Momentum distribution in nuclear matter within a perturbation approximation

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It is shown that the norm corrections, introduced to avoid the violation of the constraints on the depletion of the hole states in the standard perturbative 2p2h approach, leads in nuclear matter to a dependence of the momentum distribution with the total nucleon number. This unphysical behavior, which in turn makes the depletion to be nonextensive, arises from contributions of disconnected diagrams contained in the norm. It is found that the extensivity is again recovered when the 4p4h excitations in the ground state are included, and a reasonable value for the total number of nucleons promoted above the Fermi level is obtained.

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I. INTRODUCTION

A great amount of theoretical effort was devoted to study the influence of ground-state correlations on the nuclear physical observables. In most of them the nuclear response function is evaluated after introducing two-particle-two-hole (2p2h) admixtures within first-order perturbation into the 0p0h ground-state wave function [1-6,9]. It was found that 2p2h ground-state contributions modify significantly the mean field results for the strength function. Recently, however, the use of this perturbation procedure has been objected by Van Neck et al. [7] because it largely overestimates the effect of ground-state correlation, when the norm corrections are neglected. This happens because the perturbation adds a very large number of relatively small excited 2p2h components, with relative weights in the perturbed wave function typically large enough numerically as to strongly enhance the norm corrections. Specifically, it is reminded in Ref. [7] that a shell-model approach for occupation numbers in nuclei imposes constrains in the sense that the number of particles lifted out the Fermi sea has to be ≤ 2 . This constrain is not respected in the above-mentioned works as put in evidence by several calculations of the occupation numbers within the same perturbation approach [10].

In a recent study [8] of the Gamow-Teller strength the ground state was worked out within a finite nuclei formalism up to the first order, both in the Rayleigh-Schrödinger and Brioullin-Wigner schemes, and including the effects of the norm as was suggested in Ref. [7]. The result was the strong reduction of 2p2h contribution to the response function. In the present work we intend to implement better controlled approximations for the correlation structure of the ground state for the case of nuclear matter, which may have important consequences in the evaluation of the response function in the quasielastic region. As a first step in this direction we analyze here the momentum distribution in the ground state. The reason for that is that, as shown within a simple model by Takayanagi [9], the longitudinal response in the quasifree region is directly related to the momentum distribution of nucleons in nuclear matter. Our initial intention was to include the norm effects in the same way as we have previously done for the finite nuclei [8]. But, we soon discovered that this procedure makes the depletion of the nuclear core not to be anymore an extensive quantity because of the momentum distribution dependence on the total nucleon number A. Clearly, such a behavior is unphysical and has its origin in the contributions arising from the infinite series of disconnected Goldstone diagrams contained in the norm, as can be put in evidence by performing a perturbation expansion of the later. In order to exactly cancel these disconnected graphs one is forced to enlarge the configuration space by including npnh amplitudes with n > 2. This is done in the present work for the 4p4h components in the ground-state wave function. To go beyond this configuration space is not only a cumbersome task but hard to justify also.

II. FORMALISM

The occupation number for the single-particle state κ in the ground state $|0\rangle$ is defined as

$$n(\kappa) = \langle 0 | a^{\dagger}(\kappa) a(\kappa) | 0 \rangle, \qquad (2.1)$$

and the total number of particles promoted above the Fermi level, $N_{\rm >}$, and those remaining below the Fermi level, $N_{\rm <}$, are

$$N_{>} = \sum_{\kappa(\epsilon_{\kappa} > \epsilon_{F})} n(\kappa), \quad N_{<} = \sum_{\kappa(\epsilon_{\kappa} < \epsilon_{F})} n(\kappa) = A - N_{>}.$$
(2.2)

