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Cross section, polarization observables, and phase-shift parameters in *p-d* and *n-d* elastic scattering

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Differential cross section, polarization observables, and phase-shift and mixing parameters up to $L=4$ and $J\leq\frac{11}{2}$ are calculated for *p-d* and *n-d* elastic scattering below the breakup threshold. The method consists in an expansion of the wave function in terms of the pair correlated hyperspherical harmonic basis. The reactance matrix of the system is found by means of the Kohn variational principle. As a difference with the Faddeev technique, the inclusion of the Coulomb interaction does not give any additional difficulty. The accuracy of the method allows for a detailed comparison with experimental results and for the evaluation of Coulomb effects.

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In a recent paper [1] a variational technique to calculate scattering states below the deuteron breakup threshold has been developed for the three-nucleon *N-d* system. In this approach, the wave function of the system is written as a sum of two terms

$$\Psi=\Psi_C+\Psi_A\,. \tag{1}$$

The first term Ψ_C is responsible for the description of the system when the three nucleons are close to each other. It is decomposed in channels labeled by the angular-spin-isospin quantum numbers and the corresponding two-dimensional spatial amplitudes are expanded in terms of the pair correlated hyperspherical harmonic (PHH) basis [2]. The second term Ψ_A is a solution of the Schrödinger equation in the asymptotic region where the incident nucleon and the deuteron are well apart.

The total wave function corresponding to an asymptotic state $^{(2S+1)}L_J$ is [see Eq. (3.9) of Ref. [1]]

$$\Psi_{LSJ}=\sum_{i=1,3}\left[\Psi_C(\mathbf{x}_i,\mathbf{y}_i)+\Omega^R_{LSJ}(\mathbf{x}_i,\mathbf{y}_i)+\sum_{L'S'}{}^J\tilde{R}^{SS'}_{LL'}\Omega^I_{L'S'J}(\mathbf{x}_i,\mathbf{y}_i)\right], \tag{2}$$

where the summation is extended over the three possible choices of the Jacobi coordinates $(\mathbf{x},\mathbf{y})$. L is the relative angular momentum between the nucleon and the deuteron, S is the spin obtained by coupling the spin 1 of the deuteron with the spin $\frac{1}{2}$ of the incident nucleon, and J is the total angular momentum of the system. Except for some constant factors, ${}^J\tilde{R}^{SS'}_{LL'}$ are the reactance matrix ($\mathscr{R}$ matrix) elements and Ω^λ_{LSJ} are the regular ($\lambda\equiv R$) or the irregular ($\lambda\equiv I$) solutions of the Schrödinger equation in the asymptotic region. The explicit form of the internal part Ψ_C is

$$\Psi_C(\mathbf{x}_i,\mathbf{y}_i)=\sum_\alpha\,\phi^C_\alpha(x_i,y_i)\mathscr{Y}_\alpha(jk,i),$$

$$\mathscr{Y}_\alpha(jk,i)=\{[Y_{l_\alpha}(\hat{x}_i)Y_{L_\alpha}(\hat{y}_i)]_{\Lambda_\alpha}[s^{jk}_\alpha s^i_\alpha]_{S_\alpha}\}_{JJ_z}[t^{jk}_\alpha t^i_\alpha]_{TT_z},$$

$$\Phi^C_\alpha(x_i,y_i)=\rho^{l_\alpha+L_\alpha}f_\alpha(x_i)\left[\sum_K u^\alpha_K(\rho)^{(2)}P^{l_\alpha,L_\alpha}_K(\phi_i)\right], \tag{3}$$

where ρ,ϕ_i are the hyperspherical variables and the spatial amplitudes are expanded in terms of the PHH basis functions. The index α in the sum runs over all the channels allowed by the total angular momenta J and the antisymmetrization and parity conditions. In numerical applications the sum is truncated after including all the important channels. The unknown quantities in Eqs. (2) and (3) are the hyperradial functions $u^\alpha_K(\rho)$ and the reactance matrix, which can be found by means of the Kohn variational principle [1]. Once the reactance matrix is calculated, the phase-shift and mixing parameters can be extracted by a simple diagonalization procedure.

