Deuteron formation in expanding nuclear matter from a strong coupling BCS approach

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The process of deuteron formation in intermediate heavy ion reactions is approached within the strong coupling BCS theory assuming that the final stage of the reaction can be described as an adiabatic expansion of a piece of nuclear matter. Since the gap equation in the $^3S_1$-$^3D_1$ channel goes over into the deuteron Schrödinger equation in the low density limit, a smooth transition from the superfluid Cooper pair phase to a Bose deuteron gas is found. For a fixed entropy ranging from 0.5 to 2 units per particle the deuteron fraction, the chemical potential and temperature are reported as a function of density. For densities down to $\rho = 0.1$ fm^{-3} and lower, the deuteron-to-nucleon ratio rapidly increases from a density threshold strongly depending on the entropy. Decreasing further the density this ratio tends logarithmically to one. The possible relevance of these results for heavy ion collisions and the shortcomings of the present approach are briefly discussed.

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I. INTRODUCTION

Central heavy ion collisions at E/A in the 50 to 200 MeV range can roughly be described by the initial buildup of compressed and hot nuclear matter and by a sequential decompression. At low energies the system may pass through the spinodal instability region and may undergo a liquid-gas phase transition or, at higher energies, the temperature and the internal pressure involved may be so high that the piece of nuclear matter just vaporizes into the individual nucleons. In any case, however, the expanding gas cools down rather fast, allowing for deuterons, as well as other fragments, to form again in the final state. Actually the yields of deuterons in high energy reactions always exhibit a surprisingly high production rate [1]. Typically, e.g., the ratio of deuterons to protons is of the order of unity. At higher bombarding energies ($E/A \geq 200$ MeV) such abundances have been successfully explained from a transport model [2], where the deuteron pole in the n-p scattering matrix has been explicitly isolated. Though pure phase space considerations may account for a large fraction of the deuteron production rate [3], it still may be interesting to investigate the problem, for instance, of deuteron production from a slightly different point of view. We have in mind the very idealized situation of a piece of hot nuclear matter expanding quasistationarily with total entropy kept constant. The latter feature seems to be realized in heavy ion reactions [4]. When lowering the density, the temperature will then drop, and the combination of both effects will allow the appearance of deuterons (Mott transition) [5]. The phenomenological observation that the main components of the outgoing fragments, in this energy region, are nucleons and deuterons and α particles indicates that nuclear matter has not enough time, during the expanding phase, to get close to its slowest free energy state. In fact, at low enough densities and temperatures, nuclear matter is expected to form a gas of alpha particles. Therefore a mixture of nucleons and deuterons is a metastable system, which spontaneously decays in a gas of alpha particles. This metastable system, however, is worth studying, since it is likely to have some relevance in the final stage of heavy ion reactions. The characteristic time of the expanding phase is, therefore, assumed to be faster than the alpha formation rate, consistently with phenomenology, and slow enough, as already mentioned, to consider nuclear matter not too far from thermal equilibrium.

In this work we will be mainly concerned with the deuteron formation. An aspect we will partially develop is that nuclear matter at low density, below a certain critical temperature T_c, may show strong n-p pairing in the deuteron channel. This has been revealed in several recent studies [5–8] and, in fact, it can be anticipated since the NN force in the deuteron channel is more attractive than in the usual p-p or n-n pairing channels. At higher bombarding energies the entropy production is typically several units and this is probably too high for the superfluid phase to occur, but at lower energies ($E/A \leq 100$ MeV) the entropy per nucleon $S/A < 1$ [9] and this may well allow the approach of deuteron pair condensation. The reason why such a pair condensate may be relevant for the deuteron production an qualitatively be explained, at least, within the BCS approach to superfluidity. In this approach it has been shown [10] that in the low density limit the BCS equation for the pairing condensate goes over into the Schrödinger equation for the deuteron. In a first attempt we will try to get some insight into the physics of this process in adopting a mean field strong coupling BCS approach. We know about the weakness of this approach, which completely leaves aside the influence of fluctuations, that, in this type of situation, may be crucial, both with respect to the finiteness of the system and with respect to a possible crossover from BCS superconductivity to Bose-Einstein condensation of the deuterons. Indeed, in the limit of interparticle distance much larger than the deuteron radius a_0, i.e., $k_F a_0 \ll 1$, the deuterons behave essentially as a gas of bosons, which can undergo the usual condensation phenomenon. We may deal with this difficult problem in the future. Here we will stay within the pure BCS approach.

II. STRONG COUPLING BRUECKNER-BCS APPROACH

Proton-neutron pairing in nuclear matter has only been investigated very recently [5–7]. Using realistic bare forces in the gap equation and a Brueckner–Hartree-Fock G-matrix approach for the normal mean field, astonishing high values for the gap in the deuteron channel have been found in symmetric nuclear matter. Since the attraction in the deuteron channel, i.e., SD $T=0$ channel, is stronger than in the $T=1$ neutron-neutron or proton-proton channels, this may actually not be a complete surprise. However, the in-medium renormalization (screening) of the bare n-p force is certainly an unsolved problem, in spite of the fact that the use of bare n-n and p-p forces yields quite reasonable values for the gap Δ in the scalar isovector channel $T=1$ [11]. Bearing this difficulty in mind, we nonetheless go on using the bare Paris force as the n-p interaction in the S-D pairing channel, which in the mostly interesting low density regime may eventually not be a quite unreasonable choice.

For the normal mean field we use, as in an early paper [11], the one from the Brueckner–HF approach. The equations to be solved are therefore of the form schematically shown in Fig. 1. The intermediate two particle propagator g (the two lines in the figure) reads

$$g = \frac{(1-f_k)(1-f_{k'})}{\omega - \epsilon_k - \epsilon_{k'} + i\eta} , \qquad (1)$$

where f_k is the Fermi function, calculated at the single particle energy ϵ_k, the latter being obtained from the Brueckner–HF self-consistent potential, schematically indicated in the lower part of Fig. 1.

