PHYSICAL REVIEW C VOLUME 52, NUMBER 5 NOVEMBER 1995

# Pairing strength in neutron-rich isotopes of Zr

J. L. Durell, W. R. Phillips, C. J. Pearson, J. A. Shannon, W. Urban, and B. J. Varley
*Department of Physics and Astronomy, University of Manchester, M13 9PL, United Kingdom*

N. Rowley
*Department of Physics and Astronomy, University of Manchester, M13 9PL, United Kingdom and Department of Physics, University of Surrey, Guildford, GU2 5XH, United Kingdom*

K. Jain
*Department of Physics, University of Surrey, Guildford, GU2 5XH, United Kingdom*

I. Ahmad, C. J. Lister, L. R. Morss, K. L. Nash, and C. W. Williams
*Argonne National Laboratory, Argonne, Illinois 60439*

N. Schulz, E. Lubkiewicz,* and M. Bentaleb
*Centre de Recherches Nucléaires, IN2P3-CNRS, Université Louis Pasteur, 67037 Strasbourg, France*

(Received 1 May 1995)

Rotational bands based on two-quasi-neutron excitations have been observed for the first time in neutron-rich isotopes. The bandhead excitation energies have been used within the BCS model to determine the strength $G$ of the neutron pairing interaction. Values of $G=0.21$ and 0.19 MeV have been obtained for $^{100}$Zr and $^{102}$Zr, respectively. For these neutron-rich nuclei the strength $G$ is thus close to $20/A$ MeV, smaller than the value $23/A$ MeV used to describe even-even nuclei near stability in this region.

PACS number(s): 21.10.Re, 21.30.+y, 23.20.Lv, 27.60.+j

It has been established [1–6] that neutron-rich isotopes near mass number 100 have large stable deformations characterized by values of the quadrupole deformation parameter $\epsilon_2$ between 0.3 and 0.4. The large moments of inertia of the ground-state rotational bands in the even-even neutron-rich Sr, Zr, and Mo isotopes suggest that pairing is relatively weak in these nuclei. Indeed, it has been proposed [7] that pairing much weaker than generally accepted for nuclei near the line of stability close to this mass region is necessary for the large deformations to be attained. As yet no data exist to give a direct measure of the strength of pairing in these neutron-rich isotopes. A first indication of effects possibly related to the pairing strength comes from the experimental values of the pairing energy, determined by nuclear masses, which reduces by 28(4)% between $^{100}$Zr and $^{102}$Zr. In this Rapid Communication we present the first observation of two-quasi-neutron rotational bands in $^{100}$Zr and $^{102}$Zr, the excitation energies of which allow a direct determination of the pairing strength within the BCS model.

Partial level schemes of $^{100}$Zr and $^{102}$Zr have been established using triple coincidences between prompt $\gamma$ rays emitted following the spontaneous fission of $^{248}$Cm. The Cm source was placed at the center of the EUROGAM I array at Daresbury Laboratory. For this experiment the array consisted of 45 Compton-suppressed Ge detectors and 5 low-energy photon spectrometers. Triple coincidences between the Ge detectors (and unfolded higher multiplicity events) were placed into a $\gamma\gamma\gamma$ cube, enabling rapid creation of one-dimensional $\gamma$-ray spectra produced by supplying two gates on chosen $\gamma$ rays. The selectivity of double gating enabled detailed decay schemes to be constructed. Partial level schemes of $^{100}$Zr and $^{102}$Zr are shown in Fig. 1. The 1821 and 1981 keV levels in $^{102}$Zr have been observed [8] in the $\beta$ decay of a high-spin isomer in $^{102}$Y. The higher sensitivity of the present data has allowed the observation of three weaker $\gamma$-decay branches from the 1821 keV level.

