Reply to "Comment on 'Shape and superdeformed structure in Hg isotopes in relativistic mean field model' and 'Structure of neutron-deficient Pt, Hg, and Pb isotopes' "

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(Received 25 July 1995)

In reply to the Comment by Heyde *et al.*, we discuss the sensitivity of the results of relativistic mean field (RMF) calculations on the shape of nuclei to the choice of input parameters in the Lagrangian of the σ,ω,ρ model and to the pairing gap in the BCS theory.

PACS number(s): 21.10.−k, 21.60.−n, 27.70.+q, 27.80.+w

In our previous papers [1,2] we discussed the change of the shape of Pt, Hg, and Pb isotopes along each isotope chain based on relativistic mean field (RMF) calculations within the nonlinear σ,ω,ρ model. We have assumed the NL1 parametrization, and treated the pairing interaction in the BCS theory by approximating the gap parameters for neutrons and protons with those given by the χ^2 fitting of the even-odd mass difference of many nuclei in the wide range of the nuclear chart [3]. We have thus shown that the ground state of ^{180}Hg is predicted to be superdeformed, and that some neutron-deficient Pb isotopes are deformed in their ground states. Our calculations predicted that the ground state of some Hg isotopes are prolate. Heyde *et al.* [4] pointed out that these results contradict experimental data and questioned whether the theoretical predictions hold independently of the input parameters such as the choice of the force parameters in the relativistic Lagrangian and the choice of the pairing gap. The aim of this reply is to answer some of the criticisms raised by Heyde *et al.* [4]. In particular we carry out new calculations to test the sensitivity of our results to the choice of the interaction and the magnitude of the pairing gap.

We first clarify the situation about the position of the superdeformed state. Our calculations in Ref. [1] predicted that the ground state of ^{180}Hg is superdeformed. In order to see the dependence of this prediction on the input parameters, Fig. 1 compares the binding energies per particle of Hg isotopes for different shapes calculated by assuming the NL1 set. The open circles are the binding energy per particle for the oblate shape, i.e., the binding energy per particle for the state which has the largest binding energy among all the oblate shape configurations. Similarly, the solid circles are that for the prolate shape with a normal deformation, while the open triangles are that for the prolate shape with a large deformation corresponding to a superdeformed configuration. As in Ref. [1], these three lines were obtained by assuming the average pairing gaps for neutrons and protons given by the systematic analysis of the even-odd mass difference [3]. The solid triangles were calculated by reducing the gap parameter for neutrons by a factor of two from that in Ref. [3].

The figure shows that the relative position of the superdeformed state is very sensitive to the strength of the pairing gap parameter. This resembles the situation in the nonrelativisitic macroscopic-microscopic approach, where the position of the superdeformed state strongly depends on the gap parameter [5]. For example, the superdeformed state is predicted to be the ground state for ^{180}Hg if the gap parameter of Ref. [3] is used as it is, but becomes an excited state if it is reduced by 50%. We found that even a change of the pairing gap for neutrons of 10% significantly alters the relative positions among the oblate, the prolate with a normal deformation, and the superdeformed configurations.

We repeated similar calculations by replacing the NL1 set by the NL-SH set. In this case, it was found that the superdeformed state is always located at the highest energy position among the three configurations. These studies show that the theoretical results concerning the shape of the ground state and the relative position of the superdeformed state strongly depend on both the choice of the input parameters in the RMF Lagrangian and the strength of the pairing interaction.