The gap equation reads

$$\Delta_p = -\sum_k v_{pk} \frac{\Delta_k}{2E_k} \tanh\left(\frac{\beta E_k}{2}\right) \qquad (2)$$

with $E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta_k^2}$. As usual the density is obtained from

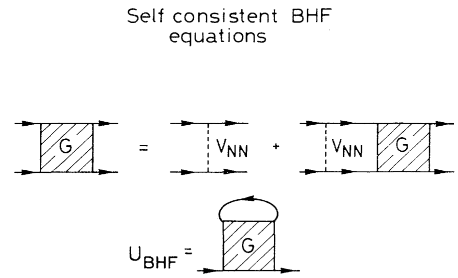

FIG. 1. Diagrammatic representation of the self-consistent BHF equations. The first line is the Brueckner-Bethe-Goldstone equation; the second line shows the BHF mean field, i.e., the single-particle potential at the one-hole line level of approximation.

$$\rho = \frac{N}{V} = \frac{1}{V} \sum_{\vec{k},\sigma,\tau} n_k ,$$
$$n_k = \frac{1}{2}\left(1 - \frac{\epsilon_k - \mu}{E_k} \tanh\frac{\beta E_k}{2}\right) , \qquad (3)$$

and the anomalous density is given by

$$\kappa_k = \frac{\Delta_k}{2E_k} \tanh\left(\frac{\beta E_k}{2}\right) . \qquad (4)$$

The expression for the entropy is

$$S = -\sum_{\vec{k},\sigma,\tau} \{f(E_k)\ln f(E_k) + [1-f(E_k)]\ln[1-f(E_k)]\} . \qquad (5)$$

Equations (2)–(4) are schematic. A more detailed account and derivation is given in the Appendix. The coupled set of nonlinear Eqs. (2) and (3) is solved, for a given total density ρ, adopting a separable form of the Paris potential [12]. The same numerical method of Ref. [7] is used in solving the gap equation for Δ_k. In the present work, however, the chemical potential μ has been also determined from the set of Eqs. (2) and (3). Once the solution for the gap function Δ_k and the chemical potential μ are found, the quantities κ and S are calculated from Eqs. (4) and (5), respectively.

III. RESULTS

A. Pairing at zero temperature

First, we discuss the results for the $^3S_1$-$^3D_1$ pairing gap in symmetric nuclear matter at zero temperature, as a function of the Fermi momentum k_F. In Fig. 2 is reported the total pairing gap, as defined in Ref. [7], at the Fermi momentum, in a range of densities around and below the saturation value, together with the corresponding S and D components. In comparison with

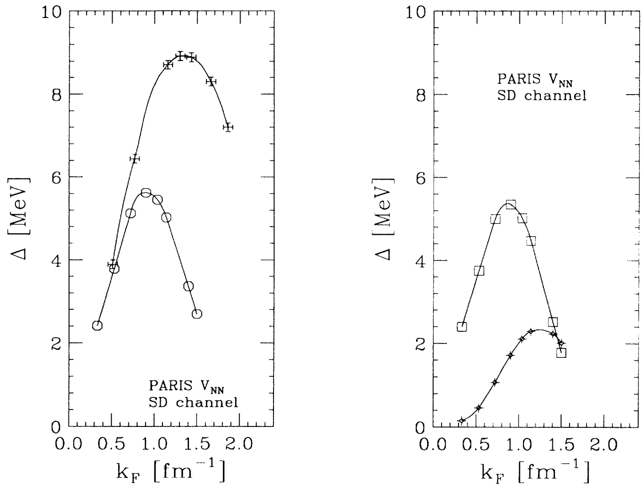

FIG. 2. (Left) Pairing gap vs Fermi momentum for symmetric nuclear matter in the S-D channel. The values indicated by the crosses and the circles correspond to the use of the free single particle spectrum and the BHF mean field, respectively. (Right) Comparison between the contribution of the S wave (squares) and of the D wave (stars) to the total gap. It has to be noticed that the total gap is not the sum of both components [Eq. (A17)].

the previous calculations of Ref. [7], a substantial reduction of the pairing gap is obtained, as a consequence of including, in the self-consistent procedure, the calculation of the chemical potential μ. It turns out, in fact, that the self-consistent value of chemical potential, due to the strong pairing gap, can be slightly different from the previously assumed weak coupling value $\mu = \epsilon_{k_F}$. Even if the deviations from the weak coupling values are small, they affect appreciably the results, since the pairing gap depends very sensitively (exponentially) on μ. For comparison displayed in the same figure is also the total pairing gap obtained neglecting the Brueckner–HF mean field, assuming for the single particle spectrum the free one, $\epsilon_k = \frac{\hbar^2 k^2}{2m}$. The reduction of the gap is mainly due to the effective mass m^* introduced by the mean field. Since $m^*/m < 1$, the kinetic energy increases and the density of state decreases, lowering the effective interaction at the Fermi energy, a well-known effect in superfluidity and superconductivity. On the other hand it is known [13] that the so-called E mass again increases m^* at ϵ_F to values around one. A consistent treatment of such effects, together with screening effects, would certainly be a very interesting investigation. Here we will refrain from such considerations and stick to the k mass only [13]. In spite of the reduction due to $m^*/m < 1$ the gap values around saturation of 3–5 MeV still appear quite large.

Since the deuteron exists in free space as a bound state and the Pauli principle in nuclear matter in the deuteron channel acts somewhat differently from the n-n and p-p channels, we investigated whether the n-p superfluidity in nuclear matter still survives when the n-p attraction is artificially lowered such that the bound state in free space disappears. We found that even in that case superfluidity exists being as usual triggered by the sharp Fermi surface.

The supermultiplet SU(4) invariance of symmetric nuclear matter implies an alpha-like structure of the ground state, but it has to be noticed that this structure is not in contrast or in competition with superfluidity, neither for the isoscalar nor for the isovector channels. In fact, genuine three- or four-body correlations are expected to be small in nuclear matter, the main components being the products of two-body correlations. Similar remarks apply to the case of symmetric medium-light nuclei, where the observed alpha structure is essentially a mean field effect [14], and therefore it has nothing to do with nucleon-nucleon correlations. The large S-D pairing gap indicates, therefore, a genuine superfluid phase of nuclear matter, at least in the density range not too far from saturation. Figure 3 (left) displays the corresponding chemical potentials as a function of density, both with and without the mean field contributing. The difference between the two values is quite close to the value $U(k_F)$ of the single particle potential at the Fermi momentum. In the present work we pay much attention to the low density region. There the mean field becomes smaller and smaller and the two chemical potentials tend to the same limit. As shown in Fig. 3 (right-hand side), this limit is nothing but one half of the deuteron binding energy, $\mu_0 = \epsilon_D/2 = -1.1$ MeV, in agreement with the considerations developed in Ref. [10]. In that reference it was shown that in the low density limit the BCS gap equation coincides with the Schrödinger equation for the deuteron. In this case, in fact, the chemical potential μ turns negative, and the pairing gap becomes very small with respect to $|\mu|$ and can be dropped out in the expression for the quasiparticle energy, i.e., $E_k \approx |\epsilon_k - \mu|$. Then it is easy to see that the gap equation (1) goes over into the Schrödinger equation, with the eigenvalue equal to 2μ, which therefore must coincide with the deuteron binding energy. To this end Eqs. (2), (3), and (4) can be combined to give

$$\frac{p^2}{m}\kappa_p + (1 - 2n_p) \int \frac{d^3k}{(2\pi\hbar)^3} v_{pk}\kappa_k = 2\mu\kappa_p \ . \qquad (2')$$

