Effects of short-range correlations on Ca isotopes

G. A. Lalazissis
Physik Department, Technische Universität München, D-85747 Garching, Germany

S. E. Massen
Nuclear Physics Laboratory, Department of Physics, University of Oxford, Oxford OX1-3RH, United Kingdom and Department of Theoretical Physics, Aristotle University of Thessaloniki, GR 54006 Thessaloniki, Greece

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The effect of short-range correlations (SRC) on Ca isotopes is studied using a simple phenomenological model. Theoretical expressions for the charge (proton) form factors, densities, and moments of Ca nuclei are derived. The role of SRC in reproducing the empirical data for the charge density differences is examined. Their influence on the depletion of the nuclear Fermi surface is studied and the fractional occupation probabilities of the shell model orbits of Ca nuclei are calculated. The variation of SRC as function of the mass number is also discussed.

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I. INTRODUCTION

Calcium nuclei have been of great experimental as well as theoretical interest. It is the only magic element for which precision measurements on isotope shifts [1,2] (see also [3]) have been carried out over a full neutron shell, namely the $1f_{7/2}$ shell between the two doubly magic isotopes ^{40}Ca and ^{48}Ca.

The empirical data for the isotope shifts [1] show an anomalous A dependence. The addition of neutrons to the ^{40}Ca core leads to an increase of the charge radii up to ^{44}Ca. Then adding more neutrons the charge radii start to decrease. The very interesting feature is that the charge radii of the two doubly magic nuclei ^{40}Ca and ^{48}Ca have practically the same value. It is noted, however, that the electron scattering experiments have shown that the charge distributions of these magic nuclei are not identical [4]. Moreover, Ca isotopes demonstrate clearly the very interesting differential effect of the odd-even staggering of nuclear radii. That is, the mean square (MS) radii of the odd neutron nuclei are smaller than the average of their even neutron neighbors.

Apart from laser spectroscopy, which provides very accurate experimental information about the isotope shifts of Ca nuclei, other experimental techniques (muon spectroscopy, electron and hadron scattering) offer additional information on the charge and mass distributions [5]. Empirical data for the form factors and their isotopic change for some even stable isotopes is available. Therefore, the rich experimental input makes Ca nuclei attractive for theoretical study.

Brown *et al.* [6] calculated the charge distribution of the Ca isotopes using a Woods-Saxon state dependent potential with a density dependent symmetry potential which was determined in a self-consistent way and using noninteger occupation probabilities for the $1d_{3/2}$, $1f_{7/2}$, and $2p_{3/2}$ states. Bhattacharya *et al.* [7] using an average one-body potential of Woods-Saxon type and experimental occupation probabilities have reproduced the parabolic variation of the charge radii of the Ca isotopes. Zamick [8] and Talmi [9], in analogy with the binding energies, assumed that the effective radius operator has a two body part as well as a one-body part. They were able to explain the odd-even staggering effect observed in Ca isotopes assuming that a mechanism which gives rise to this odd-even variation is the polarization of the core by the valence neutrons. Finally Barranco and Broglia [10], in perhaps the most fundamental approach, succeeded to explain the parabolic variation of the MS charge radii introducing collective zero-point motion.

Mean field calculations fail to reproduce the parabolic behavior of the charge radii of the Ca isotopes. This is an indication that one has to go beyond the Hartree-Fock approach taking into account nuclear correlations. There are various types of correlations. In Refs. [8–10] ground state correlations have been accounted and their effect to the reproduction of the empirical data for the isotope shifts of Ca nuclei was investigated. However, other types of nuclear correlations might also be important, such as short-range correlations (SRC) which describe the effect of the distortion of the relative two-body wave function at small distances.

Correlated charge form factors, $F_{\rm ch}(q)$, and densities of s-p and s-d shell nuclei were calculated [11–15], using Jastrow type correlations [16] for the correlated wave functions of the relative motion and employing the factor cluster expansion of Ristig, Ter Low, and Clark [17,18]. First the method was applied to the doubly closed shell light nuclei ^{4}He , ^{16}O, and ^{40}Ca and then it was extended in an approximate way to all nuclei in the region $4 \leqslant A \leqslant 40$.

