Prescission neutron multiplicities and nuclear viscosity: A systematic study

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Prescission neutron multiplicities have been calculated in the framework of a simple, dynamical model of fission. The fission trajectories have been calculated by solving Euler-Lagrange equations with dissipation generated through two-body nuclear viscosity. Systematic study of the relationship between the prescission neutron multiplicities and nuclear viscosity has been made in the range of mass 150–200 and incident energy 4–13 MeV/nucleon. The values of the viscosity coefficients which are used to predict the observed prescission neutron multiplicities follow a global relation in the region of mass and energy studied.

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In recent years there has been a lot of experimental activities to measure the prescission neutron multiplicities in heavy ion induced fusion-fission reactions over a large range of incident energies and compound nuclear masses [1–9]. Presently, it is well known that the measured prescission neutron multiplicities much overestimate the theoretical values calculated from statistical models with level density parameters adjusted to explain the observed fission excitation functions. The large excess of neutrons which are emitted before the nucleus undergoes fission is interpreted as arising from the dynamical effects in the fission decay process. The role of nuclear viscosity in the dynamical evolution of the fission decay process is a very important one. It does not allow the system to undergo rapidly the shape changes manifested in the fission decay process and thereby introduces a time delay in the temporal evolution of the system. This time delay for the evolution of the system from the equilibrium configuration to the saddle point and then to the scission point allows more prescission neutrons to be emitted. In the framework of the statistical model, the dynamical time delay is introduced in an *ad hoc* manner to reproduce the observed prescission neutron multiplicities [8]. Alternatively, the time delay is estimated from the prescission neutron multiplicities observed in excess of statistical model predictions [3,9]—emphasizing the need of a proper dynamical description of the fission decay process to explain the origin of such delay.

In recent years, there have been a few theoretical attempts to describe the fission dynamics in terms of Langevin equations with the constraint that the Langevin trajectories are allowed only for a certain period of time [10–13]. As the dynamics of the fission decay process, in its full glory, with all the associated collective degrees of freedom, is a very complex process, one has to invoke some simplifying assumptions to solve the problem. For example, the numerical calculations of Langevin trajectories have so far been restricted to one-dimensional cases only. In these calculations, different values of friction coefficients have been used in different mass systems to reproduce the observed prescission neutron multiplicities. In Ref. [12], the value of the reduced friction coefficient β used to reproduce the experimental prescission neutron multiplicities varies from 2–30 ($\times 10^{21}$ sec^{-1}). The reduced friction coefficient $\beta = 1\times 10^{21}$ sec^{-1} corresponds to two-body viscosity of 1×10^{-23} MeV sec fm^{-3} [12]. Recently, we have proposed a simple model with a different approach where the dynamical evolution of the fission process is simulated using Euler-Lagrange equations [14]. Here, the exit channel configuration has been assumed to comprise of two leptodermous spheres connected by a cylindrical neck formed by their overlapping density profiles which essentially reduces the dynamics to one dimension. This model has been quite successful in predicting the fragment yields and total average kinetic energies of the systems with $A_{cn} < 100$ (A_{cn} being the mass of the compound nucleus) where asymmetric fission of the compound system is predominant. In the present paper, this model has been extended to estimate the prescission neutron multiplicities for different fusion-fission systems at various incident energies. The objective of the present studies is to investigate, in detail, the relationship between nuclear dissipative force and prescission neutron emission, and to see whether any global systematics exists for such dissipative forces in a wide range of masses and excitation energies.

The details of the fission dynamics used in the present model are given in Ref. [14]. Therefore, in the following, the salient features of the model are described in brief. Assuming the shape of the fissioning nucleus as two leptodermous spheres connected by a cylindrical neck [15], the number of collective fission degrees of freedom reduces to only one, i.e., surface to surface separation s along the line joining the centers of the two spheres. This is related to the neck radius ρ by the following expression [15]:

$$\rho^2 = 2\bar{R}(s - s_0), \tag{1}$$

where $\bar{R}$ is the reduced radius and $s_0 = 3.74$ fm is the maximum distance beyond which the proximity interaction vanishes. The separation s is expressed in terms of the center to center distance r by the relation $s = r - r_1 - r_2$, where r_1, r_2 are the sharp surface radii of the two spheres. The radial and the tangential components of the frictional coefficients η_r and η_t, following Werner-Wheeler method for nearly irrotational hydrodynamical flow are given by [16]

