Nuclear binding effects in relativistic Coulomb sum rules

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We extend the formulation of relativistic Coulomb sum rules to account for the average effects of nuclear binding on the initial and final states of ejected nucleons. Relativistic interactions are included by using a Dirac representation adapted from a vector-scalar field theory. The scalar field reduces the effective nucleon mass M^* and increases the relativistic effects of recoil and Fermi motion. We consider two models for the off-shell behavior of the nuclear electromagnetic current and demonstrate that the sum rule is accurate for applications to data over the interesting range of M^* and three-momentum q. We further indicate that the form of the sum rule is sufficiently general to accommodate a broad class of off-shell form factor models.

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I. INTRODUCTION

The subject of this paper is the Coulomb sum rule (CSR) for inelastic electron scattering [1], in a reformulation which extends its validity to relativistic momentum transfers $q \gtrsim M$, where $q \equiv |\mathbf{q}|$ is the three-momentum transfer to the target nucleus, and M is the nucleon mass.[1] The purpose of this work is to provide practical methods for the analysis of the longitudinal (e, e') response of nuclear targets, at the higher energies now available at the Continuous Electron Beam Accelerator Facility (CEBAF) and other electron accelerators. In a previous paper [3], we derived a relativistic Coulomb sum rule (RCSR) which incorporates the relativistic effects of nucleon recoil at large q, as well as of Fermi motion. In the present work we extend this theoretical approach to include the effects of nuclear interactions on the RCSR.

In our recent article, we discussed in detail the assumptions under which a RCSR could be derived. The basic approximation is that only nucleon, i.e., as opposed to antinucleon, degrees of freedom enter the Coulomb response in the spacelike regime accessible by (e, e') experiments. We call this the "nucleons-only" approximation; it ignores effects of antinucleons, but includes fully the relativity of the nucleons. Following conventional treatments, we adopt an impulse approximation which ignores explicit contributions from the exchange of charged mesons. We include nucleon anomalous moments and elastic form factors, which are important at higher energies. The final assumption is less conventional, but absolutely necessary. In order to derive a *non-energy-weighted* sum rule, it must be possible to factor *all* dependence on the photon energy ω from the current matrix element, otherwise unwanted dynamical effects enter into the Coulomb response function and complicate the isolation of correlation effects. In this paper, we restrict our attention to form factor models which have factorable dependence on ω.

We define the nuclear Coulomb sum in terms of the Coulomb response function $W_C(\omega, \mathbf{q})$ and the proton electric form factor $G_{E,p}(Q^2)$:

$$\Sigma(\mathbf{q}) \equiv \int_{\omega_{\rm el}^+}^{q} d\omega \; \frac{W_C(\omega, \mathbf{q})}{G_{E,p}^2(Q^2)}, \tag{1.1}$$

where the lower limit $\omega_{\rm el}^+$ excludes the elastic peak, and the upper limit q restricts the integration to spacelike four-momenta. Under the assumptions of Ref. [3] mentioned above, we obtain a RCSR which can be expressed in terms of one- and two-body contributions:

$$\Sigma(\mathbf{q}) \equiv \Sigma^{(1)}(\mathbf{q}) + C(\mathbf{q}) + \Sigma_{\rm un}^{(2)}(\mathbf{q}), \tag{1.2}$$

where the one-body contribution is of the form

$$\Sigma^{(1)}(\mathbf{q}) \equiv 2 \sum_{\mathbf{p}\sigma} n_\sigma(\mathbf{p}) \; r_\sigma(\mathbf{p}, \mathbf{q}), \tag{1.3}$$

$C(\mathbf{q})$ includes two-body correlation information in momentum space, and $\Sigma_{\rm un}^{(2)}(\mathbf{q})$ is the uncorrelated two-body part which is related to the square of the nuclear elastic form factor, as discussed in Ref. [3].

In the one-body term (1.3), $n_\sigma(\mathbf{p})$ is the nucleon momentum distribution function for isospin projection σ and one spin projection, and $r_\sigma(\mathbf{p}, \mathbf{q})$ is a kinematic factor which arises due to relativistic nucleon recoil and Fermi motion in the target. In the nonrelativistic limit $(q \ll M)$, we have $r_p \to 1$ and $r_n \to 0$, which leads to $\Sigma^{(1)}(\mathbf{q}) \to Z$; then the sum rule (1.2) is the result of Ref. [1]. The relativistic effects are all in the functions $r_\sigma(\mathbf{p}, \mathbf{q})$, representing recoil of the struck nucleon and Fermi motion in the target ground state. In Ref. [3] we further showed that (1.3) does not depend strongly on the details of $n_\sigma(\mathbf{p})$, but only on the lowest momentum moments, e.g., $\langle \mathbf{p}^2 \rangle_\sigma$. This leads to a method of evaluating (1.3) accurately in a weakly model-dependent manner, and in principle permits the extraction of the

[1]For a derivation of the nonrelativistic CSR in second quantization, see Ref. [2].

correlation function $C(\mathbf{q})$ from the experimentally measured Coulomb sum (1.1), using (1.2).

In the present paper, we investigate what changes are required in the sum rule when the Coulomb sum $\Sigma(\mathbf{q})$ is modified by relativistic two-body interactions. We study the effect of the average interaction in the nucleus using the mean-field approach of quantum hadrodynamics (QHD), a relativistic field theory for nuclear physics [4]. In particular, we consider QHD-I, which includes vector and scalar isoscalar mesons only. We then reformulate the Coulomb sum rule of Ref. [3] so that results (1.1)–(1.3) have a similar structure, but with modifications reflecting the mean-field effects of these relativistic interactions.

These modifications have two main effects on the RCSR. First, there are kinematical effects resulting from mean-field interactions of the nucleons, which are represented in QHD-I by a reduction in the effective nucleon mass M^* in the medium. Consequences of a reduced effective nucleon mass M^* for the Coulomb sum have been considered previously in a Fermi gas model [5,6]. These calculations include interactions of both initial and final plane-wave nucleon states with the mean fields, through the effective mass M^*, which can be interpreted as binding in the initial state, and final state interactions of the ejected nucleon with the nucleus. Chinn, Picklesimer, and Van Orden [7,8] have studied the effects of final state interactions on the Coulomb response of a Fermi gas, using more realistic interactions, and have seen similar effects to those seen due to $M^*/M<1$. Second, the electromagnetic coupling of the nucleon in the medium may be modified by the mean fields, entering through the off-shell behavior of the nucleon elastic form factors. Both modifications introduce a degree of model dependence in the RCSR which is not present in the nonrelativistic formulation, nor in the relativistic formulation of Ref. [3]. We show how these features can be incorporated into the theory to allow the evaluation of the one-body contribution $\Sigma^{(1)}(\mathbf{q})$, and the subsequent extraction of the two-body correlation function $C(\mathbf{q})$ from the measured Coulomb response.

