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Intermittency in microscopic simulations of multifragmentation

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We investigate the intermittent nature of the mass spectra resulting from computer simulations of head-on nuclear collisions leading to multifragmentation in the frame of the quasiclassical nuclear model (QCNM). It is found that these spectra show what is called strong intermittency.

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I. INTRODUCTION

Intermittency analysis has been used in many fields of theoretical physics including turbulent flows [1], percolation models [2], and particle production in ultrarelativistic heavy-ion collisions [3]. Existence of intermittency implies scale invariance properties and it has been pointed out that it can be a signature of a phase transition. Recently this kind of analysis has been applied to nuclear multifragmentation.

The possibility that the fragment spectra resulting from intermediate energy nuclear collisions show intermittency is a topic of debate. It has been suggested recently [4] that the appearance of this effect in the analysis of experimental data is due to improper binning of events corresponding to different excitation energies or to using a multifragmentation model that predicts excessively broad multiplicity distributions. In Ref. [4] the analysis was performed using a statistical model. On the other hand, using a model with the same philosophy [5] it was found that the final spectra showed intermittency. Both calculations share the shortcoming that the final fragment spectra are obtained assuming that the system under study has reached thermodynamic equilibrium prior to fragmentation. For a recent review of intermittency analysis in nuclear fragmentation, see Refs. [6, 7].

It has recently been shown that the final fragment structure is strongly related to the earliest formed bound density fluctuations in phase space, indicating that nonequilibrium effects should play an important role in any analysis of fragmentation processes [8–10].

To explore this effect in the study of intermittency we calculate it taking as an input the mass spectra generated through a fully microscopic simulation of head-on nuclear collisions at intermediate energy using the quassiclassical nuclear model (QCNM) [11].

II. INTERMITTENCY

Intermittency is a way of extracting out the chaotic aspects of a given distribution function. The method of factorial moments is suitable for considering dynamical fluctuations without bias from statistical ones, hence it is appropriate to analyze the characteristics of the fragment distribution.

Given a set of collision experiments involving A nucleons ($A = A_t + A_p$ with A_t the target nucleons and A_p the projectile nucleons), each of the resulting final fragment spectra can be considered as a set of N fragments distributed over M bins of size $\delta y = A/M$. The factorial moment of order i is defined by [3]

$$\langle F_i \rangle \;=\; M^i \left\langle \frac{1}{M} \sum_{m=1}^{M} \frac{V_{n,i}}{V_{N,i}} \right\rangle, \tag{1}$$

where n is the number of fragments in bin m, the angular brackets denote the average over a large number of experiments, and $V_{x,i} = x(x-1)\cdots(x-i+1)$. The following alternative definition has also been suggested for the analysis of the kind of processes to be analyzed in this work:

$$\langle F_i \rangle \;=\; M^i \left\langle \frac{1}{M} \sum_{m=1}^{M} \frac{V_{n,i}}{\langle N \rangle^i} \right\rangle, \tag{2}$$

where $\langle N \rangle$ is the mean multiplicity of the sample events. Following Ref. [12], moments calculated in this way will be referred to as horizontal moments.

Intermittent behavior exists when the factorial moments follow a power law

$$\langle F_i \rangle \;\sim\; M^{\varphi_i}, \quad 0 \le \varphi_i \le i-1, \tag{3}$$

where the log-log plot slope parameter φ_i can be used to characterize the intermittency strength. The description may also be given in terms of the generalized dimensions D_i commonly used in nonlinear physical phenomena related, for $i \ge 2$, to φ_i by [13]

$$D_i = 1 - \frac{\varphi_i}{i-1}. \tag{4}$$

In the limit $i \to 0$, D_i equals Mandelbrot's similarity dimension. For homogeneous fractals $D_j = D_i$ and for inhomogeneous fractals $D_j > D_i$ for $j < i$ [14, 15].

The QCNM

The QCNM has already been described in a series of publications (see Ref. [11] and references therein) and for the sake of completeness we briefly describe its main characteristics. In this model the finite nuclear system is simulated via a set of classical particles labeled with spin and isospin labels. These particles interact via a classical two-body potential which reads

$$V = Vn + Vp + Vc, \tag{5}$$

where

$$Vp(p_{ij}, q_{ij}) = V_{0p} \exp\left[-0.5\left(\frac{p_{ij}^2}{p_0^2} + \frac{q_{ij}^2}{q_0^2}\right)\right]\delta\tau_{ij}\delta\sigma_{ij}, \tag{6}$$

where p_{ij} is the relative momentum and q_{ij} is the relative distance. $\delta\tau_{ij}$ and $\delta\sigma_{ij}$ are Dirac's deltas in isospin and the spin labels, respectively. This is the term responsible for simulating the Pauli exclusion principle, while

$$Vn(r_{ij}) = V_{0n}\left[\left(\frac{r_1}{r_{ij}}\right)^{a1} + \left(\frac{r_1}{r_{ij}}\right)^{a2}\right]\frac{1}{\exp\left(-\beta r_{ij} + \mu\right)} \tag{7}$$

is the so called nuclear term, and

$$Vc(r_{ij}) = \frac{\alpha_0}{r_{ij}} \tag{8}$$

is the Coulomb term.

