# Quartets reexamined

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Alpha spectroscopic factors for quartet wave functions are recalled for identical chains, which correspond to positive natural parity states. A generalization is presented for nonidentical chains which can give rise to negative natural parity states. Applications to the $0f$-$1p$ shell quartet structures observed in the ($^{16}$O,$^{12}$C) and ($^{6}$Li,$d$) reactions are given for the population of the first $2^+$ excited state and for the population of low-lying $3^-$ excited states.

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## I. INTRODUCTION

A microscopic understanding of four-particle transfer reaction cross sections requires a careful analysis of spectroscopic factors. Unfortunately they are not available for odd states of even-even nuclei of the $fp$ shell, mostly due to the absence of microscopic calculations of the final state wave functions. The most up-to-date $fp$ shell-model calculations for instance are done for even states and do not include odd configurations in their basis states, by including the $g9/2$ level in their scheme for instance. Such calculations are in principle feasible; however, the size of the problem becomes too large and they have not been undertaken yet [1]. In a more realistic way, but still on a macroscopic basis, we propose in the present work to obtain these spectroscopic factors through a generalization of the quartet scheme [2] to odd states.

Let us consider a quartet configuration made of two protons in a given shell-model orbital and of two neutrons either in the same or in another orbital. Each neutron-proton pair is coupled to the maximum chain spin $J_{12}$. These two chains are then recoupled to the even final spin $J$ which ranges from 0 to $2J_{12}-2$, due to the exclusion Pauli principle [2]. Such configurations are highly symmetric and correspond to very low excitation energies [3]. In alpha direct surface transfer reactions, one can assume that final states will be populated through the component of their wave functions corresponding to a quartet coupled to a core made of the ground state of the even-even target nucleus. This mechanism is strongly favored as it has been shown that at one-third of the central nuclear density, i.e., at the nuclear surface, nuclear matter has a tendency to condense into alpha particles [4].

We propose to generalize these quartet wave functions for natural negative parity states, i.e., odd spin states, as follows. This can be achieved by choosing basic configurations made of two protons in the same shell-model orbital, one neutron being in that orbital while the other one belonging to an orbital of the next major shell with opposite parity. The two proton-neutron chains are coupled to their maximum angular momenta (stretch scheme) which are then recoupled to the final odd spin of the nucleus. These wave functions are still highly symmetric and correspond to low-lying excited states such as $3^-$ states. In what follows, we will also considered direct alpha surface transfer reactions leading to odd final states described by wave function components built from the coupling of a quartet to the even-even target nucleus in its ground state.

Our paper is organized as follows. Section II presents the basic formalism to extract alpha spectroscopic factors involved in the distorted-wave Born approximation (DWBA) cross sections. In Sec. III, we discuss the experimental spectroscopic strength of alpha-transfer reactions populating the first $2^+$ excited states of various residual nuclei in the $fp$ shell. These correspond to quartet states of the first type made up of two identical chains. Finally, in Sec. IV, we consider for the $fp$ shell the case of the $3^-$ states, corresponding to the second kind of quartet wave function where nonidentical chains are recoupled.

## II. QUARTET SPECTROSCOPIC FACTORS

A quartet wave function is built from two neutrons in a $n_1,l_1,j_1$ single-particle level and two protons in another $n_2,l_2,j_2$ level. These two levels can be the same. Each neutron-proton pair is coupled to its maximum spin, $J_{12}=j_1+j_2$. The quartet wave function is defined by coupling these two identical chains of spin $J_{12}$ to a given final spin $J$ (natural positive parity spin). It can be written as

$$|Q;J\pi\rangle=\mathcal{N}[[(n_1,l_1,j_1)(n_2,l_2,j_2)]J_{12}[(n_2,l_2,j_2)(n_1,l_1,j_1)]J_{12};J\pi\rangle, \tag{1}$$

where $\mathcal{N}$ is the norm of the quartet wave function; it is given by

$$\mathcal{N}=\left(2-2\begin{bmatrix} j_1 & j_2 & J_{12} \\ j_2 & j_1 & J_{12} \\ J_{12} & J_{12} & J \end{bmatrix}\right)^{-1/2}. \tag{2}$$