Within the Hartree-Fock (HF) approximation, $|0\rangle \equiv |\text{HF}\rangle = |0p0h\rangle$, one obtains the well-known step function for $n(\kappa)$, i.e., $n(\kappa) = \theta(\epsilon_F - \epsilon_\kappa)$, where ϵ_κ and ϵ_F are, respectively, the single-particle and Fermi energies. This is the zero-order approximation for the occupation numbers distribution. When *npn*h correlations are added to the ground-state wave function, states above the Fermi level are populated with the corresponding depletion of the nuclear core, and the occupation number takes the form

$$n(\kappa) = \theta(\epsilon_{r} - \epsilon_{r}) + \delta n(\kappa), \qquad (2.3)$$

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FIG. 1. Goldstone diagrams corresponding to the second-order correction $\delta n^{(2)}(\kappa)$. Each line indicates schematically a particle or a hole state, the dots represent the residual interaction and the encircled dots correspond to the number operator.

being

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$$N_{>} = \sum_{\kappa(\epsilon_{\kappa} > \epsilon_{F})} \delta n(\kappa).$$
(2.4)

A. 2p2h correlations

The ground-state wave function, with the first-order 2p2h perturbations included to the HF state, reads

$$|0\rangle = \mathscr{N}[|\mathrm{HF}\rangle + \frac{1}{4} \sum_{p's,h's} c_{p_1 p_2 h_1 h_2} |p_1 p_2 h_1 h_2\rangle], \quad (2.5)$$

where

$$c_{p_1 p_2 h_1 h_2} = -\frac{\langle p_1 p_2 h_1 h_2 | \hat{V} | \text{HF} \rangle}{E_{p_1 p_2 h_1 h_2}}, \qquad (2.6)$$

are the 2p2h amplitudes, and

$$\mathcal{N} = \frac{1}{\sqrt{1 + \mathcal{N}_{2p2h}}},\tag{2.7}$$

is the overall normalization factor, with

$$\mathcal{N}_{2p2h} \equiv \frac{1}{4} \sum_{p's,h's} |c_{p_1 p_2 h_1 h_2}|^2.$$
(2.8)

 $|p_1p_2h_1h_2\rangle$ and $E_{p_1p_2h_1h_2}$ stand, respectively, for the 2p2h configurations and single-particle energies, and \hat{V} is the residual interaction.

The usual second-order approximation for $n(\kappa)$ is obtained by retaining only the first term in the expansion

$$\mathcal{N}^{2} = 1 - \mathcal{N}_{2p2h} + \mathcal{N}_{2p2h}^{2} - \cdots, \qquad (2.9)$$



FIG. 2. Goldstone diagrams corresponding to the expansion of the norm in the normalized second-order 2p2h approximation $\delta n^{(2N)}(\kappa)$.

and for $\delta n(\kappa)$ one gets

$$\delta n^{(2)}(\kappa) = \frac{1}{2} \sum_{p's,h's} \left(\delta_{\kappa,p_1} - \delta_{\kappa,h_1} \right) \left[\frac{V_{p_1 p_2 h_1 h_2}}{E_{p_1 p_2 h_1 h_2}} \right]^2,$$
(2.10)

where $V_{p_1p_2h_1h_2}$ stand for the antisymmetrized matrix elements of \hat{V} . The contribution of $\delta n(\kappa)^{(2)}$ is schematically shown in Fig. 1, and

$$N_{>}^{(2)} = 2\mathcal{N}_{2p2h}.$$
 (2.11)

This result is only valid when $\mathcal{N}_{2p2h} \leq 1$, condition which is not fulfilled in most of the numerical calculations performed so far. This was precisely the reason why Van Neck *et al.* [7] have proposed to use the second-order normalized approximation [with \mathcal{N} given by (2.7)]. This leads to the variation in the occupation numbers

$$\delta n^{(2N)}(\kappa) = \mathcal{N}^2 \delta n^{(2)}(\kappa) \tag{2.12}$$

and the corresponding number of promoted particles is

$$N_{>}^{(2N)} = \mathscr{N}^{2} N_{>}^{(2)} = 2 \frac{\mathscr{N}_{2\text{p2h}}}{1 + \mathscr{N}_{2\text{p2h}}} \leq 2, \qquad (2.13)$$

which is a satisfactory bound for $N_>$. Yet, $\delta n^{(2N)}(\kappa)$ contains an infinite series of disconnected diagrams, illustrated in Fig. 2. We show below how they are removed by enlarging the configuration space.

