Searching for three-nucleon resonances

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We search for three-neutron resonances which were predicted from pion double charge exchange experiments on ^{3}He. All partial waves up to $J=\frac{5}{2}$ are nonresonant except the $J^\pi=\frac{3}{2}^+$ one, where we find a state at $E=14$ MeV energy with 13 MeV width. The parameters of the mirror state in the three-proton system are $E=15$ MeV and $\Gamma=14$ MeV. The possible existence of an excited state in the triton, which was predicted from a H(^{6}He,α) experiment, is also discussed.

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I. INTRODUCTION

One of the clearest indications of the spin dependence of the nucleon-nucleon interaction is the fact that there exists a bound triplet deuteron, but not a singlet deuteron or dineutron. The possibility that by adding one or two more neutrons to an unbound dineutron one might wind up with a bound trineutron or tetraneutron has long been discussed. However, precise experiments and calculations show no indication for the existence of such bound structures [1]. Nevertheless, the existence of resonant states of these nuclei, which have observable effects, cannot be excluded.

Sperinde *et al.* found [2] that the differential cross section of the ^{3}He$(\pi^-,\pi^+)3n$ pion double charge exchange reaction strongly differs from what one would expect from a pure phase space description of the final state. The discrepancy seemed to be well explained by the assumption that there exists a three-neutron resonance at 2 MeV energy with 12 MeV width. The authors suggested that this state is probably a member of a $T=\frac{3}{2}$ isospin quartet in the $A=3$ nuclei, another member being the three-proton state found in [3]. The mechanism of the double charge exchange experiment was studied theoretically in [4]. The authors found that by taking into account the final state interaction between the outgoing neutrons, one can get good agreement with the experimental results of [2]. However, the nature of this final state interaction, whether it produces a three-neutron resonance or not, was not studied in [4].

Recently, a more thorough experimental study of the ^{3}He$(\pi^-,\pi^+)3n$ process was performed [5]. The authors pointed out that the analysis of [2] was incorrect, because a four-body phase space had been divided out of the cross section, instead of the correct three-body one. This mistake leads to a peak in the cross section at low energies. However, three-neutron structures also appear in [5] at small angles. These structures at $\sim$20 MeV three-neutron missing mass and with $\sim$20 MeV width could be caused by three-neutron resonances.

There are experimental indications of the possible existence of resonances in other $A=3$ nuclei, too. For example, in a recent H(^{6}He,α) experiment [6] the authors concluded that in order to explain their results, an excited state of the triton has to be assumed with $E^*=7.0\pm0.3$ MeV excitation energy and $\Gamma=0.6\pm0.3$ MeV width.

Theoretically the problem of a three-neutron resonance was studied by Glöckle [7], who calculated the pole position of the S matrix from the analytical continuation of the Faddeev kernel for three neutrons with pure $^1S_0$ $N-N$ interaction. He found that the pattern of the pole trajectory ruled out the possibility of a low-energy $\frac{1}{2}^+$ three-neutron resonance in his model.

The aim of this current work is to systematically investigate the possible existence of resonances in the lowest partial waves of the three-neutron system by using the complex scaling method. We shall also comment on the triton excited state.

II. COMPLEX SCALING

In coordinate space, resonance eigenfunctions, corresponding to the complex energy solutions of the Schrödinger equation

$$\hat{H}|\Psi\rangle=(\hat{T}+\hat{V})|\Psi\rangle=E|\Psi\rangle, \quad (1)$$

show oscillatory behavior in the asymptotic region with exponentially growing amplitude, $\sim\exp[i(\kappa-i\gamma)r]$ $(\kappa,\gamma>0)$. Thus the resonance eigenfunctions are not square integrable. The complex scaling method (CSM) [8] reduces the description of resonant states to that of bound states, thus avoiding the problem of asymptotics. In the CSM we define a new Hamiltonian by

$$\hat{H}_\theta=\hat{U}(\theta)\hat{H}\hat{U}^{-1}(\theta), \quad (2)$$

and solve the complex equation

$$\hat{H}_\theta|\Psi_\theta\rangle=\varepsilon|\Psi_\theta\rangle. \quad (3)$$

Here the unbounded similarity transformation $\hat{U}(\theta)$ acts, in the coordinate space, on a function $f(\mathbf{r})$ as

$$\hat{U}(\theta)f(\mathbf{r})=e^{3i\theta/2}f(\mathbf{r}e^{i\theta}). \quad (4)$$