 

The square brackets are the coefficient of the unitary transformation allowing one to change from one basis to the another one; it is defined in terms of the standard $9j$ symbol by (see Ref. [5])

$$\begin{bmatrix} j_1 & j_2 & J_{12} \\ j_3 & j_4 & J_{34} \\ J_{13} & J_{24} & J \end{bmatrix} = \sqrt{(2J_{12}+1)(2J_{34}+1)(2J_{13}+1)(2J_{24}+1)} \begin{Bmatrix} j_1 & j_2 & J_{12} \\ j_3 & j_4 & J_{34} \\ J_{13} & J_{24} & J \end{Bmatrix}. \tag{3}$$

For the case of two identical chains, the final spin $J$ is even and ranges between 0 and $2J_{12}-2$, due to the Pauli principle. The alpha spectroscopic amplitude for a natural positive parity state is then [6]

$$\langle \phi_\alpha | Q; J\pi \rangle = 2\mathcal{N} \sum_{L_1,L_2} \begin{bmatrix} j_1 & j_2 & J_{12} \\ j_1 & j_2 & J_{12} \\ L_1 & L_2 & J \end{bmatrix} \begin{bmatrix} l_1 & 1/2 & j_1 \\ l_1 & 1/2 & j_1 \\ L_1 & 0 & L_1 \end{bmatrix} \begin{bmatrix} l_2 & 1/2 & j_2 \\ l_2 & 1/2 & j_2 \\ L_2 & 0 & L_2 \end{bmatrix} \langle n_1 l_1 n_1 l_1 | N_1 L_1 00 \rangle \langle n_2 l_2 n_2 l_2 | N_2 L_2 00 \rangle$$
$$\times \langle N_1 L_1 N_2 L_2 | NL00 \rangle. \tag{4}$$

The alpha wave function $\phi_\alpha$ is a four-nucleon $L=S=T=0$ wave function where each of the four nucleons lie in the $0s1/2$ orbital. The brackets $\langle | \rangle$ are the coefficients of the Talmi-Moshinsky transformation [7]. They are different from 0 only when the parity and the energy are conserved. The above formula is valid only for an even-even target nucleus and a transfer reaction toward empty shells; otherwise, there are additional fractional parentage coefficients. In expression (4), the first pair of square brackets recouples the two proton-neutron chain to a proton pair and a neutron pair. The two other pair brackets transform the $jj$ coupling into the $\not{s}$ coupling. The first two Talmi-Moshinsky coefficients extract the $0s$ relative motion of the two neutrons and of the two protons, while the third one extracts the total center-of-mass motion from the $0s$ relative motion of the two dinucleon pairs. The factor of 2 arises from the fact that the two protons and the two neutrons occupy the same subshells. The final spin $J$, which is equal to $L$, is even.

For natural negative parity states, these formulas have to be modified as follows. Let us consider the particular case where the two protons are in a $n_1,l_1,j_1$ orbital, the first neutron being in that same level and the other one in a $n_2,l_2,j_2$ orbital belonging to a major shell of a different parity. This generalized quartet wave function can be written as

$$|Q;J\pi\rangle = \mathcal{N}[[(n_1,l_1,j_1)(n_1,l_1,j_1)]J_{11}[(n_1,l_1,j_1)(n_2,l_2,j_2)]J_{12};J\pi\rangle. \tag{5}$$

The corresponding value of the norm $\mathcal{N}$ is

$$\mathcal{N} = \left( 1 - \begin{bmatrix} j_1 & j_1 & J_{11} \\ j_1 & j_2 & J_{12} \\ J_{11} & J_{12} & J \end{bmatrix} \right)^{-1/2}. \tag{6}$$