This paper is organized as follows. In Sec. II we introduce the basic formalism to include relativistic mean-field effects in the Coulomb sum rule (RCSR). We introduce two models (F and G) for the electromagnetic charge operator, and investigate the resulting behavior connected to different off-shell assumptions for the nucleon elastic form factors. In Sec. III, we derive a modified version of the RCSR (1.1)–(1.3), concentrating on the explicit changes to the one-body term $\Sigma^{(1)}(\mathbf{q})$ in off-shell models F and G. In Sec. IV, we illustrate the operation of the sum rule in a simple nuclear system: uniform nuclear matter treated in the mean-field approximation, with nuclear binding effects incorporated using QHD-I. We examine the sensitivity of the RCSR to M^* and to the choice of off-shell model (F and G), focusing on the convergence of the moment expansion in each case. We further demonstrate that the particular form of the RCSR given here is applicable to *both* models, and argue that the same form should also be valid for a broad class of form factor models. In Sec. V, we draw conclusions, give guidelines for the application of the RCSR to data, and indicate important directions for future work.

II. FORMALISM

In this section, we review the formalism for electron scattering from nuclei, as it pertains to our development of relativistic Coulomb sum rules. We first give some standard results of the plane-wave impulse approximation (PWIA), which is used for the analysis of (e,e') experiments on nuclei, in a single-particle basis which accounts for the vector and scalar interactions of QHD-I. We then give two models for the nucleon form factors off shell, which will be used in the next section to illustrate the sensitivity of the sum rule to different off-shell assumptions.

A. PWIA in M^* basis

We begin with the differential cross section for the scattering of ultrarelativistic electrons from nuclear targets, which is commonly written in the form

$$\frac{d^2\sigma}{d\Omega' dE'} = \frac{d\sigma_M}{d\Omega'}\left[\frac{Q^4}{\mathbf{q}^4}W_C(\omega,\mathbf{q}) + \left(\frac{1}{2}\frac{Q^2}{\mathbf{q}^2}+\tan^2\frac{\theta}{2}\right)W_T(\omega,\mathbf{q})\right], \qquad (2.1)$$

where $q^\mu=(\omega,\mathbf{q})$ is the four-momentum transferred from the electron to the nucleus via virtual photon exchange, and $Q^2 \equiv |\mathbf{q}|^2-\omega^2>0$. The longitudinal contribution in (2.1) has been expressed in terms of the Coulomb response function

$$W_C(\omega,\mathbf{q}) \equiv \sum_f |\langle f|\hat{J}_0(q)|i\rangle|^2\,\delta(\omega-E_f+E_i), \qquad (2.2)$$

where $|i\rangle$ and $|f\rangle$ denote initial[2] and final nuclear many-body states, respectively.

In general, the electromagnetic current density operator $\hat{J}_\mu(q)$ may include contributions from both nucleons and charged mesons. Meson exchange current (MEC) contributions have been considered by Schiavilla *et al.* [9], for example, but are not included here. Including only nucleons with electromagnetic form factors, the current density operator can be written in the form

$$\hat{J}_\mu(q) \equiv \int d^3x\; e^{i\mathbf{q}\cdot\mathbf{x}}\;\bar{\hat{\psi}}(\mathbf{x})\Gamma_\mu(q)\hat{\psi}(\mathbf{x}), \qquad (2.3)$$

where $\hat{\psi}(\mathbf{x})$ is the (Schrödinger picture) field operator for a point Dirac nucleon, and Γ_μ represents the electromagnetic coupling at the γNN vertex. There is also a

[2]For notational simplicity, we assume a nondegenerate target ground state; the results are easily generalized to unpolarized targets with $J\neq 0$.

sum over proton and neutron isospin projections, which will be suppressed until needed. Any effects of charged mesons not included in the current operator (2.3) may be interpreted as two-body effects, as discussed in Ref. [3].

To proceed to a relativistic sum rule, we shall expand the field operators $\hat{\psi}(\mathbf{x})$ of (2.3) in a plane-wave basis, i.e., in a PWIA. In Ref. [3], we took this to be the free basis, i.e., the momentum eigenstates of the free Dirac equation. In the present paper, we shall instead expand the field operators in a basis of plane waves moving in the presence of uniform (isoscalar) Lorentz scalar and vector potentials, such as would be generated by scalar and vector mesons in a relativistic field theory of nuclei, e.g., QHD-I. With this modification, the theoretical development is formally similar to that of Ref. [3]. The consequences of the change of basis show up after the nucleons only approximation, in which antinucleon degrees of freedom are removed approximately from the spacelike Coulomb response. In the following, nucleons only will refer to nucleons in the presence of scalar and vector potentials in the target. The effect of this modification is to include efficiently the effects of these mean-field potentials in both the initial and final target states.

We therefore use the plane-wave solutions of the Dirac equation

$$\left[\gamma^{\mu}(i\partial_{\mu} - g_v V_{\mu}) - (M - g_s\phi)\right]\psi(x) = 0, \tag{2.4}$$

where the scalar and vector potentials are written in the forms $g_s\phi$ and $g_v V_{\mu}$, respectively, corresponding to the scalar and vector fields ϕ and V_{μ}, and the associated coupling constants. Here, as in the mean-field solution for uniform nuclear matter at rest, $V_{\mu} = \delta^0_{\mu} V_0$, and V_0 and ϕ are constants. The solutions to (2.4) are discussed in detail in Ref. [4]. In the case of uniform fields, the energy eigenvalues of (2.4) take the simple form $E^{(\pm)}_{\mathbf{p}} = g_v V_0 \pm E^*_{\mathbf{p}}$, where $E^*_{\mathbf{p}} \equiv \sqrt{\mathbf{p}^2 + M^{*2}}$ and the effective nucleon mass is defined as $M^* \equiv M - g_s\phi$. With the nucleons-only approximation, we need only consider positive-energy solutions to (2.4). These are plane-wave solutions of momentum $\mathbf{p}$, which obey the equation

$$\left[\gamma^0 E^*_{\mathbf{p}} - \gamma\cdot\mathbf{p} - M^*\right]u_s(\mathbf{p}) = 0. \tag{2.5}$$

Explicit forms for the interacting solutions $u_s(\mathbf{p})$ can be obtained directly from the free solutions, given in Appendix A of Ref. [3], by making the replacement $M \rightarrow M^*$. Since the vector potential $g_v V_0$ appears additively in the nucleon eigenenergy, it does not appear in (2.5) or its plane-wave spinor solutions.