[If θ is real, $\hat{U}(\theta)$ means a rotation into the complex coordinate plane, if it is complex, it means a rotation and scaling.] In the case of a many-body Hamiltonian, (4) means that the transformation has to be performed in each Jacobi coordinate. For a broad class of potentials there is the following connection between the spectra of $\hat{H}$ and $\hat{H}_\theta$ [9]: (i) the bound eigenstates of $\hat{H}$ are the eigenstates of $\hat{H}_\theta$, for any value of θ within $0 \leqslant \theta < \pi/2$; (ii) the continuous spectrum of $\hat{H}$ will be rotated by an angle 2θ; (iii) the complex generalized eigenvalues of $\hat{H}_\theta$, $\varepsilon_{\rm res} = E - i\Gamma/2$, $E,\Gamma > 0$ (where Γ is the full width at half maximum) belong to its proper spectrum, with square-integrable eigenfunctions, provided $2\theta > |\arg \varepsilon_{\rm res}|$. These complex eigenvalues coincide with the S matrix pole positions. This method was tested for three-body resonances in [10], and was applied to the low-lying spectra of the ^{6}He, ^{6}Li, and ^{6}Be nuclei in [11]. Further details and references of the method can be found there. We note that the CSM is identical to a contour rotation in momentum space [12]. That latter method was also used, for example, to study three-body resonances in the $A=6$ nuclei [13].

Up to Eq. (3) our treatment of the three-body resonances is exact. The only approximation we use here is that now we expand the wave function of Eq. (3) in terms of products of Gaussian functions with different widths, and determine the expansion coefficients from the $\langle \delta\Psi_\theta | \hat{H}_\theta - \varepsilon | \Psi_\theta \rangle = 0$ projection equation. Thus we select the square integrable solutions of Eq. (3), which are the three-body resonances, and discretize the continuum. A term of this expansion looks like $\rho_1^{l_1} \exp[-(\rho_1/\gamma_i)^2] Y_{l_1 m_1}(\hat{\rho}_1) \cdot \rho_2^{l_2} \exp[-(\rho_2/\gamma_j)^2] Y_{l_2 m_2}(\hat{\rho}_2)$, where l_1 and l_2 are the angular momenta in the two relative motions, respectively, and the widths γ of the Gaussians are the parameters of the expansion.

III. RESULTS

We use the Minnesota effective nucleon-nucleon interaction [14], together with the tensor force of [15,16]. The space exchange parameter of the tensor force is adjusted to the splitting of the ^3P_J phase shifts. As one can see in Fig. 1, this force produces $p+p$ phase shifts which are in good agreement with experiment. The higher partial waves, not shown in Fig. 1, are also close to the (practically zero) experimental values. In the three-neutron wave functions all $n+n$ partial waves up to $l=2$ are included. In some cases we checked the role of the $l=3$ partial waves and found them insignificant.

Figure 2(a) shows a typical result of our calculations, in this case for $J^\pi = \frac{3}{2}^-$. What one can see is the discretized rotated three-body cut of Eq. (3), with $E = \mathrm{Re}(\varepsilon)$ and $\Gamma = -2\,\mathrm{Im}(\varepsilon)$. The energy points are all sitting on a half-line whose angle to the E axis is roughly 2θ. If there were a resonance in this partial wave, it would be in the upper right triangle, separated from the continuum points.

In Fig. 2(b) we show the result for $J^\pi = \frac{3}{2}^+$, which is found to be the only resonant partial wave. The resonant state is shown by the circle. We find that the dominant terms in the resonant $J^\pi = \frac{3}{2}^+$ wave function are the $[(l_1 l_2)L,S] = [(11)1,\frac{3}{2}]$ and $[(11)2,\frac{1}{2}]$ LS components, where L and S are the total angular momentum and total spin, respectively. The other components have small but non-negligible effects.

