Two-pion exchange contributions to nuclear charge asymmetry

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In a nonrelativistic formalism we derive a charge-symmetry-breaking (CSB) two-pion exchange interaction $V_{nn}(2\pi)-V_{pp}(2\pi)$, which includes both CSB vertex corrections and the CSB effect of propagators arising from the mass differences of the intermediate baryons in different charge states. While the former are small, the latter gives a contribution of the sign and scale comparable to the experimental difference in the effective range parameters and the binding energy difference between ^{3}H and ^{3}He.

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I. INTRODUCTION

Traditional measures of nuclear charge asymmetry have been obtained from the positive value for the difference $\Delta a=|a_{nn}|-|a_{pp}|\approx\mathcal{O}$ (1 fm) of the NN singlet scattering lengths and the positive value for the ^{3}H-^{3}He binding energy difference $\Delta E\approx\mathcal{O}$ (100 keV). Both measures Δa and ΔE are quoted after correction of experiment for direct electromagnetic effects and are quite consistent in sign and magnitude. A positive Δa reflects an interaction between two neutrons which is more attractive than between two protons and more binding energy is provided for ^{3}H as compared to ^{3}He. The consistency in magnitude is more interesting. It has long been known from separable potential models that the $A=3$ binding energies are much more sensitive to small differences in effective ranges ($\Delta r_0=r_{nn}-r_{pp}$) than to small differences in scattering lengths Δa [1]. Recently it has been shown by sophisticated Faddeev and quantum Monte Carlo calculations with modern NN potentials that ΔE in the $A=3$ system can be explained by a charge asymmetric NN force which has been adjusted to match Δa and Δr_0 in the $A=2$ system [2–4]. These demonstrations of full consistency between $\{\Delta a,\ \Delta r_0\}$ and ΔE coupled with the empirical values themselves now allow us to evaluate the various expected theoretical contributions to charge symmetry breaking (CSB). In particular, it has been claimed that $\rho\omega$ mixing alone accounts for most of these two measures of charge asymmetry [5], leaving little room for other mechanisms such as simultaneous $\pi\gamma$ exchange or baryon mass differences in 2π exchange. This claim has been recently called into question by a variety of model calculations [6], and it is again timely to reexamine these other mechanisms.

Early calculations of 2π exchange contributions to nuclear charge asymmetry in these two- and three-nucleon systems drew quite different conclusions. Riska and Chu [7] constructed a dispersion theoretic CSB 2π potential from (crossed) empirical πN amplitudes which included a particular nucleon pole ansatz motivated by the Adler soft-pion consistency condition. With this ansatz their potential yields $\Delta a=-2.7$ fm and $\Delta E=-180$ keV [8]. If the additional pole term from their ansatz is neglected, the central part of their potential changes sign and increases so much that $\Delta a\approx+6$ fm [7]. On the other hand, Noble attempted to reconstruct their potential with another technique and suggested a positive ΔE (seemingly in the more massive $A=41$ system) of several hundred keV. No details of his calculation were given [9]. Both these estimates are rather far from experiment, even if one acknowledges that the correction of experiment for direct electromagnetic effects is not fully under control [10]. A much smaller estimate of $\Delta a=+0.30$ fm was obtained from the 2π exchange box plus crossed box field theoretical potentials of Partovi and Lomon [11] by taking into account the nucleon mass difference in intermediate states [12]. This calculation was repeated [13] a few years later to find Δa still small but negative, $\Delta a=-0.21$ fm. Although both calculations indicate a small (on the empirical scale of 1 fm) charge asymmetry from 2π exchange, recent reviews of charge asymmetry [14,15] have noted this discrepancy and have called for further calculation.

