## **Compressibility probed by linear momentum transfer**

F. Haddad, <sup>1,2</sup> J. B. Natowitz, <sup>1</sup> B. Jouault, <sup>2</sup> V. de la Mota, <sup>2</sup> G. Royer, <sup>2</sup> and F. Sébille <sup>2</sup>

<sup>1</sup>*Cyclotron Institute, Texas A & M University, College Station, Texas 77843*

<sup>2</sup>*Subatech, 4 Av A. Kastler, 44070 Nantes cedex 03, France*

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The linear momentum transfer occurring in central heavy ion collisions is studied within the Landau Vlasov model. A sensitivity to the equation of state is found for this observable. Assuming the free nucleon-nucleon cross section, a soft compressibility modulus (around 210 MeV) is required to reproduce the large amount of data available for linear momentum transfer. This result stands for local and nonlocal effective potentials.

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Heavy ion collisions offer the unique opportunity to explore nuclear matter under wide ranges of compression  $[1]$ and/or excitation energy  $[2]$ . It is then possible to drive the system far from its ground state and to study the thermodynamics of nuclear matter. Intensive work is being done both experimentally and theoretically to extract the nuclear equation of state (EOS). Particular care is devoted to determine the compressibility modulus,  $K_{\infty}$ , which governs the ability of the nuclear matter to be compressed. This special interest is further motivated by the importance of this quantity in other fields of physics such as astrophysics  $[3]$ .

The complexity of heavy ion collisions requires detailed theoretical analysis. Strong dynamical effects are present and a dual sensitivity to both, the nuclear equation of state and the in-medium nucleon-nucleon cross section,  $\sigma_{nn}^{\text{med}}$ , exists for many observables  $[4,6]$ . In addition, between 20 and several hundred AMeV they are of comparable importance which makes the extraction of the EOS difficult in this energy range.

In order to be sensitive to the EOS, an observable has to be developed at the beginning of the reaction when compression is present. It also has to be resistant to the evaporative stage of the reaction. The flow of nuclear matter  $[7]$  and the oscillation of the giant monopole resonance  $|8|$  are good examples. The linear momentum transfer in central collisions has also been suggested as a relevant observable and has been shown to be sensitive to both the equation of state and the nucleon-nucleon cross section  $[9]$ . This observable is established just after the preequilibrium stage of the reaction and contains information about the early time of the collision. It is commonly used to characterize the transfer of the incident energy to the compound nucleus  $(CN)$  formed in the reaction. In  $[9]$ , a study of this observable using quantum molecular dynamic (QMD) and local interactions has shown evidences for a soft EOS.

In this paper, the linear momentum transfer in central collisions is investigated within the Landau Vlasov model using different types of one-body effective potentials.

Our calculations were performed by solving the Landau Vlasov  $(LV)$  equation  $[10]$  which describes the time evolution of the one-body phase space distribution function,  $f(\vec{r}, \vec{p}, t)$ :

$$
\frac{\partial f(\vec{r},\vec{p},t)}{\partial t} + \{f(\vec{r},\vec{p},t),H\} = I_{\text{coll}}[f(\vec{r},\vec{p},t)],\tag{1}
$$

where  $\{ , \}$  stands for the Poisson bracket, *H* is the one-body mean field Hamiltonian, and *I*<sub>coll</sub> is the Uehling-Uhlenbeck</sub> collision term.

Two main ingredients are needed in this transport model: an effective potential and the in-medium nucleon-nucleon cross section. As already stated in the introduction, these two features act together and their relative contributions to the evolution of the colliding system are important to know.

In spite of intensive theoretical work, in-medium effects on the nucleon-nucleon cross section remain an open problem  $[11]$ . We choose to work with the energy dependent free nucleon-nucleon cross section in all our calculations. This choice is based on early work on the flow observable  $|4-6|$ . Indeed, Bao-an Li found  $[4]$  that measurements of the balance energy, where the flow vanishes, are accurately reproduced using a local Zamick interaction and the free nucleonnucleon cross section. Also, a study of flow in  $Nb+Nb$ reactions from 50 to several hundred AMeV showed a good agreement of the calculation with the data when a momentum dependent interaction was used together with the free nucleon-nucleon cross section  $[6]$ .

In order to study EOS effects, we used different types of potentials. Local Zamick interactions with an isospin dependence and  $K_\infty$ =200 MeV (soft) and  $K_\infty$ =380 MeV (stiff). Momentum dependent interactions based on the Gogny interaction [12] with soft (*D*1*G*1,  $K_{\infty}$ =228 MeV) and stiff ( $D1G3$ ,  $K_{\infty}$  = 360 MeV) compressibilities. Functional forms and coefficients of these potentials can be found in  $[6]$ .

In order to determine the linear momentum of the CN in central collisions, the first step is to look at the emission pattern of the reaction. Such a typical pattern is displayed in the upper panel of Fig. 1 for the  $O+U$  reaction at 50 AMeV and  $b=2$  fm. It corresponds to a LV calculation using a local soft interaction. It should be noted that similar results are obtained for any of the other effective forces used in this work.