In this model the parameters ($V_{0p}, p_0, q_0, V_{0n}, r1, a1, a2, \beta, \mu$) have been fitted in order to properly describe some properties of nuclear systems such as the binding energy of finite nuclei and the equation of state (EOS) of nuclear matter (for infinite nuclear matter Vc is set to 0) with a compressibility $K = 350$ MeV and a binding energy of $E_b \simeq 16$ MeV at a density of $\rho \simeq 0.17$ fm^{-3} (see Ref. [8]).

III. RESULTS

Two sets of evolutions were obtained using the above mentioned model. In these simulations two spin-isospin symmetric nuclei of mass 40 performed head-on collisions; one set (hereafter referred to as I) corresponds to the projectile boosted to an energy of 40 MeV/A, and the second one (II) to an energy of 80 MeV/A. In this way we are isolating the effects due to the dynamical evolution of the system from the geometrical ones and no uncertainty due to binning will remain.

The simulations span an extremely long time interval of 1200 fm/c, so that the final configurations considered in the calculation of the intermittency are very close to the final asymptotic ones. This property can be verified by analyzing the complete evolutions using the early cluster recognition algorithm (ECRA) [8]. The calculation of the fragment spectrum in the ECRA formalism is based on the following considerations. Given a system of nucleons (i.e., a configuration resulting from a molecular dynamics simulation of a heavy-ion collision) whose Hamiltonian is of the form $H = \sum_i \frac{p_i^2}{2m} + \sum_{i,j} V_{ij}$, then the following conditions apply.

(a) Nucleons being members of a cluster C must be bound,

$$\forall i \in C, e_i = T_{i\text{c.m.}} + \sum_j V_{ij} < 0,$$

where $T_{i\text{c.m.}}$ is the kinetic energy of particle i calculated in the c.m. of the cluster C and $\sum_j V_{ij}$ is the potential energy between particle i and all other particles belonging to C.

The A_T target and A_P projectile nucleons are grouped in clusters and free nucleons under the constraint that each cluster nucleon obeys (a).

This grouping is not unique. For a given set of A_T + A_P nucleons many partitions which obey (a) may be possible. Then an extra condition is imposed.

(b) The cluster structure of the configuration under study is the partition that maximizes the total binding energy of the system (each fragment is considered as noninteracting with the others or with free nucleons).

A subset of complete evolutions was analyzed as a function of time using the ECRA algorithm and compared with the result obtained using a simple configuration-space minimum spanning tree (MST) cluster recognition algorithm with a clusterization radius of 6 fm, which corresponds to the range of the attractive part of the nuclear potential term in the QCNM. In the MST approach a cluster is defined in the following way: let i and j denote two members of a given cluster C, then $\forall i \in C\ \exists j \in C/ \mid \mathbf{r}_i\text{-}\mathbf{r}_j \mid < R_{\text{cl}}$, where R_{cl} denotes the clusterization radius and $\mathbf{r}_k$ the spatial position of particle k.

In Fig. 1 we show the average number of fragments for three mass ranges as a function of time according to the ECRA (full lines) and the MST cluster recognition algorithm (dashed lines) (see figure caption for details). It can be seen that the population of the selected mass bins have reached their asymptotic values for times much smaller than the final evolution time. It is then appropiate to use the MST approach for the calculation of the

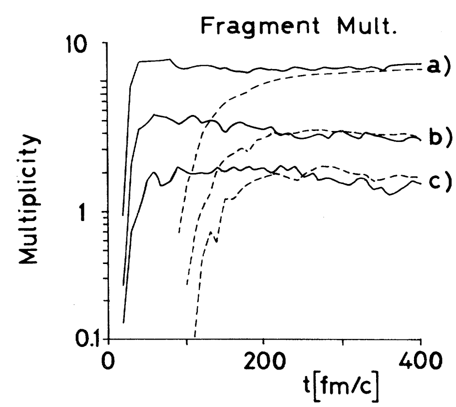

FIG. 1. Multiplicity per bin as a function of time according to ECRA analysis (full lines) and simple MST recognition algorithm (dashed lines) (notice that the ECRA results attain asymptotic values much earlier that those corresponding to the q-space analysis) for mass bins (a) $3 \leq A \leq 5$, (b) $5 \leq A \leq 7$, and (c) $7 \leq A \leq 9$.

intermittency exponents of the asymptotic fragment distribution.