To make contact with the experimental results, the *p-d* process is of particular interest since new and accurate experimental data [3,4] have been obtained in the last years, allowing for a detailed comparison with theoretical calculations. The Faddeev method has been successfully applied to calculate the ground state of the three-nucleon system and the *n-d* process; however, such method encounters some difficulties when directly applied to the *p-d* case. They are caused by the long-range behavior of the Coulomb potential. For the *p-d* process, the Faddeev equations in configuration space have been solved at zero energy using a projected Coulomb potential [5] and the corresponding solution was used as a trial input to obtain the scattering length using the Kohn variational principle. In the momentum representation an approximate treatment for the process has been given in Ref. [6] where the Coulomb t matrix was replaced by the potential itself and the final results include a renormalization procedure to correct for the long tail of the Coulomb potential.

With the PHH technique, the inclusion of the Coulomb interaction is straightforward since no partial wave decomposition of the potential is performed. The accuracy in the calculations, when including the Coulomb interaction, is the same as for the n-d case. Therefore, the difference between the p-d and n-d results gives a measure of the Coulomb effects.

Interesting quantities, to be compared with the experimental results, are the differential cross section, the nucleon analyzing power A_y, for which recent experimental results are available [3], and the phase-shift and mixing parameters, for which accurate values have been extracted from the data [4]. The agreement between experimental and theoretical results can be used as a test for the nucleon-nucleon (NN) interaction as well as for the three-nucleon interaction (TNI) terms, when included. The differential cross section is in general well reproduced by the theoretical calculations using different realistic NN potentials, nevertheless some discrepancies are observed. One important reason for them is the wrong prediction of the $^2S_{1/2}$ phase. This phase is correlated to the three-nucleon bound state energy that agrees with the corresponding experimental value only when appropriate TNI terms are included in the Hamiltonian, at least if local NN potentials are used. The A_y analyzing power is extremely sensitive to the ^3P_J NN force, as it has been observed by Witała and Glöckle [7] for the n-d process. The agreement between the theoretical prediction for A_y and the experimental data can be improved by including in the NN interaction charge symmetry breaking (CSB) terms. Such a behavior was pointed out in Refs. [7,8] and it is also corroborated in the present work. Recently [9], accurate results have been obtained for A_y in n-d scattering, but the number of experimental points is still insufficient for extracting a conclusive set of phase parameters. Finally, a detailed check can be done by comparing the experimental and theoretical phase-shift and mixing parameters at various energy values. Old phase-shift analyses [10] can be taken only qualitatively due to the rather large relative errors in the fit. On the other side, the analysis of Ref. [4] is sufficiently accurate to provide a confident check for the considered NN interaction, including TNI and CSB effects.

The investigation of the N-d phases and mixing parameters, calculated by the PHH technique for different NN potentials, has been initiated in Refs. [1,11,12]. In Ref. [11] calculations using the semirealistic potential of Malfliet and Tjon and the Argonne (AV14) interaction have been performed. In Ref. [1] the TNI models of Tucson-Melbourne (TM) and Brazil (BR) have been included. The cutoff parameter of these potentials was fixed in order to reproduce the triton experimental binding energy. In Ref. [12] a detailed comparison between the solution obtained by the PHH technique and the Faddeev equations in momentum space has been performed for elastic n-d scattering. The conclusion was that these two very different methods give extremely close results.

As mentioned above, the inclusion of CSB terms in the NN interaction can improve the agreement with the experimental data. Recently, a new NN interaction of the Argonne type, the AV18 potential [13], has been constructed including charge-dependent terms. A preliminary study of the three-nucleon system properties with such interaction has been done in Ref. [14]. The results for other quantities of interest are presented here. In order to reproduce the binding energy of the three-nucleon system in connection with the AV18 potetial we will use as TNI terms the so-called Urbana model IXb (UR) [15].