Indeed this equation looks very much like the deuteron Schrödinger equation into which it turns for a diluted system, i.e., when the chemical potential becomes negative. The results of Fig. 3 confirm that, within the BCS theory, the transition from the superfluid phase to the deuteron gas is a smooth one, also in the case the Brueckner–HF mean field is included (see also Ref. [15] on this point).

The transition from the superfluid phase to the deuteron gas is better illustrated by considering the pairing abnormal density κ_k, or its Fourier transform in coordinate space $\phi(r)$, which should smoothly merge into the deuteron wave function in the low density limit. This is displayed in Fig. 4, where the two components of $\phi(r)$ are drawn at different densities. For the lowest density at $\rho = 0.0075$ fm^{-3} a comparison is made with the deuteron

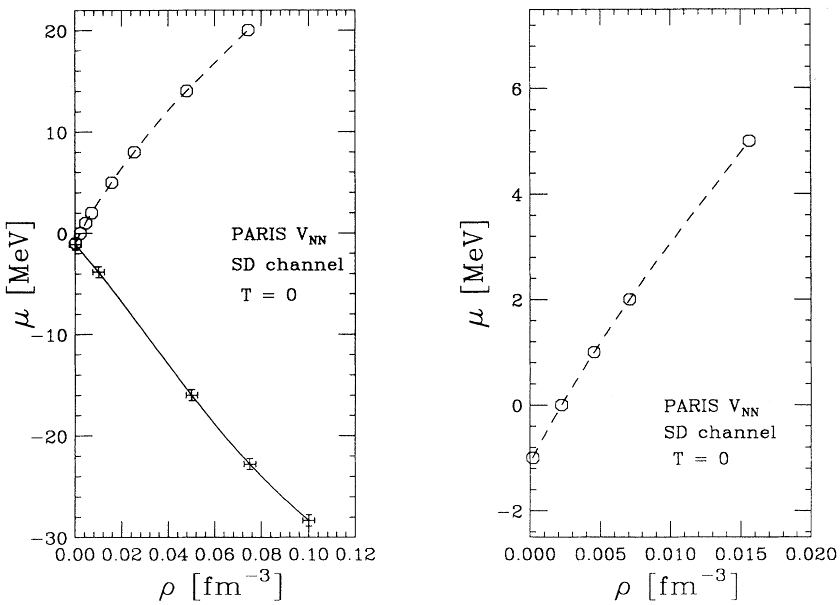

FIG. 3. Chemical potential vs density calculated self-consistently from Eqs. (2) and (3) with mean field (crosses) and without mean field (circles). The right-hand side figure is a blowup of the upper curve of the left-hand side in the low density region.

wave function. One can observe a large overlap between $\phi(r)$ and the deuteron wave function, which indicates that already at this density the BCS solution is hardly distinguishable from a gas of deuterons, despite the fact that the chemical potential is about +1 MeV still away from the deuteron limit value, and the value of the gap Δ is still a few MeV. At increasing densities the inner part of $\phi(r)$ is not so much affected, while he large r behavior changes drastically. At large distance, oscillations in $\phi(r)$ appear, in accordance with the expected long range (off diagonal) order of the superfluid phase.

From the results of the present section one can then conclude that the BCS theory describes, in the low density limit, a smooth transition from the S-D superfluid phase of symmetric nuclear matter to the Bose condensate phase of an ideal deuteron gas. In the BCS language

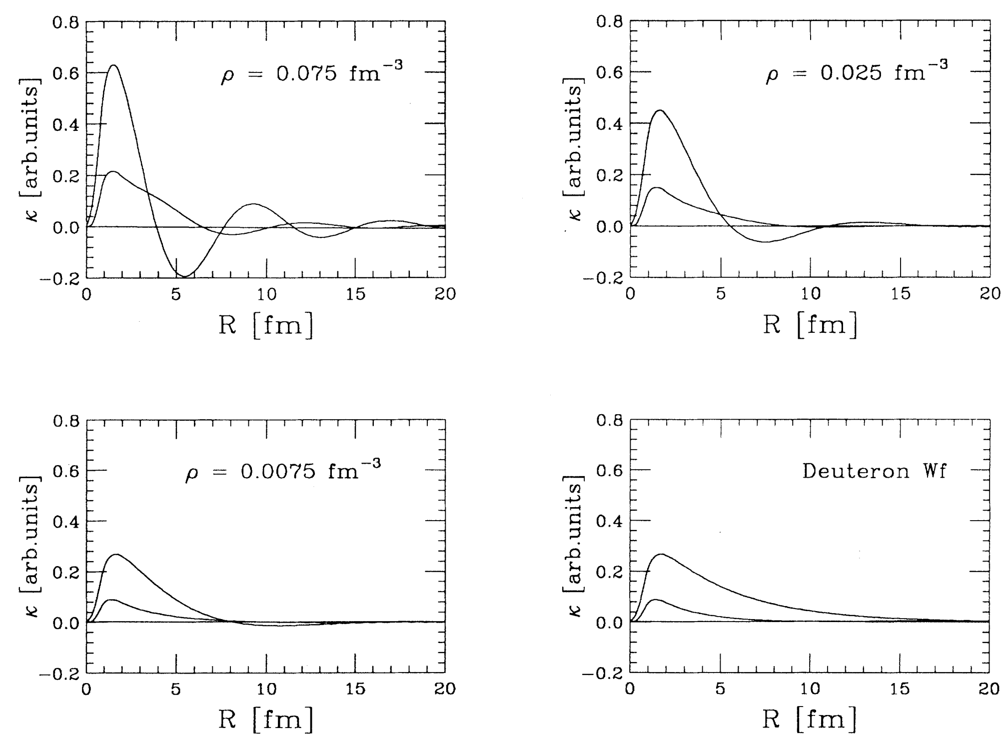

FIG. 4. The S and D components of the pairing correlation function in coordinate space for three different values of the baryonic density. The right lower figure displays for comparison the deuteron wave function.

it corresponds to a transition from the weak to the strong coupling limit. The Cooper pairs, each one of which has zero momentum, merge naturally into the zero momentum condensate of deuterons. If this simple picture is still valid, when going beyond the BS treatment, it is still an open question.

