RAPID COMMUNICATIONS

PHYSICAL REVIEW C VOLUME 52, NUMBER 3 SEPTEMBER 1995

Color fluctuations in hadrons and proton coherent diffractive dissociation on helium

M. Strikman
Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802
and Institute of Nuclear Physics, St. Petersburg, Russia

V. Guzey
Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802
(Received 5 June 1995)

The differential cross section of inelastic coherent diffractive dissociation off nuclei $p+{}^4\mathrm{He}\rightarrow X+{}^4\mathrm{He}$ is expressed in terms of the relative cumulants of the cross-section distribution $P_N(\sigma)$. The theoretical result for the ratio $r=(d\sigma_{\mathrm{diff}}/dt)^{p\mathrm{He}}_{t=0}/(d\sigma_{\mathrm{diff}}/dt)^{pp}_{t=0}=6.8\text{–}7.6$ is close to the value $r=7.1\pm0.7$ which we extracted from the FNAL data. These are the only $A>2$ data of this kind. The comparison provides the first confirmation of the color/cross-section fluctuation approach to the description of the absolute value of the inelastic diffraction cross section off nuclei. It provides also a new constraint on the first four cumulants of the cross-section distribution.

PACS number(s): 24.85.+p, 13.85.Hd, 25.40.Ve, 25.80.Hp

One of the manifestations of the composite structure of hadrons is that constituents of rather different size are present in hadrons. At high energies the transition time from one configuration to another exceeds by far the time of the interaction with the target:

$$\frac{2p_{\mathrm{lab}}}{M^2-m^2}\gg 2R. \qquad (1)$$

Here M is the mass of an inelastic cross-section eigenstate and m is the ground-state mass of the hadron; R is a longitudinal length characterizing the interaction of the target. Hence one can consider these fluctuations as frozen during the collision time and then integrate over the probability of configurations in a projectile. Since these configurations interact with very different strength (mostly because of color fluctuations in hadrons), one should take the fluctuation of the interaction strength—cross-section fluctuations—into account in a realistic picture of hadron-nucleus interactions; for a recent review see [1].

The convenient formalism to accommodate this physics—the scattering eigenstate formalism—was suggested long ago by Good and Walker [2]. The projectile wave function $|\Psi\rangle$ is expanded as a sum of eigenstates of the purely imaginary T matrix (for simplicity we consider the scattering amplitude as purely imaginary)

$$|\Psi\rangle=\sum_{\kappa} c_{\kappa}|\psi_{\kappa}\rangle, \quad \sum_{\kappa}|c_{\kappa}|^2=1, \qquad (2)$$

provided

$$\mathrm{Im}\, T|\psi_{\kappa}\rangle=T_{\kappa}|\psi_{\kappa}\rangle. \qquad (3)$$

T_{κ} is the cross section for the interaction of projectile configuration k with the target. In this basis there are no transitions between different states and this makes it possible to describe a number of physical processes in terms of a distribution over the values of the cross section $P(\sigma)$.

Namely, $P(\sigma)=\Sigma_{\kappa}|c_{\kappa}|^2\delta(\sigma-T_{\kappa})$ gives the probability that a given configuration interacts with a nucleon with a total cross section σ. It allows us to treat the projectile as a coherent superposition of scattering eigenstates, each with an eigenvalue σ. This idea gives rise to the term *hadronic cross-section fluctuations*.

In Refs. [3,4] $P(\sigma)$ was determined for pion and nucleon projectiles using data on diffraction off nucleons and deuterons, as well as the analog of the quark counting rules to fix the behavior of $P(\sigma)$ for small σ. Furthermore, $P_{\pi}(\sigma\ll\langle\sigma\rangle)$ was calculated directly in QCD [3]. More recently a similar technique was used to calculate the process of electroproduction of ρ mesons by longitudinally polarized virtual photons [5]. The predictions of Ref. [5] were very recently confirmed by the ZEUS experiment at HERA [6], thus providing indirect confirmation of the calculation of $P_{\pi}(\sigma\ll\langle\sigma\rangle)$.

It was suggested in Ref. [7] that the data on inelastic coherent diffraction off nuclei would provide a critical test of the concept of cross section fluctuations. The total cross sections of diffraction dissociation were calculated. However practically no data on the total cross section of coherent diffractive dissociation are available and only a comparison of the predicted A dependence of this cross section with the A dependence of exclusive channels measured at FNAL was possible.