TABLE I. Binding energy, kinetic energy, occupation probabilities, and scattering lengths in correspondence to the AV18 and AV18+UR potential models.

	B (MeV)	T (MeV)	$P_{S'}$ (%)	P_D (%)	P_P (%)	^2a (fm)	^4a (fm)
				^{3}H			
AV18	7.61	46.66	1.30	8.50	0.07	1.27	6.33
AV18+UR	8.49	51.34	1.05	9.31	0.13	0.63	6.33
				^{3}He			
AV18	6.91	45.62	1.53	8.46	0.06	1.17	13.6
AV18+UR	7.75	50.27	1.24	9.26	0.13	−0.02	13.7

The AV18 potential has been only recently completed in its final form, therefore it is interesting to know its prediction for the three-nucleon bound state and zero energy processes. The corresponding results are collected in Table I. For the triton bound state the value $B(^3\mathrm{H})=7.61$ MeV is in agreement with the conclusion of Ref. [16], where a value around 7.62 MeV is obtained for all the local potentials considered, fitting the N-N scattering data with a χ^2 per datum near one. In Ref. [16] it has been pointed out the need of including TNI terms when local interactions are used in order to obtain the correct three-nucleon binding. Some of the results obtained for the AV18+UR potential are listed in Table I. For the ^{3}He, ^{3}H mass difference we obtain $\Delta B = B(^3\mathrm{H}) - B(^3\mathrm{He}) = 0.735$ MeV. It has to be noticed that, in the calculation, the difference between the proton and neutron masses has been disregarded. However, it is known that this effect gives an additional contribution of the order of 0.015 MeV [17]. Summing up this contribution to the above estimated mass difference, the resulting value comes close to the experimental one (0.764 MeV). Also the doublet and quartet n-d scattering lengths are well reproduced by the AV18+UR model, while for the p-d process the problem of the extrapolation to zero energy of the experimental results arises, as it has been discussed in Refs. [1,5].

The Kohn variational principle provides a second-order estimate of the $\mathscr{R}$ matrix, which can be diagonalized using the formalism of Seyler [18] and the convention fixed in Ref. [12]. In order to compare with the best available experimental results for p-d and n-d elastic scattering, the incident nucleon energy was taken $E_N=3$ MeV. Our results for the phase-shift and mixing parameters are given in Table II, where incoming states with relative angular momentum $L \leq 4$ and total angular momentum $J \leq \frac{11}{2}$ have been taken into account. It has been checked that the phases corresponding to higher angular momenta do not contribute appreciably to the cross section and polarization observables. The results reported in Table II correspond to the AV18 potential. When using the AV18+UR potential model, some small, but appreciable differences, are observed in channels with $J=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$. The corresponding values are given in Table II in parentheses. For higher channels the effects of the TNI decrease due

TABLE II. Phase-shift and mixing parameters (in degrees) for elastic *p-d* and *n-d* scattering at 3.0 MeV. The results correspond to the AV18 potential. When appreciable differences are observed the results of the AV18+UR model are given in parentheses.