In this quartet wave function, the neutron-proton pair in the $n_1,l_1,j_1$ orbit is coupled to its maximum spin, $J_{11}=2j_1$; similarly for the other pair, $J_{12}=j_1+j_2$. The alpha spectroscopic amplitude for a natural negative parity states reads [6]

$$\langle \phi_\alpha | Q; J\pi \rangle = \sqrt{2}\mathcal{N} \sum_{L_1,L_{12}} \begin{bmatrix} j_1 & j_1 & J_{11} \\ j_1 & j_2 & J_{12} \\ L_1 & L_{12} & J \end{bmatrix} \begin{bmatrix} l_1 & 1/2 & j_1 \\ l_1 & 1/2 & j_1 \\ L_1 & 0 & L_1 \end{bmatrix} \begin{bmatrix} l_1 & 1/2 & j_1 \\ l_2 & 1/2 & j_2 \\ L_{12} & 0 & L_{12} \end{bmatrix} \langle n_1 l_1 n_1 l_1 | N_1 L_1 00 \rangle \langle n_1 l_1 n_2 l_2 | N_{12} L_{12} 00 \rangle$$
$$\times \langle N_1 L_1 N_{12} L_{12} | NL00 \rangle. \tag{7}$$

As above, we are only concerned with direct transfer reactions on even-even target nuclei toward empty shells. Otherwise, one would have to introduce appropriate fractional parentage coefficients. The spectroscopic factors are proportional to the squares of these amplitudes. The factor $\sqrt{2}$ arises from the fact that there are two identical nucleons in the same orbital. In this latter case $J$ is odd and equal to $L$.

In Fig. 1, the alpha spectroscopic factors $S(\alpha)$ are given for different shell-model configurations versus their total spin $J$. For high values of $J$, namely, for the largest angular momenta, $S(\alpha)$ increases as the final spin $J$ increases. This is due to the fact that the four nucleons have a marked tendency for high spin states and to move together in the same direction as the four nucleons of an alpha particle would do upon rotating around the target core with an angular momentum $J=L$. It can also be inferred from Fig. 1 that some natural negative parity state based for instance the configuration $\{[1p3/2]^2_{3^+}[0g9/2,1p3/2]_{6^-}\}_{J^-}$ can be as strongly populated as natural positive parity states.

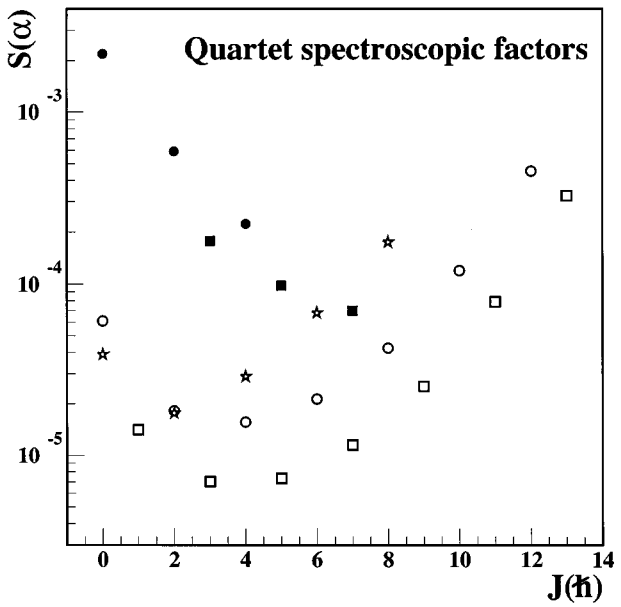


FIG. 1. Alpha spectroscopic factors for natural parity states calculated for various quartet configurations. Open circles, $(0f7/2)^4$ configurations; solid circles, $(1p3/2)^4$ configurations; stars, $(1p3/2,0f7/2)^2$ configurations (natural positive parity states); open squares, $(0f7/2)^2(0f7/2,0g9/2)$ configurations; solid squares, $(1p3/2)^2(1p3/2,0g9/2)$ configurations (negative parity states).

