Does the 3N force have a hard core?

J. A. Eden* and M. F. Gari
Institut für Theoretische Physik, Ruhr Universität Bochum, D-44780 Bochum, Germany
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The meson-nucleon dynamics that generates the hard core of the RuhrPot two-nucleon interaction is shown to vanish in the irreducible 3N force. This result indicates a small 3N force dominated by conventional light meson-exchange dynamics and holds for an arbitrary meson-theoretic Lagrangian. The resulting RuhrPot 3N force is defined in the Appendix. A completely different result is expected when the Tamm-Dancoff–Bloch-Horowitz procedure is used to define the NN and 3N potentials. In that approach (e.g., full Bonn potential), both the NN *and* 3N potentials contain nonvanishing contributions from the coherent sum of meson-recoil dynamics and the possibility of a large hard core requiring explicit calculation cannot be ruled out.

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I. INTRODUCTION

Existing descriptions of the three-nucleon force [1] essentially fall into two categories. The first is the semiphenomenological approach adopted for the calculation of the Tucson-Melbourne force [2–8], where the $\pi\pi$, $\pi\rho$, and $\rho\rho$ contributions to the 3N interaction are fixed by subtracting the forward-propagating Born amplitudes from πN-scattering, photopion production, and photon-scattering data and the off-shell extrapolation that is necessary for three-body applications is obtained from PCAC and current algebra. There is little doubt that at $Q^2=0$, this force can be regarded as essentially exact. The second approach, which has been adopted for the calculation of the Brazilian [9–13] and RuhrPot 3N forces is to make use of a particular meson-theoretic model and directly calculate the leading-order contributions. We illustrate the two schemes in Fig. 1.

One important question that remains to be addressed is this: "*Does the 3N interaction have a hard core like that found in the NN interaction?*" To address this question, we will first recall the dynamical origins of the hard core in the NN interaction, and then ask if such dynamics can produce a corresponding hard core in the three-body system.

About 5 years ago it was shown [14] that the hard core of the NN-nucleon interaction arises naturally in a boson-exchange model when the dynamics is no longer truncated to include the exchange of only a few light mesons. In particular, the RuhrPot NN interaction [14,15] introduces a closure approximation to incorporate the additional $(J^\pi;T)$-exchange dynamics that is required by completeness. Conventional light-meson-exchange dynamics still contributes to the NN interaction at long distances, but the hard core region is now completely dominated by the additional *contact* contributions. We illustrate this two-phase approach in Fig. 2.

This natural separation of the long- and short-range NN interaction dynamics has proven critical in resolving a number of problems in existing boson-exchange potentials (BEP's). For example, when the contact interactions are included to describe the hard core of the NN interaction, it is no longer necessary to adopt the artificial NN-meson cutoffs that are required [16] in conventional boson-exchange models. Instead, the RuhrPot NN interaction and associated exchange currents [17,18] use self-consistently calculated form factors [19,20] that possess an asymptotic Q^2 dependence which is consistent [21] with perturbative QCD [22]. In addition, the vector-meson couplings satisfy the $SU(3)_F$ prediction of $g^2_{NN\omega}/g^2_{NN\rho}\sim 9$, which can be compared to values near 27 that are typical [23] of conventional boson-exchange potentials.

So how do the contact interactions effect the irreducible 3N force? What happens to the three-body force when two or three of the nucleons begin to overlap? Our answers to these questions are organized as follows. In Sec. II, we summarize the Tamm-Dancoff–Bloch-Horowitz and unitary transformation projection formalisms that define the one- and two-boson-exchange interactions in the A-body system. In Sec. III we use these results to recall how the hard core of the NN interaction arises naturally when the exchange dynamics is no longer arbitrarily truncated to include only the first few light mesons. In Sec. IV we carry these ideas into the 3N

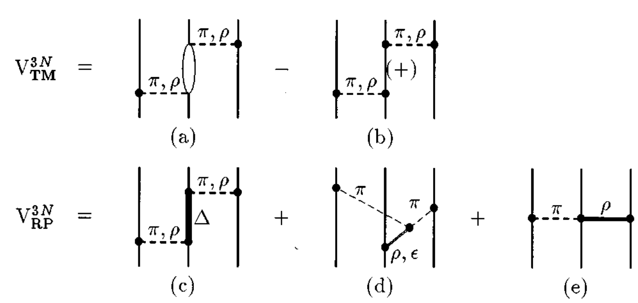

FIG. 1. The $\pi\pi$, $\pi\rho$, and $\rho\rho$ contributions to the Tucson-Melbourne interaction $V^{3N}_{\rm TM}$ are fixed by subtracting the forward-propagating Born amplitudes from πN-scattering, photopion production, and photon-scattering data. The RuhrPot interaction $V^{3N}_{\rm RP}$ is calculated from an effective meson-baryon model, as is the Brazilian force. All of these approaches involve only light-meson-exchange dynamics, so that it remains to be seen if a hard core exists in the 3N force.

*Electronic address: jamie@deuteron.tp2.ruhr-uni-bochum.de

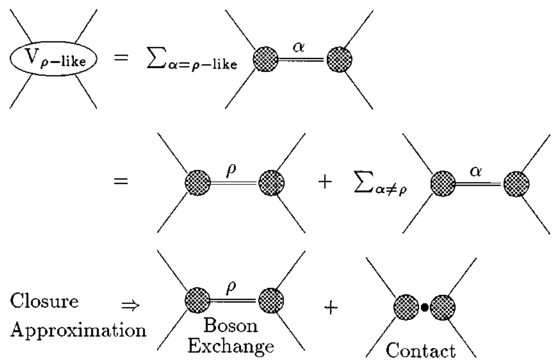

FIG. 2. The RuhrPot *NN* interaction introduces a closure approximation to incorporate the additional $(J^{\pi};T)$-exchange dynamics that is required by completeness. Conventional light-meson-exchange dynamics still contributes to the *NN* interaction at long distances, but the hard core region is now completely dominated by *contact* interactions.

system to investigate the existence of a possible hard core in the irreducible 3*N* interaction. Within the unitary transformation approach, we find that *the class of* $(J^{\pi};T)$*-exchange processes generating the hard core repulsion in the RuhrPot NN interaction vanish in the nonrelativistic limit of the irreducible 3N interaction.* This result is not confined to any particular meson-theoretic model and it eliminates the most likely source of a hard core in the 3*N* interaction. However, a completely different result is expected when the Tamm-Dancoff–Bloch-Horowitz procedure is used to define the *NN*and 3*N* potentials. In that approach (e.g., full Bonn potential) both the *NN and* 3*N* potentials contain nonvanishing contributions from the coherent sum of meson-recoil dynamics and the possibility of a large hard core requiring explicit calculation cannot be ruled out. Our conclusions are presented in Sec. V.