	p-d	*n-d*		*p-d*	*n-d*
$^4D_{1/2}$	−3.56	−3.85	$^2P_{1/2}$	−7.36	−7.49
	(−3.57)	(−3.84)		(−7.37)	(−7.50)
$^2S_{1/2}$	−32.2	−35.3	$^4P_{1/2}$	22.1	24.2
	(−27.8)	(−30.8)		(22.3)	(24.5)
$\eta_{1/2+}$	1.10	1.12	$\varepsilon_{1/2-}$	5.71	6.68
	(1.47)	(1.45)		(5.83)	(6.82)
$^4S_{3/2}$	−63.1	−69.9	$^4F_{3/2}$	0.849	0.920
	(−63.1)	(−69.7)		(0.850)	(0.921)
$^2D_{3/2}$	2.15	2.36	$^2P_{3/2}$	−7.14	−7.18
	(2.15)	(2.36)		(−7.15)	(−7.20)
$^4D_{3/2}$	−3.83	−4.14	$^4P_{3/2}$	24.2	26.0
	(−3.83)	(−4.14)		(24.2)	(26.0)
$\varepsilon_{3/2+}$	0.800	0.747	$\varepsilon_{3/2-}$	−2.20	−2.61
	(0.802)	(0.754)		(−2.23)	(−2.66)
$\zeta_{3/2+}$	1.30	1.35	$\zeta_{3/2-}$	−0.321	−0.265
	(1.30)	(1.35)		(−0.314)	(−0.256)
$\eta_{3/2+}$	−0.322	−0.363	$\eta_{3/2-}$	−3.11	−3.52
	(−0.316)	(−0.356)		(−3.11)	(−3.53)
$^4G_{5/2}$	−0.189	−0.206	$^4P_{5/2}$	23.9	26.0
				(24.1)	(26.3)
$^2D_{5/2}$	2.13	2.33	$^2F_{5/2}$	−0.433	−0.466
$^4D_{5/2}$	−4.13	−4.46	$^4F_{5/2}$	0.876	0.951
$\varepsilon_{5/2+}$	−0.350	−0.315	$\varepsilon_{5/2-}$	0.343	0.538
$\zeta_{5/2+}$	−0.699	−0.701	$\zeta_{5/2-}$	0.932	0.926
$\eta_{5/2+}$	−2.07	−2.04	$\eta_{5/2-}$	−0.343	−0.334
$^4D_{7/2}$	−3.77	−4.07	$^2F_{7/2}$	−0.427	−0.460
$^2G_{7/2}$	0.099	0.107	$^4F_{7/2}$	0.921	1.00
$^4G_{7/2}$	−0.199	−0.214	$\varepsilon_{7/2-}$	−0.143	−0.232
$\varepsilon_{7/2+}$	0.712	0.355			
$\zeta_{7/2+}$	1.14	1.14			
$\eta_{7/2+}$	−0.422	−0.459			
$^2G_{9/2}$	0.098	0.105	$^4F_{9/2}$	0.855	0.922
$^4G_{9/2}$	−0.208	−0.223			
$\varepsilon_{9/2+}$	−0.422	−0.176			
$^4G_{11/2}$	−0.190	−0.208			

to the centrifugal barrier and the two potential models are equivalent. These results can be compared with those of the Bochum group [19] obtained with the Bonn B potential (for *n-d* scattering) and with those of the Faddeev calculation of Ref. [6] obtained with the PEST16 potential [20] (for *p-d* scattering).

In Table III our results for the *p-d* scattering using the four potential models AV14, AV14+BR, AV18, and AV18+UR are compared to the phase-shift experimental analysis given in Ref. [4]. This comparison represents, together with the bound state and zero energy results, a general test that should be satisfied by a realistic *NN* interaction. Figure 1 shows the *p-d* ad *n-d* cross sections calculated with the AV18+UR, which is the most accurately determined of the four above-mentioned models. The experimental points correspond to the *p-d* scattering and are taken from Ref. [3]. Figures 2 and 3 display the neutron and proton analyzing power A_y, respectively, for all the four potential models. The experimental points are from Ref. [9] (*n-d*) and from Ref. [3] (*p-d*).

It can be seen from Tables II and III that the inclusion of the TNI terms mainly affects the $^2S_{1/2}$ phase and the corresponding mixing parameter $\eta_{1/2+}$. The changes in these quantities are in the correct direction, nevertheless the final values still differ in some measure from those directly derived from the experimental data. It can be noticed that, if the

TABLE III. The phase-shift and mixing parameters (in degrees) calculated for p-d scattering at 3.0 MeV are given together with the phase-shift analysis of Ref. [4].