Rotational bands with bandhead energies near 2 MeV have been observed in both $^{100}$Zr and $^{102}$Zr. The decays of the bandheads are complicated suggesting that the bands have a large $K$ quantum number. Since the sequences of levels built on the bandheads follow closely rotational model expectations, it is possible to restrict the $K$ values of the bands by considering the ratios of transition energies. This procedure strongly suggests that the $K$ values are 6 and 4 for $^{100}$Zr and $^{102}$Zr, respectively. The assignment of $I=4$ and 5 to the 1821 and 1981 keV levels of $^{102}$Zr is supported by the population [8] of these levels in the $\beta$ decay of $^{102}$Y.

Comparison [1] with calculated single-particle Nilsson levels and experimentally observed bands in the neighboring odd-$N$ isotopes of Zr suggest that the configurations of these bands are $I^\pi=6^+$, $\nu\frac{9}{2}^+$ [404]$\otimes\nu\frac{3}{2}^+$ [411], and $I^\pi=4^-$, $\nu\frac{5}{2}^-$ [532]$\otimes\nu\frac{3}{2}^+$ [411] for the two-quasi-particle bands in $^{100}$Zr and $^{102}$Zr, respectively. Evidence to support these assignments comes from the observed in-band branching ratios. Within the rotational model the ratio of cascade to cross-over transition intensities determines the magnitude of the parameter $(g_K-g_R)/Q_0$, where $g_K$ and $g_R$ are gyromagnetic ratios and $Q_0$ is the intrinsic quadrupole moment of the band. Values of 0.18(2) (eb)$^{-1}$ and 0.13(2) (eb)$^{-1}$ have been determined [1] for the bands based on $K^\pi=\frac{3}{2}^+$ and

*Permanent address: Jagellonian University, Krakow, Poland.

 