The purpose of this paper is to present an explicit 2π exchange (TPE) potential which arises from a rather different theoretical approach than the Partovi-Lomon reduction of covariant Feynman graphs to a nonrelativistic potential. In the present case, the TPE potential is instead based on nonrelativistic πNN and $\pi\Delta N$ vertices and the baryon mass differences are taken into account in the vertices and in the intermediate state energy denominators. The derivation of the potential has been thoroughly described in a paper by Niskanen [16] which was devoted to class IV charge symmetry breaking in elastic np scattering. A class IV CSB force (in the terminology of Henley and Miller [17]) has an effect only in the np system which is an eigenstate of the charge sym-

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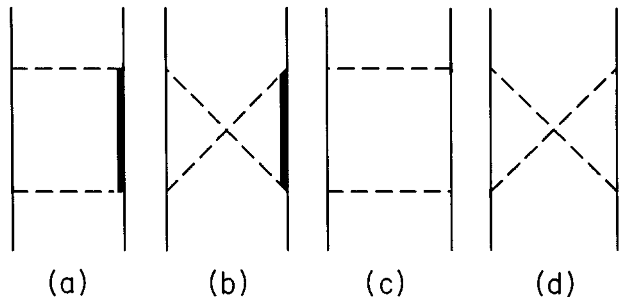

FIG. 1. Two-pion exchange diagrams of the two-pion exchange NN potential. The solid lines are nucleons, heavy solid line are Δ's, and the dotted lines are neutral and charged pions.

metry operator $P_{\rm CS}$ (a rotation by π about the two-axis of isospin space [18]). In contrast, the charge asymmetry effects of TPE we will calculate and display are in mirror systems which are *not* eigenstates of $P_{\rm CS}$. The differences $\{\Delta a, \Delta r_0\}$ and ΔE in the nn vs pp interaction arise from a charge asymmetric force termed class III which is proportional to the zeroth component of the total isospin operator $\tau_{10}+\tau_{20}$ (where $\tau_0|p\rangle=+|p\rangle$).

Baryon mass differences in the vertices of two-pion exchange give rise to both class IV and class III CSB forces, but baryon mass differences in intermediate states cause only a class III CSB force. We obtain a range of $\Delta a\sim+(0.08$–$0.15)$ fm from nucleon mass differences in the vertices. The actual values of the present calculation depend upon both the form factor of the meson-baryon-baryon vertices and the chosen charge symmetric potential used in the evaluation of Δa. Very importantly, however, the contribution from NN intermediate states is canceled by $\Delta a\sim-(0.17$–$0.27)$ fm from 2π exchange diagrams which contain one nucleon and one Δ. Thus the final result is small and in the wrong direction compared to the empirical values of $\{\Delta a, \Delta r_0\}$ and ΔE. In addition, we find many interesting cancellations among the totality of nonrelativistic box and crossed box potentials with neutral and charged pions, as did Cheung and Machleidt earlier in a study of pion mass difference contributions to charge *dependence* [19]. Each individual contribution, however, is small on the scale of the empirical Δa ($\sim$1 fm), and so the cancellations are not as delicate as in the case of charge dependence. On the other hand, the contribution to Δa from baryon mass differences in the intermediate state energy denominators is about 1 fm, comparable in size and magnitude to the empirical number. In this case the individual contributions do not cancel and the contributions from terms with one nucleon and one Δ are small compared to those of the NN crossed box terms. The latter agree in sign but are larger than found in Ref. [12] which calculated a numerical derivative (in the neutron-proton mass difference) in the multidimensional integrals of the SU(2) symmetric Partovi-Lomon potential.

We derive the potentials in Sec. II, present our results in Sec. III, and conclude with a short discussion in Sec. IV.

II. TWO-PION EXCHANGE CHARGE ASYMMETRIC POTENTIAL

The calculation of the two-pion exchange (TPE) potential is performed following Ref. [16], which was devoted to a study of this contribution in the case of isospin changing class IV interactions. Comparison with a coupled channels calculation involving Δ isobar excitations showed that especially in the low energy limit it is reasonable to use the TPE potential derived at zero energy in the static model for the baryons and parametrized in a simple form in Table IV of Ref. [16].