B. 4p4h correlations

The ground-state wave function with the 2p2h and 4p4h admixtures included is written as

$$|0\rangle = \mathcal{N}\left[|\mathrm{HF}\rangle + \frac{1}{(2!)^{2}} \sum_{p's,h's} c_{p_{1}p_{2}h_{1}h_{2}}|p_{1}p_{2}h_{1}h_{2}\rangle + \frac{1}{(4!)^{2}} \sum_{p's,h's} c_{p_{1}p_{2}p_{3}p_{4}h_{1}h_{2}h_{3}h_{4}}|p_{1}p_{2}p_{3}p_{4}h_{1}h_{2}h_{3}h_{4}\rangle\right], \quad (2.14)$$

where

$$c_{p_1 p_2 p_3 p_4 h_1 h_2 h_3 h_4} = \frac{\langle \mathrm{HF} | \hat{V} | p_1 p_2 h_1 h_2 \rangle \langle p_1 p_2 h_1 h_2 | \hat{V} | p_1 p_2 p_3 p_4 h_1 h_2 h_3 h_4 \rangle}{E_{p_1 p_2 h_1 h_2} E_{p_1 p_2 p_3 p_4 h_1 h_2 h_3 h_4}},$$
(2.15)

and

FIG. 3. Goldstone diagrams corresponding to the different contributions in the fourth order 4p4h correction $\delta n^{(4C)}(\kappa)$.

$$\mathcal{N} = \frac{1}{\sqrt{1 + \mathcal{N}_{2p2h} + \mathcal{N}_{4p4h}}},$$
 (2.16)

with

$$\mathcal{N}_{4p4h} = \frac{1}{(4!)^2} \sum_{p's,h's} |c_{p_1 p_2 p_3 p_4 h_1 h_2 h_3 h_4}|^2, \quad (2.17)$$

which is the norm factor coming from the 4p4h contributions. The occupation numbers in the ground state (2.14) read

$$\delta n(\kappa) = \mathcal{N}^2 [\delta n^{(2)}(\kappa) + \delta n^{(4)}(\kappa)], \qquad (2.18)$$

where

$$\delta n^{(4)}(\kappa) = \frac{4}{(4!)^2} \sum_{p's,h's} \left(\delta_{\kappa,p_1} - \delta_{\kappa,h_1} \right) |c_{p_1 p_2 p_3 p_4 h_1 h_2 h_3 h_4}|^2.$$
(2.19)

Now, making an expansion up to the fourth order

$$\delta n(\kappa) = [1 - \mathcal{N}_{2p2h}] \delta n^{(2)}(\kappa) + \delta n^{(4)}(\kappa), \quad (2.20)$$

and after working out the 4p4h amplitudes we get

$$\delta n^{(4)}(\kappa) = \mathscr{N}_{2\text{p}2\text{h}} \delta n^{(2)}(\kappa) + \delta n^{(4C)}(\kappa), \qquad (2.21)$$

or

$$\delta n(\kappa) = \delta n^{(2)}(\kappa) + \delta n^{(4C)}(\kappa), \qquad (2.22)$$