II. FORMALISM

The energy eigenvalue problem for an arbitrary interacting meson-baryon system is given by

$$H|\Psi\rangle=(H_0+H_I)|\Psi\rangle=E_i|\Psi\rangle, \tag{2.1}$$

where the total Hamiltonian H is separated into free and interacting parts H_0 and H_I, and $|\Psi\rangle$ denotes the complete meson-baryon state with energy E_i. This provides a relativistic time-ordered framework, rather than a manifestly covariant one. A rigorous solution of Eq. (2.1) in it present form is impossible because the wave functions contain not only nucleon degrees of freedom, but explicit meson, resonance, and antinucleonic degrees of freedom as well. However, as is well known, the problem can be reduced to a tractable form by partitioning the total Hilbert space into two parts,

$$\mathscr{H}_{\eta}=\{|N^{(+)}\rangle,|N^{(+)}N^{(+)}\rangle,\dots\},$$

$$\mathscr{H}_{\lambda}=\{\text{everything else}\}, \tag{2.2}$$

so that $\mathscr{H}_{\eta}$ is the Hilbert subspace consisting only of the positive-frequency parts of the nucleon state vectors, and $\mathscr{H}_{\lambda}$ contains everything else, e.g., $|\Delta\rangle$, $|N\pi\rangle$, $|N\Delta\pi\rho\rangle$, etc. The total wave function can now be expressed as,

$$|\Psi\rangle=\eta|\Psi\rangle+\lambda|\Psi\rangle=|\psi\rangle+|\phi\rangle, \tag{2.3}$$

where we have introduced projection operators satisfying the conventional algebra $\eta^2=\eta$, $\lambda^2=\lambda$, $\eta\lambda=\lambda\eta=0$, and $\eta+\lambda=1$ to obtain $|\psi\rangle=\eta|\Psi\rangle\in\mathscr{H}_{\eta}$ as a purely nucleonic state and $|\phi\rangle=\lambda|\Psi\rangle\in\mathscr{H}_{\lambda}$ as a state whose description requires explicit meson- and/or resonance- and/or negative-frequency degrees of freedom. We have not given explicit expressions for η and λ, nor will we need to since $|\psi\rangle=\eta|\psi\rangle$ and $|\phi\rangle=\lambda|\phi\rangle$. The point is that the energy eigenvalue problem can now be written as

$$E_i|\psi\rangle=\eta H\eta|\psi\rangle+\eta H\lambda|\phi\rangle, \tag{2.4a}$$

$$E_i|\phi\rangle=\lambda H\eta|\psi\rangle+\lambda H\lambda|\phi\rangle. \tag{2.4b}$$

There are a number of different (but equivalent [24,25]) ways in which Eq. (2.4) can be reduced when $|\phi\rangle$ vanishes in the observable states. One seeks to derive an *effective* Hamiltonian H_{eff} (or an effective interaction V_{eff}) which satisfies,

$$H_{\text{eff}}|\psi\rangle=[H_0+V_{\text{eff}}]|\psi\rangle=E_i|\psi\rangle \tag{2.5}$$

so that the matrix elements of the Hamiltonian between interacting meson-baryon wave functions $|\Psi\rangle$, as required in Eq. (2.1), can be computed as an effective Hamiltonian between conventional nucleonic wave functions $|\psi\rangle$.

Perhaps the most commonly adopted reduction scheme is found in the Tamm-Dancoff approximation [26], or in one of the many equivalent schemes like that due to Bloch and Horowitz [27]. Here one begins by noting that the free-energy Hamiltonian cannot cause transitions between the Hilbert subspaces, so $\lambda H_0\eta=0$ can be used to reduce Eq. (2.4b) to

$$(E_i-H_0)\lambda|\phi\rangle=\lambda H_I[\eta|\psi\rangle+\lambda|\phi\rangle]. \tag{2.6}$$

Collecting the $\lambda|\phi\rangle$ terms together gives [25] Sawada's result,

$$\lambda|\phi\rangle=\frac{1}{\{1-[\lambda/(E_i-H_0)]H_I\}}\frac{\lambda}{E_i-H_0}H_I\eta|\psi\rangle. \tag{2.7}$$

After inserting this into Eq. (2.4a) and noting that $\eta H_I\eta=0$, a comparison with Eq. (2.5) shows that

$$V_{\text{eff}}=H_I\frac{1}{[1-[\lambda/(E_i-H_0)]H_I]}\frac{\lambda}{E_i-H_0}H_I. \tag{2.8}$$

Equivalently, we can recognize Eq. (2.6) as a recursive definition of $|\phi\rangle$, so that

$$\lambda|\phi\rangle=\frac{\lambda}{E_i-H_0}R|\psi\rangle,\quad R=H_I+H_I\frac{\lambda}{E_i-H_0}R \tag{2.9}$$

and

$$V_{\text{eff}}=H_I\frac{\lambda}{E_i-H_0}R. \tag{2.10}$$

Expanding the interaction Hamiltonian in powers of the strong coupling,

$$H_I=\sum_{i=1}^{\infty} H_i \tag{2.11}$$

allows for the calculation of the effective interaction to any desired order. In particular, the one-boson-exchange potential (OBEP) in the A-nucleon system is given by

$$V_{\rm eff}^{(2?)}=H_1\frac{\lambda}{E_i-H_0}H_1 \tag{2.12}$$

and the corresponding two-boson-exchange potential is

$$\begin{aligned}V_{\rm eff}^{(4?)}=&H_1\frac{\lambda}{E_i-H_0}H_1\frac{\lambda}{E_i-H_0}H_1\frac{\lambda}{E_i-H_0}H_1\\&+H_2\frac{\lambda}{E_i-H_0}H_1\frac{\lambda}{E_i-H_0}H_1\\&+H_1\frac{\lambda}{E_i-H_0}H_2\frac{\lambda}{E_i-H_0}H_1\\&+H_1\frac{\lambda}{E_i-H_0}H_1\frac{\lambda}{E_i-H_0}H_2+H_2\frac{\lambda}{E_i-H_0}H_2.\end{aligned} \tag{2.13}$$

We have labeled the order of the interactions in Eqs. (2.12) and (2.13) with question marks since these expressions are obtained from Eq. (2.10) by expanding the numerator to definite order, but retaining the full (infinite order) energy dependence in the denominator.

We now appear to have effective interactions for usage in Eq. (2.5). Indeed, if we set $H_2=0$ in Eqs. (2.12) and (2.13), we obtain the OBEP and TBEP interactions used in the full Bonn potential. However, it is easy to see that the Hermiticity of these effective interactions is destroyed by an explicit dependence on the full initial-state energy E_i. In addition, from Eqs. (2.3) and (2.9) we observe that the wave functions are not orthonormal but instead satisfy

$$\delta_{fi}=\left\langle \psi_f\left|\left[\mathbb{1}+R^\dagger\frac{\lambda}{E_f-H_0}\frac{\lambda}{E_i-H_0}R\right]\right|\psi_i\right\rangle. \tag{2.14}$$

As such, the orthonormality condition depends on the order at which we truncate the interaction. These are certainly unwanted complications that will invite dubious approximation.