$$\eta_r = 18\pi\mu\bar{R}s/(s_0 - s), \tag{2a}$$

$$\eta_t = 36\pi\mu s, \tag{2b}$$

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where μ is the viscosity coefficient. The fission trajectories are calculated by solving the Euler-Lagrange equations [14], with conservative (proximity + Coulomb) forces and dissipation generated by the frictional forces given by the above Eqs. (2). The dynamical evolution starts at $r=r_{\rm min}$, where the minimum of the potential occurs. Once the trajectory reaches the saddle point or the top of the barrier, it is almost certain that it will reach the scission point.

An important feature of this model pertains to the collective energy assigned to the fission degree of freedom at the start of the dynamical evolution of the system. A random fraction of the total available energy in the minimum potential energy configuration goes into the collective degree of freedom which initiates the dynamical evolution of the system, leading eventually to fission. The total available excitation energy E^* is partitioned into two parts, i.e., E_c and E_i, the energies associated with the collective fission degree of freedom and intrinsic nucleonic motion, respectively. Each of such partition is assumed to have equal probability of occurrence and is realized by using a uniform random number distribution.

The emission of the prescission neutrons is incorporated in the present model as follows. During the temporal evolution of the fission trajectory, the intrinsic excitation of the system is calculated at each time step. Correspondingly, the neutron decay width at that instant, Γ_n, is calculated using the relation $\Gamma_n=\hbar W_n$, where the decay rate per unit time, W_n, is given by [17]

$$W_n=\int_0^{E_{\rm max}} dE\,\frac{d^2\Pi_n}{dE\ dt}. \qquad (3)$$

The rate of decay $A\rightarrow A-1+n$ in an energy interval $[E,E+dE]$ and a time interval $[t,t+dt]$, $d^2\Pi_n/dE\ dt$, is given by

$$\frac{d^2\Pi_n}{dE\ dt}=\frac{1}{\pi^2\hbar^3}E\sigma_{\rm inv}\mu_r\frac{\omega_{A-1}(E^*_{A-1})}{\omega_A(E^*_A)}. \qquad (4)$$

The quantity $\omega_A(E^*_A)$ is the level density for the nucleus with mass A and excitation energy E^*_A, $\sigma_{\rm inv}$ is the inverse cross section for the reaction $(A-1)+n\rightarrow A$, and μ_r is the reduced mass. The upper limit of integration $E_{\rm max}$ in Eq. (3) is given by

$$E_{\rm max}=E^*+B_A-(B_{A-1}+B_n), \qquad (5)$$

where B_A is the binding energy of the nucleus with mass number A and B_n is the neutron separation energy. The neutron emission at a particular instant is realized if the ratio of the neutron decay time $\tau_n(=\hbar/\Gamma_n)$ and the time step τ is greater than a random number, i.e., $\tau/\tau_n>R_n$ (R_n being the random number lying between 0 and 1). The value of the time step τ is chosen in such a way that it satisfies the condition $\tau/\tau_n\ll 1$. This procedure simulates the law of radioactive decay with half-life τ_n [12,13]. (In the present calculation, typical values of τ/τ_n were $\sim 10^{-5}$ or less.) The kinetic energy of the emitted neutron is extracted through random sampling technique. After the emission of the neutron, the intrinsic excitation energy is recalculated and the trajectory is continued. In the present calculation, the fission trajectories have also been corrected for the proton emission which has been simulated following the same procedure as for the neutrons with appropriate corrections in Eqs. (3)–(5) for the Coulomb interaction. For each angular momentum l, of the compound system, the average number of the emitted neutrons per fission event, $\langle M_n\rangle_l$, is given by the ratio of the total number of neutrons thus emitted and the total number of the fission events. The average prescission neutron multiplicity $\langle\langle M_n\rangle\rangle$ is thus calculated as

$$\langle\langle M_n\rangle\rangle=\frac{\Sigma_{l=0}^{l=l_f}\langle M_n\rangle_l(2l+1)p_l}{\Sigma_{l=0}^{l=l_f}(2l+1)p_l}, \qquad (6)$$

where l_f is the critical angular momentum for fusion. The quantity p_l is the fission probability for the angular momentum l and is defined as

$$p_l=\frac{N_l}{N}. \qquad (7)$$

Here N is the total number of trajectories used in the Monte Carlo simulation and N_l is the number of trajectories which undergo fission.