with

$$\delta n^{(4C)}(\kappa) = \frac{1}{4} \sum_{p's,h's} \frac{V_{p_1 p_2 h_1 h_2} V_{p_3 p_4 h_3 h_4}}{E_{p_1 p_2 h_1 h_2} E_{p_3 p_4 h_3 h_4}} \left\{ -(\delta_{\kappa,p_1} + \delta_{\kappa,p_2} - \delta_{\kappa,h_1} - \delta_{\kappa,h_2}) \left[\frac{V_{p_1 p_2 h_1 h_3} V_{p_3 p_4 h_2 h_4}}{E_{p_1 p_2 h_1 h_3} E_{p_3 p_4 h_3 h_4}} + \frac{V_{p_1 p_3 h_1 h_2} V_{p_2 p_4 h_3 h_4}}{E_{p_1 p_3 h_1 h_2} E_{p_2 p_4 h_3 h_4}} \right] + (\delta_{\kappa,p_1} - \delta_{\kappa,h_1}) \left[4 \frac{V_{p_1 p_3 h_1 h_3} V_{p_2 p_4 h_2 h_4}}{E_{p_1 p_3 h_1 h_3} E_{p_2 p_4 h_2 h_4}} + \frac{1}{2} \frac{V_{p p_2 h_3 h_4} V_{p_3 p_4 h_1 h_2}}{E_{p p_2 h_3 h_4} E_{p_3 p_4 h_1 h_2}} \right] \right\},$$

$$(2.23)$$

being the contribution of the connected diagrams illustrated in Fig. 3. Figures 3(a), (b), and (c) describe the different contributions that can arise from the first term, while Fig. 3(d) represents the second term. For the depletion number we now get

$$N_{>} = 2\mathcal{N}_{2p2h} - 2\mathcal{N}_{2p2h}^{2} + 4\mathcal{N}_{4p4h} = 2\mathcal{N}_{2p2h} + 4\mathcal{N}_{4p4h}^{C},$$
(2.24)

where the last term comes from $\delta n^{(4C)}(\kappa)$. Note that the result (2.24) corresponds to the expansion up to fourth order of the quantity

$$\frac{2\mathscr{N}_{2p2h} + 4\mathscr{N}_{4p4h}}{1 + \mathscr{N}_{2p2h} + \mathscr{N}_{4p4h}} \leq 4 \tag{2.25}$$

which is the total depletion number obtained from the exact occupation probabilities (2.18), and same as (2.13), is also properly bounded.

Regarding the results of this subsection, the following should be remarked.

(a) In expansion (2.9) are retained the first two terms.

(b) The contributions to $\delta n(\kappa)$ coming from the disconnected graphs brought around by \mathcal{N}_{2p2h} are not present in final result because they cancel out with topological similar

contributions arising in the wave function (2.14), i.e., the first term of $\delta n^{(4)}(\kappa)$ in Eq. (2.21).

(c) The norm term \mathcal{N}_{4p4h} does not contribute in a calculation up to the fourth order and has been ignored in the expansion of \mathcal{N}^2 .

(d) We do not include the coupling among the 2p2h states $(\langle 2p2h | \hat{V} | 2p2h' \rangle = 0)$, because it would only lead to a redistribution of the 2p2h occupation probabilities, which does not have any effect on $\delta n^{(2)}(k)$.

C. Evaluation of the occupation numbers distribution in nuclear matter

In nuclear matter the HF ground state is approximated by the Fermi gas, the single-particle quantum numbers κ are the momentum **k**, the spin projection m_s and the isospin projection m_t of the particle, and $n(\kappa)$ turns into the momentum distribution

$$n(k) = \theta(1-k) + \delta n(k), \qquad (2.26)$$

where k is measured in units of the wave number k_F that defines the Fermi surface. The corresponding depletion number now reads

$$N_{>} = 3A \int_{1}^{\infty} dk k^{2} \theta(k-1) \,\delta n(k).$$
 (2.27)