The problems associated with the Tamm-Dancoff approach can be removed [24] by expanding the energy E in the same way as we expanded the Hamiltonian in Eq. (2.11). It is then possible to obtain an interaction to a definite order and to calculate a renormalization condition for the wave functions. An equivalent solution, which was developed by Okubo [25] and which we will refer to as the *unitary transformation method*, proceeds by rewriting Eq. (2.4) in matrix form as

$$E_i\begin{pmatrix}|\psi\rangle\\|\phi\rangle\end{pmatrix}=\begin{pmatrix}\eta H\eta & \eta H\lambda\\ \lambda H\eta & \lambda H\lambda\end{pmatrix}\begin{pmatrix}|\psi\rangle\\|\phi\rangle\end{pmatrix} \tag{2.15}$$

and introducing new states $|\chi\rangle$ and $|\varphi\rangle$ through a unitary transformation,

$$\begin{pmatrix}|\psi\rangle\\|\phi\rangle\end{pmatrix}=U\begin{pmatrix}|\chi\rangle\\|\varphi\rangle\end{pmatrix}. \tag{2.16}$$

In terms of the new states, the energy eigenvalue problem reads

$$U^\dagger HU\begin{pmatrix}|\chi\rangle\\|\varphi\rangle\end{pmatrix}=E_i\begin{pmatrix}|\chi\rangle\\|\varphi\rangle\end{pmatrix}. \tag{2.17}$$

Unlike the Tamm-Dancoff reduction, where we found that Eqs. (2.7) and (2.9) related $|\psi\rangle$ and $|\phi\rangle$, we can now choose U such that it is unitary and ensures $U^\dagger HU$ is diagonal, so that $|\chi\rangle$ and $|\varphi\rangle$ are completely decoupled orthonormal states. In particular, for

$$U=\begin{pmatrix}\eta(1+A^\dagger A)^{-1/2}\eta & -\eta A^\dagger(1+AA^\dagger)^{-1/2}\lambda\\ \lambda A(1+A^\dagger A)^{-1/2}\eta & \lambda(1+AA^\dagger)^{-1/2}\lambda\end{pmatrix} \tag{2.18}$$

a short calculation shows that the nondiagonal elements of Eq. (2.17) vanish for any A which satisfies

$$\lambda(H+[H,A]-AHA)\eta=0. \tag{2.19}$$

We will derive the operators A that satisfy Eq. (2.19) in a moment. When $|\varphi\rangle$ vanishes in the observable states, the energy eigenvalue problem reduces to the conventional Schrödinger equation

$$\langle\Psi|H|\Psi\rangle=\langle\chi|H_{\rm eff}|\chi\rangle=\langle\chi|E_i|\chi\rangle \tag{2.20}$$

with the Hermitian effective interaction,

$$H_{\rm eff}=\eta(1+A^\dagger A)^{-1/2}\eta(1+A^\dagger)H(1+A)\eta(1+A^\dagger A)^{-1/2}\eta. \tag{2.21}$$

The complete meson-baryon wave functions $|\Psi\rangle$ are then described in terms of the operators A and the nucleonic wave functions $|\chi\rangle$,

$$|\Psi\rangle=(1+A)\eta(1+A^\dagger A)^{-1/2}\eta|\chi\rangle. \tag{2.22}$$

As already anticipated, we find $|\chi\rangle\in\mathscr{H}_\eta$ and $|\varphi\rangle\in\mathscr{H}_\lambda$ preserve the orthonormality of $|\Psi\rangle$,

$$\delta_{fi}=\langle\Psi_f|\Psi_i\rangle=\langle\chi_f|\eta(1+A^\dagger A)^{-1/2}\eta(1+A^\dagger)(1+A)\eta(1+A^\dagger A)^{-1/2}\eta|\chi_i\rangle=\langle\chi_f|\chi_i\rangle, \tag{2.23a}$$

$$\delta_{fi}=\langle\Psi_f|\Psi_i\rangle=\langle\varphi_f|\eta(1+AA^\dagger)^{-1/2}\eta(1-A)(1-A^\dagger)\eta(1+AA^\dagger)^{-1/2}\eta|\varphi_i\rangle=\langle\varphi_f|\varphi_i\rangle. \tag{2.23b}$$

We next derive the operators A that satisfy Eq. (2.19) by expanding A and the Hamiltonian to which we already know it is related, in powers n of the coupling constant,

$$H=H_0+\sum_{n=1}^{\infty} H_n\,,\quad A=\sum_{n=1}^{\infty} A_n\,, \tag{2.24}$$

where $A=\lambda A\eta$ shows that $A_0=0$. Similarly, we note that $\eta H_I\eta = \eta H_0\lambda = \lambda H_0\eta = 0$, so that Eq. (2.19) becomes

$$0=\sum_{n=1}^{\infty}\lambda\Bigg[H_n+[H_0,A_n]+\sum_{i=1}^{n-1}H_iA_{n-i} -\sum_{i=1}^{n-2}\sum_{j=1}^{n-i-1}A_iH_jA_{n-i-j}\Bigg]\eta. \tag{2.25}$$

We choose to further constrain A by demanding Eq. (2.25) is satisfied at each order of n. With $H_0\eta=\mathcal{E}_i\eta$, where $\mathcal{E}_i$ is the free-particle energy of the initial state, we obtain,

$$(\mathcal{E}_i-H_0)A_n=\lambda\Bigg[H_n+\sum_{i=1}^{n-1}H_iA_{n-i} -\sum_{i=1}^{n-2}\sum_{j=1}^{n-i-1}A_iH_jA_{n-i-j}\Bigg]\eta. \tag{2.26}$$

This completes the definition of A. To calculate the OBEP and TBEP interactions in the A-body system, we require

$$A_1=\frac{\lambda}{\mathcal{E}_i-H_0}H_1\eta, \tag{2.27a}$$

$$A_2=\frac{\lambda}{\mathcal{E}_i-H_0}H_2\eta+\frac{\lambda}{\mathcal{E}_i-H_0}H_1\frac{\lambda}{\mathcal{E}_i-H_0}H_1\eta, \tag{2.27b}$$

$$\begin{aligned}A_3=&\frac{\lambda}{\mathcal{E}_i-H_0}H_3\eta+\frac{\lambda}{\mathcal{E}_i-H_0}H_1\frac{\lambda}{\mathcal{E}_i-H_0}H_2\eta\\&+\frac{\lambda}{\mathcal{E}_i-H_0}H_1\frac{\lambda}{\mathcal{E}_i-H_0}H_1\frac{\lambda}{\mathcal{E}_i-H_0}H_1\eta\\&+\frac{\lambda}{\mathcal{E}_i-H_0}H_2\frac{\lambda}{\mathcal{E}_i-H_0}H_1\\&-\frac{\lambda}{\mathcal{E}_i-H_0}\frac{\lambda}{\mathcal{E}_a-H_0}H_1\eta_aH_1\frac{\lambda}{\mathcal{E}_i-H_0}H_1\eta,\end{aligned}$$

where $\mathcal{E}_a$ is the free-particle energy of an intermediate η-space state. The OBEP in the A-body system is then given by

$$V_{\rm eff}^{(2)}=\eta_f\left\{H_1\lambda\frac{(1/2)\,(\mathcal{E}_i+\mathcal{E}_f)-H_0}{(\mathcal{E}_f-H_0)(\mathcal{E}_i-H_0)}\lambda H_1\right\}\eta_i \tag{2.28}$$