The interpenetration stage begins around  $t_0$ =30 fm/*c* and from there the thermalization occurs. This is expressed in the lower panel of Fig. 1 which presents as a function of time the normalized quadrupole moment in *p* space of the full distribution. The normalized quadrupole moment reaches zero around  $t_1 = 140$  fm/*c*. This means a spherical distribution in *p* space, i.e., global thermalization is achieved. Between  $t_0$  and  $t_1$  preequilibrium emission occurs. In our cal-



FIG. 1. Emission rate (upper panel) and normalized quadrupole moment (lower panel) as a function of time for  ${}^{16}O+{}^{238}$  U at 50 A MeV.

culations, free particles are identified as those nucleons experiencing a local density lower than  $\rho_0/10$ . In this work, the CN is assumed to be formed once the preequilibrium stage is finished. All CN characteristics are then determined at  $t_1$ .

We choose to focus on central collision (impact parameter between 0 and 4 fm) of the  $16O+238U$  system at several energies (ranging from 20 to 70 AMeV). In Fig. 2, the linear momentum transfer calculated for the Zamick interaction is displayed as a function of the quantity  $\sqrt{(E-V_c)/A_p}$  where  $V_c$  represents the Coulomb barrier between the target and the projectile,  $E$  is the incident energy, and  $A_p$  is the projectile mass number. Symbols correspond to experimental data  $[14 - 25]$ . Theoretical results for central collisions are represented by thick bars. The length of these bars accounts for the different impact parameter values selected as central collisions. Results using the soft EOS are represented by black bars whereas those from the stiff EOS are represented by shaded bars.

The calculations with Zamick soft interaction follow nicely the experimental values over a wide range of incident energies. On the other hand, the calculations with the Zamick stiff interaction overestimate the momentum transfer. This finding is in accordance with that of  $[9]$  and can be compared with a previous work of Xu *et al.* [13] who found that a local stiff interaction always overestimates the CN formation cross section.



FIG. 2. Linear momentum transfer for local interactions. Calculations are represented by thick bars and data by symbols.

In order to get deeper insight into this phenomenon, the momentum transfer per nucleon of the projectile as a function of the incident energy is reported in Fig. 3. Solid symbols (points and triangles) represent our LV calculations. Data for  $14N+238U$  are from [14] and are displayed as open circles. The experimental points show a saturation of the transferred momentum per nucleon around 200 MeV/*c* for incident energy above 30 AMeV. To clearly see the decrease of momentum transfer with incident energy, a line assuming full momentum transfer is also displayed in Fig. 3. All these features are well reproduced by the Zamick soft interaction. For the Zamick stiff interaction, a saturation is also present but the momentum transfer is overestimated by  $25\%$  (see triangles).



FIG. 3. Linear momentum transfer per incident nucleon as function of the energy for local interactions. Calculations correspond to solid symbols (points and triangles) and data to open circles. The curve characterizes full momentum transfer.

symbols.

These results suggest that, assuming  $\sigma_{nn}^{\text{med}} = \sigma_{nn}^{\text{free}}$ , a soft interaction is needed in order to reproduce basic dynamical nuclear properties such as linear momentum transfer.

Before any definite statement on the value of the compressibility modulus is made, the effect of the momentum dependence of the mean field potential has to be studied. This dependence is known to have important effects on the flow observable in this energy range. Indeed the inclusion of such dependence drastically changes the conclusions on the compressibility modulus value. Use of local interactions imply a stiff EOS in  $[26]$  whereas inclusion of momentum interaction favor softer ones  $(7,6)$ .

Figure 4 summarizes calculations obtained for Gognytype interactions. Results using the *D*1*G*1 force ( $K_{\infty}$ =228) MeV) are displayed by thick black bars and those from *D*1*G*3 ( $K_{\infty}$  = 360 MeV) by thick shaded bars. For a momentum dependent interaction, the difference between the results using the two different compressibility values is smaller. *D*1*G*3 overshoots the data quite clearly. *D*1*G*1 on other hand is in agreement with the data although it slightly overestimates them at high energy. Nevertheless, the soft EOS is again favored for the momentum dependent interaction.

In conclusion, calculations within the Landau Vlasov framework have been performed to study the linear momentum transfer in a heavy ion reaction. In accordance with  $\boxed{9}$  a sensitivity of this observable to the EOS has been observed. A detailed study confirms the need of a soft EOS in order to reproduce the large amount of data available for linear momentum transfer. This conclusion is valid for local (confirming results found in  $[9]$  as well as for momentum dependent interactions. This result is in agreement with several previous calculations  $[7,8,6]$  which find a compressibility modulus

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value around 210 MeV. In-medium effects which are believed not to change by more than 20% the free cross section

FIG. 4. Linear momentum transfer for momentum dependent interactions. Calculations are represented by thick bars and data by

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[11] should only slightly modify our results.