In Ref. [3] we argued that the Coulomb response function (2.2) for spacelike ($\omega < |\mathbf{q}|$) photon exchange is dominated by nucleon (NN) contributions to the current matrix elements, and that antinucleon ($\bar{N}\bar{N}$) and pair ($N\bar{N}$) terms could be neglected. This is an exact result for a uniform free Fermi gas, and leads to the nucleons-only approximation for interacting nuclear systems. The presence of a strong scalar field in the nucleus induces mixing of free N and $\bar{N}$ states, particularly for nucleons of high momentum, as in the final states of (2.2). However, transforming to the plane-wave basis formed from the solutions of (2.5), hereafter referred to as the "M^* basis," removes this mixing by the potentials. The nucleons-only approximation is again adopted for interacting nuclei, but here refers to nucleons of mass M^*. This use of the M^* basis for final states implies substantial interaction of excited (ejected) nucleons before leaving the target, and is probably a better assumption for large nuclei than for small. Then the electromagnetic current operator takes the form[3]

$$\hat{J}_{\mu}(q) \simeq \sum_{\mathbf{p}}\sum_{ss'} \frac{\bar{u}_{s'}(\mathbf{p}+\mathbf{q})}{\sqrt{2E^*_{\mathbf{p}+\mathbf{q}}}}\Gamma_{\mu}(q)\frac{u_s(\mathbf{p})}{\sqrt{2E^*_{\mathbf{p}}}}a^{\dagger}_{\mathbf{p}+\mathbf{q}s'}a_{\mathbf{p}s}, \tag{2.6}$$

where $a^{\dagger}_{\mathbf{p}s}$ is a creation operator for a nucleon with spin projection s and momentum $\mathbf{p}$. The energy denominators in (2.6) reflect the normalization of the plane-wave spinors to $2E^*_{\mathbf{p}}$ particles/volume. The formal derivation of the RCSR now follows closely that of Ref. [3], once we have discussed the form of the electromagnetic vertex operator $\Gamma_{\mu}(q)$.

B. Off-shell nucleon form factors

It is conventional to express the γNN vertex operator in the form

$$\Gamma_{\mu}(q) = F_1\gamma_{\mu} + i\frac{\kappa}{2M}F_2\sigma_{\mu\nu}q^{\nu}, \tag{2.7}$$

where κ is the nucleon anomalous magnetic moment, M is the free nucleon mass, and F_1 and F_2 are the Dirac and anomalous form factors, respectively. In general, F_1 and F_2 are scalar functions of p, p', and q. This form is sufficiently general for matrix elements between nucleon states (antinucleons excluded) as in the nucleons-only PWIA. Since the tensor $\sigma_{\mu\nu}$ is antisymmetric, only the three-momentum $\mathbf{q}$ enters explicitly the Coulomb operator Γ_0. Any remaining dependence on the photon energy ω enters Γ_0 *only* through the form factors F_1 and F_2.

For scattering from a free nucleon, the form factors F_1 and F_2 depend only on the scalar Q^2. To ensure the correct charge and magnetic moments for free nucleons, they are normalized at $Q^2 = 0$ according to $F_{1p}(0) = F_{2p}(0) = F_{2n}(0) = 1$ and $F_{1n}(0) = 0$. The form factors F_1 and F_2 are obtained from (e,e') scattering data, usually in terms of the more convenient Sachs electric and magnetic form factors G_E and G_M:

$$\begin{aligned} F_1(Q^2) &= \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau}\,, \\ \kappa F_2(Q^2) &= \frac{G_M(Q^2) - G_E(Q^2)}{1+\tau}, \end{aligned} \tag{2.8}$$

[3]For simplicity, we discretize the sum over the momentum $\mathbf{p}$.

where $\tau \equiv Q^2/4M^2$ and $Q^2 \equiv -(k_\mu - k'_\mu)^2$, i.e., the photon four-momentum is determined by the four-momentum transfer at the γee vertex. For a single nucleon in free space, both energy and momentum are conserved at the γNN vertex as well, and we have $p'_\mu = p_\mu + q_\mu$. See Appendix A for an alternative form of (2.7) which is expressed in terms of the Sachs form factors. We use the standard parametrization of the Sachs form factors, along with the assumption $G_{En}(Q^2) = 0$, as discussed in Ref. [3]. This choice has the convenient feature that all Sachs form factors are proportional, which satisfies a condition assumed in our derivation of a *non-energy-weighted* sum rule in Ref. [3].

For interacting nucleons in a nucleus, one needs information about the form factors off their free mass shell, which is simply not known. This leaves considerable freedom to extrapolate off shell from the known on-shell forms. Given no other information, a common approach has been to assume that the dependence on Q^2 of the form factors $F_1(Q^2)$ and $F_2(Q^2)$ does not change off shell from that for a free nucleon. This implies, at least in part, a separation of the dynamics of the electromagnetic structure of the nucleon from that of the nucleus. However, even this assumption does not uniquely specify the off-shell form factors, because of the freedom to transform $\Gamma_\mu(q)$ using the Gordon identity to other operators with other form factors, each of which are equivalent on shell, but not off shell. This issue has been considered by de Forest [10], and by Chinn and Picklesimer [11], who have given examples of possible choices of extrapolation, and studied the sensitivity of the response functions and of the Coulomb sum to those choices. Our method is similar in principle, but is based specifically on the Gordon transformation which relates F_1 and F_2 to the free Sachs form factors G_E and G_M, given in (2.8) for free nucleons. We consider two possibilities: that the functional dependence on Q^2 of the functions $F_1(Q^2)$, $F_2(Q^2)$ is unchanged off shell (model F), or alternatively that the functional dependence on Q^2 of the functions $G_E(Q^2)$ and $G_M(Q^2)$ is unchanged off shell (model G). Other choices are possible. For example, in principle, the vector field V_0 may also enter the current operator. We restrict our attention to models F and G which illustrate the important issues and do not depend on V_0.

The first choice, model F, is the most common; here it is simply assumed that the Dirac and anomalous form factors $F_1(Q^2)$ and $F_2(Q^2)$ are unchanged in the nuclear environment, i.e., that they are given by (2.8) where both the Sachs form factors and the kinematic variable $\tau \equiv Q^2/4M^2$ are evaluated at the actual momentum transfer of the experiment, i.e., at the momentum $q_\mu \equiv k_\mu - k'_\mu$ found at the electron vertex. This is in some sense a minimal assumption, in that the effects of the scalar field enter explicitly the current operator $\hat{J}_\mu(q)$ *only* through the nucleon field operators. In spite of its apparent simplicity, this model has the peculiar feature that the Dirac and anomalous magnetic moments are treated differently in the nuclear medium. This follows since the Dirac moment $(e/2M^*)$ scales with the nucleon mass M^*, as can be seen from the solution of (2.4) for a nucleon at rest in the presence of a uniform magnetic field, while the anomalous moment $(\kappa e/2M)$ is unchanged in the medium. This behavior is possible for a theory with point Dirac nucleons dressed by charged meson fields, such as QHD-II [4], since the anomalous moments in such a theory are typically included explicitly by hand, following a solution of the many-body problem using point nucleons. However, this behavior is not likely for a theory with internal nucleon electromagnetic structure, such as QCD, since there one expects that the Dirac and anomalous magnetic moments may have similar origins, and therefore may respond in a similar manner to the nuclear environment.