The starting point of the model is the pion-nucleon coupling with the mass difference of the neutron and proton, $m_n-m_p=\delta(m_n+m_p)$, taken into account. In the nonrelativistic limit this vertex is of the form [17]

$$
\begin{aligned}
H_{\pi NN}&=H_0+H_1+H_2\\
&=-i\frac{f}{\mu}[(\vec{p'}-\vec{p})\cdot\vec{\sigma}\vec{\tau}\cdot\vec{\phi}+(\vec{p'}-\vec{p})\cdot\sigma\phi_0\delta\\
&\quad+i(\vec{p'}+\vec{p})\cdot\vec{\sigma}(\vec{\tau}\times\vec{\phi})_0\delta],
\end{aligned}
\tag{1}
$$

where $\vec{p}$ and $\vec{p'}$ are the initial and final nucleon momenta, and $\vec{\sigma}$ and $\vec{\tau}$ are the spin and isospin operators. Clearly, in H_1 only the neutral component of the pion field $\vec{\phi}$ is involved, whereas in H_2 only the charged pions participate. We use the pion-nucleon coupling $f^2/4\pi=0.075$ from recent analyses [20]. The $\pi N\Delta$ coupling is similar [21] except that the spin (isospin) operators become the corresponding $N\to\Delta$ transition operators $\vec{S}$ ($\vec{T}$). Because of the missing isospin operator, the H_1 term drops out, leaving

$$
\begin{aligned}
H_{\pi N\Delta}&=H_0'+H_2'\\
&=-i\frac{f^*}{\mu}[(\vec{p'}-\vec{p})\cdot\vec{S}\vec{T}\cdot\vec{\phi}+i(\vec{p'}+\vec{p})\cdot\vec{S}(\vec{T}\times\vec{\phi})_0\delta].
\end{aligned}
\tag{2}
$$

The $\pi N\Delta$ coupling constant is taken as $f^{*2}/4\pi=0.35$ from the free Δ width. The value of the neutron-proton mass difference will be used also for the Δ mass splittings between successive charge states. This can be justified by the nonrelativistic constituent quark model [21] and is well consistent with the available experimental constraints [22]. The pion-baryon vertices have each also a monopole form factor

$$
F(q)=\frac{\Lambda^2-\mu^2}{\Lambda^2+q^2}. \tag{3}
$$

The values 1000 MeV and 790 MeV are used for the parameter Λ.

We calculate the direct box and crossed box diagrams with two nucleons and one nucleon and one nucleon and one Δ in the intermediate state pictured in Fig. 1. Using the above vertices to first order in δ we get, for the $N\Delta$ intermediate state contributions in momentum space,

$$V_{2\pi}^{\rm III}(N\Delta)=\left(\frac{ff^*}{\mu^2}\right)^2\delta(\tau_{10}+\tau_{20})\left\{\left[\frac{8}{9}\left(k^2-\frac{q^2}{4}\right)^2+\frac{4}{81}q^2k^2(2\vec{\sigma}_1\cdot\vec{\sigma}_2-S_{12})+2\left(\frac{k^2}{3}-\frac{q^2}{4}\right)i\vec{q}\times\vec{P}\cdot(\vec{\sigma}_1+\vec{\sigma}_2)\right]D_B\right.$$
$$\left.+\left[\frac{8}{9}\left(k^2-\frac{q^2}{4}\right)^2-\frac{16}{9}\left(k^4-\frac{q^4}{16}\right)-\frac{4}{81}q^2k^2(2\vec{\sigma}_1\cdot\vec{\sigma}_2-S_{12})+\frac{2}{9}\left(\frac{k^2}{3}-\frac{q^2}{4}\right)i\vec{q}\times\vec{P}\cdot(\vec{\sigma}_1+\vec{\sigma}_2)\right]D_C\right\}. \quad (4)$$

The overall momentum transfer is $\vec{q}$, and $\vec{P}$ is the average relative momentum of the nucleons, $\vec{P}=\frac{1}{2}(\vec{p}_1+\vec{p}_1')=-\frac{1}{2}(\vec{p}_2+\vec{p}_2')$ in the center of mass system. The momentum $\vec{k}$ is an intermediate momentum variable and is integrated over. D_B is the sum of all direct box diagram propagators in different time orderings and D_C the same for crossed diagrams. Only the spin-orbit terms and the second spin-independent term in the crossed box diagram arise from the vertex H_2; the others originate from H_1 and have the same spin-space structure as the charge-independent force.