and the corresponding TBEP is

$$\begin{aligned}V_{\rm eff}^{(4)}=\eta_f\Bigg\{&H_1\frac{\lambda}{\mathcal{E}_f-H_0}H_1\lambda\frac{(1/2)(\mathcal{E}_f+\mathcal{E}_i)-H_0}{(\mathcal{E}_f-H_0)(\mathcal{E}_i-H_0)}\lambda H_1\frac{\lambda}{\mathcal{E}_i-H_0}H_1-\frac{1}{2}H_1\frac{\lambda}{\mathcal{E}_f-H_0}\left[\frac{\lambda}{E_a-H_0}H_1\eta_aH_1+H_1\eta_aH_1\frac{\lambda}{E_a-H_0}\right]\\
&\times\frac{\lambda}{\mathcal{E}_i-H_0}H_1+H_2\lambda\frac{(1/2)(\mathcal{E}_f+\mathcal{E}_i)-H_0}{(\mathcal{E}_f-H_0)(\mathcal{E}_i-H_0)}\lambda H_2+\frac{1}{2}(\mathcal{E}_f-\mathcal{E}_i)H_1\frac{\lambda}{\mathcal{E}_f-H_0}\Bigg[H_1\eta_aH_1\frac{\lambda}{(E_a-H_0)(\mathcal{E}_f-H_0)}\\
&+\frac{\lambda}{(\mathcal{E}_i-H_0)(E_a-H_0)}H_1\eta_aH_1-H_1\frac{\lambda}{\mathcal{E}_f-H_0}H_1\frac{\lambda}{\mathcal{E}_f-H_0}-\frac{\lambda}{\mathcal{E}_i-H_0}H_1\frac{\lambda}{\mathcal{E}_i-H_0}H_1\Bigg]\frac{\lambda}{\mathcal{E}_i-H_0}H_1\\
&-\left[\frac{\mathcal{E}_f+\mathcal{E}_i}{8}-\frac{E_a}{2}\right]H_1\frac{\lambda}{(E_a-H_0)(\mathcal{E}_f-H_0)}H_1\eta_aH_1\frac{\lambda}{(E_a-H_0)(\mathcal{E}_i-H_0)}H_1+H_2\frac{\lambda}{\mathcal{E}_f-H_0}H_1\frac{\lambda}{\mathcal{E}_i-H_0}H_1\\
&+H_1\frac{\lambda}{\mathcal{E}_f-H_0}H_2\frac{\lambda}{\mathcal{E}_i-H_0}H_1+H_1\frac{\lambda}{\mathcal{E}_f-H_0}H_1\frac{\lambda}{\mathcal{E}_i-H_0}H_2+\frac{1}{2}(\mathcal{E}_f-\mathcal{E}_i)H_2\frac{\lambda}{\mathcal{E}_f-H_0}\\
&\times\left[\frac{\lambda}{\mathcal{E}_i-H_0}H_1-H_1\frac{\lambda}{\mathcal{E}_f-H_0}\right]\frac{\lambda}{\mathcal{E}_i-H_0}H_1+\frac{1}{2}(\mathcal{E}_f-\mathcal{E}_i)H_1\frac{\lambda}{\mathcal{E}_f-H_0}\left[\frac{\lambda}{\mathcal{E}_i-H_0}H_2-H_2\frac{\lambda}{\mathcal{E}_f-H_0}\right]\frac{\lambda}{\mathcal{E}_i-H_0}H_1\\
&+\frac{1}{2}(\mathcal{E}_f-\mathcal{E}_i)H_1\frac{\lambda}{\mathcal{E}_f-H_0}\left[\frac{\lambda}{\mathcal{E}_i-H_0}H_1-H_1\frac{\lambda}{\mathcal{E}_f-H_0}\right]\frac{\lambda}{\mathcal{E}_i-H_0}H_2\Bigg\}\eta_i\,.\end{aligned} \tag{2.29}$$

By contrast to Eqs. (2.12) and (2.13), Eqs. (2.28) and (2.29) involve no references to the full energy and define OBEP and TBEP interactions to definite order.

The unitary transformation method has some significant advantages over the Tamm-Dancoff approximation.

(1) The wave functions are orthonormal, and remain so when A is truncated to a definite order.

(2) The effective Hamiltonian is Hermitian and depends only on the *free* (not full) energy of the observables states.

(3) The interaction is well defined to any definite order: the definition of the OBEP interactions does not change when higher-order interactions are introduced.

From the above discussion we realize that, given any microscopic model definition for the NN interaction, a corresponding definition of the 3N interaction follows immediately. However, we have also seen that the unitary transformation method possesses operators which are entirely absent from the Tamm-Dancoff result. Without even writing down a Lagrangian, we will later see that, in applications assuming orthogonal wave functions, the Tamm-Dancoff approximation (as used to define the Bonn model) guarantees a 3N force which possesses a hard core, whereas the unitary transformation method (as used to define the RuhrPot model) ensures the corresponding dynamics vanish in the 3N system.

III. OBEP AND THE HARD CORE OF THE *NN* INTERACTION

The characteristic behavior of the NN interaction has been known for a long time. A weak attraction results at long ranges almost entirely from the exchange of the bound-state π meson. At intermediate ranges multiple-pion exchange becomes important, so that one-boson-exchange potentials (OBEP's) generally need to account for correlated 2π and 3 π exchange by including the ρ and ω mesons. The apparently less important uncorrelated 2π exchange requires explicit calculation of a two-boson-exchange potential (TBEP). Moving towards the hard core, heavier mesons gain an obvious importance as a means of effectively describing highly correlated and complicated exchange processes.

The generic form of the NN interaction is written symbolically as

$$V_{\rm OBEP}=\sum_{\alpha_\pi=\pi\text{-like}} V_{\alpha_\pi}=V_\pi+V_{\pi'}+V_{\pi''}+\cdots$$

$$+\sum_{\alpha_\rho=\rho\text{-like}} V_{\alpha_\rho}=V_\rho+V_{\rho'}+V_{\rho''}+\cdots$$

$$+\sum_{\alpha_\omega=\omega\text{-like}} V_{\alpha_\omega}=V_\omega+V_{\omega'}+V_{\omega''}+\cdots \qquad (3.1)$$

$$\vdots, \qquad (3.2)$$

where, for each $(J^\pi;T)$, an entire spectrum of exchange processes contribute to the interaction. The problem is how to include such dynamics without introducing too many parameters.

The conventional approach in meson physics is simply to forget about everything except the mesons with masses under about 1 GeV. This certainly seems reasonable when we consider the individual contribution of any given heavy meson. After all, a large meson mass causes the meson propagator to suppress the contribution to the NN interaction at low energies, and in any event the coupling constants and form factor scales are mostly unknown.