An alternative assumption, which we call model G, is that the Sachs electric and magnetic form factors $G_E(Q^2)$ and $G_M(Q^2)$ are unchanged in the nuclear medium. To express $\Gamma_\mu(q)$ in the form (2.7), the transformation (2.8) is now performed in the M^* basis: the off-shell forms of F_1 and F_2 are now given in terms of the free Sachs form factors by

$$
\begin{aligned}
F_1(Q^2;\tilde{\tau}^*) &= \frac{G_E(Q^2) + \tilde{\tau}^* G_M(Q^2)}{1+\tilde{\tau}^*}\,, \\
\kappa F_2(Q^2;\tilde{\tau}^*) &= \frac{M}{M^*}\,\frac{G_M(Q^2) - G_E(Q^2)}{1+\tilde{\tau}^*}\,,
\end{aligned}
\tag{2.9}
$$

where the new variable $\tilde{\tau}^* \equiv [\mathbf{q}^2 - (E^*_{\mathbf{p}+\mathbf{q}} - E^*_{\mathbf{p}})^2]/4M^{*2}$. In contrast to model F described above, the off-shell choice (2.9) leads to a total nucleon magnetic moment equal to $(1+\kappa)\,e/2M^*$. In particular, the effective Dirac and anomalous magnetic moments behave similarly in the nuclear medium. [Note that the factor M/M^*, appearing on the right-hand side of F_2 in (2.9), has the effect of replacing $\kappa/2M$ by $\kappa/2M^*$ in (2.7).] Also, since $\tilde{\tau}^*$ depends *only* on the momenta $\mathbf{p}$ and $\mathbf{q}$, the photon energy ω now enters the form factors F_1 and F_2 *only* through the Sachs form factors $G_E(Q^2)$ and $G_M(Q^2)$. This can be seen by expressing Γ_0 in the alternate form given in (A5) of Appendix A. In the next section, we will see that model G leads to a relativistic Coulomb sum rule which is the direct analog of that derived in the free PWIA, but with $M \to M^*$.

III. COULOMB SUM RULES

The derivation of a RCSR based on the interacting PWIA of Sec. II is formally similar to that based on the free PWIA of Ref. [3]. We begin with the Coulomb response function (2.2), and formally evaluate the spacelike Coulomb sum (1.1) by integration over the photon energy ω. As described in Ref. [3], to arrive at a *non-energy-weighted* sum rule it must be possible to factor *all* dependence on ω (which here enters through Q^2) from the current matrix element in (2.2). This can be accomplished by requiring that the ratio $\Gamma_0(q)/G_{Ep}(Q^2)$ be independent of ω, since the plane-wave spinors appearing in (2.3) are functions only of the three-momenta $\mathbf{p}$ and $\mathbf{q}$. For a system of Dirac protons, for which $\Gamma_0 = \gamma_0$, this is satisfied trivially. In model G, it is satisfied by explicit construction, using the assumption of proportional Sachs form factors. For more complicated off-shell models, such as model F, which do involve explicit dependence on ω, the derivation of a RCSR may require further assumptions. In this article, we will assume that the above condition is satisfied, and outline the remaining steps which lead to a RCSR.

We next use closure to perform the sum over final states in the squared matrix element. In the nonrelativistic Coulomb sum rule, in which the integration is over all ω, the use of closure here is exact. In the relativistic case, the use of closure over the spacelike states alone requires certain assumptions about the spacelike nuclear excitation spectrum, as discussed in Ref. [3]. These arguments were based primarily on the example of a Fermi gas. We make similar arguments here, and assume that the spacelike spectrum is saturated by the nucleons-only response, where now "nucleons only" refers to positive-energy baryons of mass M^*. After performing the sum over final states $|f\rangle$, the momentum-space anticommutation relations are used to separate one- and two-body terms, as in (1.2). The relativistic recoil function which appears in (1.3) is of the form

$$r_\sigma(\mathbf{p},\mathbf{q}) = \frac{1}{2}\sum_{ss'} \left| j_{s's,\sigma}(\mathbf{p},\mathbf{q}) \right|^2, \tag{3.1}$$

where the matrix element is defined by

$$j_{s's,\sigma}(\mathbf{p},\mathbf{q}) \equiv \frac{\bar{u}_{s'\sigma}(\mathbf{p}+\mathbf{q})}{\sqrt{2E^*_{\mathbf{p}+\mathbf{q}}}} \frac{\Gamma_0(q)}{G_{Ep}(Q^2)} \frac{u_{s\sigma}(\mathbf{p})}{\sqrt{2E^*_{\mathbf{p}}}}, \tag{3.2}$$

and $u_{s\sigma}(\mathbf{p})$ represents a nucleon with momentum $\mathbf{p}$, spin s, and isospin projection σ. The above expressions are identical in form to those obtained in the free PWIA of Ref. [3], but with the replacement $M \to M^*$.

Although the one-body term, evaluated using (3.1) and (3.2), would account *exactly* for the relativistic effects of nucleon recoil and Fermi motion, it requires knowledge of the nucleon momentum distribution $n_\sigma(\mathbf{p})$ in the target ground state, which is not normally known precisely. We therefore expand the recoil factor $r_\sigma(\mathbf{p},\mathbf{q})$ in powers of the nucleon momentum $\mathbf{p}$, as in Ref. [3]. Assuming spherical symmetry, the one-body term takes the form[4]

$$\Sigma^{(1)}(\mathbf{q}) = \sum_\sigma N_\sigma \, r_{i\sigma}(\mathbf{q}) \, \langle \mathbf{p}^i \rangle_\sigma, \tag{3.3}$$

where the momentum moments are defined as

$$\langle \mathbf{p}^i \rangle_\sigma \equiv \frac{2}{N_\sigma} \sum_{\mathbf{p}} n_\sigma(\mathbf{p}) \, \mathbf{p}^i, \tag{3.4}$$

and $N_\sigma = Z, N$ for $\sigma = p, n$, respectively. In Ref. [3], we explained that only the first few moments are required to obtain an accurate result for the one-body term. In the next section, we will study how the convergence properties of this moment expansion depend on the parameter M^*, and on the particular choice of off-shell model. We now give the precise forms of the recoil factor $r_{i\sigma}(\mathbf{q})$ in models F and G.

A. Sum rules in model G

We first consider model G, in which we assume that the medium dependence of the nucleon electromagnetic coupling is described by the off-shell form factors (2.9). By construction the form factors F_1 and F_2 depend on Q^2 only through their linear dependence on the free Sachs form factors, which are assumed to be proportional. This ensures that the ratio $\Gamma_0/G_{Ep}(Q^2)$ is independent of the photon energy ω. With this condition satisfied, and since the baryon spinors are also independent of ω, the matrix element $j_{s's,\sigma}(\mathbf{p},\mathbf{q})$ is a function only of three-momenta and the separation of one- and two-body terms in $\Sigma(\mathbf{q})$ proceeds as described above, leading to (1.3).

