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Exact methods for expectation values in canonical fragmentation models

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We introduce for canonical fragmentation models an exact method for computing expectation values which exclude the largest cluster. This method allows for the computation of the reduced multiplicity and other quantities of interest introduced by Campi, and a comparison shows that the percolation model and a recent canonical model differ mostly only in small respects in these ensemble averages.

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Campi and Krivine [1,2] introduced a method for distinguishing fragmentation models from one another. By comparing various expectation values in which the largest cluster is excluded but the particle number and fragment multiplicity are held fixed, they showed that the percolation model has a distinctly different behavior than many competing nuclear fragmentation models.

In this paper we analyze another statistical weight we have been using recently and show that it shares many of the same properties as percolation theory, a point already apparent from a consideration of its critical exponents [3]. The method used in this paper is unusual in that it is an exact computational method: Monte Carlo sampling is avoided by exploiting some properties of the partition function which enable the exact evaluation of the reduced quantities.

We begin by assuming that each fragmentation outcome happens with a probability proportional to the Gibbs weight [4–7]

$$W(\vec{n})=\prod_{k\geq 1}\frac{x_k^{n_k}}{n_k!}, \tag{1}$$

where n_k is the number of fragments of size (or charge) k and x_k is a parameter associated with k sized fragments. We then define the microcanonical partition function as

$$Z_A^{(m)}(\vec{x})=\sum_{\pi_m(A)} W(\vec{n}), \tag{2}$$

where $\pi_m(A)$ is the set of partitions of A nucleons into m fragments, i.e., $\Sigma_k k n_k=A$, $\Sigma_k n_k=m$. These partition functions satisfy the identity

$$\frac{\partial Z_A^{(m)}}{\partial x_k}=Z_{A-k}^{(m-1)}(\vec{x}), \tag{3}$$

which allows for the computation of the partition functions recursively from $\Sigma_k\langle n_k\rangle=m$, since

$$\langle n_k\rangle=x_k\frac{\partial}{\partial x_k}\ln Z_A^{(m)}=x_k\frac{Z_{A-k}^{(m-1)}}{Z_A^{(m)}}. \tag{4}$$

Campi defined reduced moments as moments in which the largest cluster is excluded from the measure, i.e.,

$$M_s(\vec{n})\equiv\sum_k k^s n_k-k_{\max}^s, \tag{5}$$

where $k_{\max}$ is the size of the largest cluster. This suggested the definition of the reduced variance γ_2 [1,2] for a fragmentation event should be

$$\gamma_2(\vec{n})=\frac{M_2M_0}{M_1^2}=(m-1)\frac{\Sigma_k k^2 n_k-k_{\max}^2}{(A-k_{\max})^2}. \tag{6}$$

Its expectation value can be computed by breaking events into classes specified by $k_{\max}$, and summing the expectation values over those classes with the appropriate weight, i.e.,

$$\langle\gamma_2\rangle=(m-1)\sum_{k_{\max}}\Pr(k_{\max})\frac{\Sigma_k k^2\langle n_k\rangle(k_{\max})-k_{\max}^2}{(A-k_{\max})^2}, \tag{7}$$

where $\Pr(k_{\max})$ is the probability of $k_{\max}$ being the largest cluster size and $\langle n_k\rangle(k_{\max})$ is the expectation value of n_k when $k_{\max}$ is fixed.

To compute expectation values in which the largest cluster size is fixed we need to compute the partition function for such ensembles. Clearly this partition function is given by all the terms in the microcanonical partition function which have $x_{k_{\max}}$ as the highest x_k in the term. Consider

$$\Delta Z_A^{(m)}(k_{\max})\equiv Z_A^{(m)}(x_1,\ldots,x_{k_{\max}},0,\ldots,0)-Z_A^{(m)}(x_1,\ldots,x_{k_{\max}-1},0,\ldots,0). \tag{8}$$