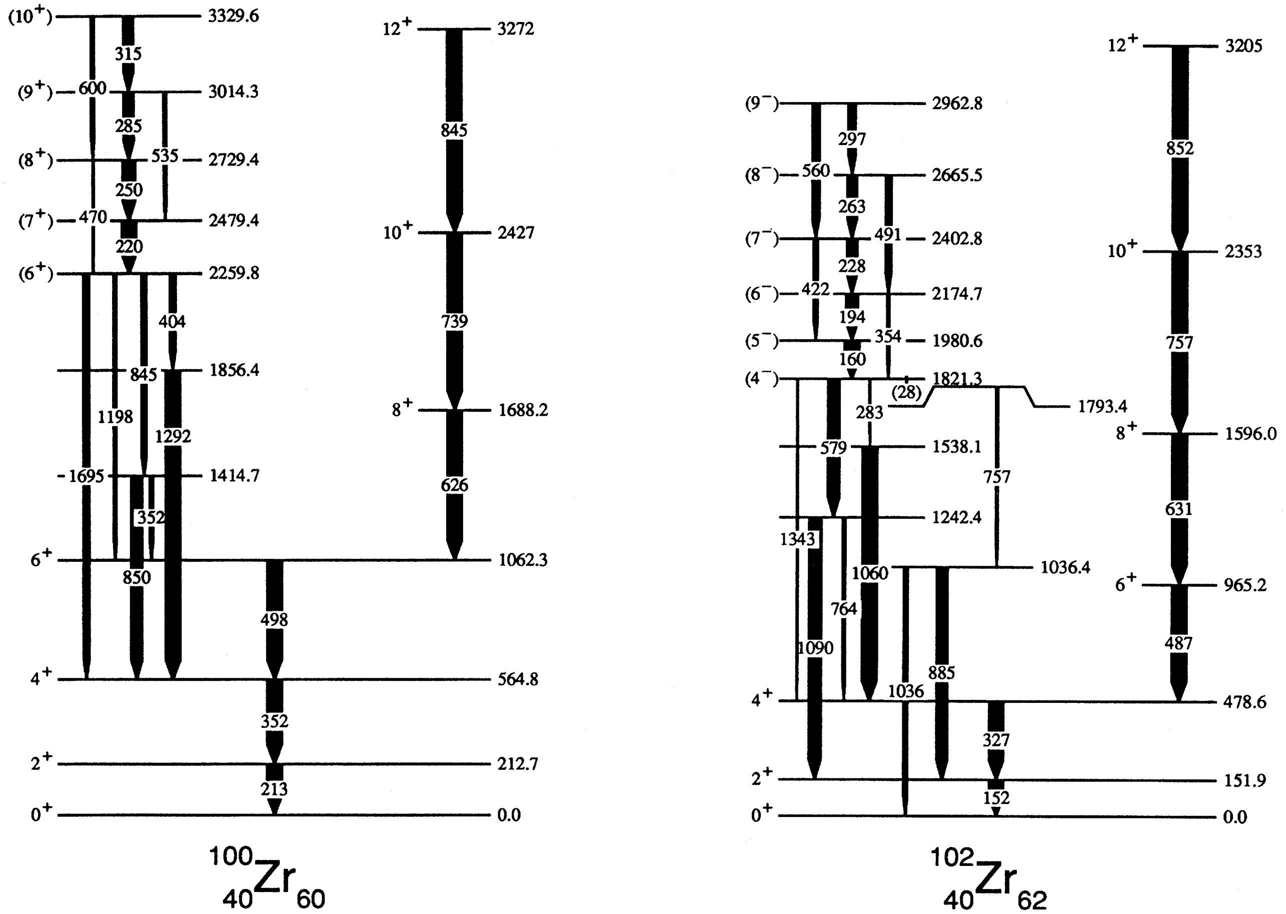


FIG. 1. Partial level and decay schemes for $^{100}$Zr and $^{102}$Zr. The thickness of the arrows indicate the branching ratios of decays from a level.

$K^{\pi}=\frac{5}{2}^{-}$ states, respectively. In the case of a two-quasiparticle band, the magnitude of $(g_K-g_R)/Q_0$ can be predicted from values of the single quasiparticle components, and the prediction for the proposed $K^{\pi}=4^{-}$ band is 0.15(2) $(\mathrm{eb})^{-1}$. This is in excellent agreement with the value of 0.14(1) $(\mathrm{eb})^{-1}$ obtained from the present data, supporting the proposed assignment to the 1821 keV bandhead. No $K^{\pi}=\frac{9}{2}^{+}$ band is known in the odd-$A$ isotopes of Zr, however the $\nu\frac{9}{2}^{+}[404]$ quasineutron has a large $g$-factor $g_K$ and hence the cascade transitions in a rotational band including this single-particle configuration would dominate over the crossover transitions. This is seen to be so in the $^{100}$Zr excited band where the crossover transitions have been observed only weakly if at all. We therefore assign these bands as two-quasi-neutron excitations based upon the configurations suggested above. These assignments are supported by the BCS calculations discussed below, as, at the known deformations of $^{100}$Zr and $^{102}$Zr, the suggested configuration for each nucleus is the lowest-lying two-quasi-neutron state.

The excitation energy of a two-quasiparticle band is determined by the strength of the pairing interaction and the energies of the contributing single-particle states relative to the Fermi level. The single-particle energies are, for a fixed $Z$ and $N$, dependent upon the deformation of the nucleus and the parameters of the potential in which the nucleons move. Therefore, if the nuclear deformation and the potential parameters are known, it is possible within a model description of pairing to determine the strength of the pairing interaction from the bandhead energies. The deformations of $^{100}$Zr and $^{102}$Zr have been calculated, within the rotational model, from the measured [2–4] lifetimes of the first-excited $I^{\pi}=2^{+}$ states. Nilsson potential parameters for the neutron-rich Zr region have been presented in the literature [9]. In order to examine the strength $G$ of the pairing interaction it only remains to specify the pairing model to be used.