B. Finite temperature and thermodynamics

One of the main problems of the BCS theory at finite temperature is the implicit assumption that the only thermal excitations of the system are due to the pair-breaking mode. The thermal center-of-mass motion of the pairs, and therefore, of the deuterons, is entirely neglected. The inclusion of the thermal pair motion [16] can modify quite a lot the low density and low temperature behavior of the system, where the critical temperature, in the strong coupling limit, can differ from the corresponding BCS value. As an illustration we report in Fig. 5 the entropy per particle at the temperature $T = 1$ MeV as a function of density (i) as calculated from BCS, Eq. (5), and (ii) as obtained assuming a deuteron Bose gas at the same density and μ. As one can see, according to the results of Sec. III A, in the density range where it is meaningful to speak of a deuteron gas, the entropy due to boson thermal motion is comparable to the one of pair breaking. Therefore, the inclusion of thermal motion of the pairs in the theory is supposed to be a very important but theoretically difficult step with which we may deal in a future publication. At the moment we will ignore this complication and stick to the pure thermal BCS mean field description.

As mentioned in the introduction, in heated expanding nuclear matter formed in heavy ion reactions the ratio of the number of deuterons vs the total number of nucleons

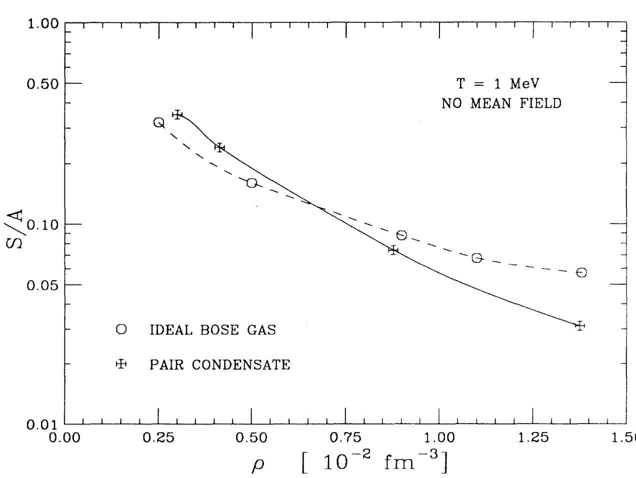

FIG. 5. Entropy per nucleon vs density from BCS theory at $T = 1$ MeV. For comparison is also reported S_A of an ideal Bose gas (dashed line).

as a function of the density of the system is of interest. This ratio, according to standard BCS theory [17], is given by

$$\frac{\rho_S}{\rho} = 1 - \frac{\hbar^2}{2m^*}\frac{1}{3\pi^2\rho}\beta \int dk k^4 \left[1 - \tanh^2\left(\frac{\beta E_k}{2}\right)\right] , \quad (6)$$

where m^* is the effective mass. As we mentioned already we suppose here that the expansion process occurs, since it is in an open system, isentropically, an assumption which has been confirmed to hold approximately true in BUU type transport calculations [9]. In this pilot study we also neglect any dynamical effect and mimic the expansion quasistatically. We therefore show in Fig. 6 the ratio of Eq. (6), for fixed values of the entropy per particle, as a function of the total density.

For each value of the entropy S/A there is a well-defined value of the density ρ from where on the existence of the Cooper pairs sets in and the number of deuterons steeply rises as the density is lowered further. As the density increases, for a fixed value of the entropy, the temperature increases, and the critical temperature for the superfluid phase is readily reached. In the same figure is drawn also the line (dashed line) through the points where the condition $\mu = U(k_F)$ is fulfilled. The corresponding densities can be considered the critical ones for which deuterons start to be formed (Mott transition). If the single particle mean field is neglected, these points mark the densities for which the chemical potential turns negative, i.e., two particle bound states appear. The uprising trend of N_s/N in the final stage of the expansion is due to the fact that the final infinitely diluted stage is always at zero temperature where, within the pure BCS theory, everything is paired ($N_s/N = 1$). This behavior can be seen more clearly in the blowup of Fig. 6 (right) for low densities. The approach towards the value $N_S/N = 1$ is not yet so apparent even at the lowest values of the density for which the numerical accuracy of the calculation is still reliable. For too low densities, it is difficult numerically to solve the gap equation for a well-defined value of the entropy. The asymptotic behavior of N_S/N or $\rho \to 0$ is actually extremely slow. As shown in the Appendix one has

$$\frac{\rho_S}{\rho} \sim 1 - S_A \frac{E_0}{\mu_0} \ln^{-1}\left(\frac{2\rho S_A}{\rho_c}\right) , \quad (7)$$

where E_0 is the quasiparticle energy at zero momentum and ρ_c is a reference density whose explicit expression is given in the Appendix.

Equation (7) is valid in the limit $\rho \ll \rho_c/2S_A$ and $T \ll |\mu_0|$. It has to be noticed that the ratio ρ_S/ρ reaches 1 with an infinite slope at $\rho = 0$.