In the present work the method is expanded to the study of the isotopes of the closed shell nuclei. The isotopic chain of Ca nuclei has been chosen for the study, due to its special interest. The calculated values of the differences of the charge density distributions of even Ca nuclei are compared with the available experimental data [19,4]. Our aim is to examine the effect of SRC on Ca nuclei as well as their variation with the mass number.

High resolution electron scattering experiments have shown deviations from the mean field picture [20,21]. The quantum states, especially those near the Fermi surface, appear to be depleted. This is a clear demonstration that the single-particle orbits are partially occupied because of the nucleon-nucleon correlations [21]. The depletion of the oc-

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cupied states can be attributed to a coupling of the Hartree-Fock ground state to low-lying collective modes and to SRC due to hard collisions between nucleons at small distances. Calculations for nuclear matter including SRC have shown [22,23] that the depletion of the otherwise filled orbits is 10–20 %.

Recently, a simple method has been proposed for the determination of the fractional occupation probabilities of the shell model orbits of the ground state wave function as well as the total depletion of the nuclear Fermi sea [24,25] (see also [15]). The correlated densities are used as an input and the connection is done by means of the "natural orbital" representation [26], which allows us to keep the simplicity of the shell model picture. Here, employing this method, we determine the occupation numbers of the shell model orbits and the total depletion of the nuclear Fermi sea. The latter reflects the influence of SRC to the deviation from the mean field approach. The variation of these quantities with the mass number is also studied.

The paper is organized as follows. In Sec. II the relevant formalism of SRC for the closed shell nuclei is shown. In Sec. III the expansion to the isotopes of the closed shell nuclei is presented. In Sec. IV the method for the determination of the occupation numbers is briefly discussed. In Sec. V the numerical results are reported and discussed. Finally, Sec. VI summarizes our main conclusions.

II. CORRELATED CHARGE FORM FACTOR, DENSITIES, AND MS RADII FOR CLOSED SHELL NUCLEI

Expression for the correlated charge form factors, $F_{\rm ch}(q)$, of the closed s-p and s-d shell nuclei were derived [11–15] in the framework of the factor cluster expansion of Ristig, Ter Low, and Clark [17,18] using the Jastrow ansatz for the correlated wave functions. This type of correlations is characterized by the correlation parameter λ_{nlS} which enters in the normalized correlated wave functions of the relative motion:

$$\psi_{nlS}(r)=N_{nlS}[1-\exp(-\lambda_{nlS}r^2/b^2)]\phi_{nl}(r), \qquad (1)$$

where N_{nlS} are the normalization factors, $\phi_{nl}(r)$ are the harmonic oscillator (HO) wave functions, and $b=\sqrt{2}b_1$ ($b_1=\sqrt{\hbar/m\omega}$) is the harmonic oscillator parameter for the relative motion. In this approach the expression for the point proton form factor, $F(q)$, takes the form

$$F(q)=F_1(q)+F_2(q), \qquad (2)$$

where

$$F_1(q)=\frac{1}{Z}\exp\left[-\frac{b_1^2q^2}{4}\right]\sum_{k=0}^{2} N_{2k}\left(\frac{b_1q}{2}\right)^{2k} \qquad (3)$$

is the contribution of the one-body term to $F(q)$ with

$$N_0=2(\eta_{1s}+\eta_{2s}+3\eta_{1p}+5\eta_{1d}),$$
$$N_2=-\tfrac{4}{3}(2\eta_{2s}+3\eta_{1p}+10\eta_{1d}), \qquad (4)$$
$$N_4=\tfrac{1}{3}(4\eta_{2s}+8\eta_{1d})$$

and η_{nl} is the occupation probability (0 or 1 in the present case) of the nl state.

The contribution of the two-body term, $F_2(q)$, to the form factor $F(q)$ can be expressed in a rather simple way in closed form by means of the matrix elements:

$$A_{nlS}^{n'l'S'}(j_k)=\langle\psi_{nlS}|j_k(qr/2)|\psi_{n'l'S'}\rangle.$$

These are simple polynomials and exponential functions of q^2 [11–15]. In our study the correlation parameter λ_{nlS} is taken state independent ($\lambda_{nlS}=\lambda$). It is noted that it has been shown in [15] that the effect of the state dependence of the short-range correlations is small.