Using the form factors (2.9) in (2.7) to evaluate the matrix element $j_{s's,\sigma}(\mathbf{p},\mathbf{q})$ we obtain

$$r_\sigma(\mathbf{p},\mathbf{q}) = \epsilon_\sigma^2 \, r_E(\mathbf{p},\mathbf{q}) + \mu_\sigma^2 \, r_M(\mathbf{p},\mathbf{q}), \tag{3.5}$$

where we have defined the electric (E) and magnetic (M) recoil functions

$$r_E(\mathbf{p},\mathbf{q}) \equiv \frac{1}{1+\bar{\tau}^*} \frac{(E^*_{\mathbf{p}+\mathbf{q}} + E^*_{\mathbf{p}})^2}{4E^*_{\mathbf{p}} E^*_{\mathbf{p}+\mathbf{q}}} \tag{3.6}$$

and

$$\begin{aligned} r_M(\mathbf{p},\mathbf{q}) &\equiv \frac{1}{1+\bar{\tau}^*} \frac{\bar{\tau}^*(E^*_{\mathbf{p}+\mathbf{q}} + E^*_{\mathbf{p}})^2 - (1+\bar{\tau}^*)\mathbf{q}^2}{4E^*_{\mathbf{p}} E^*_{\mathbf{p}+\mathbf{q}}} \\ &= \frac{1}{1+\bar{\tau}^*} \frac{1}{4M^{*2}} \frac{|\mathbf{p}\times\mathbf{q}|^2}{4E^*_{\mathbf{p}} E^*_{\mathbf{p}+\mathbf{q}}}. \end{aligned} \tag{3.7}$$

The functional forms of r_E and r_M can alternatively be obtained by writing Γ_μ of (2.7) in terms of G_E and G_M, as explained in Appendix A. In (3.5) we have also defined the nucleon electric charge ϵ_σ (in units of the proton charge) and the total magnetic moment μ_σ (in nuclear magnetons):

$$\begin{aligned} \epsilon_\sigma &\equiv \frac{G_{E\sigma}(Q^2)}{G_{Ep}(Q^2)} = \begin{cases} 1 & \text{for } \sigma = p \\ 0 & \text{for } \sigma = n \end{cases}, \\ \mu_\sigma &\equiv \frac{G_{M\sigma}(Q^2)}{G_{Ep}(Q^2)} = \begin{cases} 1+\kappa_p & \text{for } \sigma = p \\ \kappa_n & \text{for } \sigma = n. \end{cases} \end{aligned} \tag{3.8}$$

The functional form of the recoil function (3.5) is identical to that obtained in the free PWIA of Ref. [3], but with $M \to M^*$. This was ensured by choosing to include the factor M/M^* in (2.9). As described above, we expand $r_\sigma(\mathbf{p},\mathbf{q})$ in powers of the nucleon momentum $\mathbf{p}$, which with (1.3) leads to an expression for the one-body term $\Sigma^{(1)}(\mathbf{q})$, as in (3.3). The expansion coefficients are given in Appendix B, and are identical to those obtained

[4]We have changed notation slightly from Ref. [3], in naming the expansion coefficients $r_{i\sigma}(\mathbf{q})$.

in the free PWIA of Ref. [3], but with $M \to M^*$. In this model, therefore, we can include the relativistic effects of a strong scalar field, in a straightforward extension of the RCSR derived in the free PWIA.

B. Sum rules in model F

We now return to model F, in which we assume that the nucleon form factors are given by (2.8). Now the photon energy ω enters not only through Q^2, which appears in the free Sachs form factors, but also through $\tau \equiv Q^2/4M^2$, which enters in the transformation (2.8). This additional dependence on ω can *not* be removed by simply dividing by an overall factor $G_{Ep}(Q^2)$, as in model G, without some further approximation.

In order to proceed, we will first calculate the matrix element appearing in (3.1), using the vertex operator (2.7) and the form factors (2.8). Since division by $G_{Ep}(Q^2)$ is not sufficient to render the matrix element a function only of the momenta $\mathbf{p}$ and $\mathbf{q}$, this function will not yet lead to the one-body term of the Coulomb sum rule. However, it does serve as a useful starting point to illustrate the important issues. We obtain

$$r_\sigma(\mathbf{p},\mathbf{q};Q^2) = \left[\frac{G^*_{E\sigma}(Q^2,\tilde{\tau}^*)}{G_{Ep}(Q^2)}\right]^2 r_E(\mathbf{p},\mathbf{q}) + \left[\frac{G^*_{M\sigma}(Q^2,\tilde{\tau}^*)}{G_{Ep}(Q^2)}\right]^2 r_M(\mathbf{p},\mathbf{q}), \qquad (3.9)$$

where we have emphasized explicitly the dependence on Q^2. The form of (3.9) is similar to (3.5) for model G, but with "effective" Sachs electric and magnetic form factors, defined by

$$G^*_{E\sigma}(Q^2,\tilde{\tau}^*) \equiv F_{1\sigma}(Q^2) - \kappa_\sigma \frac{M^*}{M}\tilde{\tau}^* F_{2\sigma}(Q^2),$$
$$G^*_{M\sigma}(Q^2,\tilde{\tau}^*) \equiv F_{1\sigma}(Q^2) + \kappa_\sigma \frac{M^*}{M} F_{2\sigma}(Q^2), \qquad (3.10)$$

which reduce to the free Sachs form factors if $M^* = M$. Inserting $F_1(Q^2)$ and $F_2(Q^2)$ of (2.8) into (3.10), we have

$$\frac{G^*_{E\sigma}(Q^2,\tilde{\tau}^*)}{G_{Ep}(Q^2)} = \epsilon_\sigma \left[\frac{1+\frac{M^*}{M}\tilde{\tau}^*}{1+\tau}\right] + \mu_\sigma \left[\frac{\tau-\frac{M^*}{M}\tilde{\tau}^*}{1+\tau}\right],$$
$$\frac{G^*_{M\sigma}(Q^2,\tilde{\tau}^*)}{G_{Ep}(Q^2)} = \epsilon_\sigma \left[\frac{1-\frac{M^*}{M}}{1+\tau}\right] + \mu_\sigma \left[\frac{\tau+\frac{M^*}{M}}{1+\tau}\right], \qquad (3.11)$$

which illustrates the "mixing" of (free) electric and magnetic contributions in this model. This mixing enters as a result of the reduced effective nucleon mass, and is distinct from the usual Lorentz mixing which occurs for a moving particle. Effective form factors of the sort (3.11) can also be seen in Ref. [6], for example.

To continue the derivation of a *non-energy-weighted* sum rule, we must make some assumption about the remaining ω dependence, which arises through the variable τ in (3.11). A simple method is based on the excitation energy of a uniform Fermi gas in QHD-I, which is expressible simply in terms of the three-momenta $\mathbf{p}$ and $\mathbf{q}$: $\omega = E^*_{\mathbf{p}+\mathbf{q}} - E^*_{\mathbf{p}}$. We therefore make the replacement in (3.11)

$$\tau(Q^2) \to \tau(Q^2)\Big|_{\omega=E^*_{\mathbf{p}+\mathbf{q}}-E^*_{\mathbf{p}}} = \tilde{\tau}^*\left(\frac{M^*}{M}\right)^2, \qquad (3.12)$$

to obtain energy-independent effective form factors. The factor $(M^*/M)^2$ arises directly from the definition of τ following (2.8). With the replacement (3.12) in (3.11), the recoil function (3.9) is now independent of the photon energy ω, and the derivation of a non-energy-weighted Coulomb sum rule for model F proceeds as for model G.