The second issue is the shape in the ground state of Hg isotopes. As seen in Fig. 1, our calculations predict that the ground state of ^{184}Hg and ^{186}Hg is prolate. Heyde *et al.* point out that this contradicts a large body of experimental

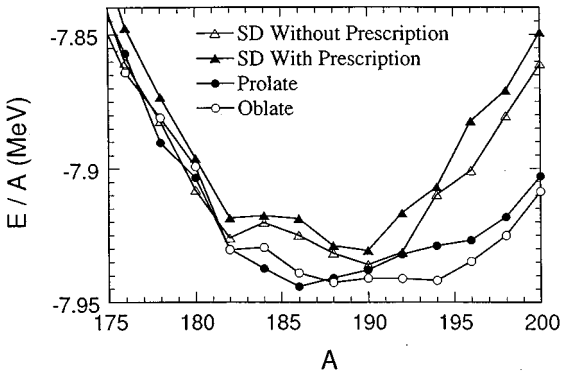

FIG. 1. Binding energies per particle for Hg isotopes as functions of mass number. The open circles, solid circles, and open triangles are those for the oblate, prolate with a normal deformation, and prolate with a superdeformation, respectively. The solid triangles are those for the prolate shape with a superdeformation when the pairing gap parameter for neutrons is reduced by 50%. The NL1 set was assumed.

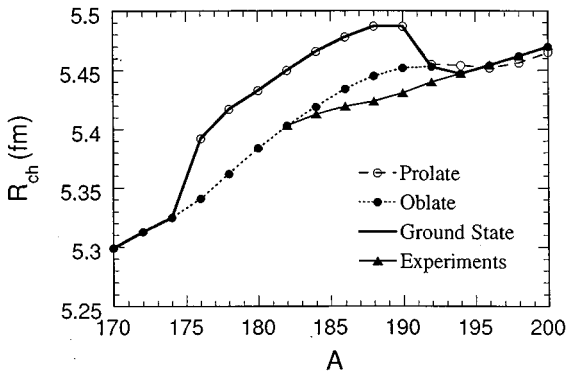

FIG. 2. The root mean square charge radii for the oblate and prolate configurations of Hg isotopes as functions of mass number. The value for the ground-state configuration for each isotope is connected by a thick solid line. The NL-SH set has been used. The solid triangles are data taken from Ref. [16].

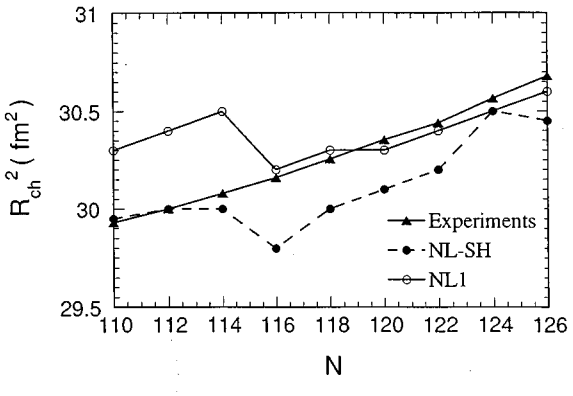

FIG. 3. Comparison of the mean square charge radii for Pb isotopes calculated for the NL-SH and NL-1 sets in the RMF approximation with experimental data taken from Ref. [16]. The gap parameters in Ref. [3] have been used for the neutrons and protons.

data, such as the systematics of the hyperfine structure splitting of odd mass isotopes and isotope shift [6,7], the prolate-oblate energy difference [8], the energy spectra of adjacent odd mass nuclei, and the $B(E2)$ values extracted from life time measurements, all of which indicate that the ground state of Hg isotopes is weakly oblate or nearly spherical.

From the theoretical side, a number of old calculations [9–12] predicted that a shape transition from oblate towards prolate shapes in the ground state might occur with decreasing mass number. To the contrary, all the more recent calculations based on the use of the Strutinsky+BCS approach [13–15] predict an oblate ground-state shape. The latter calculations are consistent with the experimental data for the neutron-deficient Hg nuclei.