Then the charge form factor, $F_{\rm ch}(q)$, is written

$$F_{\rm ch}(q)=f_p(q)\times f_{\rm CM}(q)\times F(q), \qquad (5)$$

where $f_p(q)$ and $f_{\rm CM}(q)$ are the corrections due to the finite proton size [11] and the center of mass motion [27], respectively. The correlation parameter λ and the HO parameter b_1 are determined by fitting formula (5) to the experimental data of $F_{\rm ch}(q)$.

An interesting feature of this method is the possibility of finding an analytic form for the correction to the uncorrelated charge (proton) density distribution by means of a Fourier transform of $F_2(q)$. Thus the correlated proton density distribution is written

$$\rho_{\rm cor}(r)=\rho_1(r)+\rho_2(r). \qquad (6)$$

Analytic expressions can also be derived for the various moments of the charge (proton) density distribution. The moments have the form

$$\langle r^k\rangle=\langle r^k\rangle_1+\langle r^k\rangle_2, \qquad (7)$$

where $\langle r^k\rangle_1$ and $\langle r^k\rangle_2$ are the contributions of the one- and two-body density $\rho_1(r)$ and $\rho_2(r)$, respectively.

A very satisfactory approximate expression for the MS charge radius is also derived,

$$\langle r^2\rangle_{\rm ch}=C_{\rm HO}\left(1-\frac{1}{A}\right)b_1^2+C_{\rm SRC}b_1^2\lambda^{-3/2}+r_p^2+\frac{N}{Z}r_n^2, \qquad (8)$$

where r_p^2 and r_n^2 are the proton and neutron MS charge radii, respectively. For the latter the value $r_n^2=-0.116$ fm^2 is used [28]. The constants $C_{\rm HO}$ and $C_{\rm SRC}$ for ^{40}Ca are $C_{\rm HO}=3$ and $C_{\rm SRC}=12.4673$. It is noted that the values obtained with formula (8) differ less than 0.08% from those obtained with the exact expression for the radii and therefore formula (8) is suitable for practical use.

The merit of the approach is mainly the simplicity, as most of the calculations are analytic. Another advantage is the possibility of obtaining approximate expressions of the two body term of various quantities by expanding the expression of the form factor in powers of λ. This allows the study of the open shell nuclei in the region $4\leqslant A\leqslant 40$ [13,14].

III. EXTENSION TO THE ISOTOPES OF CLOSED SHELL NUCLEI

The method described in the previous section is exactly applicable to doubly closed s-p and s-d shell nuclei and approximately to open shell nuclei in the region $4 \leqslant A \leqslant 40$. Here, we extend the method to the study of the isotopes of the closed shell nuclei.

In an isotopic chain all nuclei have the same atomic number Z. Thus we assume that the correlated charge (proton) form factors, densities, and moments of the isotopes can be described by the same formulas (2), (6), and (7) where, however, different values for the parameters λ and b_1 are used. The correlation parameters as well as the size parameters for the isotopes could be easily determined if empirical data for the charge form factors (especially for high momentum transfers) of all the isotopes were available. However, usually this is not the case and therefore one has to try other possibilities.

First, the correlation parameter is written in a more convenient way:

$$\mu = \sqrt{\frac{b_1^2}{\lambda}}. \tag{9}$$

Then we assume that the correlation parameters $\mu(A_c+n)$ and the HO parameters $b_1(A_c+n)$ of the isotopes can be written

$$\mu(A_c+n) = \mu(A_c) + \delta\mu(A_c+n), \tag{10}$$

$$b_1(A_c+n) = b_1(A_c) + \delta b_1(A_c+n), \tag{11}$$

where $\mu(A_c)$ and $b_1(A_c)$ are the parameters of the corresponding closed shell nucleus (A_c). The differences $\delta\mu$ and δb_1 express the change in the parameters due to the addition of extra neutrons (n).

Using (10) and (11) expression (8) for the MS charge radius, $\langle r^2\rangle_{\rm ch} = r^2$, can be written in the following way:

$$r^2(A_c+n) = r^2(A_c) + \delta r^2(A_c+n)$$
$$= C_{\rm HO}\left(1-\frac{1}{A_c+n}\right)[b_1(A_c)+\delta b_1(A_c+n)]^2 + C_{\rm SRC}\frac{[\mu(A_c)+\delta\mu(A_c+n)]^3}{b_1(A_c)+\delta b_1(A_c+n)} + r_p^2 + \frac{A_c-Z+n}{Z}r_n^2. \tag{12}$$

For the closed shell nuclei experimental data is available in most of the cases. The parameters $\mu(A_c)$ and $b_1(A_c)$ can be determined by a fit to the data. Thus the problem is reduced to the determination of the differences δb_1 and $\delta\mu$. For this, additional input is necessary. In the present approach the differences are determined using the empirical data for the isotope shifts and isospin dependent theoretical expressions for the oscillator parameters.