At this point, it is possible to expand (3.9) about $\mathbf{p}=\mathbf{0}$ and obtain a moment expansion for $\Sigma^{(1)}(\mathbf{q})$, as in (3.3). However, the resulting coefficients $r_{i\sigma}(\mathbf{q})$ are much more complicated than those which arise from (3.5) in model G, due to the dependence on $\mathbf{p}$ of the effective form factors (3.11). In addition, we will see that the essential physics of Fermi motion enters (3.9) only through the electric and magnetic recoil functions $r_E(\mathbf{p},\mathbf{q})$ and $r_M(\mathbf{p},\mathbf{q})$, and can be ignored in the effective form factors themselves. It is therefore useful to consider an approximation scheme which will allow a more efficient evaluation of the one-body term: setting $\mathbf{p}=\mathbf{0}$ in (3.11) after applying (3.12), we can write

$$r_\sigma(\mathbf{p},\mathbf{q}) = \epsilon^{*2}_\sigma(\mathbf{q})\, r_E(\mathbf{p},\mathbf{q}) + \mu^{*2}_\sigma(\mathbf{q})\, r_M(\mathbf{p},\mathbf{q}), \qquad (3.13)$$

where we have defined the "effective" nucleon charges and magnetic moments

$$\epsilon^*_\sigma(\mathbf{q}) \equiv \left.\frac{G^*_{E\sigma}(Q^2,\tilde{\tau}^*)}{G_{Ep}(Q^2)}\right|_{\mathbf{p}=\mathbf{0}} = \epsilon_\sigma\left[\frac{2M^2+M(E^*_{\mathbf{q}}-M^*)}{2M^2+M^*(E^*_{\mathbf{q}}-M^*)}\right] + \mu_\sigma\left[\frac{(M^*-M)(E^*_{\mathbf{q}}-M^*)}{2M^2+M^*(E^*_{\mathbf{q}}-M^*)}\right],$$
$$\mu^*_\sigma(\mathbf{q}) \equiv \left.\frac{G^*_{M\sigma}(Q^2,\tilde{\tau}^*)}{G_{Ep}(Q^2)}\right|_{\mathbf{p}=\mathbf{0}} = \epsilon_\sigma\left[\frac{2M(M-M^*)}{2M^2+M^*(E^*_{\mathbf{q}}-M^*)}\right] + \mu_\sigma\left[\frac{2MM^*+M^*(E^*_{\mathbf{q}}-M^*)}{2M^2+M^*(E^*_{\mathbf{q}}-M^*)}\right], \qquad (3.14)$$

following (3.8). We will refer to expression (3.13) as the "factored moment expansion." The effective form factors now enter (3.13) as functions only of the momentum transfer $\mathbf{q}$, but do not complicate its expansion in $\mathbf{p}$. That this simplification is a valid approximation will be shown numerically in the next section.

IV. SUM RULES FOR RELATIVISTIC NUCLEAR MATTER

We now apply the sum rules derived in the previous section to a model nuclear system: uniform nuclear matter in the relativistic mean-field approximation, as given by QHD-I [4]. For this system the effects of nuclear structure on the Coulomb sum $\Sigma(\mathbf{q})$ enter through the effective nucleon mass M^* and through the nuclear electromagnetic current, as we have discussed in Sec. III. We are interested in the dependence of the Coulomb sum on M^* and on the choice of off-shell model (F or G) for the current. We then demonstrate how to apply the sum rule methods of Sec. III to the model system, as if it were measured Coulomb response data. The analysis will necessarily be model dependent, through the choice of M^* and the off-shell model (F or G), as well as through the moments $\langle \mathbf{p}^i \rangle_\sigma$ of the nucleon momenta in the target. We shall examine the accuracy of the moment expansion for the "best-case" analysis, that is, for which the sum rule parameters match those of the assumed nuclear model. This will illustrate the convergence properties of the moment expansion in each model, and the validity of the factored moment expansion in model F.

A. Test in Fermi gas model

We begin by computing the Coulomb sum $\Sigma(\mathbf{q})$ for a uniform Fermi gas of nucleons moving in uniform vector and scalar potentials. This has been studied previously [5,6]. The result can be written in the form

$$\Sigma(\mathbf{q}) = \int \frac{d^3p}{(2\pi)^3} \sum_\sigma n_\sigma(\mathbf{p})\left[1-n_\sigma(\mathbf{p}+\mathbf{q})\right] \times \sum_{ss'} \left| j_{s's,\sigma}(\mathbf{p},\mathbf{q}) \right|^2, \tag{4.1}$$

where the matrix element $j_{s's,\sigma}(\mathbf{p},\mathbf{q})$ is defined in (3.2). We will evaluate (4.1) by numerical integration over p and θ, the angle between $\mathbf{p}$ and $\mathbf{q}$. The result is independent of V_0, for reasons given following (2.5). The factor $n_\sigma(\mathbf{p}) \equiv \theta(p_{F\sigma} - |\mathbf{p}|)$ restricts the sum over initial states to include only those states which are occupied in the nuclear ground state, and the factor $\left[1-n_\sigma(\mathbf{p}+\mathbf{q})\right]$ ensures that final states for which $|\mathbf{p}+\mathbf{q}| \leq p_{F\sigma}$ are excluded. This Pauli exclusion is the only source of two-particle correlations in this simple model.

We shall consider two particular values of M^* which arise in QHD-I, as well as $M^* = M$. In the mean-field theory (MFT), static polarization of the baryon vacuum in the nuclear ground state (due to the scalar field ϕ) is ignored in the coupled field equations. In the relativistic Hartree approximation (RHA), static vacuum polarization is included at the one-loop level.[5] The particular values of M^* are obtained from the self-consistent solutions for uniform nuclear matter at saturation. Taking $Z = N$ and using the input parameters $p_F = 1.42\ \mathrm{fm}^{-1}$ and $E_B/N = -15.75$ MeV/nucleon, leads to the values $M^*/M = 0.556$ in the MFT, and $M^*/M = 0.718$ in the RHA. Although it may be argued that the model assumptions of MFT are technically more consistent with the nucleons-only approximation, it may also be true that the resulting effective mass M^* is too small due to the simplicity of the model, as compared to optical potential phenomenology (see, e.g., Ref. [12]). In the calculations to follow, we will show results using both the MFT and RHA values of M^* to illustrate the kinds of effects which can be expected in this range.

For applications to data, it is convenient to cast the RCSR in a slightly different form, so that the saturation of the sum rule is more apparent. As in Ref. [3], we define the "modified" Coulomb sum[6]

$$S_A(\mathbf{q}) \equiv \frac{1}{Z} \frac{\Sigma(\mathbf{q})}{r_A(\mathbf{q})}, \qquad A = \mathrm{I}, \mathrm{II}, \mathrm{III}, \ldots, \tag{4.2}$$

where the recoil "correction" factors are defined as

$$r_{\mathrm{I}}(\mathbf{q}) \equiv \left[r_{0p} + \frac{N}{Z} r_{0n} \right], \tag{4.3}$$

$$r_{\mathrm{II}}(\mathbf{q}) \equiv r_{\mathrm{I}}(\mathbf{q}) + \left[r_{2p} \langle \mathbf{p}^2 \rangle_p + \frac{N}{Z} r_{2n} \langle \mathbf{p}^2 \rangle_n \right], \tag{4.4}$$

and so on for higher orders.[7] The expansion coefficients $r_{i\sigma}(\mathbf{q})$ are given in Appendix B. The moments $\langle \mathbf{p}^i \rangle_\sigma$ are defined in (3.4). For applications of the RCSR to $\Sigma(\mathbf{q})$ of (4.1), we will assume the sharp distribution $n_\sigma(\mathbf{p}) = \theta(p_{F\sigma} - |\mathbf{p}|)$ for the evaluation of the momentum moments. Integrating this distribution over the Fermi sphere of radius $p_{F\sigma}$ leads to the ith moment, given by

$$\langle \mathbf{p}^i \rangle_\sigma = \frac{3}{3+i}\, p_{F\sigma}^i. \tag{4.5}$$

This is consistent with the PWIA and the evaluation of the RCSR on (4.1). For applications to actual data, however, it would be preferable to use the most accurate values available for these moments, e.g., using experimentally determined $n_\sigma(\mathbf{p})$ from $(e, e'p)$.

