Comment on "Energies and widths of low-lying levels in ^{11}Be and ^{11}N"

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Definitions of the energy and width of an unbound level, which have more justification than those used by Fortune *et al.* [Phys. Rev. C **51**, 3023 (1995)], are discussed and applied to low-lying levels of ^{11}Be and ^{11}N.

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Fortune *et al.* [1] (hereafter referred to as FKL) fitted the observed energies of the low-lying levels of ^{11}Be in a ^{10}Be $+n$ potential model by adjusting the potential depth, and used the same potential parameters to predict the energies of the analog levels in ^{11}N. Spectroscopic factors obtained from ^{11}Be data were used to calculate the widths of the ^{11}N levels.

The lowest $\frac{1}{2}^+$ and $\frac{1}{2}^-$ levels in ^{11}Be are bound, but the $\frac{5}{2}^+$ level and all the ^{11}N levels are unbound. While the energy of a bound level is defined uniquely, many different definitions have been used for the energy of an unbound level, and also for its width. FKL do not say how they define the energy of an unbound level, but give two definitions for the width Γ, which is evaluated either from the formula $4/\Gamma = |d(\sin 2\delta)/dE|$, where δ is the total nuclear phase shift, or as the energy interval over which δ changes from $\pi/4$ to $3\pi/4$. These definitions of Γ imply that the level energy is taken as the energy at which δ passes through $\pi/2$. There appears to be little justification for using these definitions of level energy and width in terms of δ, except in special cases [2]. For a narrow level, all definitions are expected to give essentially the same values of energy and width, but the values could differ greatly for a broad level. FKL predict single-particle widths of the ^{11}N levels of order 1–2 MeV, so that it is not obvious that different definitions would give the ^{11}N level energies the same within FKL's estimated uncertainty of 50–100 keV.

A model similar to that of FKL was used by Sherr and Bertsch [3] in discussing low-lying levels of ^{9}Be and ^{9}B, and of other mirror pairs of p-shell nuclei. They used four definitions or prescriptions for the energy of an unbound level. These definitions were discussed by Barker [4], who also introduced two other definitions based on the one-level approximation of the R-matrix theory of nuclear reactions [5], for cases where only one channel is open. These were the resonance energy E_r, at which the resonant nuclear phase shift β passes through $\pi/2$, and the peak energy E_m, where the density-of-states function ρ reaches a maximum. Here $\beta = \delta + \phi$, where $-\phi$ is the hard-sphere phase shift, and $\rho \propto \sin^2\beta/P$, where P is the penetration factor. Since ϕ and P are functions of the channel radius a, the values of E_r and E_m also depend on the choice of a.

We have made calculations with a potential model similar to that of FKL, using a central Woods-Saxon potential cut off at $r = a$, with conventional values of the radius and diffuseness parameters and depth adjusted to fit the energy of the ^{11}Be levels, and taking either E_r or E_m as the energy of an unbound level. The corresponding single-particle widths $\Gamma_{sp,r}$ and $\Gamma_{sp,m}$ are defined, respectively, as the energy interval over which β increases from $\pi/4$ to $3\pi/4$, and the FWHM of ρ. Resultant energy and width values for the unbound levels of ^{11}Be and ^{11}N are given in Table I for a range of values of a about the conventional minimum value $a = 1.45(10^{1/3}+1)$ fm $= 4.57$ fm. For comparison the results of FKL are included in Table I; for the ^{11}N levels, these are the values of E, Γ_{sp}, and $\Gamma_{\rm pred}$ from their Table III.

Our cutoff potential is closest to that of FKL for large a. For $a = 6.0$ fm, our values of $\Gamma_{sp,r}$ and $\Gamma_{sp,m}$ for the "narrow" $\frac{5}{2}^+$ level of ^{11}Be are close to their value of Γ_{sp}, since ϕ varies by only about 1° over the resonance. For the ^{11}N levels, our values of E_r and E_m, and also those of $\Gamma_{sp,r}$ and $\Gamma_{sp,m}$, are smaller than the corresponding values of FKL. This is because both the value and the energy variation of ϕ are significant in these cases. Actually, for the $\frac{1}{2}^+$ level of ^{11}N, we find that δ never reaches $\pi/2$, so that it is not clear how FKL obtained their energy and width values in this case.