The corresponding result for nucleonic intermediate states is

$$V_{2\pi}^{\rm III}(NN)=\left(\frac{f}{\mu}\right)^4\delta(\tau_{10}+\tau_{20})\left\{\left[2\left(k^2-\frac{q^2}{4}\right)^2-\frac{2}{9}q^2k^2(2\vec{\sigma}_1\cdot\vec{\sigma}_2-S_{12})\right]D_B+\left[2\left(k^2-\frac{q^2}{4}\right)^2-8\left(k^4-\frac{q^4}{16}\right)\right.\right.$$
$$\left.\left.+\frac{2}{9}q^2k^2(2\vec{\sigma}_1\cdot\vec{\sigma}_2-S_{12})+4\left(\frac{k^2}{3}-\frac{q^2}{4}\right)i\vec{q}\times\vec{P}\cdot(\vec{\sigma}_1+\vec{\sigma}_2)\right]D_C\right\}. \quad (5)$$

In this case D_B has only the propagators of those ''stretched'' diagrams which have at least one pion in flight all the time. Other TPE box diagrams are two-body reducible and are taken into account in iterations of the one-pion exchange potential (OPEP). Otherwise the results are similar. Even in the $N\Delta$ case a large part of the interaction could be treated by coupled channels and would be reducible in a more general two-baryon state space.

After integration over $\vec{k}$ the potential will be only a function of $\vec{q}$ (and $\vec{P}$ in the spin-orbit term), if the baryon energies are omitted in the propagators (static model). As shown in Ref. [16] these local potentials calculated at zero energy can be very well fitted with simple Yukawa-type potentials with form factor modifications, and we shall use this parametrization for the CSB two-pion exchange. The coupled channels method for including the isobars generates also the energy dependence of the intermediate states. In particular, the $N\Delta$ threshold effects arise in a natural way. As interesting as a study of the energy dependence of class III forces would be in the Δ threshold region, in a calculation of low energy scattering parameters, the use of a more exact coupled channels treatment is unnecessary. In Ref. [16] it was shown that at low energies the two approaches are reasonably close to each other, so that the potential parametrization can be justifiably used at least for elastic two-nucleon scattering, which has no external probe on the intermediate states.

The above contributions arise from the CSB spin-isospin structure of the pion-baryon vertices. In the difference of the nn and pp interactions there arises also an effect of different intermediate state energy denominators. Shortly, this means that, relatively speaking, in pp scattering an intermediate nn state (with mesons) has a higher excitation energy than the pp intermediate state in nn scattering. This difference arises only in the exchange of charged mesons $\pi^{\pm}$. For the nucleonic intermediate state this contribution is possible only in the crossed diagram of Fig. 1(d), since the direct box is basically an iteration of neutral pion exchange. In the $N\Delta$ case it is also easy to see that in the box diagrams the excitation energies are the same in both nn and pp scattering, if the quark model is used as a guide to relate the mass splitting of the Δ quadruplet and the nucleon doublet. Therefore, it is sufficient to study only the crossed meson contributions of Figs. 1(b) and 1(d).