The systems typically chosen for the present work are ^{16}O $+^{142}$Nd, ^{24}Mg$+^{134}$Ba, ^{32}S$+^{126}$Te, ^{50}Ti $+^{108}$Pd, ^{18}O$+^{150}$Sm, ^{19}F$+^{181}$Ta, and ^{28}Si$+^{170}$Er in the mass range of 150–200. All the systems are above the Businaro-Gallone point, and symmetric fission is the predominant mode of decay. The prescission neutron multiplicities have been calculated by varying the viscosity coefficient. The calculated values of prescission multiplicities and the viscosity coefficients used are tabulated along with the respective experimental data for comparison in Table I. It is seen from Table I that the values of the viscosity coefficients in the whole range of masses and the incident energies studied varies between 2–4 ($\times 10^{-23}$ MeV sec fm^{-3}) or 0.03–0.06 TP ($1TP=6.24\times 10^{-22}$ MeV sec fm^{-3}). The values of the viscosity coefficients in the similar range have also been reported in the literature [18,19]. In the Langevin description of fission dynamics [12,13], the values of the reduced friction coefficient β needed to reproduce the prescission neutron multiplicity data were found to have a strong system dependence [β = 3, 7, 20 ($\times 10^{21}$ sec^{-1}) for ^{200}Pb, ^{251}Es, and ^{158}Er, respectively]. However, using an improved version of the model [12], it had been found that the prescission neutron multiplicities for light and medium heavy nuclei (fissility parameter $X<0.8$) could be reasonably well explained with $\beta=2\times 10^{21}$ sec^{-1} (which corresponds to two body viscosity $\mu=2\times 10^{-23}$ MeV sec fm^{-3}). This is similar to the values of the viscosity coefficients obtained in the present calculation for lighter system at lower incident energies. With the increase in bombarding energy, the calculations of Ref. [12] systematically underpredict the experimental prescission neutron multiplicities. In the present calculation also similar trends are clearly visible where it is observed that for fixed mass of the compound system, the coefficient of viscosity increases with the increase of the incident energy per nucleon (E/A). This may be indicative of the fact that the viscosity coefficient is excitation energy or temperature dependent. It is further observed from Table I that the viscosity coefficient increases slowly with the increase in the mass of

TABLE I. The calculated and the experimental prescission neutron multiplicities $\langle\langle M_n\rangle\rangle$ and the respective values of the viscosity coefficients μ used in the calculation.

System	Incident energy (MeV)	μ (MeV sec fm^{-3})	$\langle\langle M_n\rangle\rangle$ Calc.	$\langle\langle M_n\rangle\rangle$ Expt.	Av. kin. energy $\langle E_K\rangle$ (MeV)
^{24}Mg+^{134}Ba	180.0	3.0	2.61	2.5±0.5	112.45
^{32}S+^{126}Te	180.0	2.7	1.80	1.7±0.5	111.60
^{50}Ti+^{108}Pd	216.0	1.8	0.53	0.3±0.3	110.96
^{16}O+^{142}Nd	100.0	2.6	1.31		111.57
	190.0	3.3	3.71		111.57
	207.0	3.4	4.19	3.6±0.5	111.57
^{18}O+^{150}Sm	108.0	2.6	1.57	1.5±0.25	112.87
	110.0	2.7	1.76	1.6±0.25	112.95
	122.0	3.2	2.52	2.7±0.30	113.00
	166.0	3.9	4.26	5.0±0.90	116.25
^{28}Si+^{170}Er	135.0	3.1	1.53	1.7±0.30	145.47
	150.0	4.1	2.58	2.75±0.25	146.61
	165.0	4.2	3.02	3.0±0.25	147.91
^{19}F+^{181}Ta	95.0	3.2	1.63	1.8±0.20	145.02
	107.0	3.9	2.53	2.8±0.20	145.71
	116.0	4.0	2.84	3.0±0.20	146.34
	126.0	4.2	3.27	3.4±0.20	146.91
	135.0	4.2	3.45	3.7±0.20	147.57

the system. This is in agreement with the trend observed in [12], where it was shown that the difference between the experimental and predicted neutron multiplicities gradually increases with the mass number if one uses a fixed value of β over the whole mass range, indicating a higher value of β for heavier systems.

