PHYSICAL REVIEW C VOLUME 53, NUMBER 2 FEBRUARY 1996

Comment on "Shape and superdeformed structure in Hg isotopes in relativistic mean field model" and "Structure of neutron-deficient Pt, Hg, and Pb isotopes"

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(Received 1 February 1995)

We point out that the results of relativistic mean field calculations for neutron-deficient Pt, Hg, and Pb isotopes by S. K. Patra *et al.* [Phys. Rev. C **50**, 1924 (1994)] and S. Yoshida *et al.* [Phys. Rev. C **50**, 1398 (1994)] contradict the large body of experimental data on these nuclei. In particular, we question their predictions of deformed ground states in the Pb isotopes and prolate and superdeformed ground states in the Hg isotopes.

PACS number(s): 21.10.−k, 21.60.−n, 27.70.+q, 27.80.+w

Neutron-deficient Pt, Hg, and Pb exhibit a variety of structures that put stringent tests on theoretical models. In particular, the Hg and Pb nuclei exhibit excellent examples of shape coexistence [1]. The first fingerprint of large prolate deformations in this mass region was the large isotope shifts, observed by Bonn *et al.* [2] in the neutron-deficient odd-mass Hg nuclei. Evidence for coexisting deformed structures has come mainly from radioactive decay studies of excited states; see, e.g., Refs. [3,4].

For the even-even Hg isotopes, compelling evidence (see, e.g. [3,4]) has been presented for oblate ground states and excited prolate bands in ^{180,182,184,186,188,190}Hg. Figure 1 displays the excitation energies of coexisting states in the even-even Hg isotopes. Experimental data on $B(E2)$ values are discussed in Ref. [1]; the deduced quadrupole deformations $|\beta_2|$ are shown in Fig. 2. They are 0.10–0.12 and ~0.25 for oblate and prolate band structures, respectively. Additional information about deformations in this mass region comes from isotope shifts [5].

In recent studies [6,7] based on a relativistic mean field (RMF) approach using the NL1 parametrization, predictions were made for binding energies, deformations, and radii in the neutron-deficient Pt, Hg, and Pb isotopes. In particular, calculations were made which predicted (i) prolate ground-state shapes in ^{178,182,184,186}Hg, (ii) a superdeformed ground state in ^{180}Hg, and (iii) deformed ground states in the even-even Pb isotopes with $184 \leqslant A \leqslant 196$. In this Comment, we wish to point out that (i) the results of the RMF calculations are in striking disagreement with experiment and (ii) the comparison with experiment done in Refs. [6,7] is based on incorrect experimental numbers. In the following we briefly summarize the main points of our criticism.

(a) The experimental prolate-oblate energy difference in the Hg isotopes has been extracted by Dracoulis *et al.* in Ref. [8]. According to this analysis, the ground-state configuration corresponds to an oblate shape.

(b) As seen in Fig. 2, for ^{184,186}Hg there are states corresponding to *two* bands with regard to the values of $|\beta_2|$ deduced from measured $B(E2)$ values, the smaller values of $|\beta_2| \sim -0.11$ are associated [9] with moderately deformed oblate ground states while the larger values, $|\beta_2| \sim 0.25$, correspond to prolate *intruder* bands. In their Fig. 2, Patra *et al.* [6] ignored the data for oblate bands.

(c) Experimental data for the nuclear charge radii r_{ch} in the Hg isotopes indicate a smooth increase in r_{ch} with neutron number [5]. In contrast, the RMF calculations predict a dramatic increase in r_{ch} below $A=188$, and Patra *et al.* notice this as "a puzzle as to why the charge radius does not reflect the behavior of the quadrupole deformation." As discussed above, experimental ground-state quadrupole deformations are consistent with the smooth behavior of charge radii. (For more discussion regarding this point, see Ref. [10].)