In a very recent publication [29] new improved expressions for the oscillator spacing $\hbar\omega$ were derived. These expressions have the advantage of being isospin dependent. They were obtained by employing new expressions for the MS radii of nuclei, which fit the experimental MS radii and the isotope shifts much better than other frequently used relations. In the present work the following formula for $\hbar\omega$ taken from [29] is used:

$$\hbar\omega = 38.6A^{-1/3}[1+1.646A^{-1}-0.191(N-Z)A^{-1}]^{-2}. \tag{13}$$

The derivation of the HO parameters of the isotopes by means of (13) is straightforward:

$$b_1(A_c+n) = \sqrt{\frac{\hbar^2}{38.6m}A_c^{1/6}\left(1+\frac{n}{A_c}\right)^{1/6}\left[1+\frac{1.646}{A_c}\times\left(1+\frac{n}{A_c}\right)^{-1} - 0.191\frac{n}{A_c}\left(1+\frac{n}{A_c}\right)^{-1}\right]}. \tag{14}$$

Formula (14) is used for the determination of the differences $\delta b_1(A_c+n)$. Finally, the isotopic changes $\delta\mu(A_c+n)$ of the correlation parameters are adjusted to reproduce the empirical data of the isotope shifts, $\delta r^2(A_c+n)$, using Eq. (12). Hence, in the present approach, the parameters $b_1(A_c+n)$ and $\mu(A_c+n)$, calculated from (10) and (11), respectively, are determined using the empirical data for the charge radii of the isotopic chain and the information obtained from the experimental charge form factor of the closed shell isotope.

IV. THE METHOD FOR THE DETERMINATION OF THE OCCUPATION NUMBERS

The correlated proton densities $\rho_{\rm cor}(r)$ can be used as an input for the determination of the fractional occupation probabilities of the shell model orbits of the ground state wave function [24]. The connection with short range correlations is done by employing the "natural orbital" representation [26]. The natural orbital approach has already been applied for nuclear studies in the past [30] (see also [31]). Recently it was also employed [32] within a variational Jastrow-type correlation method for the study of quantum liquid drops.

For spherical symmetric systems the density distribution in the natural orbital representation $\rho_{\rm n.o.}(r)$ takes the simple form

$$\rho_{\rm n.o.}(r) = \frac{1}{4\pi}\sum_{nl}(2j+1)\eta_q|\phi_q(r)|^2, \tag{15}$$

where η_q is the occupation probability ($\eta_q \leqslant 1$) of the $q(=nlj)$ state. The "natural orbitals" ϕ_q are approximated by the radial part of the single-particle wave functions $\{R_{nl}\}$ of a harmonic oscillator potential. The occupation probabilities are determined by assuming $\rho_{\rm cor}(r) = \rho_{\rm n.o.}(r)$. That is, the correlated proton density distribution, $\rho_{\rm cor}(r)$, in which the effect of short-range correlations is taken into account, equals with density distribution $\rho_{\rm n.o.}(r)$, corresponding to the natural orbital representation. We demand the first few moments of $\rho_{\rm cor}(r)$ to be equal to those of $\rho_{\rm n.o.}(r)$ distribution,

TABLE I. The values of the HO parameters b_1 (in fm) and the SRC parameters μ (in fm) for the Ca nuclei.