We see that $\Delta Z_A^{(m)}(k_{\max})$ is the partition function for ensembles with fixed maximum cluster size $k_{\max}$, since the first term collects all terms with $x_{k_{\max}}$ or lower, and the second term eliminates those terms which do not have an $x_{k_{\max}}$. From this result we can determine $\Pr(k_{\max})$ and $\langle n_k\rangle(k_{\max})$, which are given by

$$\Pr(k_{\max})=\frac{\Delta Z_A^{(m)}(k_{\max})}{Z_A^{(m)}(x_1,\ldots,x_A)}, \tag{9}$$

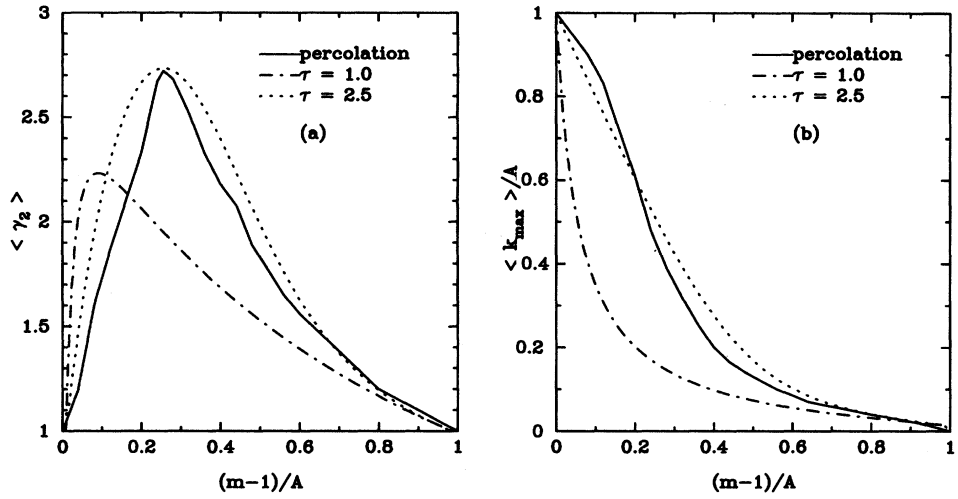

FIG. 1. Expected reduced multiplicity $\langle \gamma_2 \rangle(m)$ (a) and largest cluster size $k_{\max}/A$ (b) vs $(m-1)/A$ for $\tau=1.0,2.5$ and the percolation model at $A=125$.

$$\langle n_k \rangle(k_{\max}) = \begin{cases} 0, & k > k_{\max}, \\ \dfrac{Z_{A-k}^{(m-1)}(x_1, \ldots, x_{k_{\max}}, 0, \ldots, 0)}{\Delta Z_A^{(m)}(k_{\max})}, & k = k_{\max}, \\ \dfrac{\Delta Z_{A-k}^{(m-1)}(k_{\max})}{\Delta Z_A^{(m)}(k_{\max})}, & k < k_{\max}. \end{cases} \tag{10}$$

This method is quite general and can be applied to other models. For example, equipartitioning models, which have weights given by

$$W(\vec{n}) = \prod_{k \geq 1} z_k^{n_k} \tag{11}$$

can also be analyzed by this method with some minor modifications. For example, $z_k = 1$ is the model used by Sobotka and Moretto [8].

With these identities there is sufficient information to compute $\langle \gamma_2 \rangle$ and other reduced moments for any Gibbs model specified by x_k and Eq. (1). We use $x_k = x/k^{\tau}$ for a variety of reasons discussed elsewhere [9]. Campi and Krivine [2] following Mekjian [4] considered this model with $\tau = 1.0$ and showed that its reduced variance and other related expectation values had a distinctly different behavior than the percolation model. Plotting the expected reduced variance vs $(m-1)/A$, $\langle \gamma_2 \rangle(m)$ has a single peak. The location, height, and width of this peak for the two models (and other models they considered) are completely different, suggesting the usefulness of this plot in distinguishing fragmentation models. The choice $\tau = 0$ was considered by Gross *et al.* [10–12], and a different model was analyzed by Pan and Das Gupta [13].