(d) The prediction of a superdeformed (SD) ground state in ^{180}Hg by Patra *et al.* contradicts experimental data of Ref. [8] and the smooth systematic behavior shown in Fig. 1. Experimentally, moments of inertia of SD bands in the $A=192$ mass region are of the order of $90\hbar^2$/MeV [11]. This would correspond to a first excited $I^\pi=2^+$ state at about 33 keV. It is worth noting that in the systematic calculations based on the macroscopic-microscopic approach [12] the

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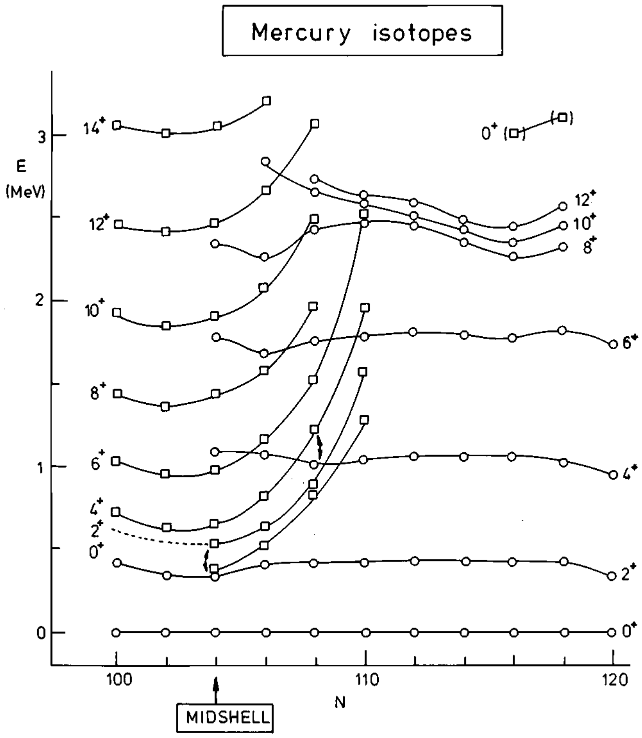

FIG. 1. The systematics of deformed bands in the even-even Hg nuclei. The 0^+ states near 3 MeV in ^{196,198}Hg are identified as proton-pair excitations from their strong population in the (^{3}He,n) reaction. (From Ref. [1].)

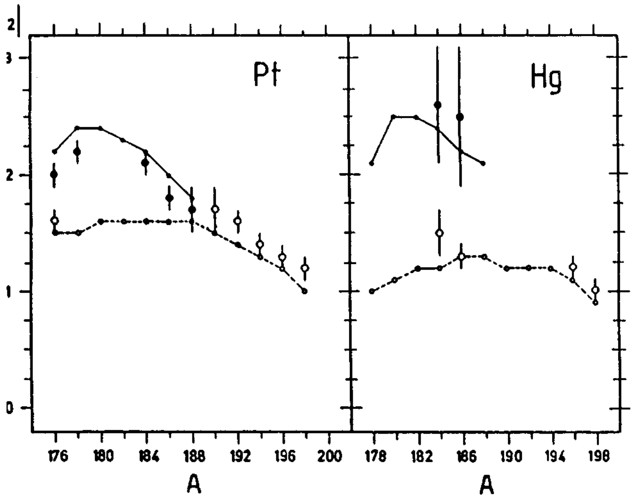

FIG. 2. Experimental quadrupole deformations β_2 extracted from the $B(E2)$ values in Pt and Hg isotopes. They are compared to the theoretical values calculated with the Woods-Saxon potential. The open circles relate to the oblate band, the solid circles to the prolate band. (From Ref. [1].)

reflection-asymmetric hyperdeformed band in ^{180}Hg is predicted at $E^* \sim 6.5$ MeV.

(e) The predicted deformed ground states in the neutron-deficient Pb isotopes contradict experimental data concerning energy spectra and charge radii (see Refs. [1,12,13]). In particular, the very smooth behavior in the nuclear charge radii [5] gives no indication of shape change.

(f) The experimental binding energies of ^{176,180}Pt, ^{174,180,184,188,190,192}Hg, and ^{178,180,184,188,192,194,196,198}Pb are unknown. The "experimental" values quoted in Refs. [6,7] are not measurements: They are extrapolated from systematic trends [14].

(g) Both works [6,7] contain many conceptual errors related to theoretical aspects. For instance, in Ref. [6] it is stated that, contrary to the RMF model, a drawback of nonrelativistic calculations is that the parameters, such as the force parameters, are determined phenomenologically from properties of stable nuclei. Actually, the RMF approach is based on an effective Lagrangian whose parameters are adjusted to properties of known nuclei (masses, radii, etc.). The fact that the model is relativistic is irrelevant. All theoretical models describing physics of exotic nuclei have to involve dramatic extrapolations. It is impossible to say that a model can describe the properties of nuclei far off stability, as the authors qualify the RMF approach, since the (dis)agreement with experiment cannot be assessed (for more discussion regarding this point, see Ref. [15]). The statement that the available model space used in practical nonrelativistic calculations is too small to discuss SD states is incorrect. Usually, the calculations based on the macroscopic-microscopic approach, such as those of Ref. [16], involve $N_{\max}=14-16$ oscillator shells. This can be compared with the number $N_{\max}=12$ used in Refs. [6,7].