A	b_1	μ
40	1.860	0.499
41	1.857	0.504
42	1.855	0.548
43	1.853	0.534
44	1.851	0.566
45	1.849	0.543
46	1.848	0.545
47	1.846	0.527
48	1.845	0.528

$$\langle r^k\rangle_{\rm cor}=\langle r^k\rangle_{\rm n.o.}. \qquad (16)$$

The details of the calculations are described in Refs. [24,25]. The merit of this approach is that by "mapping" the correlated density distributions to those calculated with the "natural orbitals" a relationship is established between the fractional occupation probabilities and the short-range correlations. The effect of short-range correlations is taken into account in an effective way and it is absorbed in the values of the calculated occupation numbers and the size parameters $b_{\rm n.o.}$. It is a suitable way of keeping the simplicity and visuality of the single-particle picture. It should be noted, however, that this relationship is not completely clear because one is not able to distinguish the correction to the charge form factor $F_{\rm ch}(q)$ for large q because of short-range correlations from the one due to meson exchange currents. In addition, the use of harmonic oscillator wave functions is a simplification. Proper linear combinations of oscillator wave functions could be used instead. In such a case, however, the method loses its simplicity.

The method was improved [25] by considering different oscillator parameters for the hole states and those above the Fermi sea (FS). Specifically the $\rho_{\rm n.o.}(r)$ was divided in two parts:

$$\rho_{\rm n.o.}(r)=\rho_{\rm n.o.}^{<\rm FS}(r)+\rho_{\rm n.o.}^{>\rm FS}(r), \qquad (17)$$

where each part is expressed by a harmonic oscillator basis characterized by the oscillator parameters b and $\tilde{b}$, respectively. In addition, it was assumed that for $\rho_{\rm n.o.}^{<\rm FS}(r)$ the occupancy of the states above the Fermi level is practically zero, while for $\rho_{\rm n.o.}^{>\rm FS}(r)$ only the states above the Fermi sea have occupancies which appreciably differ from zero. The two parts of $\rho_{\rm n.o.}(r)$ in (17), somehow, reflect nuclear characteristics which are sensitive to the low and high momentum component of the charge form factor, respectively. In the present work the formalism of Ref. [25] has been adopted.

V. NUMERICAL RESULTS AND DISCUSSION

In Table I the correlation parameters $\mu(A_c+n)$ and the HO parameters $b_1(A_c+n)$ for all Ca isotopes considered in this approach are shown. Their calculation has been done with the aid of formulas (11) and (10). The parameters $\mu(A_c)$ and $b_1(A_c)$ of the closed shell nucleus (core nucleus A_c = 40) were determined by fitting the theoretical expression of the charge form factor, $F_{\rm ch}(q)$, of ^{40}Ca to the experimental data. The differences δb_1 are determined using formula (14) while $\delta\mu$ are adjusted so that expression (12) for the MS charge radii to reproduce the empirical data of the isotope shifts, δr^2, of Ca nuclei [1].

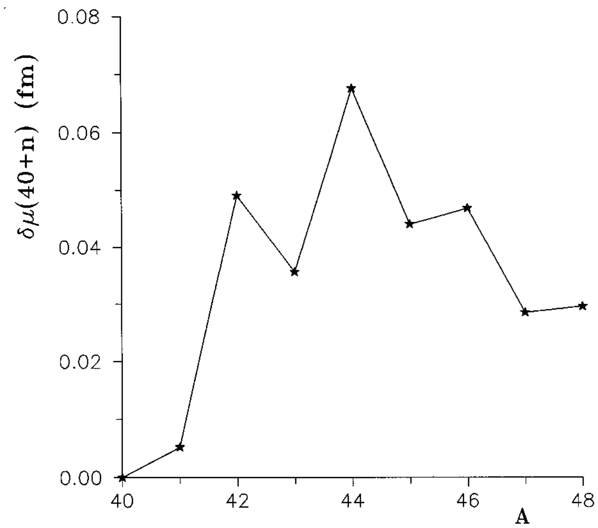

FIG. 1. The isotopic change $\delta\mu$ of the correlation parameter μ as function of the mass number A.

In Fig. 1 the isotopic change $\delta\mu$ of the correlation parameter μ as function of the mass number is shown. It is observed that the parameter μ, which expresses the strength of the short-range correlations, is increasing up to ^{44}Ca and then starts decreasing following the same variation with the isotope shifts (see for example Fig. 49 of [1]). This indicates that there is a proportion between the strength of SRC and the size of the nucleus, that is when SRC become stronger the charge radii become larger. It is interesting to note that, if for the determination of b_1 isospin independent expressions of $\hbar\omega$ (see for example Refs. [33–35]) were used instead, the two quantities do not show the same variation. In this