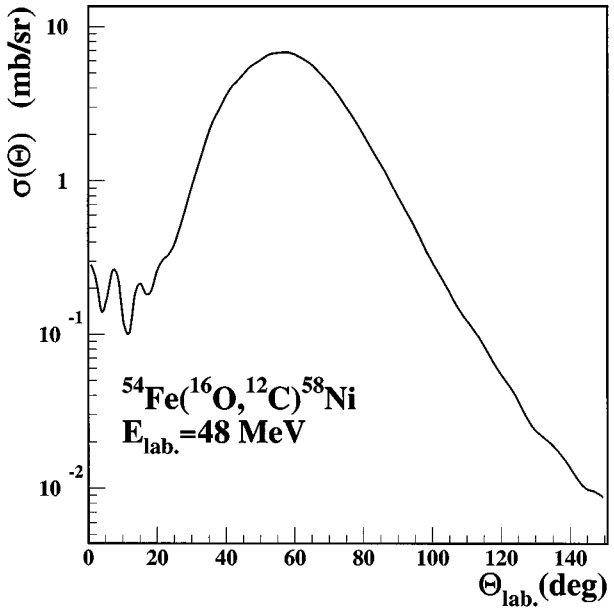


FIG. 2. Reduced DWBA angular distribution for the $^{54}$Fe($^{16}$O,$^{12}$C)$^{58}$Ni reaction populating the first $2^+$ excited state located at 1.454 MeV, calculated at 48.0 MeV incident energy.

### III. SYSTEMATICS OF THE FIRST EXCITED $2^+$ STATE CROSS SECTIONS IN THE *fp* SHELL

References [3] and [8] have reported measurements of differential cross sections for the first excited $2^+$ states of direct alpha-transfer ($^{16}$O,$^{12}$C) reactions on various *fp* shell target nuclei performed at 48.0 MeV incident energy and at 40° laboratory angle. We have extracted spectroscopic factors from these experimental data by computing the DWBA reduced cross sections with the code PTOLEMY [9]. The optical parameters which we used are given in Table I and are average parameters for this range of nuclei [10]. The alpha bound state parameters are 1.25 fm for the reduced radius and 0.65 fm for the diffusivity. The product of the entrance spectroscopic factor by the residual nucleus one is given by the usual relationship: $\sigma_{\text{expt}}=S_iS_f\sigma_{\text{DWBA}}$. In principle we can obtain only an order of magnitude for the spectroscopic factor since the theoretical DWBA differential cross section varies rapidly with the *Q* value of the reaction and with the angle. However, at 48 MeV $^{16}$O incident energy, the grazing angle is not too far from 40.0° laboratory as shown in Fig. 2 where the DWBA cross section for the $^{54}$Fe($^{16}$O,$^{12}$C)$^{58}$Ni reaction populating the first $2^+$ excited state is plotted in the laboratory frame.

In Table II, the spectroscopic factors for the residual nuclei populated by the ($^{16}$O,$^{12}$C) alpha-transfer reactions [8] are given for a large number of *fp* shell nuclei. The entrance spectroscopic factor is the same for all reactions. For the $^{44}$Ti residual nucleus, the experimental spectroscopic factor is higher than for all the other Ti isotopes. This leads us to conclude that the $[0f7/2]^4_{2^+}$ quartet configuration with some admixture of the $[1p3/2^4]_{2^+}$ component must be strongly excited. The values of these quartet spectroscopic factors are $S([0f7/2]^4_{2^+})=0.18\times10^{-4}$ and $S([1p3/2]^4_{2^+})=0.59\times10^{-3}$, respectively (see Fig. 1). Also, for a $2^+$ state, the calculated spectroscopic factor value is much higher for the $1p3/2$ orbital than for $0f7/2$. An increasing neutron number

TABLE I. Optical model parameters for an $^{16}$O projectile at 48.0 MeV incident energy on various $0f$-$1p$ shell nuclei.

| | |
|---|---|
| *V* (MeV) | 50.0 |
| *W* (MeV) | 20.0 |
| *r* (fm) | 1.250 |
| *a* (fm) | 0.550 |