We consider the usual monopole pairing force of the form

$$H_{\mathrm{pair}}=-G\sum a_{\nu}^{+}a_{\bar{\nu}}^{+}a_{\bar{\mu}}a_{\mu}, \tag{1}$$

where $G$ is the strength in the BCS [10] procedure and $a$ and $a^{+}$ are particle annihilation and creation operators. For the nuclei in question, the single-particle level densities are rather low and it is therefore essential to take into account the blocking effect of the unpaired nucleons. For a two-quasi-particle state the particle number $N$ is reduced by two and the blocked states are removed from the calculation (see Ref. [11]). In addition to blocking, a further effect which must be taken into account is the Gallagher-Moszkowski

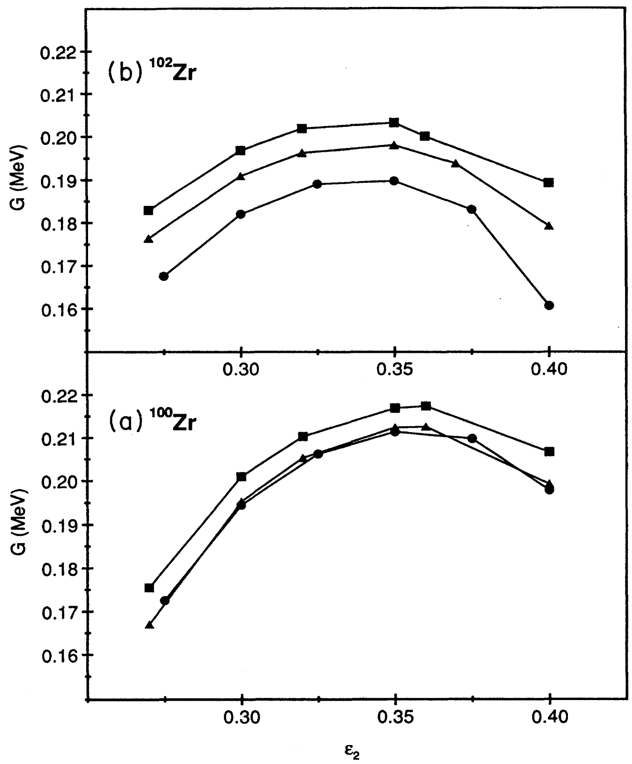


FIG. 2. (a) The pairing strength $G$ determined as a function of the quadrupole deformation parameter $\epsilon_2$ by fitting the bandhead in $^{100}$Zr; (b) similar curves for $^{102}$Zr. Solid squares show the results for $\epsilon_4$ equal to zero, with no spin-spin shift; triangles show results for $\epsilon_4$ equal to zero with the spin-spin shift taken into account; circles show results for G including the spin-spin shift and the effect of the theoretically [14] predicted values of $\epsilon_4$.

(GM) spin-spin shift of the two-quasi-particle states [12]. From two single-quasi-particle levels with $K=K_1$ and $K=K_2$ it is possible to make two two-quasi-particle states with $K_{\text{total}}=K_1 \pm K_2$ and with bandhead spins $I=K_{\text{total}}$. The two corresponding energy levels are split by the spin-spin force. The low-spin members of these doublets have not been observed in this mass region but, from other mass regions [13], a shift of around 200 keV appears to be typical corresponding to a difference of 400 keV between the two members of the doublet. We shall present results of calculations which either ignore or take account of this correction.

The deformation of $^{100}$Zr has been inferred from a precise [3] lifetime measurement, but in the case of $^{102}$Zr there is a large uncertainty in the experimental deformation. We have therefore checked the sensitivity of our results to variations of these deformations around their experimental and theoretically predicted [14] values. Since in Ref. [14] theoretical values of the hexadecapole deformation parameter $\epsilon_4$ are given, we shall also test the sensitivity of our calculations to this parameter. It should be noted that the experimental and theoretical quadrupole deformations of $^{100}$Zr are in excellent agreement.