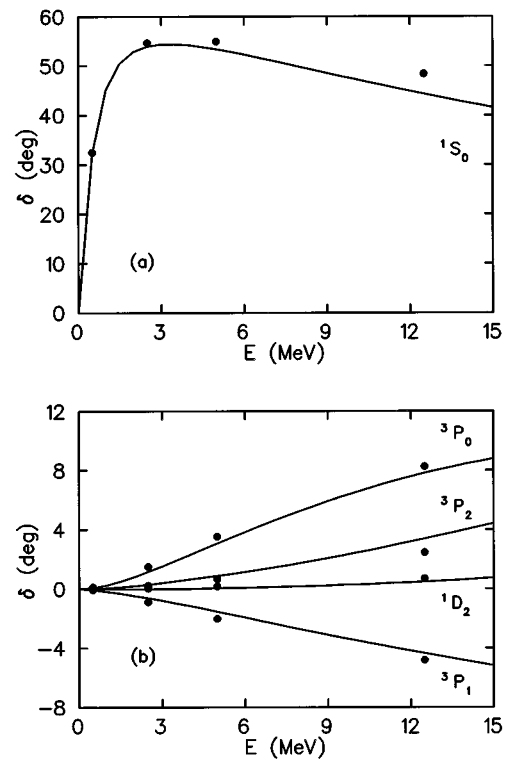

FIG. 1. S-wave (a) and P- and D-wave (b) $p+p$ phase shifts in the center-of-mass frame. Experimental data are taken from [17].

Because we solve Eq. (3) approximately, the resonance parameters depend slightly on θ. To find the optimal θ value we use the method based on the complex virial theorem [18]. We repeat the calculations with several different θ values, and find a region where the resonance parameters are most insensitive to the change of θ. To reduce computer time, we perform these calculations with a wave function which contains only the above mentioned two most important LS components. In Fig. 3 one can see that the optimal θ angle is somewhere between 0.3 and 0.35 rad. Using such a θ value in the full model the parameters of the resonance are $E=14$ MeV and $\Gamma=13$ MeV.

What can one say about possible resonances in other $A=3$ nuclei? The effect of the Coulomb force in the mirror three-proton system is trivial, pushing the $\frac{3}{2}^+$ resonance toward higher energy and increasing its width accordingly. The resonance parameters are $E=15$ MeV and $\Gamma=14$ MeV.

The situation is more complex in the cases of ^{3}H and ^{3}He, because these nuclei have two-body channels below the three-nucleon thresholds. Recently an experiment, studying the H(^{6}He,α) reaction, found indications of an excited state of the triton at $E^* = 7.0 \pm 0.3$ MeV excitation energy with $\Gamma = 0.6 \pm 0.3$ MeV width [6]. This state would be situated between the $d+n$ and $n+n+p$ thresholds. It was suggested in [6] and [19] that the dominant configuration in this state is the $(nn)p$ one, i.e., the resonance originates from the closed three-body channel.

Unfortunately our current N-N interaction is not appropriate for a calculation which contains all the relevant angular momentum configurations for the triton. This is because the

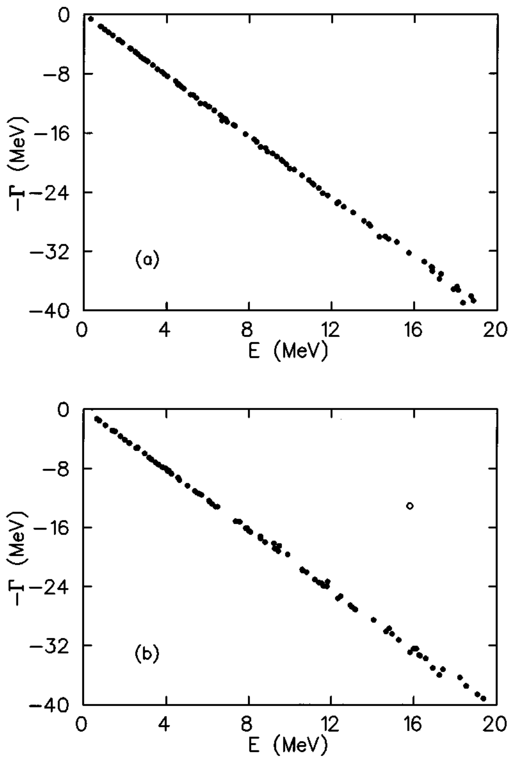

FIG. 2. Energy eigenvalues of the complex scaled Hamiltonian of the (a) $\frac{3}{2}^-$, and (b) $\frac{3}{2}^+$ three-neutron states; $E=\mathrm{Re}(\varepsilon)$, $\Gamma=-2\,\mathrm{Im}(\varepsilon)$. The dots are the points of the rotated discretized continua, while the circle is a three-neutron resonance. The rotation angle is 0.4 rad.