A ^{11}N level observed [6] in the ^{14}N(^{3}He, ^{6}He)^{11}N reaction with a mass excess of 25.23 ± 0.10 MeV (corresponding to $E = 2.24 \pm 0.10$ MeV [7]) and width 0.74 ± 0.10 MeV has been identified as the $\frac{1}{2}^-$ level [6]. The energy from such a reaction is most appropriately taken as a value of E_m and, following the procedure of FKL and others, the corresponding width Γ_m may be taken as a value of $\mathcal{S}\Gamma_{sp,m}$, where $\mathcal{S}$ is the ^{10}C + p spectroscopic factor. Agreement between calculation and experiment is obtained for $a = 4$–5 fm, which encompasses the conventional value 4.57 fm, and $\mathcal{S} = 0.80 \pm 0.11$. An apparently similar calculation by Bertsch (see Ref. [6]) gave $\mathcal{S} = 0.7 \pm 0.1$. These values of $\mathcal{S}$ may be compared with shell model values of 0.60 [8] and 0.66 [9], and experimental values for the analog ^{11}Be level of 0.63 ± 0.15 [10] and 0.96 [11] obtained from stripping reactions.

We consider the values of E_m and Γ_m for the other levels, for $a = 4.57$ fm. For the $\frac{5}{2}^+$ level of ^{11}Be, interpolation of the values in Table I gives $\Gamma_{sp,m} = 0.155$ MeV. The measured width of this level is 0.10 ± 0.02 MeV [7], giving $\mathcal{S} = 0.65 \pm 0.13$. This is in reasonable agreement with the experimental stripping value of 0.50 [11], and in good agreement with the shell model value 0.67 [12]. The $\frac{1}{2}^+$ ^{11}N level is predicted at 1.40 MeV with a width of 1.01 ± 0.07 MeV

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TABLE I. Energies and widths of ^{11}Be and ^{11}N levels calculated from a potential model.

	J^π	a (fm)	E_r (MeV)	$\Gamma_{sp,r}$ (MeV)	E_m (MeV)	$\Gamma_{sp,m}$ (MeV)	Γ_m (MeV)
^{11}Be	$\frac{5}{2}^+$	4.0	1.275 [a]	0.139	1.275 [a]	0.136	
		5.0	1.275 [a]	0.169	1.275 [a]	0.165	
		6.0	1.275 [a]	0.178	1.275 [a]	0.175	
		4.57			1.275 [a]	0.155	0.10 [a]
		FKL			1.275 [a]	0.177	0.10 [a]
^{11}N	$\frac{1}{2}^+$	4.0	1.83	4.83	1.53	1.54	
		5.0	1.54	2.41	1.33	1.17	
		6.0	1.40	1.45	1.25	0.98	
		4.57			1.40	1.31	1.01 ± 0.07
		FKL			1.60 ± 0.22	$2.1^{+1.0}_{-0.7}$	$1.58^{+0.75}_{-0.52}$
	$\frac{1}{2}^-$	4.0	2.38	1.31	2.30	0.94	
		5.0	2.24	1.02	2.16	0.89	
		6.0	2.15	0.87	2.09	0.83	
		4.57			2.21	0.91	0.74 [a]
		FKL			2.48	1.45	0.91 ± 0.22 [b]
	$\frac{5}{2}^+$	4.0	4.00	0.73	3.97	0.68	
		5.0	3.86	0.77	3.82	0.74	
		6.0	3.78	0.75	3.74	0.77	
		4.57			3.88	0.72	0.47 ± 0.09
		FKL			3.90	0.88	0.50 ± 0.10

[a]Fitted value.
[b]0.73 ± 0.17 at the measured energy of 2.24 MeV.

(using $\mathscr{S}=0.77\pm0.05$, based on experimental values for ^{11}Be of 0.73 ± 0.06 [10] and 0.77 [11] and the shell model value 0.82 [12]), and the $\frac{5}{2}^+$ level at 3.88 MeV with width 0.47 ± 0.09 MeV (using $\mathscr{S}=0.65\pm0.13$ from above).

So far these calculations have used the single-particle potential model, which is justified only if $\mathscr{S}\approx1$. In the present case, the evidence suggests $\mathscr{S}$ values somewhat less than one for all levels. This can be taken into account in a model, based on the many-channel formulas of R-matrix theory [5], which has been used previously to discuss low-lying levels of $A=13$ nuclei [13] and $A=9$ nuclei [4]. We consider this approach here, using the one-level approximation for all J^π values.