From Ref. [16] we get the CSB difference between the nn and pp TPE potentials arising from the propagators to be

$$V_{N\Delta}(nn-pp)=\left(\frac{ff^*}{\mu^2}\right)^2\frac{8}{9}\left[(k^2-q^2/4)^2-\frac{1}{18}q^2k^2(2\vec{\sigma}_1\cdot\vec{\sigma}_2-S_{12})\right](D_{nn}^{N\Delta}-D_{pp}^{N\Delta}) \quad (6)$$

and

$$V_{NN}(nn-pp)=\left(\frac{f}{\mu}\right)^4 4\left[(k^2-q^2/4)^2+\frac{1}{9}q^2k^2(2\vec{\sigma}_1\cdot\vec{\sigma}_2-S_{12})\right](D_{nn}^{NN}-D_{pp}^{NN}). \quad (7)$$

TABLE I. The two-pion exchange (TPE) contributions to Δa, Δr_0, and ΔE, the binding energy difference between ^{3}He and ^{3}H; baryon mass differences in the vertices. The charge asymmetric TPE potentials are distinguished by the monopole form factor parameter Λ. The Reid soft-core (Reid) and de Tourreil–Rouben–Sprung (dTRS) potentials are the charge symmetric potentials V(CS) used in the calculation of Δa, Δr_0, and the estimate of $\Delta E_{\rm GS}$ based on these effective range parameters. Another estimate $\Delta E_{\rm FF}$ is obtained from the ''model-independent'' hyperspherical formula described in the text.

V(CS), Λ	Δa (fm)	Δr_0 (fm)	$\Delta E_{\rm GS}$ (keV)	$\Delta E_{\rm FF}$ (keV)
Reid, 1000 MeV	–0.27	+0.0052	–19	–9
dTRS, 1000 MeV	–0.23	+0.0045	–16	–9
Reid, 790 MeV	–0.10	+0.0017	–12	–1
dTRS, 790 MeV	–0.08	+0.0012	–5	–1

These equations are basically the same as Eqs. (11) and (13) in Ref. [16], except that the neutral pion contribution has been subtracted from the isospin factor and a factor of 2 has been included to account for the possibility of either of the nucleons being excited to a Δ. The quantity $D_{nn}^{BB'}$ is the sum of the nonrelativistic propagators of the crossed diagrams involving intermediate baryons BB' and two charged pions with different time orderings in nn scattering (respectively, in pp scattering).

The CSB contribution due to the baryon mass splittings in the propagators appears surprisingly large as compared with that coming from the CSB vertex effects. However, this can be directly understood since the vertex corrections must be of the order of $\delta=(M_n-M_p)/(M_n+M_p)$, whereas in the propagators the neutron-proton mass difference (actually taken twice) is to be compared with the exchanged meson energies and the Δ excitation energy, which are significantly smaller than the nucleon mass. The nucleonic intermediate state is further favored over the $N\Delta$, because it does not have the additional Δ-nucleon mass difference, which is of the order of two-pion masses. In addition, it can be seen in Eqs. (6) and (7) that the only spin-dependent term surviving in the $^1S_0$ state interferes destructively with the positive spin-independent part in the $N\Delta$ case but constructively in the NN case (note that the q^2 gives a negative contribution in coordinate space).

III. RESULTS

Before presenting our results we briefly discuss the experimental evidence for the charge asymmetry scales cited in the Introduction. The proton-proton $^1S_0$ low energy scattering parameters are very accurately known, but the direct electromagnetic (em) interaction must be subtracted and this subtraction is model dependent [10]. The corrected values we will use are [18]

$$a_{pp}=-17\pm 0.2 \text{ fm}, \quad r_{pp}=2.83\pm 0.02 \text{ fm};$$

similar suggested values will be found in the recent review literature [14,15,23] (see also [4]). The experimental values of a_{nn} found from $\pi^- d\to\gamma nn$ in which only the photon was detected [24] are