However there exist previously overlooked unique data on the the process of proton inelastic diffractive scattering off ^{4}He which were obtained nearly 15 years ago in the FNAL jet target experiment [8]. So in this paper we will analyze these data to obtain another test of the discussed approach. The specific feature of the ^{4}He nucleus is that its radius is small: so, in deference to the approach used in Ref. [7], we cannot neglect the slope of the diffractive amplitude β, as compared to the slope of the ^{4}He form factors.

Diffractive scattering occurs when the final state has the same quantum numbers as the incident hadron h, that is, whenever it overlaps any $|\psi_{\kappa}\rangle$. Thus, subtracting the elastic contribution, we can write

$$\left(\frac{d\sigma_{\rm diff}}{dt}\right)^{h\rm He}=\frac{1}{16\pi}\sum_{\kappa}|c_\kappa|^2T_\kappa^2-\left(\sum_{\kappa}|c_\kappa|^2T_\kappa\right)^2. \quad (4)$$

Here T_κ is the elastic scattering amplitude for a hadron cross-section eigenstate $|\psi_\kappa\rangle$ scattering off a nucleus of ^{4}He. This formula enables us to investigate the relation between the differential cross section, which can be extracted from experimental data, and the moments of the distribution $P(\sigma)$, which describe the cross-section fluctuations and contain information about the hadronic structure.

When the instantaneous configuration can be considered frozen, the scattering process should be calculated first for the particular configuration and then integrated over all configurations which satisfy Eq. (1), weighted by the probability of the configuration. In doing so one essentially uses the completeness of the intermediate and final states.

Let us suppose that the incident hadron in state κ scatters off the nucleus of ^{4}He in states ι, $\jmath$, ℓ, m of its four nucleons. Then, to determine the scattering amplitude of this process, $F^\kappa_{\iota\jmath\ell m}$, we employed the Glauber method. This method requires knowledge of the amplitude of the hadron-hadron scattering and the wave function of ^{4}He. The parametrization of the hadron-hadron amplitude was taken as

$$f_\iota^\kappa(\vec{k}-\vec{k}')=\frac{\iota}{4\pi}\,\sigma_\iota^\kappa e^{-\frac{\beta}{2}(\vec{k}-\vec{k}')2}. \quad (5)$$

Here σ_ι^κ is the total scattering cross section for the hadron and nucleon in configurations κ and ι respectively; β is a parameter whose numerical value will be discussed further below. The wave function of ^{4}He and Φ was taken in a simple form [9]:

$$\Phi=B\prod_{\iota=1}^{4}\exp(-\alpha\cdot p_\iota^2)\cdot\delta\left(\sum_{\iota=1}^{4}p_\iota\right), \quad (6)$$

with $\alpha=23\ ({\rm GeV}/c)^2$. It leads to the single nucleon form factor $F_{^4{\rm He}}(q^2)=\exp(-\frac{3}{8}\alpha q^2)$.

This form of the wave function allows us to reproduce well the total cross section of p^4He scattering as well as the elastic cross section at small t.

For given instantaneous configurations of the projectile and the target, and at zero transverse momentum, the Glauber method leads to $F^\kappa_{\iota\jmath\ell m}$

$$\mathrm{Im}F^\kappa_{\iota,\jmath,\ell,m}=\frac{\sigma_\iota^\kappa}{\pi}-\frac{\sigma_\iota^\kappa\sigma_\jmath^\kappa}{16\pi^2(\alpha+\beta)}+\frac{\sigma_\iota^\kappa\sigma_\jmath^\kappa\sigma_\ell^\kappa}{48\pi^3(\alpha+\beta)^2}-\frac{\sigma_\iota^\kappa\sigma_\jmath^\kappa\sigma_\ell^\kappa\sigma_m^\kappa}{1024\pi^4(\alpha+\beta)^3}. \quad (7)$$

One can see from Eq. (7) that in the ^{4}He case one cannot neglect the slope of the rescattering amplitude as compared to the slope of the nuclear many body form factor, $\beta/\alpha\approx 0.5$ for the case of a nucleon projectile. At the same time the parameter $\langle\beta\rangle+\alpha$ is sufficiently large as compared to the possible changes of β related to the fluctuations of σ. We can thus neglect fluctuations of β.