Following (1.2), the above definitions lead to an *approximate* sum rule of the form

$$S_A(\mathbf{q}) \simeq 1 + \frac{1}{Z} \frac{1}{r_A(\mathbf{q})} \left[C(\mathbf{q}) + \Sigma^{(2)}_{\mathrm{un}}(\mathbf{q}) \right], \tag{4.6}$$

as discussed in Sec. V of Ref. [3]. The Fermi gas model includes only Pauli correlations, which vanish identically for $q \geq 2p_F$. Consequently, an application of the RCSR

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[5] "MFT" and "RHA" are used here in the conventional sense defined in Ref. [4].

[6] Note the factor $1/Z$, which is not present in S_A as defined in Ref. [3].

[7] In this notational scheme, $r_A(\mathbf{q})$ includes terms through $O(\mathbf{p}^{2A-2})$.

to (4.1) will be considered "accurate" if the modified Coulomb sum $S_A(\mathbf{q})$ tends to unity for momenta beyond the range of these correlations, i.e., if $S_A(\mathbf{q}) \to 1$ for $q \geq 2p_F$. This is frequently referred to in the literature as "saturation" of the sum rule.

B. Numerical results

In the following examples, we take $Z = N$. To demonstrate the importance of the effects of a reduced effective mass M^*, we first consider a system of Dirac protons, for which $\epsilon_p = \mu_p = 1$ and $\epsilon_n = \mu_n = 0$. In Ref. [3], we showed for $M^*/M = 1$ that the lowest-order sum rule, $S_{\rm I}(\mathbf{q})$, is accurate to within $\sim 1\%$ for this case (see Fig. 2 in Ref. [3]). In Fig. 1, the dotted and dot-dashed curves show the Coulomb sum $\Sigma(\mathbf{q})$ calculated with $M^*/M = 1$ and $M^*/M = 0.556$, respectively. In this example, we see a 10–20 % reduction in the Coulomb sum over this momentum range, due to the reduced effective mass M^*. This effect has been noted previously for lower momenta [5,6]. The dashed and solid curves show the lowest-order sum rule $S_{\rm I}(\mathbf{q})$ applied to $\Sigma(\mathbf{q}, M^*)$ in two different ways: In the dashed curve, the recoil factor $r_{\rm I}(\mathbf{q})$ has been evaluated at the free nucleon mass M, while the solid curve shows a consistent application of the sum rule, in which $r_{\rm I}(\mathbf{q})$ has been evaluated at the reduced mass M^*. The dashed curve demonstrates that, if the effective nucleon mass is substantially reduced in the medium, then one must account for this in the application of the RCSR. The solid curve demonstrates that even when the effective mass is greatly reduced, e.g., $M^*/M = 0.556$, the lowest-order sum rule $S_{\rm I}(\mathbf{q})$ applied to a system of Dirac nucleons is accurate to within $\sim 1\%$, *if* the appropriate value of M^* is used.

In Fig. 2, we show the RCSR applied to a uniform system of nucleons with anomalous magnetic moments,

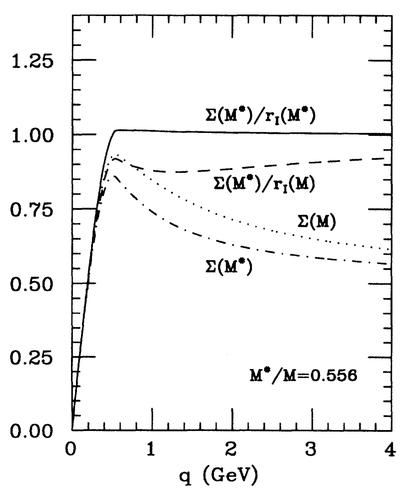

FIG. 1. $\Sigma(\mathbf{q})$ for a uniform system of Dirac protons, at two values of M^*. Two different evaluations of the lowest-order RCSR are shown as indicated.

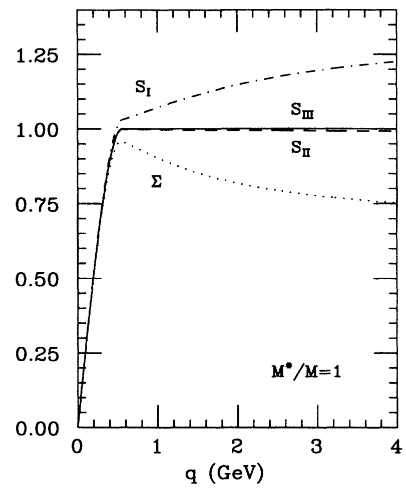

FIG. 2. RCSR evaluated for a uniform system of nucleons with anomalous magnetic moments, for $M^*/M = 1$. The Coulomb sum $\Sigma(\mathbf{q})$ in (4.1) and $S_A(\mathbf{q})$ in (4.2) are shown as indicated.

for $M^*/M = 1$. (Here models F and G are identical.) A similar figure was shown in Ref. [3], but it is useful to review the main results here for comparison. The unmodified Coulomb sum $\Sigma(\mathbf{q})$ is shown by the dotted curve, and indicates an enhancement relative to the free Dirac result. This is a consequence of the nucleon anomalous moments, as explained in Ref. [3]. The lowest-order sum rule $S_{\rm I}(\mathbf{q})$ overshoots the "correct" result, i.e., saturation, increasingly with higher momentum, and indicates the importance of Fermi motion effects when anomalous moments are included. In Ref. [3] we explained that this is because the magnetic contribution is enhanced relative to the electric by a factor $\mu_p^2 + \mu_n^2 \simeq 11.4$. This prevents the cancellation of electric and magnetic Fermi motion effects, which occurs nearly exactly for Dirac nucleons. The striking feature is that the $O(\mathbf{p}^2)$ result $S_{\rm II}(\mathbf{q})$ is accurate to within $\sim 1\%$ over this momentum range. We also show the $O(\mathbf{p}^4)$ result $S_{\rm III}(\mathbf{q})$, which is accurate to better than 0.1% over the same range. This we interpret as numerical evidence of convergence of the moment expansion in this model. The results shown here will be modified for $M^*/M < 1$, and will also depend on the choice of form factor model (F or G). We are interested in the accuracy of sum rules designed to account for these modifications.