$$a_{nn}=-18.5\pm 0.4 \text{ fm}, \quad r_{pp}=2.80\pm 0.11 \text{ fm},$$

in excellent agreement with the kinematically complete determination of $a_{nn}=-18.7\pm 0.6$ fm from the same reaction [25]. The measured ^{3}H-^{3}He binding energy difference is 764 keV. The direct electromagnetic contributions to this number (the static Coulomb force between the proton pair in ^{3}He plus smaller em effects) have been estimated in a ''model-independent'' manner discussed later [26] by direct Faddeev calculation with many potential models [3] and by a combination of the two [2]. These estimates are in basic agreement; the corrections are 671 ± 29 keV, 671 ± 6 keV, and 683 keV, respectively [27]. Finally we must correct these results further for the difference in the kinetic energy of the nn and pp systems due to the neutron-proton mass difference. The latter correction is estimated to be -0.2 fm for a_{pp} and $+0.2$ fm for a_{nn} [18] and calculated to be $+(11–12)$ keV in the $A=3$ bound state [3,26]. All charge asymmetric effects other than these are then ascribed to charge asymmetry of the NN system. The characteristic measures of this aspect of charge asymmetry then become

$$\Delta a_{\rm expt}\equiv(|a_{nn}|-|a_{pp}|)\approx+1.1\pm 0.6 \text{ fm},$$

$$\Delta r_{0\ \rm expt}\equiv(r_{nn}-r_{pp})\approx-0.02\pm 0.11 \text{ fm},$$

$$\Delta E_{\rm expt}\equiv(^3\text{H}-{}^3\text{He})\approx 76\pm 24 \text{ keV},$$

against which the effects of theoretical models can be compared [27].

Now we turn to a presentation of these effects from the two-pion exchange potentials of Sec. II. Table I summarizes the total effect of baryon mass differences in the vertices and Table II demonstrates the dominance of mass differences in the NN intermediate state denominators over the $N\Delta$ intermediate states. The shifts in a and r are obtained by adding a model for $\Delta V=V_{nn}-V_{pp}$ to a model for the charge symmetric reaction. We chose the Reid soft-core potential [28] which despite its name has a large repulsion at small r, and the de Tourreil–Rouben–Sprung (dTRS) potential [29] which has a ''super soft core'' and a meson-theoretic outer region. The values of Δa and Δr_0 were obtained with the variable phase method. Our codes give $a=-17.10$ (-17.37) fm and $r_0=+2.79$ $(+2.84)$ fm for the Reid (dTRS) potential alone. Since both potentials were fitted to the experimental pp scattering parameters, the variation in the ''pure nuclear'' quantities is a specific example of the model dependence of the Coulomb subtraction.

The columns on the right of Table I display estimates of ΔE made in two different ways. The first, labeled $\Delta E_{\rm GS}$, relies upon the known relationship between a, r_0, and the

TABLE II. Two-pion exchange (TPE) contributions to Δa, Δr_0, and ΔE; baryon mass differences in intermediate energy state denominators. The intermediate states are labeled as *NN* (nucleon-nucleon) and $N\Delta$ (nucleon-delta); other notations as in Table I.

	V(CS), Λ	Δa (fm)	Δr_0 (fm)	$\Delta E_{\rm GS}$ (keV)	$\Delta E_{\rm FF}$ (keV)
NN	Reid, 1000 MeV	+1.53	–0.027	104	55
NN	dTRS, 1000 MeV	+1.28	–0.023	88	55
NN	Reid, 790MeV	+1.01	–0.017	68	36
NN	dTRS, 790 MeV	+0.83	–0.014	56	36
$N\Delta$	Reid, 1000 MeV	+0.12	–0.002	8	10
$N\Delta$	dTRS, 1000 MeV	+0.24	–0.005	18	10
$N\Delta$	Reid, 790 MeV	+0.10	–0.002	7	7
$N\Delta$	dTRS, 790 MeV	+0.16	–0.003	11	7
Sum	Reid, 1000 MeV	+1.67	–0.029	113	65
Sum	dTRS, 1000 MeV	+1.56	–0.028	107	65
Sum	Reid, 790 MeV	+1.11	–0.019	75	43
Sum	dTRS, 790 MeV	+1.01	–0.018	69	43