After averaging over the configurations of the target,

$$\mathrm{Im}T_\kappa=4\pi\sum_{\iota\jmath\ell m}|c_\iota|^2|c_\jmath|^2|c_\ell|^2|c_m|^2\mathrm{Im}F^\kappa_{\iota\jmath\ell m}, \quad (8)$$

we obtain the elastic amplitude for a hadron cross-section eigenstate $|\psi_\kappa\rangle$ scattering off ^{4}He:

$$\mathrm{Im}T_\kappa=4\sigma_\kappa-\frac{3}{4\pi(\alpha+\beta)}\sigma_\kappa^2+\frac{1}{12\pi^2(\alpha+\beta)^2}\sigma_\kappa^3-\frac{1}{256\pi^3(\alpha+\beta)^3}\sigma_\kappa^4. \quad (9)$$

Thus we expressed the eigenvalues of the scattering eigenstate $|\psi_\kappa\rangle$ for the interaction with ^{4}He in terms of the eigenvalues σ_κ of the interaction of $|\psi_\kappa$ with a nucleon. Therefore, from Eq. (4) we see that the differential cross section is given by

$$\left(\frac{d\sigma_{\rm diff}}{dt}\right)^{h\rm He}_{t=0}=\frac{1}{16\pi}\left\{16(\langle\sigma^2\rangle-\langle\sigma\rangle^2)-\frac{6\langle\sigma\rangle^3}{\pi(\alpha+\beta)}\left(\frac{\langle\sigma^3\rangle}{\langle\sigma\rangle^3}-\frac{\langle\sigma^2\rangle}{\langle\sigma\rangle^2}\right)+\frac{59\langle\sigma\rangle^4}{48\pi^2(\alpha+\beta)^2}\left(\frac{\langle\sigma^4\rangle}{\langle\sigma\rangle^4}-\frac{27}{59}\frac{\langle\sigma^2\rangle^2}{\langle\sigma\rangle^4}-\frac{32}{59}\frac{\langle\sigma^3\rangle}{\langle\sigma\rangle^3}\right)-\frac{5\langle\sigma\rangle^5}{32\pi^3(\alpha+\beta)^3}\left(\frac{\langle\sigma^5\rangle}{\langle\sigma\rangle^5}-\frac{4}{5}\frac{\langle\sigma^3\rangle\cdot\langle\sigma^2\rangle}{\langle\sigma\rangle^5}-\frac{1}{5}\frac{\langle\sigma^4\rangle}{\langle\sigma\rangle^4}\right)\right\}. \quad (10)$$

Here we neglected terms proportional to $\langle\sigma\rangle^6/(\alpha+\beta)^4,\ldots,\langle\sigma\rangle^8/(\alpha+\beta)^6$ with an accuracy of 3%.

Similarly, the differential cross section of hadron-hadron scattering can be written in terms of averaging over internal configurations of the hadron [10]

$$\left(\frac{d\sigma_{\rm diff}}{dt}\right)^{hh}_{t=0}=\frac{1}{16\pi}\{\langle\sigma^2\rangle-\langle\sigma\rangle^2\}. \quad (11)$$

It is convenient to introduce the ratio of differential cross sections r

$$r=\left(\frac{d\sigma_{\rm diff}}{dt}\right)^{p\rm He}_{t=0}\Bigg/\left(\frac{d\sigma_{\rm diff}}{dt}\right)^{pp}_{t=0}. \quad (12)$$

Defining a factor γ and the second cumulant κ_2 as

$$\gamma=\frac{\langle\sigma\rangle}{\pi(\alpha+\beta)},\quad \kappa_2=\frac{\langle\sigma^2\rangle-\langle\sigma\rangle^2}{\langle\sigma\rangle^2}, \quad (13)$$

we can write the ratio r as

$$r=16-\frac{6\gamma}{\kappa_2}\left(\frac{\langle\sigma^3\rangle}{\langle\sigma\rangle^3}-\frac{\langle\sigma^2\rangle}{\langle\sigma\rangle^2}\right)+\frac{59}{48}\frac{\gamma^2}{\kappa_2}\left(\frac{\langle\sigma^4\rangle}{\langle\sigma\rangle^4}-\frac{27}{59}\frac{\langle\sigma^2\rangle^2}{\langle\sigma\rangle^4}-\frac{32}{59}\frac{\langle\sigma^3\rangle}{\langle\sigma\rangle^3}\right)-\frac{5}{32}\frac{\gamma^3}{\kappa_2}\left(\frac{\langle\sigma^5\rangle}{\langle\sigma\rangle^5}-\frac{4}{5}\frac{\langle\sigma^3\rangle\langle\sigma^2\rangle}{\langle\sigma\rangle^5}-\frac{1}{5}\frac{\langle\sigma^4\rangle}{\langle\sigma\rangle^4}\right). \quad (14)$$