It must be emphasized at this point that when the fragment spectra are calculated at very long times their structure does not depend strongly on the cluster recognition algorithm employed. At this stage of the evolution, the system can be viewed as a dilute mixture of free nucleons plus some small fragments. These fragments form compact structures in both q space and p space and are well separated from one another or from the free nucleons. As a consequence, the result does not depend strongly on the size of the clusterization radius as long as it is neither too small (i.e., using $R = 3$ fm the same fragment spectra are obtained), nor too big. On the other hand, it is also important to note that for short times (i.e., $t < 200$ fm/c, see Fig. 1) MST analysis does not provide a proper view of the fragmentation process. In fact, in recent publications [9, 10] it has been shown that fragments are already formed in phase space before they can be recognized in configuration space. Then, for early times, cluster analysis of the ECRA type is necessary. This fact seems to have been overlooked in recent works [16].

Both sets I and II consisted of 150 collisions, which turn out to provide good statistics as can be seen from the dispersion of the calculated moments (see below).

The colliding nuclei being of mass 40, we have to analyze fragment distributions with A=80. Factorial moments of order i=2–10 were calculated and averaged over the set of simulations. The generalized dimensions D_i were extracted for bin numbers M=5–80 (smaller bin numbers are of no interest since the largest fragment mass is less than 20, so for bin numbers M=1–4 scale invariance is trivial). Finally D_0 was extrapolated from the data. For the sake of completeness normal and horizontal factorial moments were calculated with no noticeable difference in the final results.

Figure 2 shows the log-log plot of the factorial moments vs the number of bins M for different moment orders i for the two sets of collisions that were analyzed. In all cases the linear dependence of F_i upon M is evident. Through

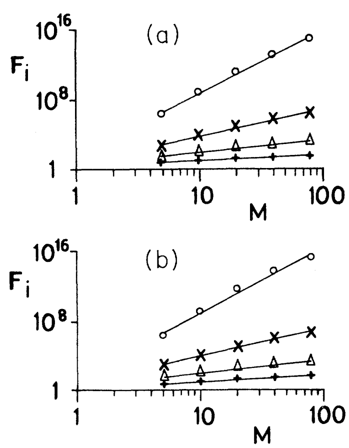

FIG. 2. log-log plot of the factorial moments (F_i) vs number of bins (M) for order i = 2 (+), 3 (triangles), 5 (diamonds), and 10 (open circles), for (a) case I and (b) case II. Straight lines denote fitting in order to get the corresponding dimension (see text for details).

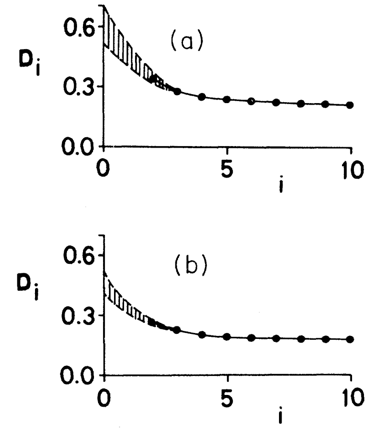

FIG. 3. Generalized dimension (D_i) vs moment order (i) (dots). The dashed line denotes extrapolation to order 0 for (a) case I and (b) case II.

a standard least squares fit the slopes φ_i of these graphs were determined and then the dimensions D_i were calculated. The results are summarized in Fig. 3 where D_i is plotted vs i. A strong intermittency pattern can be noticed since all dimensions are lower than 0.35. While the F_i values within a set have a relative dispersion of 1%, the relative error in the values of the dimensions arising from the linear fit is below 10% for both cases. Since $D_j \neq D_i$ for different i, j the underlying physical phenomena must be multifractal in nature. The relation between D_i and i can be suitably fitted by an exponential function for set II whereas for set I this relation seems to be more complex. From these data the respective values of D_0 were obtained via extrapolation; the resulting values are 0.45 ∓ 0.05 for set II and 0.6 ∓ 0.1 for set I .

These results suggest that the mass spectra in multifragmentation may eventually show intermittent behavior even if events of a given energy and impact parameter are isolated from the experimental data. It also follows that a question should be raised regarding the validity of thermodynamic models for the analysis of this kind of process, i.e., the assumption of thermal equilibration prior to fragmentation. Once the possibility of finding intermittent behavior in fully dynamical simulations of fragmentation processes has been established, one more question remains: are we in the presence of critical phenomena related to a phase transition? According to [6] intermittency is not only a signature of critical phenomena but also a way of characterizing it; on the other hand, in a recent work [17] a word of caution has been raised calling attention to many possible sources of spurious intermittency signals, amongst which finite size effects seem to be crucial. More work should be done in this direction, for example, analyzing the intermittency signal in expanding infinite periodic systems, a domain in which we are currently working.

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