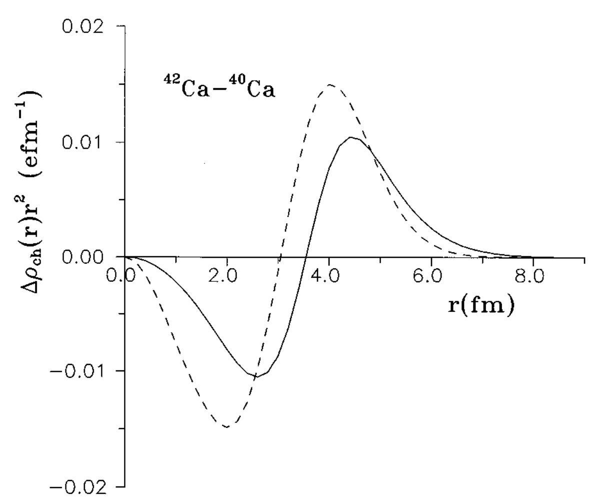

FIG. 2. The difference of the charge distributions of ^{42}Ca − ^{40}Ca, multiplied by r^2, (dashed line) calculated in the present approach together with the experimental data (solid line) taken from Ref. [19].

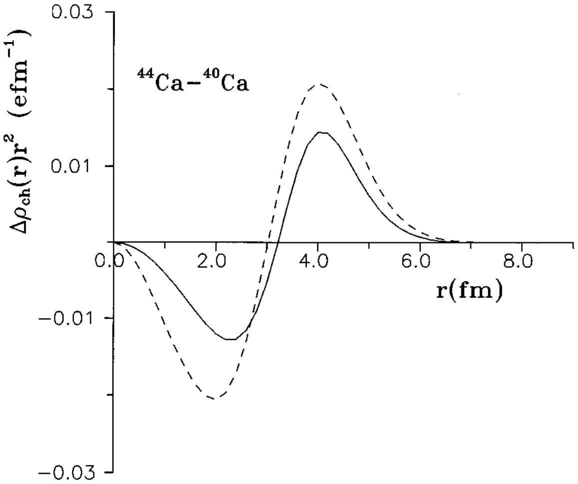

FIG. 3. The same as in Fig. 2, for the charge distribution difference of ^{44}Ca − ^{42}Ca.

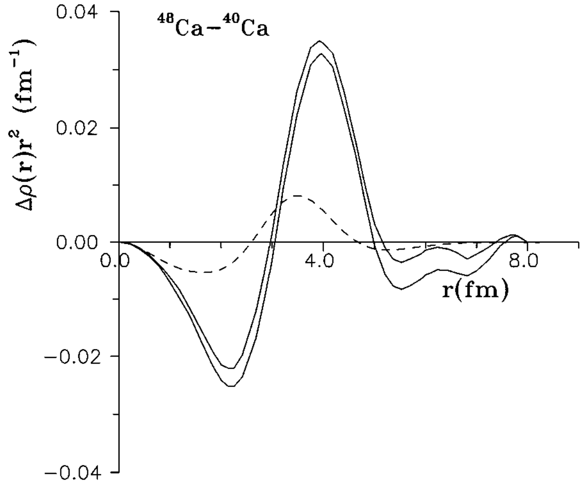

FIG. 4. The difference of the point proton distributions of ^{48}Ca − ^{40}Ca multiplied by r^2 (dashed line) calculated in the present approach together with the empirical data taken from Ref. [4]. The two solid lines correspond to the upper and lower values of the experimental proton density difference.

case, as the mass number increases the correlation parameters have opposite variation compared with the one of the charge radii. On the other hand the use of such expressions for $\hbar\omega$ does not provide a satisfactory description of the empirical data for the charge density difference of ^{48}Ca − ^{40}Ca.