	AV14	AV14+BR	AV18	AV18+UR	Fit (Ref. [4])
Phase shifts					
$^4S_{3/2}$	−63.70	−63.63	−63.11	−63.10	−63.95±0.28
$^2S_{1/2}$	−31.41	−27.70	−32.17	−27.78	−24.87±0.35
^4P	23.83	24.05	23.41	23.54	23.37±0.11
$^4P_{5/2}$-$^4P_{3/2}$	−0.48	−0.37	−0.27	−0.15	0.01±0.06
$^4P_{3/2}$-$^4P_{1/2}$	1.77	1.60	2.10	1.95	2.49±0.11
^2P	−7.29	−7.30	−7.25	−7.26	−7.11±0.24
$^2P_{3/2}$-$^2P_{1/2}$	0.23	0.22	0.22	0.22	−0.14±0.25
^4D	−3.90	−3.90	−3.82	−3.82	−3.74±0.09
$^4D_{7/2}$-$^4D_{5/2}$	0.39	0.38	0.36	0.36	0.31±0.06
$^4D_{5/2}$-$^4D_{3/2}$	−0.32	−0.32	−0.30	−0.30	−0.28±0.03
$^4D_{3/2}$-$^4D_{1/2}$	−0.29	−0.30	−0.27	−0.28	−0.16±0.02
^2D	2.19	2.19	2.14	2.14	1.99±0.09
$^2D_{5/2}$-$^2D_{3/2}$	−0.03	−0.03	−0.02	−0.02	0.0 (fixed)
Mixing parameters					
$\eta_{1/2+}$	1.26	1.67	1.10	1.47	2.00±0.10
$\zeta_{3/2+}$	1.39	1.38	1.30	1.30	1.20±0.10
$\eta_{3/2+}$	−0.34	−0.33	−0.32	−0.32	−0.28±0.10
$\varepsilon_{1/2-}$	6.22	6.40	5.71	5.83	5.73±0.13
$\varepsilon_{3/2-}$	−2.29	−2.36	−2.20	−2.23	−2.47±0.05
$\zeta_{3/2-}$	−0.33	−0.32	−0.32	−0.31	−1.25±0.54
$\eta_{3/2-}$	−3.34	−3.36	−3.11	−3.11	−3.13±0.23
$\zeta_{5/2-}$	1.00	0.99	0.93	0.92	0.62±0.20
$\eta_{5/2-}$	−0.37	−0.37	−0.34	−0.34	−0.45±0.04
$\varepsilon_{3/2+}$	0.83	0.84	0.80	0.80	2.15±0.24
$\varepsilon_{5/2+}$	−0.37	−0.37	−0.35	−0.35	−0.75±0.12

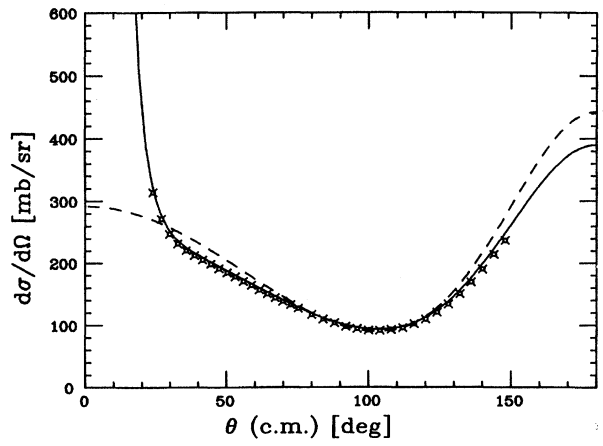

FIG. 1. p-d (solid line) and n-d (dashed line) cross sections calculated with the AV18+UR NN potential model. The experimental points are from Ref. [3].