On the other hand, since the entropy is kept constant, also T tends to zero as $\rho \to 0$, with a trend similar to the one of Eq. (7), as explicitly shown in the Appendix. We therefore show in Fig. 7 the behavior of N_s/N as a function of the temperature for various Fermi momenta. Clearly $N_s/N(T = 0) = 1$ for all values of μ. Finally, in Fig. 8 the entropy S_A vs temperature for various values of the chemical potential is displayed. We see that at constant temperature the entropy increases with decreasing

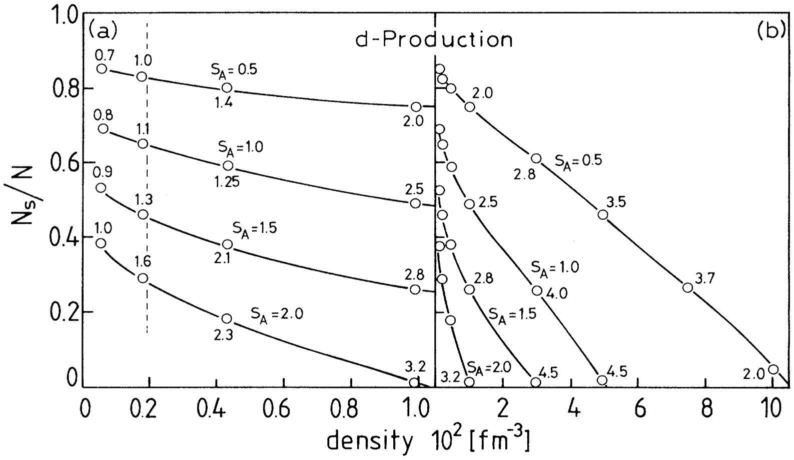

FIG. 6. (a) Deuteron-to-nucleon ratio N_S/N vs density for four values of the entropy. Also indicated is the $\mu = U(k_F)$ line (dashed line). (b) The same ratio but in an enlarged scale at low densities. In some cases the corresponding temperatures are indicated.

μ. We also indicate in this figure the critical line which separates superfluid and normal fluid domains. It may seem surprising that the chemical potential turns negative only at such extremely low densities where the interparticle distance is of order of 8 fm. One should bear in mind, however, that the deuterons are only weakly bound and that therefore corresponding wave function reaches out appreciably to distances beyond 8 fm as can be seen in Fig. 4. The Pauli principle therefore is still active. An interparticle distance 2–3 times the deuteron diameter can therefore seem reasonable for the Mott transition. We must of course always be aware of the weakness of our present approach where we completely neglect thermal and quantal fluctuations which may be very important at such low densities. For instance, the fact already mentioned, that BCS theory even at finite temperature imposes that the deuterons are at rest, seems to be a drastic assumption, which should be relaxed in a more realistic approach. Nevertheless our study may set qualitative indications.

C. Possible relevance for deuteron production in heavy ion collisions

As mentioned in the introduction, we have in mind a central ion-ion collision at $\sim$ 100 MeV/u, with a compression-decompression dynamics. To illustrate this point, we show in Fig. 9 the nuclear matter isentropics, calculated from the Skyrme force SkIII [18]. Three

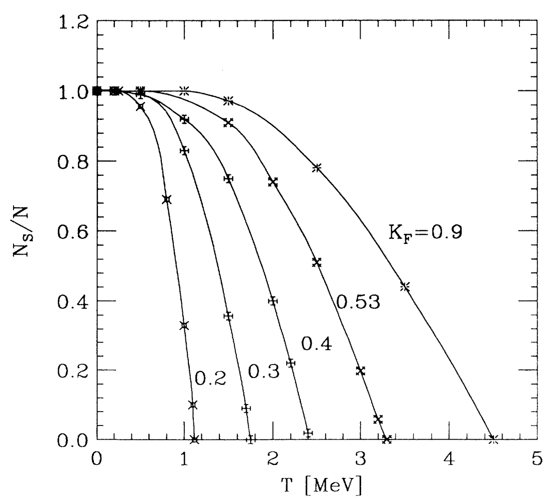

FIG. 7. Deuterons to nucleons fraction vs temperature for a fixed value of the Fermi momentum k_F. The crossing of the curves with the T axis signals the occurrence of the phase transition.

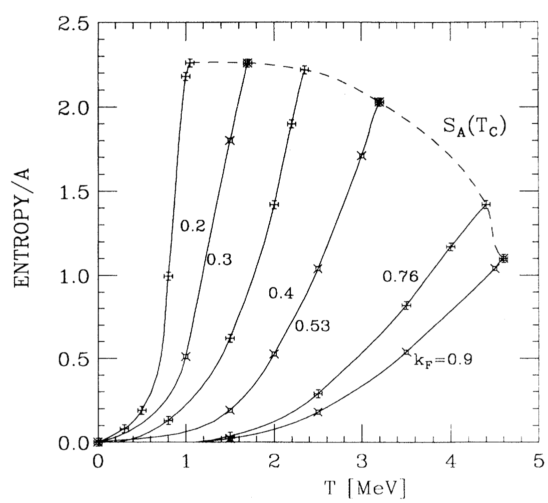

FIG. 8. Entropy per nucleon vs temperature for constant k_F. The dashed line separates the superfluid phase from the normal one in the plane entropy-temperature.

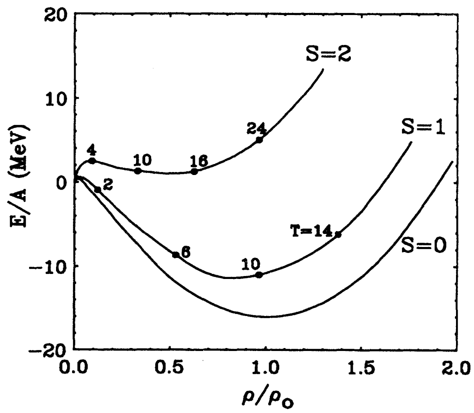

FIG. 9. Equation of state of nuclear matter for the Skyrme force SkIII. Three isentropics have ben plotted with some values of temperature.

values of the entropy per particle are considered, $S = 0$, 1, and 2, and along the curves few values of the temperature T are also reported. This equation of state is similar to the one reported in Ref. [19]. One can see that nuclear matter cools down quite fast in the decompression phase. If nuclear matter is initially compressed at a density $\rho/\rho_0 \sim 1.5$, even with an entropy as low as $S = 1$ it will be able to expand into the vapor phase. At a density $\rho/\rho_0 \sim 0.04$ the temperature becomes so low that deuteron formation can set in, according to the conclusions of the preceding paragraph.