The residual interaction is generically expressed in the form [6]

$$\hat{V}(\mathbf{q}) = \sum_{I} V^{I}(q) \mathbf{O}_{1}^{I}(\hat{\mathbf{q}}) \cdot \mathbf{O}_{2}^{I}(-\hat{\mathbf{q}}), \qquad (2.28)$$

where $V^{I}(q)$ are the interaction strengths and $I \equiv T, S, J$ encompasses the isospin, spin, and total angular momentum quantum numbers of the operators $\mathbf{O}^{I}(\hat{\mathbf{q}})$, defined as

$$\mathbf{O}^{000}(\hat{\mathbf{q}}) = 1, \quad \mathbf{O}^{010}(\hat{\mathbf{q}}) = i(\hat{\mathbf{q}} \cdot \boldsymbol{\sigma}), \quad \mathbf{O}^{011}(\hat{\mathbf{q}}) = (\hat{\mathbf{q}} \times \boldsymbol{\sigma}),$$
$$\mathbf{O}^{100}(\hat{\mathbf{q}}) = \boldsymbol{\tau}, \quad \mathbf{O}^{110}(\hat{\mathbf{q}}) = i(\hat{\mathbf{q}} \cdot \boldsymbol{\sigma})\boldsymbol{\tau}, \quad \mathbf{O}^{111}(\hat{\mathbf{q}}) = (\hat{\mathbf{q}} \times \boldsymbol{\sigma})\boldsymbol{\tau}.$$
(2.29)

The exchange contributions to $\delta n(k)$ will be dropped out since, as pointed out Van Order and Donnelly [11], they are small in comparison with the direct ones. In this way we get

$$\delta n^{(2)}(k) = \frac{1}{4} \int d\mathbf{q} [\theta(k-1)\theta(1-|\mathbf{k}+\mathbf{q}|)\mathscr{F}(\mathbf{k}\cdot\mathbf{q}+q^2,q) + \theta(1-k)\theta(|\mathbf{k}-\mathbf{q}|-1)\mathscr{F}(\mathbf{k}\cdot\mathbf{q},q)] \\ \times \sum_{I} [v^{I}(q)]^2(2T+1)2^{J}, \qquad (2.30)$$

for the second-order correction to n(k),

$$\mathcal{N}_{2p2h} = \frac{3A}{8\pi} \int d\mathbf{k} d\mathbf{q} \theta(k-1) \theta(1-|\mathbf{k}+\mathbf{q}|) \mathscr{F}(\mathbf{k} \cdot \mathbf{q} + q^2, q) \sum_{I} [v^{I}(q)]^2 (2T+1) 2^{J}$$
(2.31)

for second-order correction to the norm, and

$$\delta n^{(4C)}(k) = -\frac{1}{16} \int d\mathbf{l} \int d\mathbf{p} \int d\mathbf{q} \,\theta (1 - |\mathbf{l} + \mathbf{q}|) \,\theta(\mathbf{t} - 1) [\,\theta(k - 1)] \,\theta(k - 1) \,\mathcal{G}(\mathbf{k}, \mathbf{l}, \mathbf{p}, \mathbf{q}) + \theta(1 - k) \,\mathcal{G}(\mathbf{k} - \mathbf{q}, \mathbf{l}, \mathbf{p}, \mathbf{q})],$$
(2.32)

for the fourth-order correction to n(k). In the above equations

$$v^{I}(q) = \frac{2k_{F}^{3}}{(2\pi)^{3}\epsilon_{F}}V^{I}(q), \qquad (2.33)$$

$$\mathscr{F}(\alpha,q) = \int \frac{\theta(x-1)\,\theta(1-|\mathbf{x}+\mathbf{q}|)}{(\alpha+\mathbf{q}\cdot\mathbf{x})^2} d\mathbf{x} = \frac{2\pi}{q} \Big\{ \Big[-1+\ln 2+\ln \left| \frac{q^2-\alpha+q}{q^2-2\alpha} \right| - \frac{\alpha}{q^2} \ln \left| \frac{q^2-\alpha+q}{q-\alpha} \right| \Big] \theta(2-q) + \Big[\ln \left| \frac{q^2-\alpha+q}{q^2-\alpha-q} \right| - \frac{\alpha}{q^2} \frac{q^2-\alpha+q}{q^2-\alpha-q} + \frac{2}{q} \Big] \theta(q-2) \Big\},$$