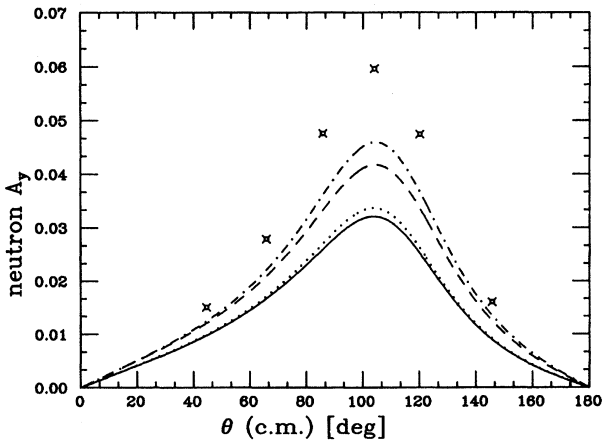

FIG. 2. The neutron analyzing power A_y calculated with the four potential models used in the present work: AV14 (solid line), AV14+BR (dotted line), AV18 (dashed line), and AV18+UR (dash-dotted line). The experimental points are from Ref. [9].

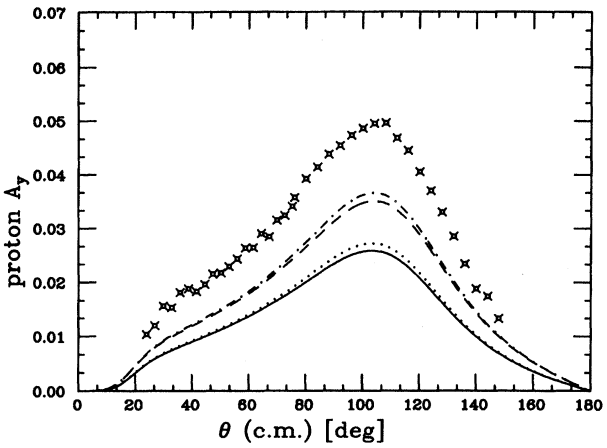

FIG. 3. The same as Fig. 2 for the proton analyzing power. The experimental points are from Ref. [3].

energy approaches zero, the $^2S_{1/2}$ phase goes to the doublet scattering length. It follows from Table I, for the AV18+UR model, that the corresponding doublet scattering length value is in agreement with the experimental one (for n-d scattering), but at $E_N=3.0$ MeV the results are no longer in agreement (we are referring now to the p-d process due to the lack of sufficient n-d experimental data). Correspondingly, the calculated mixing parameter $\eta_{1/2+}$ results to be small suggesting a stronger coupling between the S and D waves.

The P-wave parameters are slightly influenced by the TNI, but the corresponding changes in the analyzing power are of interest. By inspection of Table III it is evident that the average values for the ^4P and ^2P phases are in good agreement with those from the experimental data, but the differences $^4P_{5/2}$-$^4P_{3/2}$ and $^2P_{3/2}$-$^2P_{1/2}$ appear to be wrong. For the AV18 and AV18+UR models these differences are reduced, with a corresponding appreciable improvement of the analyzing power A_y. The strong dependence of A_y with respect to the P_J phases and, more particularly, with respect to the ^3P_J NN forces is a well-known fact [7,8]. This sensibility can be used as a fine tune for realistic NN interactions, as it has been done, for example, in Ref. [19].

Finally, it is worthwhile to notice that the AV18+UR model and the Bonn B potential, which are very different from each other, yield very similar phase shifts and mixing parameter values.

Let us summarize the main conclusions of the present paper. First of all, it is possible, at present, to calculate with a nice accuracy the phase shift and mixing parameters for the N-d elastic scattering. Detailed comparisons between the predictions of a given NN interaction model for the cross section and polarization observables with the corresponding experimental values can be done. Such comparisons will be useful to improve our knowledge and to evaluate the merits (and possible limits) of realistic NN potentials within the frame of the standard approach to the nuclear structure problem.

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