triton binding energy. This relationship, in the context of charge dependence, was numerically explored with central separable potentials in the early days of exact Faddeev calculations [1,30]. Then it was learned (and perhaps forgotten since) that the triton energy is more sensitive to r_0 than to a. The underlying physical explanation was demonstrated by Thomas [31] and reviewed by Bethe and Bacher [32] some time ago. Gibson and Stephenson [33] applied this idea to a dedicated study of the dependence of ΔE upon Δa and Δr_0. For their central separable potentials producing the correct ^{3}He binding the results can be very well fitted by

$$\Delta E_{\rm GS} = (40\Delta a - 1600\Delta r_0)\,\text{keV/fm}. \tag{8}$$

Far more sophisticated potentials and their concomitant few-body calculations support this simple prescription, but indicate that it furnishes a modest overestimate of ΔE. For example, a modification of the static Bonn one-boson-exchange-potential (Q space) (OBEPQ) [34] to produce $\Delta a = +1.31$ fm and $\Delta r_0 = -0.02$ fm (and the experimental a_{np} to ensure proper charge dependence) yields a *calculated* ΔE = 59 keV [2]. This is a little less than the estimate from Eq. (8), $\Delta E_{\rm GS} \approx 84$ keV, and presumably reflects the replacement of the central separable potentials used to establish the relation by local potentials with the tensor force. In another modern calculation [3], a component of the theoretical charge asymmetric force from $\rho\omega$ mixing [5] was artificially altered so that Δa = + 1.5 fm. Reference [3] did not indicate the value of Δr_0 which resulted from their alteration but instead stated that "the value of Δa is crucial for CSB." In the absence of information on that Δr_0, we have made a similar alteration to the $\rho\omega$ force and found (see also Table I of [5]) that $\Delta a \approx +1.5$ fm implies $\Delta r_0 \approx -0.03$ fm for both the Reid and dTRS charge symmetric potentials. Thus the prediction of $\Delta E_{\rm GS} \approx 102$ keV is again larger than the value of 75 ± 7 keV found in Ref. [3]. Finally, the new Argonne v_{18} *NN* potential, with $\Delta a = +1.65$ fm and $\Delta r_0 = -0.031$ fm, has an expectation value of $\Delta E = 66$ keV [4], again smaller than the prediction of 116 keV from Eq. (8). We will use the estimate from (8) because it summarizes a more detailed study, but realize that the values $\Delta E_{\rm GS}$ are an overestimate.

The "model-independent" estimate labeled $\Delta E_{\rm FF}$ is a direct perturbation theory estimate based upon the "hyperspherical formula" derived by Friar [35] and Fabre de la Ripelle [36]. They observed that the Coulomb contributions to the ^{3}H-^{3}He binding energy difference can be calculated using the experimental charge form factors. The hyperspherical formula works very well (to about 1%) when compared to exact Faddeev calculations [37] but is a good deal more problematic for shorter ranged potentials. The caveats to be applied to this method of estimating ΔE from a given two-body ΔV are discussed in Refs. [5,26] and the details of the experimental charge form factor input are given in [5]. The estimate in Table I labeled $\Delta E_{\rm FF}$ reflects even larger cancellations between positive *NN* and negative $N\Delta$ two-pion exchange contributions than obtained in the preceding three columns.

The contributions to Δa, Δr_0, and ΔE listed in Table I are relatively small on the scale of the empirical quantities. This is partly due to the cancellations between the *NN* and $N\Delta$ contributions and partly due to the smallness of the parameter δ which sets the scale for the CSB vertices. This factor is not present in Eqs. (6) and (7) based on the *difference of the intermediate state propagators*. Table II shows both the *NN* and $N\Delta$ contributions from this source. As anticipated, the *NN* intermediate state contributes much more than does the $N\Delta$, which is in turn of the order of the total vertex correction. There is no cancellation and consequently the sum of the two terms is much larger than the totals of Table I.