TABLE II. First $2^+$ experimental alpha spectroscopic factors defined as $S_iS_f=\sigma_{\text{expt}}/\sigma_{\text{DWBA}}$ for the ($^{16}$O,$^{12}$C) reaction. The optimum *Q* value for which these transfer reactions are dynamically favored is equal to $-6$ MeV. The transferred angular momentum $L_T$ favored at this *Q* value is equal to 3. In the last column are listed the ($^6$Li,*d*) spectroscopic factors. It can be noted that they have the same relative behavior as the ones of the ($^{16}$O,$^{12}$C) reactions.

| Final nuclei | *Q* (MeV) | $\sigma_{40^\circ\ \text{lab}}$ ($\mu$b) | $S_iS_f$ ($\times10^3$) | $S(^6\text{Li},d)$ |
|---|---|---|---|---|
| $^{44}$Ti | −3.12 | 80.0 | 2.36 | 0.32 |
| $^{46}$Ti | −0.045 | 9.2 | 1.50 | 0.071 |
| $^{48}$Ti | +1.30 | 1.35 | 0.34 | 0.052 |
| $^{52}$Ti | −0.53 | 3.75 | 0.49 | 0.013 |
| $^{52}$Cr | +0.76 | 0.88 | 0.50 | — |
| $^{54}$Cr | −0.07 | 7.80 | 2.56 | 0.075 |
| $^{58}$Ni | −2.22 | 31.0 | 7.80 | 0.058 |
| $^{60}$Ni | −2.30 | 18.5 | 4.80 | 0.042 |
| $^{62}$Zn | −4.74 | 72.0 | 33.02 | 0.072 |
| $^{64}$Zn | −4.19 | 58.0 | 17.57 | 0.088 |
| $^{66}$Zn | −3.62 | 30.0 | 9.52 | 0.061 |
| $^{68}$Zn | −2.90 | 7.0 | 2.41 | — |

for the Ti nuclei yields a smaller spectroscopic factor, with the exception of $^{52}$Ti which corresponds to the shell closure of the $^{48}_{20}\mathrm{Ca}_{28}$ double magic target nucleus. For that particular residual nucleus, the quartet state is mostly a $[1p3/2]^4_{2^+}$ configuration coupled to the $^{48}$Ca target ground state core, which has a large spectroscopic factor (see Fig. 1). In this systematics of Ti isotopes, there is a neutron blocking effect associated with the filling of the $0f7/2$ level.

A similar pattern can be observed for Cr isotopes where the neutron core shell closure at $N=28$ produces for $^{54}$Cr a spectroscopic factor larger than that of $^{52}$Cr. Indeed, $^{54}$Cr can be represented as a magic $^{50}_{22}\mathrm{Ti}_{28}$ core plus a quartet such as $[1p3/2]^4_{2^+}$.

In case of the target core $^{54}$Fe, the neutron $0f7/2$ shell is filled so that the first $2^+$ state of the $^{58}$Ni nucleus is mainly a $[1p3/2]^4_{2^+}$ quartet state coupled to the target ground state, which yields a large spectroscopic factor, $S([1p3/2]^4_{2^+}) = 0.59\times10^{-3}$, as can be seen from Fig. 1.

For the $^{60}$Ni final nucleus, the experimental spectroscopic factor is smaller than that of $^{58}$Ni, due to the excess of two neutrons in the $1p3/2$ orbit; this results from the same neutron blocking effect.

For the zinc isotopes, the spectroscopic factors decrease significantly as the neutron excess increases. The $0f7/2$ is now completely filled up and the transfer reactions occur toward the higher orbits for which the quartet spectroscopic factors are $S([0f5/2]^4_{2^+})=0.98\times10^{-5}$, $S([1p3/2]^4_{2^+}) = 0.59\times10^{-3}$, and $S([1p3/2,0f5/2]^2_{2^+})=0.24\times10^{-5}$. This is a definite signature of neutron blocking as the higher $fp$ neutron shells are now filled.