In order to examine whether the results of RMF calculations depend on the choice of the parameter set, we repeated the same calculations by replacing the NL1 set by NL-SH set. For this parameter set, the isotopes with mass number between 180 and 188 are prolate. The oblate and the prolate configurations are almost degenerate for $A=196$, 192, 190, 178, 176, and 170. All the other nuclei between $A=170$ and 200 are oblate. We then repeated our calculations by changing the pairing gap parameter for neutrons by 10%. The prolate state always stayed lower than the oblate state by more than 1 MeV in this mass range. We also calculated the shape of ^{184}Hg by reducing the pairing gap as much as 50% for both neutrons and protons simultaneously, and by expanding the single-particle basis up to the $N=20$ major shell. These calculations introduced only a very small change in the energy splitting between the prolate and oblate configurations.

Figure 2 compares the experimental values of the charge radius for Hg isotopes (the solid triangles) [16] with those calculated for the prolate (the open circles) and the oblate (the solid circles) configurations by assuming the NL-SH parametrization and the pairing gap in Ref. [3]. The charge radii for the ground-state configurations for different isotopes are connected by a thick solid line. The theoretical prediction has a large deviation from the data for those nuclei, for which the RMF calculations predict a prolate deformation for their ground states. A simlar irregular change of the charge radius occurs in the calculations using NL1 set (see Fig. 5 in Ref. [1]). These figures indicate that the prolate ground-state prediction of the present RMF calculations does not describe the observed variation of the charge radius, and that there remain serious problems to be settled in order to apply the RMF calculations to describing nuclei far from the stability line.

We made an extensive calculation also for Pb isotopes. For ^{186}Pb as an example, for which there are debates concerning the shape and the magicity of $Z=82$ [17–19], we found that the ordering between the prolate and the oblate configurations is very sensitive to the input parameters in the RMF Lagrangian as well as the choice of the pairing gap. This is similar to the situation for Hg isotopes. The spherical shape, however, always appears as an excited state as long as the gap parameter is varied within 10% of the values in Ref. [3]. In this connection, we wish to mention that Tajima *et al.* have shown that all the Pb isotopes whose mass number is larger than 186 are spherical if one assumes SKIII [20], while Girod *et al.* [21] have shown that some of the Pb isotopes are deformed in their ground state if one uses Gogny force. The HFB calculations in Ref. [22] also suggest that $Z=82$ becomes a nonmagic number when the number of neutrons is near 114.

We have calculated also the charge radius of Pb isotopes. The results are shown in Fig. 3, where the mean square charge radius calculated for the NL-SH set (the solid circles) and for the NL1 set (the open circles) are compared with the experimental data (the solid triangles) [16]. Our RMF calculations predict that the deformation sets in for the isotopes lighter than $A=196$. Though the RMF results fairly well reproduce the experimental data for spherical isotopes, they show a noticeable change at the isotope where a deformation sets in as the neutron number decreases, while the experimantal data show a monotonic dependence on the neutron number. This again points out serious difficulties of RMF calculations in reproducing the smooth behavior of the charge radius which is consistent with an almost spherical shape of the ground state for a much wider range of the mass number.

Last but not least, we admit that we made mistakes in Refs. [1,2] in quoting the binding energies of Pt, Hg, and Pb isotopes from Ref. [23]. As Heyde *et al.* [4] correctly pointed out, some numbers which we quoted in Refs. [1,2] as data

are not experimental data, but have been estimated from systematics.

In conclusion, though the RMF theory offers an attractive framework, more careful optimization of the input parameters including the pairing interaction [24] is needed for it to provide consistent results with the existing data for Hg and Pb isotopes, and to be used to extrapolate to nuclei far from the stability line. Also, calculations allowing a triaxial deformation would be required for these transitional nuclei, where the oblate and prolate configurations almost degenerate and different shapes coexist.

We will report details of our study in a separate paper.

We are very grateful to H. Flocard, Nguyen van Giai, J. Sauvage, P. Möller, D.M. Brink, H. Sagawa, M. Honma, N. Tajima, N. Onishi, and K. Matsuyanagi for very useful discussions. This work is supported by a Grant-in-Aid for General Scientific Research, Contract No. 06640368, and a Grant-in-Aid for Scientific Research on Priority Areas, Contract No. 05243102, from the Japanese Ministry of Education, Science and Culture.

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