Of course, once the system enters the spinodal zone, instability can occur, and fragments of intermediate mass can be formed [20]. This process of fragment formation does not necessarily forbid the deuteron production by the mechanism outlined in this paper. That these two processes may coexist can be seen in the computer simulation of Fig. 10, where, in the final state of the expanding nucleus, one clearly sees emerging some fragments embedded in a background of very low density nuclear matter, the latter being the environment of Cooper pairs. The fragments themselves can be also a source of deuteron emission, and therefore it is not simple experimentally to disentangle the two possible mechanisms. Recently [21], however, it has been found that above 50 MeV/u an appreciable fraction of the multifragmentation cross section can go in events where only light particles ($Z \leq 2$) are present, the so-called *vaporization* events. In this set of events the alpha particles are the dominant component, but the fraction of deuterons steeply increases at increasing beam energy and reaches values compatible with the ones reported in Fig. 6, a fact which could be an indication of the relevance of the mechanisms for deuteron formation outlined in this paper.

As can be seen from Fig. 9, for entropies ≥ 1.0 the temperatures at the lowest densities increase steeply, and therefore the deuteron formation should be strongly hindered. The precise value of the entropy production in heavy ion collisions is not well known; however, it is generally believed that $S_A \sim 1.0$ is a typical value for the rations we have in mind [22].

IV. SUMMARY AND CONCLUSIONS

In this work we considered the possibility of deuteron formation in an isentropically expanding piece of nuclear

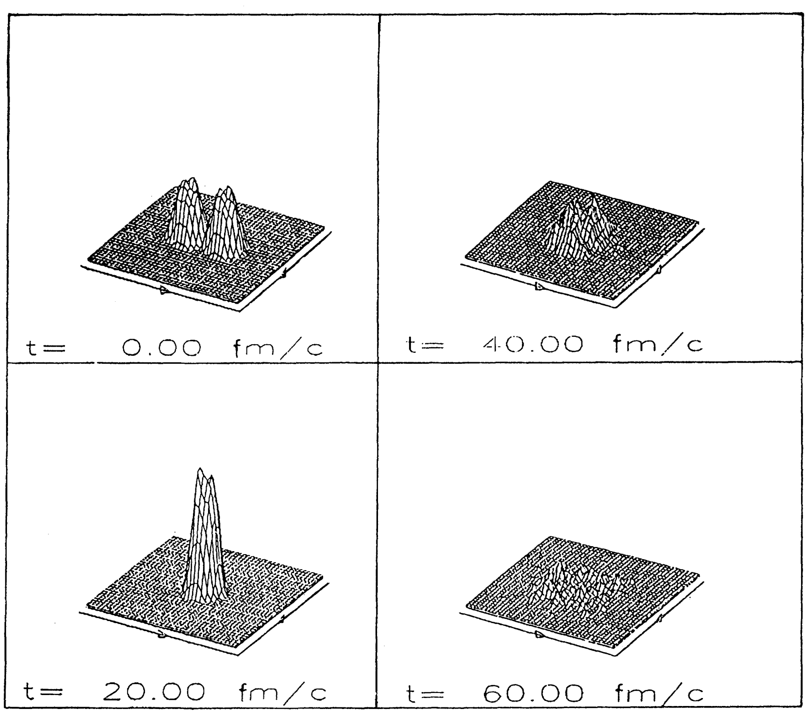

FIG. 10. Results from a BUU calculation for central collision of $^{40}Ca+^{40}Ca$ at $E/A = 100$ MeV (we are grateful to E. Suraud for providing the figure).

matter. The scenario we have in mind is that in a central collision of two heavy ions a compressed and hot blob of nuclear matter is formed which subsequently expands into the gaseous phase. A scenario of this type is depicted in Fig. 10 where a typical result from a BUU calculation is shown. The constancy of entropy is supported by the fact that we are dealing with an open system and also from numerical investigation using the transport equation. In the gas phase the pairing phenomenon of BCS theory may occur. Since deuterons also exist in free space there then may appear a crossover of the pairing from positive to negative chemical potentials (Mott transition) from where on the deuterons exist in the gas environment as bound particles.

Our approach though strongly simplified allows for the first time to follow in a continuous way deuteron formation in the medium from high to freeze-out densities. However it still suffers from several shortcomings. First, we neglect dynamic effects and treat the expansion quasistatically; it is clearly in contradiction with Fig. 10 where we see that the expansion is actually rather fast. Dynamic effects are planned to be included in forthcoming investigation (BUU plus BCS). On the theoretical side the BCS approach is also deficient: Its main drawback is that it is implicitly assumed that even at finite temperature the deuterons in the gas are at rest. This is clearly in contradiction with basic physics. One should however keep in mind that the inclusion of thermal motion of the pairs into BCS theory is a highly nontrivial theoretical problem [16]. We also intend to investigate this problem in the near future via a coupling of the present approach with a quasiparticle RPA calculation [15,23]. A further problem which we will address in a future work is to estimate the influence of pairing on the s.p. properties of nuclear matter. This would be particularly sizable at low density, as we can see from the comparison in Fig. 11 between the low density EOS in the Brueckner–Hartree-Fock limit and the one corrected by the pairing condensation energy. In spite of these shortcomings our work reveals interesting qualitative trends. It is seen that only in the extremely low density phase where the interparticle distance reaches 2–3 times the deuteron diameter the chemical potential turns negative. At these densities even if the initial reaction was very energetic and consequently the entropy and temperature very high, the temperature has dropped very much and the formation of deuterons becomes possible. Our results (Fig. 6) indeed indicate that the number of deuterons may strongly increase in the late stage of the reaction. However, even at higher densities, say $\rho_0/3$ to $\rho_0/5$, which are usually considered for deuteron formation in the coalescence model, we still get an appreciable amount of deuterons (see Fig. 6) present in the form of deuteron condensate if the entropy has typical values $S_A \sim 1.0$. Our scenario is opposite to the usual, where in most cases the deuterons are supposed to be formed through particle emission through the surface of a source (a big cluster) and subsequent final state interaction. Our study hints to the possibility that the pairing formation of deuterons may coexist with the latter process. Experimentally the existence of a deuteron Bose condensate should give signatures in the deuteron singles distributions as well as, more importantly, in the deuteron-deuteron correlation functions. The detailed balance between both production mechanisms is, however, a very difficult question and needs more work in the future.