Minnesota force reproduces the $^3S_1$ $n+p$ phase shift and the deuteron binding energy without the tensor force. However, we can estimate the role of the $n+n+p$ and $d+n$ channels in a restricted model, which contains $^3D_1$ states just between the d and n but not inside the deuteron. The Minnesota force is shown [20] to give a good overall description for the $d+p$, and supposedly for the $d+n$, scattering in such a model. It is important to note that, if there is a resonance in the $(nn)p$ configuration, it must appear in a one-channel $d+n$ model, too. The reason is that any J^π state that can be formed in the $(nn)p$ configuration can be built up from $d+n$ one, too, and this $d+n$ channel is nonorthogonal to the $(nn)p$ configuration. The $E^*=7.0$ MeV excitation energy and $\Gamma=0.6$ MeV width would imply an S matrix pole at $\varepsilon=-1.5-i0.3$ MeV in the $n+n+p$ three-body channel. If this pole has a three-body origin, and is not on the Riemann sheet, which is adjacent to the physical sheet, then we should use $\theta>1.5$ rad in the three-body channel. For such large rotation angles our method is rather unstable, and the localization of the pole is hopeless. If, however, this pole is present in the $d+n$ scattering, we can use a variety of methods to localize it. Here we use the analytic continuation of the S matrix to complex energies [16], which is feasible for a two-body case. It is basically equivalent with the CSM, but is more precise than that, especially for broad resonances. A $\frac{1}{2}^+$ pole is found at $E=1.4$ MeV $d+n$ center-of-mass energy with a width of 9 MeV. The $n+n+p$ channel can be taken into account approximately, by using a few discretized continuum states of (nn) with positive energies and square-integrable wave functions. The inclusion of this $n+n+p$ channel does not change the pole position significantly. The dominant configuration of the wave function is $d+n$, in contrast to the case of [19].

The quality of our $^3S_1$ and $^3D_1$ N-N forces makes it impossible to reach a firm conclusion concerning the existence of a narrow excited state of the triton. Our main result is that the picture of [6] and [19], namely that this state originates from the $(nn)p$ channel, is questionable. As such a state would be nonorthogonal to the $d+n$ channel, this latter configuration must be important.

Finally, we note that the use of realistic interactions together with a full model space (including $l\geqslant 3$) may change the parameters of the $\frac{3}{2}^+$ resonance. This is why we have made no attempt to further optimize our basis (expansion length, Gaussian widths, etc.) and find more precise resonance parameters. Our aim was to show the existence of such a state.

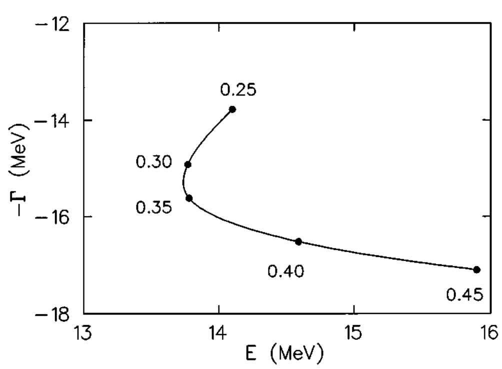

FIG. 3. The θ trajectory of the $\frac{3}{2}^+$ three-neutron resonance with a two-component wave function. The θ values are indicated.

IV. CONCLUSION

In summary, we have searched for three-neutron resonances using the complex scaling method. We have found a $J^\pi=\frac{3}{2}^+$ resonant state at $E=14$ MeV energy with $\Gamma=13$ MeV width. All other partial waves up to $J=\frac{5}{2}$ were found to be nonresonant. The parameters of the mirror $\frac{3}{2}^+$ three-proton resonance are $E=15$ MeV and $\Gamma=14$ MeV. Our results are inconclusive regarding the narrow excited state of the triton, but show the importance of the $d+n$ channel. Calculations using other methods, e.g., Refs. [12,13] would be useful.

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