Using the values of Table I the charge (proton) form factors, density distributions as well as the differences of the density distributions $\Delta\rho(40+n)=[\rho(40+n)-\rho(40)]$ can be easily calculated. In Figs. 2 and 3 the quantity $\Delta\rho_{\rm ch}(40+n)r^2$ for the charge distribution differences of ^{42}Ca − ^{40}Ca and ^{44}Ca − ^{40}Ca, respectively, are compared with the empirical data (solid lines). The same is also in Fig. 4 for the difference ^{48}Ca − ^{40}Ca. In this case the available experimental values correspond to the proton density distributions. The two solid lines correspond to the upper and lower values of the proton density difference. It is seen that the theoretical curves (dashed lines) show the correct trend. The calculated $\Delta\rho_{\rm ch}(40+n)r^2$ reproduce the behavior of the data. That is, the charge flows from the center (and the outer skin in ^{48}Ca) into a region around the half-density radius. The comparison is not very good in all cases. Especially in Fig. 4 for the difference ^{48}Ca − ^{40}Ca, where the maximum is not reproduced well. However, in the present approach SRC are only accounted. This indicates that additional correlations, as for example surface vibrations, are necessary to improve the agreement with the experiment. In this analysis only a small part of their effect, ''hidden'' in the empirical data of the isotope shifts (used for the determination of $\delta\mu$), might be accounted. It should be noted, however, that in this approach the parameters μ and b_1 are not free. If one or both of them were free, then one could expect better agreement. In such a case the effect of other type correlations would have been taken into account effectively. Thus, for example, the parameters μ and b_1 could be determined by direct fits to the experimental charge form factors. This procedure was not followed because experimental data for the charge form factors of all Ca nuclei is not available. Moreover, this data do not cover the region of the high momentum transfers ($q>3$ fm^{-1}).

Next, the calculated proton density distributions $\rho_{\rm cor}(r)$

TABLE II. The calculated harmonic oscillator parameters, b and $\tilde{b}$ (in fm) together with the occupation probabilities of the shell model orbits of Ca nuclei.

A	40	41	42	43	44	45	46	47	48
b	1.893	1.892	1.892	1.887	1.889	1.884	1.883	1.882	1.880
$\tilde{b}$	1.734	1.732	1.729	1.726	1.724	1.723	1.721	1.720	1.719
$1s$	1.000	0.999	0.860	0.983	0.709	0.911	0.884	0.999	1.000
$1p$	0.822	0.815	0.677	0.643	0.625	0.639	0.677	0.682	0.657
$1d$	0.565	0.550	0.592	0.643	0.625	0.638	0.636	0.606	0.631
$2s$	0.565	0.550	0.480	0.448	0.486	0.458	0.462	0.458	0.450
$1f$	0.424	0.438	0.479	0.448	0.486	0.458	0.462	0.458	0.449
$2p$	0.058	0.059	0.105	0.097	0.134	0.107	0.111	0.086	0.092
Depl. %	31.45	32.56	36.72	34.24	38.05	35.26	35.63	34.68	34.23

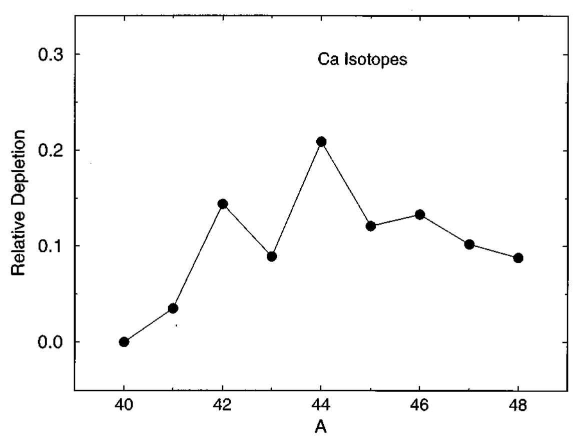

FIG. 5. The variation with the mass number A of the relative depletion of Ca nuclei.

are used for the determination of the occupation numbers of the shell model orbits of Ca nuclei following the procedure described in Refs. [24,25]. In Table II the calculated size parameters b, and $\tilde{b}$ of the two natural orbital bases are shown together with the occupation probabilities of Ca nuclei. The total depletion of the nuclear Fermi sea, which reflects the effect of the short-range correlations and gives a measure of the deviation from the mean field picture is also shown.

In Fig. 5 the relative depletion of Ca isotopes is plotted against the mass number A. As relative depletion we define the quantity

$$\text{relative depletion}=[\text{depl.}(^{40+n}\text{Ca})-\text{depl.}(^{40}\text{Ca})]/\text{depl.}(^{40}\text{Ca}).$$

A clear parabolic shape analogous to that of Fig. 1 is observed. This is because the depletion of the nuclear Fermi sea expresses the effect of SRC and thus a similar variation with the correlation parameters μ should be expected.