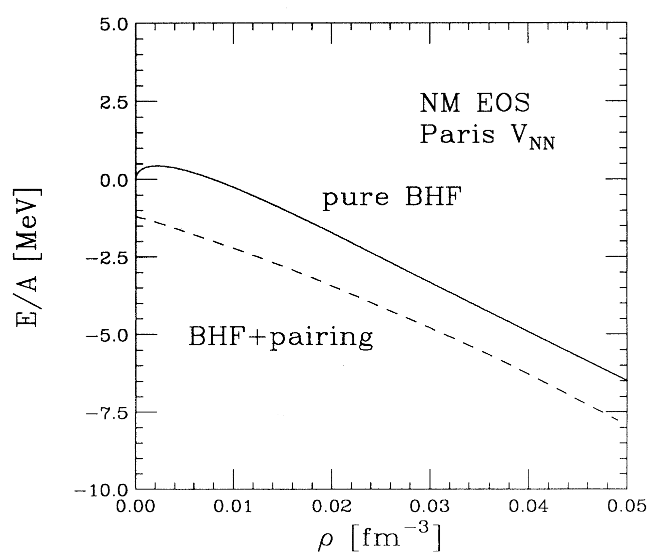

FIG. 11. Low density equation of state in the pure Brueckner–Hartree-Fock limit (solid line) and the same corrected by the pairing condensation energy in the BCS approximation (dashed line).

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APPENDIX: EXTENSION OF BCS THEORY TO THE JST PAIRING

We derive the BCS equations extended to arbitrary JST paring states in the framework of the Gorkov approach to superfluity. This extension has also been performed on the basis of the generalized Bogolyubov transformations [8]. In both cases one assumes that superfluid pairing states belonging to different JST channels are uncoupled.

We start from the model Hamiltonian defined in the grand canonical ensemble as follows:

$$H = \sum_{\vec{k},\alpha} \tilde{\epsilon}_{\vec{k}} a^\dagger_{\vec{k}\alpha} a_{\vec{k}\alpha} + \frac{1}{2} \sum_{\vec{k}} \sum_{\alpha,\alpha'} [\Delta^\dagger_{\alpha',\alpha}(\vec{k}) a_{-\vec{k}\alpha'} a_{\vec{k}\alpha} + \Delta_{\alpha,\alpha'}(\vec{k}) a^\dagger_{\vec{k}\alpha} a^\dagger_{-\vec{k}\alpha'}] , \quad (A1)$$

which is equivalent to singling out the pairing correlations in the two-body Green's function. The gap function is defined as

$$\Delta_{\alpha,\alpha'}(\vec{k}) = \sum_{\vec{k}'} \sum_{\beta,\beta'} \langle \vec{k}\alpha, -\vec{k}\alpha' | V | \vec{k}'\beta, -\vec{k}'\beta' \rangle \langle a_{-\vec{k}'\beta'} a_{\vec{k}'\beta} \rangle . \quad (A2)$$

The angular brackets must be interpreted as ensemble averages in the grand canonical ensemble. The greek labels indicate s.p. spin and isospin, $\tilde{\epsilon} = \epsilon_k - \mu, \mu$ being the chemical potential and ϵ_k the single particle spectrum. The latter is calculated in the Brueckner–Hartree-Fock approximation from the same NN realistic potential giving the pair interaction V_{NN}. The interaction V_{NN} to be used in the gap equations is, in principle, an effective interaction, which should be calculated in the framework of many-body theory. It has to be noticed however that a Brueckner G matrix cannot be used for V_{NN}, since the gap equation already sums up ladder diagrams, and thus this would introduce a double counting [24]. The main physical many-body effect is the "screening" of the NN potential due to the induced interaction [25].

Starting from the model Hamiltonian defined by Eq. (A1) the equations of motion for the s.p. propagators at finite temperature can be easily derived within the imaginary time formalism. We obtain the Gorkov equations for the propagator $G(k,\omega_n)$ and the anomalous propagator $F(k,\omega_n)$ which can be written as

$$(i\hbar\omega_n - \tilde{\epsilon}_k) G_{\alpha,\alpha'}(\vec{k},\omega_n) = \sum_{\alpha''} \Delta_{\alpha,\alpha''}(\vec{k}) F^\dagger_{\alpha'',\alpha'}(\vec{k},\omega_n) + \hbar\delta_{\alpha,\alpha'} , \quad (A3)$$

$$(i\hbar\omega_n + \tilde{\epsilon}_k) F^\dagger_{\alpha,\alpha'}(\vec{k},\omega_n) = \sum_{\alpha''} \Delta^\dagger_{\alpha,\alpha''}(\vec{k}) G_{\alpha'',\alpha'}(\vec{k},\omega_n) , \quad (A4)$$

where $\omega_n = (2n+1)\pi/\beta\hbar$ are the Matsubara frequencies for fermions. If the ground state is assumed to be time-reversal invariant, one can easily check the properties

$$\Delta_{\sigma\tau,\sigma'\tau}(\vec{k}) = -(-1)^{\sigma+\sigma'} \Delta^\dagger_{-\sigma'\tau,-\sigma\tau}(\vec{k}) , \quad (A5)$$

$$\Delta_{\sigma\tau,\sigma'\tau'}(\vec{k}) = -(-1)^{\sigma+\sigma'} \Delta^\dagger_{-\sigma\tau',-\sigma',\tau}(\vec{k}) , \quad (A6)$$

for diagonal and nondiagonal matrix elements in the isospin space, respectively. It follows that the gap function has the structure of a "unitary triplet" state, much the same way as the two phases of superfluid ^{3}He, namely, considering the gap function as a 2×2 matrix in spin space $\hat{\Delta}$, one has

$$\hat{\Delta}\hat{\Delta}^\dagger = \hat{I} \det(\Delta) , \quad (A7)$$

where $\hat{I}$ is the identity matrix and $\det(\Delta)$ is the determinant of $\hat{\Delta}$ in spin space. This property enables us to solve algebraically the two coupled equations of motion, and we obtain for the two propagators the usual form

$$G_{\alpha,\alpha'}(\vec{k},\omega_n) = -\delta_{\alpha,\alpha'} \frac{i\omega_n + \tilde{\epsilon}_{\vec{k}}}{\omega_n^2 + E_{\vec{k}}^2} , \quad (A8)$$

$$F^\dagger_{\alpha,\alpha'}(\vec{k},\omega_n) = \frac{\Delta^\dagger_{\alpha,\alpha'}(\vec{k})}{\omega_n^2 + E_{\vec{k}}^2} , \quad (A9)$$

where $E_{\vec{k}}^2 = \tilde{\epsilon}_{\vec{k}}^2 + D_{\vec{k}}^2$ is the quasiparticle energy, being $D_{\vec{k}}^2 = \det(\hat{\Delta})$. From these explicit expressions we can calculate by the usual procedure of summing up over the Matsubara frequencies the two propagators

$$F_{\alpha,\alpha'}(\vec{k},0^+) = \frac{\Delta_{\alpha,\alpha'}}{2E_{\vec{k}}} \tanh\left(\frac{\beta E_{\vec{k}}}{2}\right) , \quad (A10)$$

$$G_{\alpha,\alpha}(\vec{k},0^-) = \frac{1}{2}\left[1 - \frac{\tilde{\epsilon}_{\vec{k}}}{E_{\vec{k}}} \tanh\left(\frac{\beta E_{\vec{k}}}{2}\right)\right] . \quad (A11)$$