IV. DISCUSSION

The two-pion exchange CSB potentials displayed here are weak enough that their effect on Δa can simply be added to a good approximation (see Table II). One finds by combining the results in Tables I and II that the total CSB effect from Eqs. (4)–(7) with a monopole form factor with $\Lambda = 1000$ MeV is

$$\Delta a_{2\pi}\equiv(|a_{nn}|-|a_{pp}|)\approx+1.37\pm0.04 \text{ fm},$$

$$\Delta r_{0\ 2\pi}\equiv(r_{nn}-r_{pp})\approx-0.024 \text{ fm},$$

$$\Delta E_{2\pi}\equiv(^3\text{ H}-{}^3\text{He})\approx 93\pm 2 \text{ keV},$$

where we have taken the average of the two results and the uncertainty is due to the short range nature of the selected charge symmetric potential (Reid or de Tourreil-Rouben-Sprung). The corresponding result for a choice of $\Lambda=790$ MeV, long advocated by one of us [38] and now incorporated into new Bonn-Juelich NN potentials [39], and found necessary to explain the decay width of the Δ [40], is

$$\Delta a_{2\pi}\equiv(|a_{nn}|-|a_{pp}|)\approx+0.97\pm0.04 \text{ fm},$$

$$\Delta r_{0\ 2\pi}\equiv(r_{nn}-r_{pp})\approx-0.017 \text{ fm},$$

$$\Delta E_{2\pi}\equiv(^3\text{ H}-{}^3\text{He})\approx 65\pm 1 \text{ keV}.$$

The direct estimate of $\Delta E_{2\pi}$ based upon the hyperspherical formula depends, of course, only on the experimental charge form factors and not upon Δa and Δr_0. These estimates are $\Delta E_{2\pi}=56$ keV for $\Lambda=1000$ MeV and $\Delta E_{2\pi}=42$ keV for $\Lambda=790$ MeV, perhaps more reasonable than the contributions estimated with the aid of Eq. (8).

These results are consistent in sign with those of the first calculation [41] of nucleon mass differences in the energy denominators of the two-pion exchange SU(2) symmetric Partovi-Lomon potential. The latter was obtained from a nonrelativistic reduction of covariant Feynman graphs, in contrast to the present CSB two-pion exchange potential based on nonrelativistic πNN and $\pi\Delta N$ vertices. However, the effects of the present potential on Δa are a factor of 3–5 larger than the earlier estimate. This cannot be considered a satisfactory theoretical situation. We note that neither the covariant calculation nor the nonrelativistic calculation of two-pion exchange NN potential have a clear chiral symmetric character. Recently, Weinberg and van Kolck have emphasized the utility of an explicit consideration of chiral symmetry in the analysis of isospin violating interactions [42]. Leading-order chiral two-pion exchange NN potentials were first calculated by Ordóñez, Ray, and van Kolck [43] and subsequently verified by Friar and Coon [44]. We expect that these potentials should provide an alternative foundation for studies like the present one and hope to report on progress in a future work.

If two-pion exchange has such a large effect, what is the role of the other mechanisms suggested to account for nuclear charge asymmetry? Fortunately, it has been recently shown that, to leading order, the simultaneous exchange of a pion and photon does not produce nuclear charge asymmetry (but does give a charge-dependent force) [45]. The addition to our results of effects of $\rho\omega$ mixing, according to the traditional treatment [5] or to the latest studies of charge asymmetry from vector meson exchange [46], would lead to an overfulfillment of the experimental quota. On the other hand, if $\rho\omega$ mixing has a minimal CSB effect as claimed [6], then two-pion exchange is left as the dominant mechanism of nuclear charge asymmetry. Adjudicating the role of vector meson exchange in nuclear charge asymmetry lies beyond this study. For the present, results with the NN potentials of Sec. II indicate that the two-pion exchange contributions to class III nuclear charge asymmetry are of the same scale as the empirical measures and indeed can account for the low energy data.

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