In Fig. 6 the variation with the mass number of the occupation probabilities of the shell model orbits of Ca nuclei is shown. For the very deep states the occupation probabilities (see also Table II) are very close to one while the surface

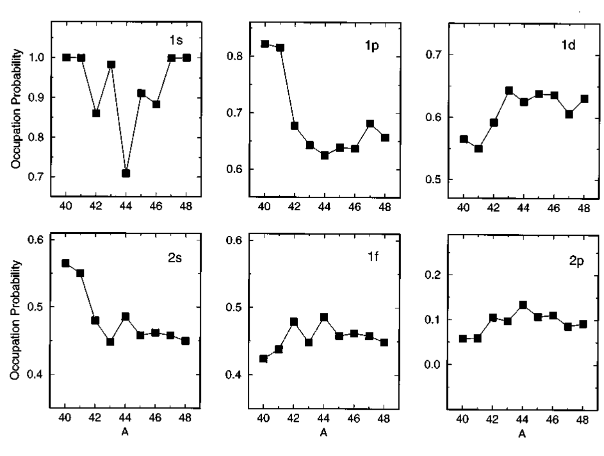

FIG. 6. The variation with the mass number A of the occupation probabilities of the shell model orbits of Ca nuclei.

TABLE III. Comparison of the occupation probabilities of ^{40}Ca calculated in this work (η_q) together with "experimental" and theoretical values from other studies.

nl	η_q	Expt. [36].	Expt. [37].	[38,39]	[40]
$1s$	1.000	0.820	0.750	0.970	0.990
$1p$	0.822	0.767	0.950	0.975	0.986
$1d$	0.565	0.720	0.770	0.884	0.962
$2s$	0.565	0.740	0.650	0.870	0.960
$1f$	0.424	0.307		0.071	0.030
$2p$	0.058	0.100		0.035	0.010

levels deviate significantly from unity, manifesting thus the effect of SRC. Their variation with the mass number shows the correct behavior and it is consistent with Fig. 1 for the correlation parameters μ, which measures the strength of SRC. That is for stronger SRC a larger "fraction" of protons is moved above the Fermi level.

It is noted that the occupation probabilities are not directly measured quantities. The experimental occupation probabilities are usually obtained by extrapolating the experimental spectral functions by means of a Gaussian fit. In general they are of limited accuracy and the comparison with the theory varies in the various models. The "experimental" information about the occupation probabilities of Ca isotopes is limited to the magic nucleus ^{40}Ca. On the other hand, there are no theoretical predictions in the literature (to our knowledge) for the other Ca nuclei. In Table III the occupation probabilities of ^{40}Ca, calculated in this work, together with the "experimental" values [36,37] and those from other theoretical analyses [38–40] are shown for the sake of comparison.

Finally we note that in Figs. 1, 5, and 6 a kind of odd-even effect is also observed. The effect of SRC appears to be weaker for the odd nuclei compared with their even partners. It should be noted, however, that in the framework of this simple approach, one cannot draw easily conclusions about the odd isotopes, where additional effects have to be taken into account. One could say that the correlation parameters $\delta\mu$ are adjusted to reproduce the isotopic changes of the charge radii and therefore are somewhat "forced" to follow such a variation.

VI. SUMMARY

In the framework of a simple phenomenological model theoretical expressions for the correlated charge (proton) form factors, densities, and moments of the isotopes of closed shell nuclei are derived. SRC are accounted using Jastrow type wave functions for the correlated wave functions of the relative motion. In the present work the isotopic chain of Ca nuclei is studied and the influence of SRC on Ca isotopes is examined by comparing with the available empirical data for the charge (proton) density differences. The calculated values for the differences of the density distributions show the correct trend. However, the present study indicates that additional correlations could improve the description of the experimental data.

The role of SRC on the depletion of the Fermi sea as well as its variation with the mass number is discussed. The occupation probabilities of the shell model orbits of Ca nuclei are calculated. One should keep in mind, however, the large uncertainties concerning their experimental determination and the model dependence of the various theoretical analyses.

Concluding we would like to mention that the main advantage of the present analysis is its simplicity. The method can be applied to other isotopic chains to provide predictions for the charge form factors, charge density differences, and other quantities for which the effect of SRC is important.

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