$$(2.34)$$

and

$$\mathscr{F}(\mathbf{k},\mathbf{l},\mathbf{p},\mathbf{q}) = [\theta(1-|\mathbf{l}+\mathbf{p}|)\mathscr{F}(\mathbf{l}\cdot\mathbf{p}+l^{2},l) + \theta(1-|\mathbf{k}+\mathbf{p}|)\mathscr{F}(\mathbf{k}\cdot\mathbf{p}+l^{2},l) + \theta(|\mathbf{l}+\mathbf{q}+\mathbf{p}|-1)\mathscr{F}(-\mathbf{l}\cdot\mathbf{p}-\mathbf{q}\cdot\mathbf{p},l) \\ + \theta(|\mathbf{k}+\mathbf{q}+\mathbf{p}|-1)\mathscr{F}(-\mathbf{k}\cdot\mathbf{p}-\mathbf{q}\cdot\mathbf{p},l)]\frac{\theta(1-|\mathbf{k}+\mathbf{q}|)}{(\mathbf{k}\cdot\mathbf{q}+q^{2}+\mathbf{l}\cdot\mathbf{q})^{2}}\sum_{I} [v^{I}(q)]^{2}(2T+1)2^{J}\sum_{I'} [v^{I'}(l)]^{2}(2T'+1)2^{J'}.$$

$$(2.35)$$

In Eq. (2.32) we do not include the contributions arising from the second term in Eq. (2.23), because, as discussed later on, they are relatively small.

From Eqs. (2.11), (2.13), and (2.31) it can be easily seen that $N_{>}^{(2)}$ fulfills the extensively condition, but $N_{>}^{(2N)}(k)$ does not. This unphysical behavior of $N_{>}^{(2N)}$ is due to the disconnected diagrams contained in $\delta n^{(2N)}(k)$ and is circumvented by including the 4p4h excitations. This lead us to the result in Eq. (2.24), which again satisfies the requirement on extensivity.

III. NUMERICAL RESULTS AND CONCLUSIONS

The momentum distribution n(k) has been evaluated for the normal nuclear matter density, i.e., $k_F = 1.36$ fm, and by parametrizing the residual interaction (2.28) as follows:

$$V^{000}(q) = C_{\pi}(q)f, \quad V^{100}(q) = C_{\pi}(q)f', \quad (3.1)$$
$$V^{010}(q) = V^{011}(q) = C_{\pi}(q)g,$$
$$V^{110}(q) = C_{\pi}(q) \left(g' - \frac{q^2}{q^2 + m_{\pi}^2}\right), \quad V^{111}(q) = C_{\pi}(q)g'.$$

Here f, f', g, and g' are the Landau-Migdal parameters, $C_{\pi}(q) = f_{\pi}^2 / \mu_{\pi}^2 \Gamma_{\pi}^2(q)$, $m_{\pi} = \mu_{\pi} / k_F$, and

$$\Gamma_{\pi}(q) = \frac{\Lambda_{\pi}^2 - m_{\pi}^2}{\Lambda_{\pi}^2 + q^2},$$
(3.2)

is the πNN form factor.



FIG. 4. Plot of the second-order 2p2h approximation for $N_>/A$, as function of the cutoff energy E_c (measured in MeV) for 2p2h excitations.

We do not incorporate explicitly the ρ meson since, as pointed out by Dickhoff [12], it produces too much suppression of the tensor force. Yet, the contribution of heavier mesons is taken into account empirically by fixing the values of g' and Λ_{π} (as done by Alberico *et al.*, [2]) to reproduce the effective tensor and central components of the force derived by Dickhoff from a *G*-matrix calculation ([12,13]). This procedure yields g'=0.5 and $\Lambda_{\pi}=800 \text{ MeV}/(\hbar ck_F)$. On the other hand, for f, f', and g we have found more appropriate to adopt the values obtained by Bäckman *et al.* [14] from the analysis of stability conditions and empirical values of physical observables, i.e., f=-0.12, f'=0.39, and g=0.

