\left(\frac{8}{3}\right)+A_2\left(\frac{14}{3}\right)-2A_2\left(\frac{11}{3}\right)\right],\end{aligned} \tag{D13}$$

$$\begin{aligned}F_{ci}=&\frac{6\pi^3a^3}{5b^2}\rho_{0n}^{7/3}\left(\frac{1}{2}\right)^{1/3}\left[6A_2\left(\frac{8}{3}\right)-\pi^2\right]\left[C\left(d_1-\frac{7\rho_iN_0^0}{3P_0^0}\right)+\frac{10C_iN_0^0}{9P_0^0}\right]+\frac{3\pi^2h^2}{3m}\rho_{0n}^{4/3}\left[6A_2\left(\frac{5}{3}\right)-\pi^2\right]\left[\frac{5N_0^0}{9P_0^0}+d_{2i}-\frac{4\rho_iN_0^0}{3P_0^0}\right]\\&-\frac{36\pi^3a^5}{5b^2}\rho_{0n}^{7/3}\left(\frac{1}{2}\right)^{1/3}\left[A_2\left(\frac{8}{3}\right)+A_2\left(\frac{14}{3}\right)-2A_2\left(\frac{11}{3}\right)\left(\frac{10}{9}C_i-\frac{7}{3}C\rho_i\right)\right]-\frac{3\pi\hbar^2}{3m}\rho_{0n}^{2/3}\rho_i\left(\frac{3}{4\pi}\right)^{1/3},\end{aligned} \tag{D14}$$

and

$$\begin{aligned}a_{ci}=&\frac{6\pi^3a^3}{5b^2}\rho_{0n}^{7/3}\left(\frac{1}{2}\right)^{1/3}\left[6A_2\left(\frac{8}{3}\right)-\pi^2\right]\left[C\left(d_{3i}-\frac{7}{3}d_1\rho_i-\frac{7}{3}d_2\rho_t\right)+\frac{10}{9}C_i\left(d_1-\frac{7\rho_tN_0^0}{3P_0^0}\right)\right]\\&+\frac{2\pi^2m}{27h^2}\left(\frac{1}{3\pi^2}\right)^{1/3}\left[6A_2\left(\frac{1}{3}\right)-\pi^2\right]\left(\frac{N_0^0}{P_0^0}\right)+\frac{56\pi^3a^5}{3b^2}C_i\rho_t\rho_{0n}^{7/3}\left(\frac{1}{2}\right)^{1/3}\left[A_2\left(\frac{8}{3}\right)+A_2\left(\frac{14}{3}\right)-2A_2\left(\frac{11}{3}\right)\right]\\&+\frac{3\pi^2h^2}{3m}\rho_{0n}^{4/3}\left[6A_2\left(\frac{5}{3}\right)-\pi^2\right]\left[d_{3i}-\frac{4}{3}\rho_id_i-\frac{4}{3}\rho_td_{2i}+\frac{5}{9}\left(d_1-\frac{4\rho_tN_0^0}{3P_0^0}\right)\right],\end{aligned} \tag{D15}$$

where

$$d_1=[(N_0^1/N_0^0)-(P_0^{1/}/P_0^0)]/(P_0^0)^2,$$

$$d_{2i}=[(N_i^0/N_0^0)-(P_i^0/P_0^0)]/(P_0^0)^2,$$

and

$$d_{3i}=(N_0^1/P_0^0)[(N_i^0/N_0^0)-(P_i^0/P_0^0)]-[N_0^0P_0^1/(P_0^0)^2][(P_1^1/P_0^0)-(N_i^1/N_0^0)]$$
$$+(N_0^0/P_0^0)[(N_i^0/N_0^0)-(P_i^0/P_0^0)][(N_0^1/N_0^0)-(P_0^1/P_0^0)].$$