But is it reasonable to neglect the *summed* contribution of *all* of the additional exchange processes? For simplicity, let us focus on the π-like contributions to OBEP, i.e., the part of the NN interaction characterized by $(J^\pi;T)=(0^-;1)$ exchange with a (not necessarily sharp) mass m_{α_π}. This does not restrict us to consider only bound-state heavy meson exchanges, but it does imply a common operator structure $\mathscr{M}_{\pi\text{-like}}$, so that the NN interaction takes the form

$$V_{\pi\text{-like}}=\mathscr{M}_{\pi\text{-like}}\sum_{\alpha_\pi=\pi\text{-like}}\frac{g^2_{NN\alpha_\pi}}{4\pi}\frac{F^2_{NN\alpha_\pi}(Q^2)}{m^2_{\alpha_\pi}+Q^2}$$

$$=\mathscr{M}_{\pi\text{-like}}\frac{g^2_{NN\pi}}{4\pi}\frac{F^2_{NN\pi}(Q^2)}{m^2_\pi+Q^2}$$

$$+\mathscr{M}_{\pi\text{-like}}\sum_{\alpha_\pi\neq\pi}\frac{g^2_{NN\alpha_\pi}}{4\pi}\frac{F^2_{NN\alpha_\pi}(Q^2)}{m^2_{\alpha_\pi}+Q^2}, \qquad (3.3)$$

where we have separated out the lightest meson contribution from the remaining processes. Note that the additional $(J^\pi;T)=(0^-;1)$ exchange contributions are required by completeness and necessarily add coherently with *no possibility for cancellation*.

From Eq. (3.3) it appears that we have conventional OBEP and a summation over heavy meson-exchange processes. If the summation α over the $(J^\pi;T)=(0^-;1)$ exchanges could be truncated at sufficiently low order, this interpretation might suffice. If not, then Eq. (3.3) introduces a semiphenomenological description of all $(J^\pi;T)=(0^-;1)$ exchange dynamics, not just meson exchanges.

In the RuhrPot NN interaction these additional contributions are retained by writing the NN interaction as

$$V_{\pi\text{-like}}\sim\mathscr{M}_{\pi\text{-like}}\frac{g^2_{NN\pi}}{4\pi}\frac{F^2_{NN\pi}(Q^2)}{m^2_\pi+Q^2}$$

$$+\mathscr{M}_{\pi\text{-like}}\Sigma_{\pi\text{-like}}F^2_{NN\pi\text{-like}}(Q^2), \qquad (3.4)$$

where the additional "contact" term introduces an effective constant $\Sigma_{\pi\text{-like}}$ through a closure approximation. The RuhrPot NN interaction also includes analogous $\Sigma_{\rho\text{-like}}$, $\Sigma_{\omega\text{-like}}$, and $\Sigma_{\epsilon\text{-like}}$ contact terms.

The contact interactions completely dominate in the region of the hard core, but are essentially vanishing at the larger distances where the meson-exchange dynamics take over. It is important to note that the meson-exchange contributions in the RuhrPot model are heavily cut down in the region of the hard core. This results because the meson-nucleon vertices are dressed with form factors that obey the asymptotic Q^2 dependence predicted by perturbative QCD [22]. In particular, the Dirac and Pauli form factors used in the RuhrPot model are given by [21]

$$F^{(1)}=\frac{\Lambda_1^2}{\Lambda_1^2+\hat{Q}^2}\frac{\Lambda_2^2}{\Lambda_2^2+\hat{Q}^2}, \qquad (3.5a)$$

$$F^{(2)}=F^{(1)}\frac{\Lambda_2^2}{\Lambda_2^2+\hat{Q}^2}, \qquad (3.5b)$$

where

$$\hat{Q}^2=Q^2\log\left[\frac{\Lambda_2^2+Q^2}{\Lambda^2_{\rm QCD}}\right]\bigg/\log\left[\frac{\Lambda_2^2}{\Lambda^2_{\rm QCD}}\right]. \qquad (3.6)$$

The meson scales Λ_1 have been obtained from direct calculation of the meson-baryon form factors [19,20], and the

QCD scales Λ_2 and $\Lambda_{\rm QCD}$ are obtained from a fit to the nucleon electromagnetic form factors at Q^2 ranging to about 30 GeV^2.

The RuhrPot NN interaction [15] fits the scattering phases with χ^2/datum=1.6 and the deuteron observables $E_D=-2.224\,575(9)$ MeV, $Q_D=0.2860(15)$ fm^2, $A_S=0.8846(8)$ fm$^{1/2}$, $D/S=0.0272(4)$, and $r_D=1.9560(68)$ fm (none of which are fitted) are all reasonably predicted as $E_D=-2.224$ MeV, $Q_D=0.276$ fm^2, $A_S=0.882$ fm$^{1/2}$, $D/S=0.025$, and $r_D=1.932$ fm. The SU(3)$_F$ result of $g^2_{NN\omega}/g^2_{NN\rho}=9$ is retained, which can be compared to values of around 27 required in conventional BEP [23]. This clearly has consequences for a consistent specification of the $\rho\pi\gamma$- and $\omega\pi\gamma$-exchange currents [17,18] required in the calculation of electromagnetic observables.

IV. CONTACT INTERACTIONS IN THE 3N SYSTEM

In the previous section we recalled how the RuhrPot NN interaction retains contact terms to include the summed $(J^\pi;T)$ exchange dynamics. This led naturally to a hard core in the OBEP NN interaction. But what about the 3N force? What happens when the TBEP dynamics of the 3N interaction is extended to include contact interactions? This simple question needs to be answered with some care.

In Sec. II we obtained two equivalent and model-independent definitions of the TBEP. In the *Tamm-Dancoff* approximation, as is used to define the full Bonn potential for example, the wave functions necessarily violate the conventional orthonormality requirement and the explicit energy dependence destroys the Hermiticity of the effective interaction. When the energy of the A-particle system is assumed to be conserved in all intermediate states the need for nonorthogonal wave functions remains, but the non-Hermiticity of the effective interaction is no longer apparent and the resulting TBEP in the three-body system is given by

$$V^{(4?)}_{\rm eff}=\eta\Bigg\{H_1\frac{\lambda}{E-H_0}H_1\frac{\lambda}{E-H_0}H_1\frac{\lambda}{E-H_0}H_1 \quad (4.1a)$$

$$+H_2\frac{\lambda}{E-H_0}H_1\frac{\lambda}{E-H_0}H_1 \quad (4.1b)$$

$$+H_1\frac{\lambda}{E-H_0}H_2\frac{\lambda}{E-H_0}H_1 \quad (4.1c)$$

$$+H_1\frac{\lambda}{E-H_0}H_1\frac{\lambda}{E-H_0}H_2\Bigg\}\eta, \quad (4.1d)$$

where E is the full (including binding) energy of the three-nucleon system.