The following three different ways of adding the groundstate correlations to the Fermi gas distribution will be numerically compared.

(1) The usual 2p2h second-order approximation, i.e., Eq. (2.30) is employed in the evaluation of $\delta n(k)$.

(2) The above approach is modified by including the effect of the norm through Eqs. (2.12), (2.7), and (2.31).

(3) Fourth order approximation within the 2p2h+4p4h subspace, in which $\delta n(k)$ is given by Eqs. (2.22) and (2.32).

The momentum space integrals have been evaluated for energies up to 300 MeV. This limit has been established by examining the behavior of $N_>/A$ [with $\delta n(k)$ given by (2.30)] as function of the cutoff energy. The results are displayed in Fig. 4.

The resulting momentum distributions n(k) are shown in Fig. 5. In approximation (2), where the results are *A* dependent, we have chosen, as an example, the ⁴⁰Ca nucleus. For this case the empirical occupation numbers are available from an optical potential analysis of experimental cross sections [15] (see also [4]). The second-order approach produces a pronounced high-momentum tail above the Fermi level with a corresponding strong reduction of the low-momentum part as compared with the Fermi gas. In the normalized case (2), the previous variation of momentum distribution $\delta n(k)$ is just rescaled by the factor $\mathcal{N}^2 = 0.11$ and this diminishes drastically the effect of the ground-state correlations. (Note that for $A = 90 \mathcal{N}^2 = 0.05$.) Within approximation (3) the depletion of the momentum distribution is quite significant, but still relatively small in comparison with



FIG. 5. Momentum distribution $n(\kappa)$ for the different approximations: Fermi gas (thin full lines), second-order approximation with 2p2h correlations $[\delta n(\kappa) = \delta n^{(2)}(\kappa)]$ (long-dashed lines), normalized second-order approximation for ⁴⁰Ca $[\delta n(\kappa) = \delta n^{(2N)}(\kappa)]$ (short-dashed lines), and fourth-order approximation with 2p2h+4p4h correlations, $[\delta n(\kappa) = \delta n^{(2)}(\kappa) + \delta n^{(4C)}(\kappa)]$ (thick full lines).

that obtained in the plain 2p2h approach. This effect can be interpreted as the reduction of the 2p2h occupation probabilities by the coupling with 4p4h states. It is worth noting that the effect of the last term in Eq. (2.23) cannot be distinguished visually in Fig. 5.

The calculated numbers of particles shifted above the Fermi level, for the implemented approximations, turn out to be $N_{>}=16.8, 1.8$, and 1.9, respectively. The last result is not only physically sound, but also consistent with the above mentioned study [15], which yields $N_{>}({}^{40}Ca) \cong 2.7$.

We summarize our conclusions as follows.

The plain second-order approximation for the momentum distribution grossly overestimates the effect of the 2p2h ground-state correlation. This is put in evidence by the very large result obtained for the number of particles shifted above the Fermi level, as already pointed out [7].

The introduction of norm corrections, following the recipe given in Ref. [7], leads unavoidably to an *A*-dependent momentum distribution, which is engendered by the disconnected diagrams present in the norm. As a consequence the number depletion becomes a nonextensive quantity for the nuclear matter, which is very strong drawback of the approximation.

We have found out that the just-mentioned unphysical contributions of the disconnected diagrams are cancelled by the inclusion of 4p4h correlations. Besides, the contribution of the 4p4h connected diagrams strongly hinder the effect of 2p2h correlations, yielding a momentum distribution with properly bounded depletion number.

The present results seem to indicate that the 2p2h and 4p4h correlations are the dominant degrees of freedom to be considered in the description of the nuclear ground state. In addition, we feel that the evaluation of the momentum distribution up to fourth order is a reasonably good approximation and that is not crucial the inclusion of higher-order correlations beyond the 4p4h ones.

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