Since that time our interest has turned to the choice $\tau = 2.5$ because of similarities with percolation theory and Bose condensation. Namely, the sudden appearance of an infinite cluster in the infinite A limit and the presence of

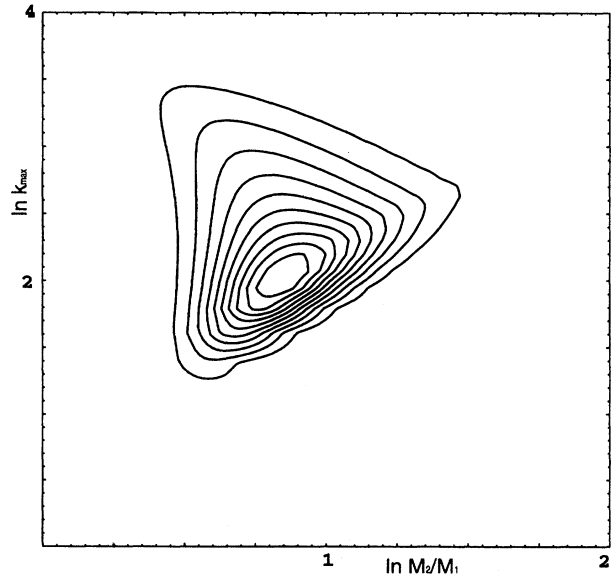

FIG. 2. Campi probability contour plot for $Z=79$, $\tau=2.5$ at the critical point $x=x_c$. The axes are logarithmic, with the largest cluster size on the y axis, and the ratio of the reduced moments M_2/M_1 on the x axis. The central rings are higher in probability than the outer rings.

condensation phenomena. As such, we have recomputed the Campi plots for this model and have discovered that they duplicate the percolation model results in many respects. Figure 1(a) shows the results. The height and location of the peak of the reduced variance $\langle \gamma_2 \rangle$ are the same in both models. The only significant difference is the width of the peak which is larger in the Gibbs model than in the percolation model. Plots of $\langle k_{\max} \rangle(m)$ vs $(m-1)/A$ given in Fig. 1(b) are also very similar for both models, and the scaling behavior of the position, width, and height with changing A also agree.

Another plot suggested by Campi [14] is to divide the event space by the maximum cluster size of the event $k_{\max}$ and the reduced second moment M_2 and plot the probability of the canonical model being at any particular point on the graph. This can also be done exactly for Gibbs models in a way completely analogous to the way given above. Define the partition function $Z_A(m_2;\vec{x})$ as the sum of the Gibbs weight Eq. (1) over all partition vectors $\vec{n}$ which satisfy $\Sigma_k k n_k = A$, $\Sigma_k k^2 n_k = m_2$. This can be computed by the following recursion,

$$Z_A(m_2;\vec{x}) = \frac{1}{A}\sum_k k x_k Z_{A-k}(m_2 - k^2;\vec{x}) \qquad (12)$$

with $Z_0(m_2;\vec{x}) = \delta_{m_2,0}$. If we define ΔZ as in Eq. (8) we find again the partition function conditioned on $k_{\max}$ being fixed, which is proportional to the probability of having an event with both m_2 and $k_{\max}$. Figure 2 plots this probability profile as a contour plot, which reveals the events are centered on a particular region in this phase space. The slopes of the edges of this region are related to the critical exponents according to Campi [14].

Clearly there are differences between percolation theory and a Gibbs model, but the differences are not as large as originally suggested by early computations. Indeed the reduced variance might not reliably distinguish percolation from a simple Gibbs model. A different method is needed to distinguish these models. However, the idea of excluding the largest cluster from the ensemble averages is a standard procedure in percolation theory [15], and this new technique for doing that analytically in the Gibbs models shows a particular advantage of these models over percolation models, which we hope will encourage further interest in them.

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