Alternatively, from Sec. II we recall that a Hermitian interaction requiring orthonormal wave functions can be obtained from the *unitary transformation* procedure, as has been done to define the RuhrPot NN and 3N interactions. Adopting energy conservation for comparison with Eq. (4.1), the resulting TBEP contributions to the three-body system are given by

$$V^{(4)}_{\rm eff}=\eta\Bigg\{H_1\frac{\lambda}{\mathcal{E}-H_0}H_1\frac{\lambda}{\mathcal{E}-H_0}H_1\frac{\lambda}{\mathcal{E}-H_0}H_1 \quad (4.2a)$$

$$+H_2\frac{\lambda}{\mathcal{E}-H_0}H_1\frac{\lambda}{\mathcal{E}-H_0}H_1 \quad (4.2b)$$

$$+H_1\frac{\lambda}{\mathcal{E}-H_0}H_2\frac{\lambda}{\mathcal{E}-H_0}H_1 \quad (4.2c)$$

$$+H_1\frac{\lambda}{\mathcal{E}-H_0}H_1\frac{\lambda}{\mathcal{E}-H_0}H_2 \quad (4.2d)$$

$$-\frac{1}{2}H_1\frac{\lambda}{\mathcal{E}-H_0}\Bigg[\frac{\lambda}{\mathcal{E}-H_0}H_1\eta H_1 +H_1\eta H_1\frac{\lambda}{\mathcal{E}-H_0}\Bigg]\frac{\lambda}{\mathcal{E}-H_0}H_1\Bigg\}\eta \quad (4.2e)$$

where $\mathcal{E}$ is the free energy of the three-body system.

Equation (4.2a) describes meson-recoil, vector-meson decay, and baryon resonance contributions to the 3N interaction, whereas Eqs. (4.2b)–(4.2d) describe contributions involving less than four vertices, at least one of which is of second order. These operators are already different in the Tamm-Dancoff and unitary transformation schemes because of the appearance of full and free-particle energies, respectively. However, the most obvious cost in achieving Hermiticity and orthonormality in the unitary transformation scheme is found in the explicit appearance of the wave function reorthonormalization contributions of Eq. (4.2e). These are entirely absent in the Tamm-Dancoff scheme.

So does Eq. (4.2) imply that the 3N interaction has a hard core? It is important to realize that the three-body force should include all $(J^\pi;T)$-exchange dynamics. Nothing in Eq. (4.2), or indeed the projection formalisms described in Sec. II, indicates that we can arbitrarily truncate the dynamics to include only the lightest exchange processes. As in the NN interaction, our task is to include these additional contributions without introducing too many parameters.

Consider the 3N interactions involving the exchange of two arbitrary mesons, say α and β. Equations (4.2a) and (4.2e) involve a product of coupling constants $g^2_{NN\alpha}g^2_{NN\beta}$, or if an arbitrary baryon resonance N^* is excited, $g_{NN\alpha}g_{NN^*\alpha}g_{NN^*\beta}g_{NN\beta}$. Conversely, Eqs. (4.2b)–(4.2d) involve $g_{NN\alpha}g_{NN\alpha\beta}g_{NN\beta}$. As such, when we sum over all mesons α and/or β, only the contributions from Eqs. (4.2a) and (4.2e) without nucleon resonances are certain to form a coherent sum with no possibility for cancellation. This identifies the most likely source of a possible hard core in the 3N force.

The situation is, however, fundamentally different to the NN interaction because Eq. (4.2) involves a linear combination of meson recoil and wave function reorthonormalization processes and these are of opposite sign. In Fig. 3 we illustrate these contributions for the exchange of arbitrary mesons α and β. The meson-recoil and reorthonormalization graphs are shown only for the time-ordered topologies $a^\dagger_\beta(3)a^\dagger_\alpha(2)a_\beta(2)a_\alpha(1)$ and $a^\dagger_\beta(3)a_\beta(2)a^\dagger_\alpha(2)a_\alpha(1)$, re-

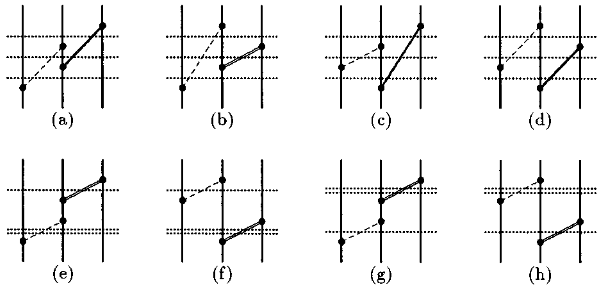

FIG. 3. The RuhrPot model adopts the unitary transformation definition of the effective 3N interaction, so that the recoil (a)–(d) and wave function reorthonormalization (e)–(h) contributions cancel in the nonrelativistic limit. The dynamics producing the hard core in the RuhrPot NN interaction therefore vanishes in the RuhrPot 3N force. By contrast, potentials defined within the Tamm-Dancoff–Bloch-Horowitz definition (e.g., the full Bonn potential) do not include wave function reorthonormalization contributions, so that the surviving recoil dynamics sums coherently to create a 3N force that is expected to be large when two or three of the nucleons are at small separations.

spectively, since all other time orderings can be reached through time reversal and permutations of the nucleon numbers.

Denoting the common operator structure for these contributions as $\mathscr{M}$, the nonrelativistic contribution to the potential energy from the meson-recoil processes of Figs. 3(a)–3(d) are

$$V_{\rm recoil}^{3N}=\mathscr{M}\sum_{\alpha\beta}\left[\frac{-2}{\omega_\alpha(\omega_\alpha+\omega_\beta)\omega_\beta}+\frac{-1}{\omega_\alpha(\omega_\alpha+\omega_\beta)\omega_\alpha}+\frac{-1}{\omega_\beta(\omega_\alpha+\omega_\beta)\omega_\beta}\right] \tag{4.3}$$

and the corresponding contribution from the wave function reorthonormalization processes of Figs. 3(e)–3(h) are

$$V_{\rm renorm}^{3N}=-\frac{1}{2}\mathscr{M}\sum_{\alpha\beta}\left[\frac{-2}{\omega_\alpha^2\omega_\beta}+\frac{-2}{\omega_\alpha\omega_\beta^2}\right]. \tag{4.4}$$

In other words, for the RuhrPot 3N interaction we have

$$V_{\rm recoil}^{3N}+V_{\rm renorm}^{3N}=0\Rightarrow \text{no hard core}. \tag{4.5}$$

Although this cancellation of wave function reorthonormalization and meson recoil terms does *not* hold in the NN system, in the 3N system it holds for all mesons, regardless of their mass and quantum numbers. This result is not dependent on the details of any meson-theoretical model and it eliminates the most likely source of a hard core in the 3N interaction. The result is good news for existing definitions of the 3N interaction [2–13] and rigorous applications [28–33] because it lends support to the notion that the 3N force can be reasonably described with only light meson-exchange dynamics. Preliminary applications using a consistent definition of the RuhrPot NN [15] and 3N (see the Appendix) interactions have reported [34] a noteworthy agreement with experiment. In particular, while the triton binding energy calculated with the NN interaction alone gives $E_B=-7.64$ MeV, when the consistently defined 3N interaction is included the result becomes $E_B=-8.34$ MeV. This compares favorably with the experimental result of $E_B=-8.48$ MeV.

We stress that the unitary transformation procedure described in Sec. II is central to our definition of the RuhrPot 3 N interaction. It ensures only Hermitian and energy independent operators arise and that they are to be taken between orthonormal wave functions. It generates wave function reorthonormalization terms that cancel the recoil dynamics, and we have seen that this is central to eliminating the most likely source of a hard core in the 3N interaction.