[1] F. Tabakin and K. Amos, Nucl. Phys. **A100**, 574 (1967).
[2] X. Campi and S. Stringari, Nucl. Phys. **A337**, 313 (1980).
[3] M. Abd-Alla, S. Ramadan, and M. Y. M. Hassan, Phys. Rev. C **36**, 1565 (1987).
[4] M. Y. M. Hassan, S. S. Montasser, and S. Ramadan, J. Phys. G **6**, 1229 (1980).
[5] D. G. Ravenhall *et al.*, Phys. Rev. Lett. **28**, 978 (1979).
[6] G. Pertsih and P. J. Siemens, Phys. Lett. **126B**, 9 (1983).
[7] C. J. Pethick and D. G. Ravenhall, Nucl. Phys. **A471**, 190 (1987).
[8] D. Bandyopadhyay *et al.*, Phys. Lett. B **218**, 391 (1989).
[9] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Reading, MA, 1969), Vols. I and II.
[10] G. Baym, H. A. Bethe, and C. J. Pethick, Nucl. Phys. **A172**, 225 (1971).
[11] H. A. Bethe, G. E. Brown, J. H. Applegate, and J. M. Lattimer, Nucl. Phys. **A324**, 487 (1979).
[12] J. M. Lattimer, Annu. Rev. Nucl. Part. Sci. **31**, 337 (1981).
[13] H. A. Bethe, G. E. Brown, J. Cooperstein, and J. R. Wilson, Nucl. Phys. **A403**, 625 (1983).
[14] G. Fuller, Astrophys. J. **252**, 741 (1982).
[15] M. Prakash, J. Wambach, and Z. Y. Ma, Phys. Lett. **128B**, 141 (1983).
[16] A. D'Arrigo *et al.*, Phys. Lett. B **262**, 1 (1991).
[17] G. Sauer, H. Chandra, and U. Mosel, Nucl. Phys. **A264**, 221 (1976).
[18] P. Bonche, S. Levit, and D. Vautherin, Nucl. Phys. **A436**, 265 (1985).
[19] P. Bonche and S. Levit, Nucl. Phys. **A437**, 42 (1985).
[20] M. Brack and P. Quentin, Phys. Lett. **52B**, 159 (1974).
[21] U. Mosel, P. G. Zint, and K. H. Passler, Nucl. Phys. **A236**, 236 (1974).
[22] P. Bonche, S. Levit, and D. Vautherin, Nucl. Phys. **A427**, 278 (1984).
[23] D. Bandyopayadhyay *et al.*, Nucl. Phys. A **511**, 1 (1990).
[24] S. Hongqui *et al.*, Chin. Phys. Lett. **7**, 117 (1990).
[25] M. Baldo, Nucl. Phys. **A519**, 243c (1990).
[26] W. Stocker and J. Burzlaff, Nucl. Phys. **A202**, 265 (1973).
[27] W. A. Kupper, G. Wegmann, and E. Hilf, Ann. Phys. (N.Y.) **88**, 454 (1974).
[28] M. Brack, V. Gul, and H. B. Hokanson, Phys. Rep. **123**, 275 (1985).
[29] J. Bartel, M. Brack, and M. Durand, Nucl. Phys. **A445**, 263 (1985).
[30] M. Brack, Phys. Rev. Lett. **53**, 119 (1984); **54**, 581 (1985).
[31] X. Vinas, M. Pi, and M. Baranco, J. Phys. G **9**, 1193 (1983).
[32] B. Behera, J. Phys. G **10**, 1731 (1984).
[33] H. Krivine and J. Treiner, Phys. Lett. B **177**, 259 (1986).
[34] J. Treiner and H. Krivine, Ann. Phys. (N.Y.) **170**, 406 (1986).
[35] H. A. Bethe, Phys. Rev. **50**, 332 (1936).
[36] W. J. Swiatecki, Proc. Phys. Soc. London A **64**, 226 (1951).
[37] M. Barranco, M. Pi, and X. Vinas, Phys. Lett. **124B**, 131 (1981).
[38] X. Campi and S. Stringari, Z. Phys. A **309**, 239 (1983).
[39] M. Barranco and J. Treiner, Nucl. Phys. **A351**, 269 (1981).
[40] F. D. Mackie and G. Baym, Nucl. Phys. **A258**, 322 (1977).
[41] C. D. Bennett and D. G. Ravenhall, Phys. Rev. C **10**, 2058 (1974).
[42] K. Kolehmainen *et al.*, Nucl. Phys. **A439**, 535 (1985).
[43] Y. H. Chu, B. K. Jennings, and M. Brack, Phys. Lett. **68B**, 407 (1977).
[44] B. Grammaticos, Z. Phys. A **312**, 99 (1983).
[45] M. Farine, Z. Phys. A **320**, 337 (1985); M. Farine and J. M. Pearson, Phys. Lett. **167B**, 259 (1986).
[46] V. Weisskopf, Phys. Rev. **52**, 295 (1937).
[47] H. A. Bethe, Rev. Mod. Phys. **9**, 53 (1937).
[48] A. M. Zebelman *et al.*, Phys. Rev. C **11**, 1280 (1975).
[49] W. Stocker, Phys. Lett. **142B**, 319 (1984).
[50] G. R. Huizenga and L. G. Moretto, Annu. Rev. Nucl. Sci. **22**, 427 (1972).
[51] S. A. Moszkowski, Phys. Rev. C **2**, 402 (1970).
[52] R. G. Seyler and C. H. Blanchard, Phys. Rev. **124**, 227 (1961).
[53] R. G. Seyler and C. H. Blanchard, Phys. Rev. **131**, 355 (1963).
[54] B. Grammaticos and A. Voros, Ann. Phys. (N.Y.) **123**, 539 (1979).
[55] B. Grammaticos and A. Voros, Ann. Phys. (N.Y.) **129**, 153 (1980).
[56] O. Bohigas *et al.*, Phys. Lett. **64B**, 381 (1967).
[57] J. P. Blaizot and B. Grammaticos, Nucl. Phys. **A355**, 115 (1981).
[58] C. Guet *et al.*, Phys. Lett. B **205**, 427 (1988).
[59] B. W. Allardyce *et al.*, Nucl. Phys. **A209**, 1 (1973).
[60] D. K. Sirvastava, Phys. Lett. **112B**, 289 (1982).
[61] R. A. Wiess and A. G. W. Cameron, Can. J. Phys. **47**, 2171 (1969).
[62] H. A. Bethe, Annu. Rev. Nucl. Sci. **21**, 93 (1971).
[63] M. Dwozecka, Acta Phys. Pol. **29**, 783 (1966).