Figure 2(a) shows the values of $G$ required to fit the two-quasi-particle state in $^{100}$Zr as a function of the deformation parameter $\epsilon_2$ in the Nilsson model. The other Nilsson model parameters $\kappa$ and $\mu$ were taken from Ref. [9] and $\epsilon_4$ was set at zero. Near the expected deformation of $\epsilon_2=0.33$, $G$ is well defined at around 0.21 MeV, and a small reduction is obtained by including the GM shift. Although the GM shift is around 10% of the two-quasi-particle energy, it yields only a 2% reduction in $G$. Inclusion of the predicted $\epsilon_4$ deformation in the calculation gives a further small reduction in $G$. The final value of $G$ for $^{100}$Zr is determined to be close to 0.21 MeV. Figure 2(b) shows the same quantities for the $^{102}$Zr two-quasi-particle state. The theoretical [14] value of $\epsilon_2$ is 0.33, which again corresponds to the situation where $G$ has a stable value. The experimental [2] value of 0.37 is larger but it has a 10% error. In this case the best value of $G$ from the two-quasi-particle energy is around 0.19 MeV, when account is taken of the predicted $\epsilon_4=0.027$.

Although our primary aim has been to fit the energies of the newly observed two-quasi-particle states in order to determine the pairing strength, it is interesting to see if our results are consistent with the pairing energy $P_N(A)$ defined in terms of the relevant neutron separation energies. This is usually accepted as being the quantity which is formally equivalent to the BCS pair gap

$$\Delta=G\sum (j+\tfrac{1}{2})\, u_j v_j , \tag{2}$$

where $u_j$ and $v_j$ are the vacancy and occupancy of the single-particle state of angular momentum $j$. However, in a region of low level density the correspondence between $\Delta$ and $P_N(A)$ is not necessarily a good one and to demonstrate this we shall calculate both $\Delta$ and the theoretical value of the pairing energy, $P_N^{\text{th}}(A)$, as evaluated using nuclear binding energies obtained from the BCS calculation. For $^{100}$Zr, in the region of the expected deformation, the calculated $\Delta$ is around 1.5 MeV, or about 40% larger than the experimental [15] value of 1.08(2) MeV for $P_N(100)$. However, by calculating $P_N^{\text{th}}(100)$ itself in the blocked BCS framework, this discrepancy reduces to 18%. The introduction of the small predicted [14] $\epsilon_4=0.007$ further reduces the discrepancy to around 10%. The supposed approximate equality of $P_N(A)$ and $\Delta$ is thus violated in this region of low level density. Numerical calculations confirm that the equality is restored for higher level densities. In the case of $^{102}$Zr, there remains a discrepancy of slightly more than 20% between $P_N^{\text{th}}(102)$ and the experimental [15] value of 0.78(4) MeV for $P_N(102)$. Despite these small discrepancies between the experimental and theoretically calculated values of the pairing energy, it is notable that with the derived values of $G$, obtained by fitting the two-quasi-particle states, a reduction of 20% in the pairing energy from $^{100}$Zr to $^{102}$Zr is reproduced theoretically. The above results have been obtained using BCS calculations without number projection. Since $P_N$ changes from 1.08 MeV to 0.78 MeV with the addition of just two neutrons these BCS calculations may be inadequate, since the BCS ground-state wave function is a linear superposition of states with different, even, particle numbers. Thus both $N=100$ and 102 are contained in the BCS calculations for both nuclei. Given the observed rapid variations with $N$ it is possible that particle-number-projected BCS calculations with variation after projection [16] may improve the agreement between the calculated pairing energy and that determined from separation energies.

In summary this Rapid Communication has presented the first data on two-quasi-neutron bands in neutron-rich nuclei near mass 100, and has used the observed positions of the bandheads to determine the strength of neutron pairing. A consistent treatment within the BCS method shows the strength to be close to 20/*A* MeV, smaller than the average value of 23/*A* MeV derived [10] from even-even nuclei with neutron to proton ratios appropriate to stability. We have also shown that in a region of low level density the usually accepted equivalence between the pairing energy determined from separation energies and the energy gap $\Delta$ deduced from a BCS calculation needs careful examination.

This work was supported by the Science and Engineering Research Council of the U.K. under Grant No. GRH71161 and by the U.S. Department of Energy under Contract No. W-31-109-ENG-38. The authors are also indebted for the use of the $^{248}$Cm to the Office of Basic Energy Sciences, U.S. Department of Energy, through the transplutonium element production facilities at the Oak Ridge National Laboratory.

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