Had we adopted a procedure like the Tamm-Dancoff approximation, as has been done to define the full Bonn potential, our results would be changed completely. From Sec. II we realize that such operators would be non-Hermitian and energy dependent and that they would need to be computed between nonorthogonal wave functions. Moreover, since there would be no wave function reorthonormalization terms, nothing would cancel the recoil dynamics and a hard core in the 3N interaction would result from the summed recoil dynamics. In other words, for a 3N interaction consistent with the full Bonn potential,

$$V_{\rm recoil}^{3N}\neq 0\Rightarrow \text{hard core exists}. \tag{4.6}$$

Both the NN and 3N Tamm-Dancoff effective interactions become consistent with the unitary transformation results only when Hermiticity is restored by removing the spurious energy dependence and the matrix elements are computed with reorthonormalized wave functions. Until such rigor is introduced into three-body applications that use Tamm-Dancoff effective interactions, it is to be hoped that any discrepancies with the (already rigorous) unitary transformation results will not be interpreted in terms of model Lagrangians.

V. CONCLUSIONS

The dynamics that generates the hard core in the RuhrPot NN interaction has been considered in the irreducible three-body force. After presenting a detailed summary of the formal definitions of OBEP and TBEP, we recalled that the hard core of the NN interaction arises naturally when the $(J^\pi;T)$-exchange dynamics is no longer truncated to include only the lightest few mesons. We explored the effect of introducing such dynamics into the 3N system. We isolated those contributions that necessarily add coherently and therefore cannot possibly be expected to cancel. Finally, we showed that these contributions to the irreducible 3N force vanish identically in the static limit. This is a model-independent result. It lends support to the conventional assumption that the 3N force can reasonably be described using only light-meson-exchange dynamics.

A completely different result is expected when the Tamm-Dancoff–Bloch-Horowitz procedure is used to define the NN and 3N potentials. In that approach (e.g., full Bonn potential), both the NN *and* 3N potentials contain nonvanishing contributions from the coherent sum of meson-recoil dynamics and the possibility of a large hard core requiring explicit calculation cannot be ruled out.

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APPENDIX: THE RUHRPOT 3N INTERACTION

The RuhrPot 3N interaction is obtained from the unitary transformation result of Eq. (2.29). We present here the leading-order contributions shown in Figs. 1(c)–(e) in the nonrelativistic limit. The meson recoil contributions are neglected because they are exactly canceled by the wave function reorthonormalization terms, the latter being absent in the Tamm-Dancoff definition of TBEP. All coupling constants and form factors are taken to be consistent with the RuhrPot NN interaction [15].

For the excitation of a Δ-resonance mediated by the exchange of π- or ρ-mesons we have,

$$V_{\Delta\pi\pi}=-\frac{2}{9}\frac{g_{NN\pi}^2 g_{N\Delta\pi}^2}{(2\pi)^6(2m)^4}\frac{F_{NN\pi}(\vec{k}_1^2)F_{N\Delta\pi}(\vec{k}_1^2)F_{NN\pi}(\vec{k}_2^2)F_{N\Delta\pi}(\vec{k}_2^2)}{(\vec{k}_1^2+m_\pi^2)(\vec{k}_2^2+m_\pi^2)(m_\Delta-m)}\times(\vec{\sigma}_1\cdot\vec{k}_1)(\vec{\sigma}_2\cdot\vec{k}_2)[4(\vec{k}_1\cdot\vec{k}_2)(\vec{\tau}_1\cdot\vec{\tau}_2)-\vec{\sigma}_3\cdot(\vec{k}_1\times\vec{k}_2)(\vec{\tau}_1\times\vec{\tau}_2)\cdot\vec{\tau}_3], \tag{A1}$$

$$\begin{aligned}V_{\Delta\rho\pi}=&-\frac{2}{9}\frac{g_{NN\pi}g_{N\Delta\pi}g_{NN\rho}g_{N\Delta\rho}}{(2m)^4(2\pi)^6}\frac{F_{NN\pi}(\vec{k}_1^2)F_{N\Delta\pi}(\vec{k}_1^2)G_{NN\rho}^M(\vec{k}_2^2)G_{N\Delta\rho}^M(\vec{k}_2^2)}{(m_\Delta-m)(\vec{k}_1^2+m_\pi^2)(\vec{k}_2^2+m_\rho^2)}\\&\times\{4(\vec{\sigma}_1\cdot\vec{k}_1)\vec{\tau}_1\cdot\vec{\tau}_2[(\vec{\sigma}_2\cdot\vec{k}_1)(\vec{k}_2)^2-(\vec{\sigma}_2\cdot\vec{k}_2)(\vec{k}_1\cdot\vec{k}_2)]\\&-(\vec{\sigma}_1\cdot\vec{k}_1)(\vec{\tau}_1\times\vec{\tau}_2)\cdot\vec{\tau}_3[(\vec{\sigma}_2\times\vec{\sigma}_3)\cdot\vec{k}_1(\vec{k}_2)^2-(\vec{\sigma}_2\cdot\vec{k}_2)\vec{\sigma}_3\cdot(\vec{k}_1\times\vec{k}_2)]\}\\&-\frac{2}{9}\frac{g_{NN\pi}g_{N\Delta\pi}g_{NN\rho}g_{\mathrm{N}\Delta\rho}}{(2m)^4(2\pi)^6}\frac{F_{NN\pi}(\vec{k}_2^2)F_{N\Delta\pi}(\vec{k}_2^2)G_{NN\rho}^M(\vec{k}_1^2)G_{N\Delta\rho}^M(\vec{k}_1^2)}{(m_\Delta-m)(\vec{k}_1^2+m_\rho^2)(\vec{k}_2^2+m_\pi^2)}\\&\times\{4(\vec{\sigma}_2\cdot\vec{k}_2)\vec{\tau}_1\cdot\vec{\tau}_2[(\vec{\sigma}_1\cdot\vec{k}_2)(\vec{k}_1)^2-(\vec{\sigma}_1\cdot\vec{k}_1)(\vec{k}_1\cdot\vec{k}_2)]\\&+(\vec{\sigma}_2\cdot\vec{k}_2)(\vec{\tau}_1\times\vec{\tau}_2)\cdot\vec{\tau}_3[(\vec{\sigma}_1\times\vec{\sigma}_3)\cdot\vec{k}_2(\vec{k}_1)^2+(\vec{\sigma}_1\cdot\vec{k}_1)\vec{\sigma}_3\cdot(\vec{k}_1\times\vec{k}_2)]\},\end{aligned} \tag{A2}$$