(h) Since the RMF calculations have been constrained to axial shapes, the excited minima predicted in an adiabatic approach often correspond to saddle points and, therefore, should not be associated with actual physical configurations.

In conclusion, we stress the fact that the large body of existing experimental data [band structures as obtained through gamma decay studies, isotope shifts, $B(E2)$ values, etc.] points towards a consistent picture of oblate and spherical ground states in the Hg and Pb isotopes, respectively, and deformed intruder structures corresponding to cross-shell excitations. In contrast to the RMF-NL1 approach, nonrelativistic studies based on deformed mean field theory give an overall good description of the above observations [1,12,17]. In our opinion, the failure of the RMF-NL1 approach in describing properties of neutron-deficient Pt, Hg, and Pb nuclei has nothing to do with the fact that this approach is relativistic, but rather with the particular parametrization of the relativistic Lagrangian and a very schematic treatment of pairing.

The authors are grateful to many colleagues for discussions on the theme of coexisting bands in the very light Hg nuclei. Two of the authors (K.H., J.L.W.) thank NATO for research Grant No. NATO CRG 92/0011/R. K.H., C.D.C., P.V.D., and M.H. thank the NFWO for financial support. This work was supported by the U.S. Department of Energy through Grants Contracts No. DE-FG05-87ER 40330 (J.W.), and DE-FG05-93ER40770 and DE-AC05-84OR21400 (W.N.).

[1] J.L. Wood, K. Heyde, W. Nazarewicz, M. Huyse, and P. Van Duppen, Phys. Rep. **215**, 101 (1992).

[2] J. Bonn, G. Huber, H.J. Kluge, L. Kugler, and E. Otten, Phys. Lett. **38B**, 308 (1972).

[3] J.D. Cole *et al.*, Phys. Rev. Lett. **37**, 1185 (1976).

[4] J.H. Hamilton *et al.*, Phys. Rev. Lett. **35**, 562 (1975).

[5] E. Otten, in *Treatise on Heavy-Ion Science*, edited by D.A. Bromley (Plenum, New York, 1989), Vol. 8, p. 517.

[6] S.K. Patra, S. Yoshida, N. Takigawa, and C.R. Praharaj, Phys. Rev. C **50**, 1924 (1994).

[7] S. Yoshida, S.K. Patra, N. Takigawa, and C.R. Praharaj, Phys. Rev. C **50**, 1398 (1994).

[8] G.D. Dracoulis *et al.*, Phys. Lett. B **208**, 365 (1988).

[9] R. Bengtsson, T. Bengtsson, J. Dudek, G. Leander, W. Nazarewicz, and J.-Y. Zhang, Phys. Lett. B **183**, 1 (1987).

[10] F. Buchinger, J.E. Crawford, A.K. Dutta, J.M. Pearson, and F. Tondeur, Phys. Rev. C **49**, 1402 (1994).

[11] R.V.F. Janssens and T.L. Khoo, Annu. Rev. Nucl. Part. Sci. **41**, 321 (1991).

[12] W. Nazarewicz, Phys. Lett. B **305**, 195 (1993).

[13] J. Heese, K.H. Maier, H. Grawe, H. Kluge, M. Schramm, R. Schramm, R. Schubart, K. Spohr, W. Meczyński, M. Janicki, J. Styczeń, and J. Grębosz, Phys. Lett. B **302**, 390 (1993).

[14] G. Audi and A.H. Wapstra, Nucl. Phys. **A565**, 1 (1993).

[15] J. Dobaczewski and W. Nazarewicz, Phys. Rev. Lett. **73**, 1869 (1994).

[16] W. Satuła, S. Ćwiok, W. Nazarewicz, R. Wyss, and A. Johnson, Nucl. Phys. **A529**, 289 (1991).

[17] S. Aberg, H. Flocard, and W. Nazarewicz, Annu. Rev. Nucl. Sci. **40**, 439 (1990).