The relative behavior of the alpha spectroscopic factors extracted from the ($^6$Li,$d$) reaction [11] is very similar to the one of the ($^{16}$O,$^{12}$C) reaction discussed above. This is illustrated by the last column of Table II. This ($^6$Li,$d$) reaction trend supports entirely the previous discussion on ($^{16}$O,$^{12}$C) reactions. Moreover, it provides us with a further justification of our analysis of the spectroscopic factors for the residual nuclei independent of the entrance channel.

## IV. SYSTEMATICS OF SOME EXCITED $3^-$ STATE CROSS SECTIONS IN THE $fp$ SHELL

At the time the experiments were done (see [3] and [8]), it was not understood why $3^-$ states were so strongly populated by the ($^{16}$O,$^{12}$C) and ($^6$Li,$d$) reactions [11]. Actually, we can build quartet wave functions with individual nucleons belonging to different major shells as derived in Sec. II, and thus obtain natural negative parity states. Let us take as an example $^{44}$Ti as a residual nucleus; it has a $3^-$ state at an excitation energy of 3.942 MeV which is strongly populated by both the ($^{16}$O,$^{12}$C) and the ($^6$Li,$d$) reactions. The experimental spectroscopic factor, in this latter case, is equal to 0.037 [11]. The corresponding quartet configuration for this $3^-$ excited state can be $\{[0f7/2]^2_{7^-}[0f7/2,0g9/2]_{8^-}\}_{3^-}$ with some admixture of the $\{[1p3/2]^2_{3^-}[1p3/2,0g9/2]_{6^-}\}_{3^-}$ configuration. The theoretical quartet spectroscopic factors are $0.702\times10^{-5}$ and $0.177\times10^{-3}$, respectively. Consequently, for $fp$ shell nuclei, the most favored configuration which can be substantially populated is the $p^3g$ where one adds one neutron into the odd $g9/2$ level.

TABLE III. The $3^-$ experimental alpha spectroscopic factors from the work of Fulbright *et al.* The corresponding quartet configurations for these excited states can be $f^3g$ with some admixture of $p^3g$ configuration with various weights. The corresponding theoretical spectroscopic factors for these $3^-$ configurations are given in the text. The $p^3g$ quartet configuration is predicted to be strongly favored in these direct alpha-transfer reactions.

| Final nuclei | Exc. (MeV) | $S(3^-)$ |
|---|---|---|
| $^{44}$Ti | 3.942 | 0.037 |
| $^{56}$Fe | 4.512 | 0.022 |
| $^{58}$Ni | 4.475 | 0.043 |
| $^{58}$Ni | 6.742 | 0.014 |
| $^{58}$Ni | 6.847 | 0.034 |
| $^{60}$Ni | 4.045 | 0.059 |
| $^{62}$Zn | 3.19 | 0.078 |
| $^{64}$Zn | 2.983 | 0.082 |
| $^{66}$Zn | 2.830 | 0.022 |

There is another reason, of a dynamical nature, to expect strongly populated $3^-$ levels in ($^{16}$O,$^{12}$C) direct reactions. At 48 MeV incident energy for these $fp$ shell nuclei, there is a perfect matching of the transferred angular momentum $L_T=3$ between the entrance grazing wave angular momentum and the exit grazing wave one. The $^{44}$Ti nucleus deserves special mention since there is some alpha clustering in its ground state and its first natural parity excited levels. This has been recently calculated and evidenced by Buck *et al.* [14].

Experimental alpha spectroscopic factors for various $3^-$ excited states populated by the ($^6$Li,$d$) direct transfer reaction are listed in Table III. Let us note that the $3^-$ quartet strength can be fragmented on several levels. In the quartet picture, one expects the $3^-$ levels to be populated through their $p^3g$ components. The actual amplitude depends upon the shell filling of the orbitals and also of a possible neutron blocking effect. We shall discuss now in detailed the results of Table III.