$$\begin{aligned}V_{\Delta\rho\rho}=&-\frac{2}{9}\frac{g_{NN\rho}^2 g_{N\Delta\rho}^2}{(2m)^4(2\pi)^6}\frac{G_{NN\rho}^M(\vec{k}_1^2)G_{N\Delta\rho}^M(\vec{k}_1^2)G_{NN\rho}^M(\vec{k}_2^2)G_{N\Delta\rho}^M(\vec{k}_2^2)}{(m_\Delta-m)(\vec{k}_1^2+m_\rho^2)(\vec{k}_2^2+m_\rho^2)}\\&\times\{4\vec{\tau}_1\cdot\vec{\tau}_2[(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{k}_1)^2(\vec{k}_2)^2-(\vec{\sigma}_1\cdot\vec{k}_1)(\vec{\sigma}_2\cdot\vec{k}_1)(\vec{k}_2)^2\\&-(\vec{\sigma}_1\cdot\vec{k}_2)(\vec{\sigma}_2\cdot\vec{k}_2)(\vec{k}_1)^2+(\vec{\sigma}_1\cdot\vec{k}_1)(\vec{\sigma}_2\cdot\vec{k}_2)(\vec{k}_1\cdot\vec{k}_2)]\\&-(\vec{\tau}_1\times\vec{\tau}_2)\cdot\vec{\tau}_3[(\vec{\sigma}_1\times\vec{\sigma}_2)\cdot\vec{\sigma}_3(\vec{k}_1)^2(\vec{k}_2)^2-(\vec{\sigma}_1\cdot\vec{k}_1)(\vec{\sigma}_2\times\vec{\sigma}_3)\cdot\vec{k}_1(\vec{k}_2)^2\\&+(\vec{\sigma}_2\cdot\vec{k}_2)(\vec{\sigma}_1\times\sigma_3)\cdot\vec{k}_2(\vec{k}_1)^2+(\vec{\sigma}_1\cdot\vec{k}_1)(\vec{\sigma}_2\cdot\vec{k}_2)\vec{\sigma}_3\cdot(\vec{k}_1\times\vec{k}_2)]\}.\end{aligned} \tag{A3}$$

For the $\rho\rightleftharpoons\pi\pi$, $\epsilon\rightleftharpoons\pi\pi$, and $\omega\rightleftharpoons\pi\rho$ contributions we have

$$V_{\rho\pi\pi}=\frac{g_{NN\pi}^2 g_{NN\rho}^2}{(2\pi)^6 2m^3}\frac{F_{NN\pi}(\vec{k}_1^2)F_{NN\pi}(\vec{k}_2^2)G_{NN\rho}^M(\vec{k}_3^2)F_{\rho\pi\pi}(\vec{k}_1^2,\vec{k}_2^2,\vec{k}_3^2)}{(\vec{k}_1^2+m_\pi^2)(\vec{k}_2^2+m_\pi^2)(\vec{k}_3^2+m_\rho^2)}(\vec{\sigma}_1\cdot\vec{k}_1)(\vec{\sigma}_2\cdot\vec{k}_2)\vec{\sigma}_3\cdot(\vec{k}_1\times\vec{k}_2)(\vec{\tau}_1\times\vec{\tau}_2)\cdot\vec{\tau}_3, \tag{A4}$$

$$V_{\epsilon\pi\pi}=\frac{g_{NN\pi}^2 g_{NN\epsilon}f_{\epsilon\pi\pi}}{(2\pi)^6 2m^2}\frac{F_{NN\pi}(\vec{k}_1^2)F_{NN\pi}(\vec{k}_2^2)F_{NN\epsilon}(\vec{k}_3^2)F_{\epsilon\pi\pi}(\vec{k}_1^2,\vec{k}_2^2,\vec{k}_3^2)}{(\vec{k}_1^2+m_\pi^2)(\vec{k}_2^2+m_\pi^2)(\vec{k}_3^2+m_\epsilon^2)}(\vec{\sigma}_1\cdot\vec{k}_1)(\vec{\sigma}_2\cdot\vec{k}_2)(\vec{\tau}_1\cdot\vec{\tau}_2), \tag{A5}$$

$$\begin{aligned}V_{\omega\pi\rho}=&\frac{g_{NN\pi}g_{NN\rho}g_{NN\omega}g_{\pi\rho\omega}}{(2\pi)^6 2m^2 m_\rho}\frac{F_{NN\pi}(\vec{k}_1^2)G_{NN\rho}^M(\vec{k}_2^2)F_{NN\omega}(\vec{k}_3^2)F_{\rho\pi\omega}(\vec{k}_1^2,\vec{k}_2^2,\vec{k}_3^2)}{(\vec{k}_1^2+m_\pi^2)(\vec{k}_2^2+m_\rho^2)(\vec{k}_3^2+m_\omega^2)}[(\vec{\sigma}_2\cdot\vec{k}_1)\vec{k}_2^2-(\vec{\sigma}_2\cdot\vec{k}_2)(\vec{k}_1\cdot\vec{k}_2)](\vec{\sigma}_1\cdot\vec{k}_1)(\vec{\tau}_1\cdot\vec{\tau}_2)\\&+\frac{g_{NN\pi}g_{NN\rho}g_{NN\omega}g_{\pi\rho\omega}}{(2\pi)^6 2m^2 m_\rho}\frac{F_{NN\pi}(\vec{k}_2^2)G_{NN\rho}^M(\vec{k}_1^2)F_{NN\omega}(\vec{k}_3^2)F_{\rho\pi\omega}(\vec{k}_1^2,\vec{k}_2^2,\vec{k}_3^2)}{(\vec{k}_2^2+m_\pi^2)(\vec{k}_1^2+m_\rho^2)(\vec{k}_3^2+m_\omega^2)}[(\vec{\sigma}_1\cdot\vec{k}_2)\vec{k}_1^2-(\vec{\sigma}_1\cdot\vec{k}_1)(\vec{k}_1\cdot\vec{k}_2)](\vec{\sigma}_2\cdot\vec{k}_2)(\vec{\tau}_1\cdot\vec{\tau}_2),\end{aligned} \tag{A6}$$

and for the $\rho\pi$ terms involving an $NN\pi\rho$ vertex on nucleon three we have

$$V_{\pi\rho}=-\frac{g_{NN\pi}^2 g_{NN\rho}^2}{(2\pi)^6 4m^3}\frac{F_{NN\pi}^2(\vec{k}_1^2)G_{NN\rho}^M(\vec{k}_2^2)F_{NN\rho}^{(1)}(\vec{k}_2^2)}{(\vec{k}_1^2+m_\pi^2)(\vec{k}_2^2+m_\rho^2)}(\vec{\sigma}_1\cdot\vec{k}_1)(\vec{\sigma}_2\times\vec{\sigma}_3)\cdot\vec{k}_2(\vec{\tau}_1\times\vec{\tau}_2)\cdot\vec{\tau}_3$$
$$+\frac{g_{NN\pi}^2 g_{NN\rho}^2}{(2\pi)^6 4m^3}\frac{F_{NN\pi}^2(\vec{k}_2^2)G_{NN\rho}^M(\vec{k}_1^2)F_{NN\rho}^{(1)}(\vec{k}_1^2)}{(\vec{k}_2^2+m_\pi^2)(\vec{k}_1^2+m_\rho^2)}(\vec{\sigma}_2\cdot\vec{k}_2)(\vec{\sigma}_1\times\vec{\sigma}_3)\cdot\vec{k}_1(\vec{\tau}_1\times\vec{\tau}_2)\cdot\vec{\tau}_3\,. \qquad (A7)$$

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