Applying the self-consistent Eq. (A2) we get from Eq. (A10) the gap equation in the standard form

$$\Delta_{\sigma,\sigma'}(\vec{k}) = -\sum_{\vec{k}'} \sum_{\sigma_1,\sigma_1'} \langle \vec{k}\sigma, -\vec{k}\sigma' | V | \vec{k}'\sigma_1, -\vec{k}'\sigma_1' \rangle \times \frac{\Delta_{\sigma_1,\sigma_1'}(\vec{k}')}{2E_{\vec{k}'}} \tanh\left(\frac{\beta E_{\vec{k}}}{2}\right) , \quad (A12)$$

and summing up over all s.p. states in Eq. (A11) we get the condition on the average number of particles

$$N = \sum_{\vec{k},\sigma} \left[1 - \frac{\tilde{\epsilon}_{\vec{k}}}{2E_{\vec{k}}} \tanh\left(\frac{\beta E_{\vec{k}}}{2}\right)\right] . \quad (A13)$$

Equations (A12) and (A13) are the basic ingredients for numerical investigation of the superfluidity in nuclear matter. For a fixed density and temperature these two coupled equations provide for the corresponding chemical potential and the gap function. If only S-D pairing is present, the gap function is expanded according to

$$\Delta_{\sigma,\sigma'}(\vec{k}) = \sum_{lm} C\left(\frac{1}{2}\sigma, \frac{1}{2}\sigma' | 1\sigma+\sigma'\right) C(1\sigma+\sigma', lm | 1m + \sigma + \sigma') Y_{lm}(\hat{k}) \Delta_{lM}(k) , \quad (A14)$$

where $M = \sigma + \sigma' + m$ is the projection of the total angular momentum $J = 1$ of the pair, and 1=0, 2. The C's are the Clebsh-Gordan coupling coefficients and the Y's are the spherical harmonics. The isospin indices and the coupling at $T = 0$ have been omitted. Inserting the expansion Eq. (A14) in Eq. (A12), one gets a set of coupled equations for the quantities $\Delta_{lM}(k)$, which is the generalized gap equation for the coupled channels $l = 0$ and $l = 2$. These equations can be simplified by using an angular average of $D(\vec{k})^2$ in the energy denominator

$$d(k)^2 = \frac{1}{4\pi} \int d\hat{k} D(\vec{k})^2 \; . \qquad \text{(A15)}$$

It has been shown (see Fig. 3 in Ref. [11]) that, in the case of $^3P_2$ superfluidity, this is an excellent approximation, which introduces an error of at most few percent. In the present case it should work even better, since the energy denominator is positive definite and has a weaker angular dependence. Then the gap components $\Delta_{lM}(k)$ turn out to be independent of M. The state of equations reduces to two coupled equations for the S and D gap components Δ_0 and Δ_2, respectively. They read

$$\Delta_l(k) = -\sum_{k'} \sum_{l'} V_{ll'}(k,k') \frac{\Delta_{l'}(k')}{2\sqrt{\tilde{\epsilon}^2_{k'} + d(k')^2}} \; , \qquad \text{(A16)}$$

where $V_{ll'}$ is the interaction in the partial wave representation. Equation (A15) gives

$$d(k)^2 = \frac{3}{8\pi}(\Delta_0^2 + \Delta_2^2) \qquad \text{(A17)}$$

which indicates that the set of equations (A16) is nonlinear. At zero temperature the energy shift due to pair correlations in the BCS limit is given by

$$\begin{aligned} \Delta E &= (U_s + \mu_s N) - (U_n + \mu_n N) \\ &= \sum_{\vec{k},\alpha} \left(\epsilon_k n^s_{\vec{k}} - \frac{d(k)^2}{2E_{\vec{k}}} - \epsilon_k n^n_{\vec{k}} \right) \; , \end{aligned} \qquad \text{(A18)}$$

where $U_s(U_n)$ is, to the leading order, the expectation value of the Hamiltonian in the superfluid (normal) ground state; $\mu_s(\mu_n)$ and $n^s_{\vec{k}}(n^n_{\vec{k}})$ are the chemical potential and the occupation numbers, respectively, of superfluid (normal) state. According to the discussion made in Sec. III it is instructive to perform the dilute gas limit. In this case we can expand ΔE in powers of $\phi_{\vec{k}} = d_{\vec{k}}/E_{\vec{k}} \ll 1$

$$\Delta E = \mu_s \sum_{\vec{k},\alpha} |\phi_{\vec{k}}|^2 + \frac{1}{2} \sum_{\vec{k},\alpha} (\mu_s - 2\epsilon_{\vec{k}}) |\phi_{\vec{k}}|^4 \; .$$

Taking into account that $N = \sum_{\vec{k},\alpha} |\phi_{\vec{k}}|^2$ to the lowest order the energy shift per nucleon is exactly equal to the nuclear binding energy in the deuteron.

According to the discussion of Sec. III B it is interesting to investigate the low density and temperature limit of the deuteron production along an isentropic line. This can be done combining both the asymptotic expressions for ρ_S and for the entropy which can be found in the textbooks [17]

$$S_A \sim \frac{1}{2} \left(\frac{kT}{E_0} \right)^{1/2} \frac{\rho_c}{\rho} e^{-E_0/kT} \; , \qquad \text{(A19)}$$

$$\frac{\rho_S}{\rho} \sim 1 + \left(\frac{kT}{\mu_0} S_A \right) \; , \qquad \text{(A20)}$$

where $\rho_c = (2m^* E_0/\hbar^2 |\mu_0| \pi)^{3/2}$. Eliminating the temperature between Eqs. (A19) and (A20), one ends up with Eq. (7) of the text.

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