In case of the $^{52}$Cr($^6$Li,$d$)$^{56}$Fe alpha-transfer reaction, for which the target nucleus has a closed neutron shell, the final $3^-$ excited state located at 4.512 MeV has a strong experimental spectroscopic factor of 0.022 [11]. The likely quartet configuration is $\{[1p3/2]^2_{3^-}[1p3/2,0g9/2]_{6^-}\}_{3^-}$ coupled to the magic neutron target core.

For the $^{54}$Fe($^{16}$O,$^{12}$C)$^{58}$Ni transfer reaction, there are three $3^-$ excited states located at 4.475, 6.024, and 7.521 MeV, respectively. All of them are strongly populated. As the neutron shell is closed for $^{54}$Fe target nucleus, the final state could have the following quartet configuration: $\{[1p3/2]^2_{3^-}[1p3/2,0g9/2]_{6^-}\}_{3^-}$ coupled to the $^{54}$Fe target core. The spins of these three states have been identified by $(\alpha,\alpha')$ on a $^{58}$Ni target nucleus (see Refs. [12] and [13]). Large alpha spectroscopic factors have been also observed in the $^{54}$Fe($^6$Li,$d$)$^{58}$Ni reaction for the $3^-$ excited states located at 4.475, 6.742, and 6.847 MeV. Their values are 0.043, 0.014, and 0.034, respectively [11].

This general behavior is similar to that observed for the $^{56}$Fe($^{16}$O,$^{12}$C)$^{60}$Ni reaction, however, with smaller yield, de-

spite the fact that there are two extra neutrons outside the closed shell. The $3^-$ excited states located at 4.045, 6.16, and 6.53 MeV are also strongly populated. Thus the same kind of $3^-$ quartet configurations as for the $^{58}$Ni nucleus can be attributed for these excited states. Furthermore, the $3^-$ excited states at 4.045 MeV have a very large quartet spectroscopic factor of 0.059 obtained from the analysis of the $^{56}$Fe($^6$Li,$d$)$^{60}$Ni differential cross section measured at 28 MeV incident energy [11].

In the $^{58}$Ni($^{16}$O,$^{12}$C)$^{62}$Zn transfer reaction the $3^-$ excited states located at 3.19 MeV is very strongly populated and has a large ($^6$Li,$d$) spectroscopic factor: 0.078 [11]. The previous $3^-$ quartet configuration $\{[1p3/2]^2_{3^-}[1p3/2,0g9/2]_{6^-}\}_{3^-}$ coupled to the $^{58}$Ni target ground state accounts for this feature. A similar behavior is also encountered for the 2.983 MeV $3^-$ state of $^{64}$Zn level with a spectroscopic factor of 0.082 [11]. A neutron blocking effect is also found with the filling of the neutron shells for the last ($^6$Li,$d$) alpha-transfer reactions on $^{62}$Ni target nuclei. Indeed, the spectroscopic factors of this first $3^-$ state is much smaller. Its experimental values for the 2.830 MeV $3^-$ state of $^{66}$Zn final states is 0.022 [11].

## V. CONCLUSION

From the comparison between theoretical alpha spectroscopic factors for quartet states and experimental data, we have shown in this work that odd as well as even angular momenta of natural parity states can be strongly populated by alpha direct surface transfer reaction. In the target nucleus, neutron excess has a marked tendency to reduced experimental transfer cross sections. Transfer reactions towards the $1p3/2$ orbitals are favored with respect to transfer reaction toward the $0f7/2$ one. Our present analysis corroborates the quartet structure evidenced in the $fp$ shell by the authors of Refs. [3] and [8] for the natural positive parity states and explained for the first time the quartet nature of the natural negative parity states such as the $3^-$ levels.

A complete shell-model calculation for odd states in $fp$ shell nuclei implies an admixture of basis states built on $fg$ and $pg$ components. Such calculations are still beyond the present state of the art of these calculations [1]. Thus our analysis presents a valuable step toward a microscopic understanding of the odd states populated by four-particle transfer reactions.

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