For numerical analysis we consider the case of a proton projectile for which data are available. We will use the distribution function $P_N(\sigma)$ in the form proposed in [3,4]

$$P_N(\sigma)=N(a,n)\frac{\sigma/\sigma_0}{\sigma/\sigma_0+a}e^{-(\sigma-\sigma_0)^n/(\Omega\sigma_0)^n}. \quad (15)$$

This distribution function has been considered for the three cases $n=2,6,10$ and fitted to the characteristic value $\kappa_2=0.25$. The quantities $\langle\sigma^3\rangle/\langle\sigma\rangle^3$, $\langle\sigma^4\rangle/\langle\sigma\rangle^4$, and $\langle\sigma^5\rangle/\langle\sigma\rangle^5$ which enter into (14) are significantly different from 1. For example, for $n=2$, $\langle\sigma^3\rangle/\langle\sigma\rangle^3=1.82$, $\langle\sigma^4\rangle/\langle\sigma\rangle^4=2.97$, and $\langle\sigma^5\rangle/\langle\sigma\rangle^5=5.3$.

To reduce the influence of the specific type of the distribution function, it is handy to introduce the relative cumulants [11]:

$$\kappa_3=\frac{\langle(\sigma-\langle\sigma\rangle)^3\rangle}{\langle\sigma\rangle^3},\quad \kappa_4=\frac{\langle(\sigma-\langle\sigma\rangle)^4\rangle}{\langle\sigma\rangle^4},\quad \kappa_5=\frac{\langle(\sigma-\langle\sigma\rangle)^5\rangle}{\langle\sigma\rangle^5}. \quad (16)$$

It allows us to rewrite the expression for r in a form which is less sensitive to the particular choice of the distribution function $P_N(\sigma)$, since the relative cumulants are small:

$n=2$, $\Omega=1.5$, $a=1$	$n=6$, $\Omega=1.1$, $a=0.1$	$n=10$, $\Omega=11$, $a=1$
$\kappa_3=0.07$	$\kappa_3=0$	$\kappa_3=0$
$\kappa_4=0.19$	$\kappa_4=0.13$	$\kappa_4=0.13$
$\kappa_5=0.15$	$\kappa_5=0.02$	$\kappa_5=0$

Making use of the cumulants, r can be presented as

$$r=16-12\gamma+4.25\gamma^2-0.875\gamma^3+\kappa_2(-0.563\gamma^2+0.375\gamma^3)-\frac{\kappa_3}{\kappa_2}(6\gamma-4.25\gamma^2+1.34\gamma^3-0.125\kappa_2\gamma^3)+\frac{\kappa_4}{\kappa_2}(1.23\gamma^2-0.75\gamma^3)-0.156\gamma^3\frac{\kappa_5}{\kappa_2}. \quad (17)$$

At this point let us examine the experimental data. We use here information on proton diffractive dissociation from ^{4}He [8] and proton [12] targets at small momentum transfer and average energy 300 GeV. These are the only experimental data on inclusive coherent diffraction of nuclei existing at the moment. To the best of our knowledge they were overlooked for many years. Using proposed exponential parametrization for the cross sections at small momentum transfer, we extrapolated data to $t=0$. In both cases the differential cross section $d^2\sigma/dtdM^2$ was integrated over the region 2.5 (GeV)$^2<M^2<8$ (GeV)^2 for which experimental data are available. The value of the ratio of differential cross sections, r, extracted from experimental data, is found to be

$$r=\int_{2.5(\mathrm{GeV})^2}^{8(\mathrm{GeV})^2}\left(\frac{d\sigma^{p\mathrm{He}}}{dM^2dt}\right)_{t=0}dM^2\Bigg/\int_{2.5(\mathrm{GeV})^2}^{8(\mathrm{GeV})^2}\left(\frac{d\sigma^{pp}}{dM^2dt}\right)_{t=0}dM^2=7.1\pm0.7. \quad (18)$$

The main error comes from the procedure of extrapolation to $t=0$.

Note that the theoretical value of r implies the integration over all diffractive masses M^2. The available data cover most of the interval. The error coming from the fact that the integrations in Eq. (18) is performed over the interval excluding the part of diffractive masses M^2 is small. Moreover, we checked that the ratio $(d\sigma^{p\mathrm{He}}/dM^2dt)_{t=0}/(d\sigma^{pp}/dM^2dt)_{t=0}$ depends on M^2 weakly. Hence the ratio of the small corrections to the cross sections $(d\sigma^{p\mathrm{He}}/dM^2dt)_{t=0}$ and $(d\sigma^{pp}/dM^2dt)_{t=0}$ originating from the region not covered experimentally is approximately equal to r which makes the error even smaller. Thus the correction related to the inclusion of all diffractive masses in the integrals in Eq. (18) is small as compared to the main error.