[64] M. Y. M. Hassan and S. S. Montasser, Acta Phys. Pol. B **11**, 567 (1980).
[65] H. Krivine and J. Triener, Phys. Lett. **124B**, 127 (1983).
[66] J. Triener *et al.*, Nucl. Phys. **A452**, 93 (1986).
[67] M. Farine, J. Cote, and J. M. Pearson, Nucl. Phys. **A338**, 86 (1980).
[68] M. Farine, J. M. Pearson, and B. Ruben, Nucl. Phys. **A304**, 317 (1978).
[69] H. J. Krappe, J. R. Nix, and A. J. Sierk, Phys. Rev. C **20**, 992 (1979).
[70] A. Gilbert and A. G. W. Cameron, Can. J. Phys. **43**, 1446 (1965).
[71] W. D. Myers and W. J. Swiatecki, Ann. Phys. (N.Y.) **55**, 359 (1969).
[72] P. Moller, W. D. Myers, W. J. Swiatecki, and J. Treiner, Report No. Amco-7, Darmstadt-Seeheim, 1984.
[73] G. Batko, O. Civitarese, and A. L. De Paoli, Z. Phys. A **327**, 329 (1987).
[74] O. Civitarese and V. S. Ramamurthy, lectures at the ''Workshop on Applied Nuclear Theory and Nuclear Model Calculations for Nuclear Technology Applications,'' ICTP, Trieste, Italy, 1988.
[75] S. Ramadan, seminar given at the ''Workshop on Applied Nuclear Theory and Nuclear Model Calculations for Nuclear Technology Applications,'' ICTP, Trieste, Italy, 1988.
[76] S. Henss *et al.*, Phys. Rev. Lett. **60**, 11 (1988).
[77] A. H. Blin and M. Brack, Nucl. Phys. **A504**, 300 (1989).
[78] A. Lejeune *et al.*, Nucl. Phys. **A453**, 189 (1986).
[79] R. Hasse and P. Schuck, Phys. Lett. B **179**, 313 (1986).