For each J^π, the energies of analog states in ^{11}Be and ^{11}N are related by the Coulomb displacement energy ΔE_C, which is calculated as the sum of an internal contribution ΔH^c, involving matrix elements of the charge-dependent interaction, and the boundary-condition contribution ΔL, due to different external wave functions. The necessary formulas are simple modifications of those given in Refs. [4,13]. Due to the Okamoto-Nolen-Schiffer anomaly, this type of calculation may not give absolute values of ΔE_C accurately, but relative values for different J^π values should be more reliable, provided contributions to ΔH^c from the point-Coulomb interaction and the electromagnetic spin-orbit interaction are included [13]. We calculate the point-Coulomb contribution and the spectroscopic factors for ΔL using the shell model code OXBASH [14], with the MWK interaction in the psd space, and with the oscillator length parameter $b=1.653$ fm obtained from the standard formula [14] $\hbar^2/M_p b^2=(45A^{-1/3}-25A^{-2/3})$ MeV. For the $A=10$, $T=1$ ground-state channel, the $\mathscr{S}$ values are 0.74, 0.76, and 0.70 for the $\frac{1}{2}^+$, $\frac{1}{2}^-$, and $\frac{5}{2}^+$ states, respectively. The electromagnetic spin-orbit contribution is calculated for the same value of b, using simplified L - S coupled wave functions; for the normal-parity $\frac{1}{2}^-$ states we use $\Psi(1s^4 1p^7[421]\frac{3}{2}\frac{1}{2}1,\frac{1}{2})$, which comprises 90% of the eigenfunction of the lowest $T=\frac{3}{2}$ state for the POT interaction of Cohen and Kurath [15], and for the non-normal–parity $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states the wave functions $\Psi((1s^4 1p^6[42]100,nlj)\frac{3}{2}j)$, with $nlj=20\frac{1}{2}$ or $12\frac{5}{2}$, where the core state comprises 85% of the lowest $A=10$, $0^+,T=1$ state. Values of ΔE_C, calculated for a range of values of a, are given in Table II; from these, values of E_r are obtained for the ^{11}N states. For each a, we adjust the value of E_r for the $\frac{1}{2}^-$ state to make $E_m=2.24$ MeV, as observed, and use the relative values of E_r for different J^π to calculate E_m for $J^\pi=\frac{1}{2}^+$ and $\frac{5}{2}^+$. The values of E_m and Γ_m are given in the last two columns of Table II, and again the FKL values are included for comparison.

For the conventional value of the channel radius $a=4.57$ fm, interpolated values of E_m and Γ_m are given in Table II. The corresponding value of Γ_m for the $\frac{5}{2}^+$ level of ^{11}Be is 0.11 MeV, in good agreement with the experimental value 0.10 ± 0.02 MeV. For ^{11}N($\frac{1}{2}^-$), the predicted value of

TABLE II. Calculated Coulomb displacement energies and predicted ^{11}N energies and widths.

J^π	E_r (^{11}Be) (MeV)	ΔH^c (MeV) Coul.	s.o.	a (fm)	ΔL (MeV)	ΔE_C (MeV)	E_r (^{11}N) (MeV)	E_m (^{11}N) (MeV)	Γ_m (^{11}N) (MeV)
$\frac{1}{2}^+$	-0.503	7.350	0.000	4.0	-0.762	6.588	1.426	1.60	1.40
				5.0	-0.682	6.730	1.568	1.59	1.36
				6.0	-0.555	6.795	1.633	1.55	1.19
				4.57				1.60	1.39
				FKL				1.60 ± 0.22	$1.58^{+0.75}_{-0.52}$
$\frac{1}{2}^-$	-0.183	7.464	-0.065	4.0	-0.691	6.708	1.866	2.24[a]	0.56
				5.0	-0.494	6.905	2.063	2.24[a]	0.68
				6.0	-0.336	7.063	2.221	2.24[a]	0.72
				4.57				2.24[a]	0.64
				FKL				2.48	0.91 ± 0.22[b]
$\frac{5}{2}^+$	1.275	7.280	-0.051	4.0	-0.326	6.903	3.519	3.91	0.42
				5.0	-0.246	6.983	3.599	3.80	0.48
				6.0	-0.158	7.071	3.687	3.73	0.50
				4.57				3.84	0.46
				FKL				3.90	0.50 ± 0.10

[a]Fitted value.

[b]0.73 ± 0.17 at the measured energy of 2.24 MeV.

Γ_m is 0.64 MeV compared with the experimental value 0.74 ± 0.10 MeV. For ^{11}N($\frac{5}{2}^+$), the values of E_m and Γ_m in Table II agree well with those in Table I and also with those of FKL. For ^{11}N($\frac{1}{2}^+$), the values of E_m and Γ_m in Table II are higher than those in Table I, by about 200 and 400 keV, while the agreement with the FKL values is surprising, in view of the uncertain way in which they were obtained.

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