The mass and energy dependence of the viscosity coefficient may be expressed in terms of a nonlinear function of E/A and mass of the compound system (A_{cn}) of the following form:

$$\mu(E/A,A_{cn})=aE/A+bA_{cn}^3. \quad (8)$$

The values of the parameters $a=0.180\pm0.023$ and $b=0.357\times10^{-6}\pm0.26\times10^{-7}$ have been obtained through least square fitting of the viscosity coefficients for all the systems studied and the value of χ^2 per degree of freedom has been found to be $\simeq0.12$. The total average kinetic energies of the fragments, $\langle E_K\rangle$, have also been calculated using the present model and are given in Table I. It is seen that the values of $\langle E_K\rangle$ are almost constant and have a very weak dependence on the incident energy. The predicted values of $\langle E_K\rangle$ are in good agreement with those obtained from Viola systematics [20] ($\langle E_K\rangle_{\mathrm{Viola}}$ being 109.0 ± 2.4, 112.9 ± 2.5, 144.5 ± 2.8, 144.0 ± 2.8 MeV for $A_{cn}=158$, 168, 198, 200, respectively).

In Fig. 1 we plot the viscosity coefficient μ as a function of E/A for various compound nuclear masses; A_{cn} = 158 [Fig. 1(a)], 168 [Fig. 1(b)], 198 [Fig. 1(c)], and 200 [Fig. 1(d)], respectively. The solid curves in Fig. 1 correspond to the values of viscosity coefficient μ obtained from the global relation [Eq. (8)] and different symbols correspond to the values of μ which have been found to reproduce the observed prescission neutron multiplicities for different entrance channels. It is seen from Fig. 1 that the global relation Eq. (8) quite successfully reproduces the trend for the values of μ used for predicting the neutron prescission multiplicities in the range of mass and energy studied.

To conclude, we have developed a dynamical model for fission where fission trajectories are generated through solving Euler-Lagrange equations of motion with dissipative

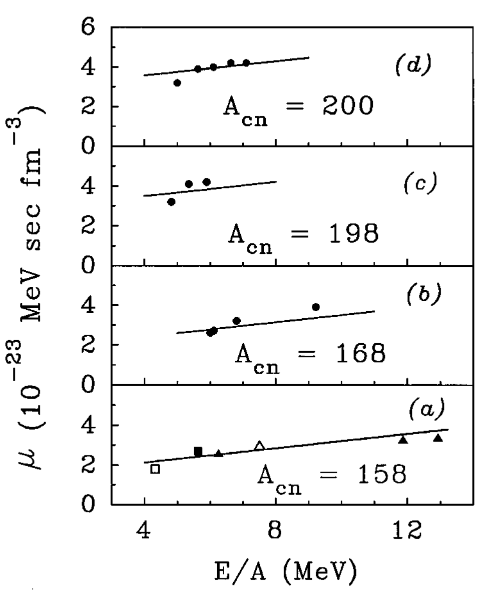

FIG. 1. Variation of viscosity coefficient μ as a function of bombarding energy per nucleon (E/A) for various compound nuclear mass systems. The solid curves correspond to the values of μ obtained from the global relation (see text). In (a), filled squares, open squares, filled triangles, and open triangles correspond to S + Te, Ti + Pd, O + Nd, and Mg + Ba, respectively [6]. Filled circles in (b), (c), and (d) correspond to O + Sm, Si + Te, and F + Ta systems, respectively [4].

forces derived from Werner-Wheeler prescription. Prescission neutron emission along the fission trajectory has been simulated through Monte-Carlo simulation technique. The evolution of the fission trajectories have been corrected for prescission proton emission, which has been simulated in a similar way as it was done for neutron emission. The emission of other complex particles (d, t, ^{4}He, etc.) and their effects on prescission neutron emission has been neglected in the present calculation. As the multiplicities of the complex particles are expected to be quite small, it may not significantly affect the present results. The relationship between the nuclear viscosity and prescission neutron multiplicities have been studied using the present model for a number of systems over a wide range of mass and incident energy. Theoretical predictions for prescission neutron multiplicities have been compared with the corresponding experimental data to extract the optimum value of the viscosity coefficient in each case. The values of the viscosity coefficients thus obtained are found to follow a global relation given by Eq. (8) in the range of mass and energy studied.

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