The extracted value of r should be compared with $r=16$ which one would expect in the impulse approximation.

For the available region of M^2 the parameter β describing the amplitude of diffractive scattering was found to be 8 ± 1 (GeV/c)$^{-2}$. We should use this value together with $\beta=13\pm0.5$ (GeV/c)$^{-2}$ corresponding to elastic proton-proton scattering because the process of interest includes both types of nucleon interaction. We notice that the diagrams of the studied process contain an equal number of vertices of both types. Hence, it seems natural to use for β the mean value of these two values.

We found $\beta=10.5\pm0.6$ (GeV/c)$^{-2}$ and $\gamma=1.01\pm0.02$. With these results we can now present r as (we do not give here the errors for the coefficients since the errors are correlated)

$$r=7.32-0.19\kappa_2-\frac{\kappa_3}{\kappa_2}(3.1-0.13\kappa_2)+0.51\frac{\kappa_4}{\kappa_2}-0.16\frac{\kappa_5}{\kappa_2}. \quad (19)$$

One can see from Eq. (19) that for the small values of cumulants the result is mainly sensitive to the value of κ_3/κ_2. Thus our theoretical predictions for r,

$$n=2, \quad r=6.79\pm0.13,$$

$$n=6, \quad r=7.53\pm0.13,$$

$$n=10, \quad r=7.54\pm0.13, \tag{20}$$

differ mainly because of the different values of κ_3/κ_2 for $n=2$ and for $n=6,10$. The range given by Eq. (20), $r=6.8$–7.6, is consistent with $r=7.1\pm0.7$ extracted from the data.

We calculated the differential cross section $d\sigma/dt$ of the coherent diffractive dissociation of protons off ^{4}He at zero momentum transfer and at high energy in terms of the relative cumulants of the distribution $P_N(\sigma)$. We found that the data are sensitive to the moments $\int\sigma^n P_N(\sigma)d\sigma$ up to $n=5$. Current models of $P_N(\sigma)$ describe the ^{4}He data with an accuracy of about 10%. The data support the small value of the ratio κ_3/κ_2 already indicated by an analysis [4] of the deuteron data as well as a rather large dispersion of $P_N(\sigma)$ around the mean value σ. Clearly new measurements of the t dependence of the cross section of diffractive dissociation at small t using ^{4}He and ^{3}He are necessary using modern jet targets. They would allow one to separate different terms in Eq. (10) and would significantly improve sensitivity to details of the distribution $P_N(\sigma)$.

We thank L. Frankfurt and G. A. Miller for useful discussions. This work was partially supported by the U.S. DOE.

[1] L.L. Frankfurt, G.A. Miller, and M.I. Strikman, Annu. Rev. Nucl. Part. Phys. **44**, 501 (1994).
[2] M. Good and W. Walker, Phys. Rev. **120**, 1957 (1960).
[3] B. Blättel, G. Baym, L. Frankfurt, and M. Strikman, Phys. Rev. Lett. **71**, 896 (1993).
[4] B. Blättel, G. Baym, L. Frankfurt, and M. Strikman, Phys. Rev. D **47**, 2761 (1993).
[5] S. Brodsky, L. Frankfurt, J.F. Gunion, A.H. Mueller, and M. Strikman, Phys. Rev. D **50**, 3134 (1994).
[6] M. Derrick *et al.*, Exclusive ρ^0 production in deep inelastic electron-proton scattering at Hera, Report No. DESY-95-133.
[7] L. Frankfurt, G.A. Miller, and M. Strikman, Phys. Rev. Lett. **71**, 2859 (1993).
[8] A. Bujak *et al.*, Phys. Rev. D **23**, 1911 (1981).
[9] E. Levin and M. Strikman, Sov. J. Nucl. Phys. **23**, 216 (1976).
[10] H. Miettinen and J. Pumplin, Phys. Rev. D **18**, 1696 (1978).
[11] A. Bulgac and L. Frankfurt, Phys. Rev. D **48**, R1984 (1993).
[12] A. Kuznetsov *et al.*, Sov. J. Nucl. Phys. **30**, 529 (1979).