We now consider $M^*/M < 1$. In Figs. 3(a) and 3(b) we show results for the RCSR in model G for RHA and MFT values of M^*, respectively. As for Fig. 2, the Coulomb $\Sigma(\mathbf{q})$ is enhanced relative to that for Dirac nucleons, due to Fermi motion effects, when anomalous moments are included. This model also has the rather peculiar feature that for $M^*/M \simeq 1/2$ we have $\Sigma(\mathbf{q}) \simeq 1$ for $q > 2p_F$. This value is coincidental (see Sec. IV C for further discussion). For still lower values of M^* the Coulomb sum can be significantly greater than unity. This is contrary to the common notion that relativistic effects necessarily suppress the Coulomb sum relative to unity. Note that the moment expansion converges more slowly for smaller M^*, as can be seen by comparing $S_{\rm II}(\mathbf{q})$ in Figs. 3(a) and 3(b). Still, the RCSR to $O(\mathbf{p}^2)$ is reasonably accurate, even for $M^*/M \sim 1/2$. The very high accuracy of $S_{\rm III}(\mathbf{q})$ is interpreted as numerical evidence of convergence in this model. Indeed, for $q > 2p_F$, $S_{\rm III} = 1.00$ for the RHA mass and $S_{\rm III} = 1.01$ for the MFT mass, at $q = 4$ GeV.

In Figs. 4(a) and 4(b) we show analogous results for a nuclear system under the off-shell assumption of model F. The Coulomb sum $\Sigma(\mathbf{q})$ behaves qualitatively differently in this model, with a minimum developing around $q \sim 2$ GeV as M^* decreases to the MFT value. The differences originate from the additional q dependence in the effective form factors $G^*_{E\sigma}/G_{Ep}$ and $G^*_{M\sigma}/G_{Ep}$, which are constants in model G. As in model G, $S_{\rm I}(\mathbf{q})$ is not adequate, and illustrates the importance of Fermi motion effects when anomalous moments are included. Also as for model G, $S_{\rm II}(\mathbf{q})$, shown by the dashed curve, is reasonably accurate over this range of momenta and M^*. The improvement in $S_{\rm III}(\mathbf{q})$ is interpreted as numerical evidence of convergence of the moment expansion. The most important feature of the sum rule results just presented for model F is the success of the factored moment expansion, which allows the use of the *same* expansion coefficients as in model G, i.e., those given in Appendix B. This approximation neglects only the Fermi motion effects which enter through the ratios $[G^*/G_{Ep}]^2$ in (3.9). To illustrate its accuracy, we have evaluated $S_{\rm II}(\mathbf{q})$ in a "consistent" moment expansion which correctly includes to $O(\mathbf{p}^2)$ additional Fermi motion effects entering through these ratios. This result is shown by a dot-dashed curve, which is nearly identical to the "factored" result, shown by the dashed curve. These are small corrections on the scale of the other kinematical effects, lending support to the factored moment expansion in model F.

In summary, we have shown that the relativistic effects of nucleon recoil, Fermi motion and a reduced effective

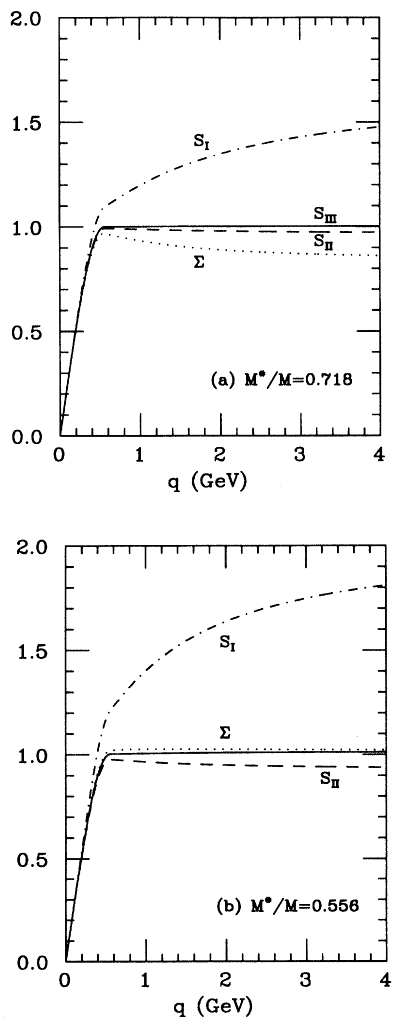

FIG. 3. Same as Fig. 2, but for $M^*/M < 1$ in model G. Results are shown for (a) $M^*/M = 0.718$ and (b) $M^*/M = 0.556$.

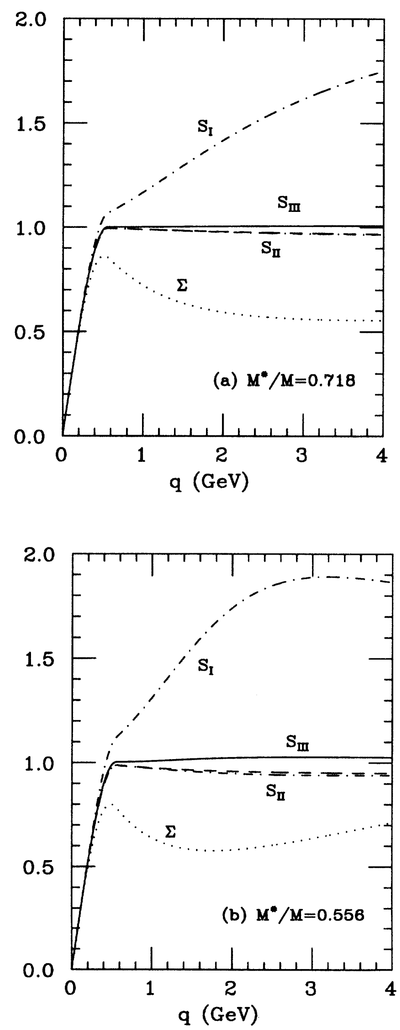

FIG. 4. Same as Fig. 3, but in model F. Note that $S_{\rm II}(\mathbf{q})$ is calculated in two ways: the "factored" (dashed) and "consistent" (similar dot-dashed) moment expansions.

mass M^* can be accounted for accurately in both models F and G. Keeping only terms through $O(\mathbf{p}^2)$ gives $\sim 5\%$ accuracy, which is small on the scale of the dominant kinematical effects. Keeping terms through $O(\mathbf{p}^4)$ gives $\sim 1\%$ accuracy. We emphasize that in both models F and G, the RCSR was evaluated using the *same* moment expansion coefficients (those in Appendix B), but in model F the effective charges and magnetic moments depend on q and M^*. This simplification requires using a factored moment expansion, in which Fermi motion effects entering though the effective form factors $G^*_{E\sigma}$ and $G^*_{M\sigma}$ are ignored. In the cases studied this is an excellent approximation, as explained further below.

C. Discussion of results

One can understand the results just presented by looking individually at the electric and magnetic recoil functions r_E and r_M, defined in (3.6) and (3.7), as functions of M^*. We shall look at the variations of $\epsilon^*_\sigma(\mathbf{q})$ and $\mu^*_\sigma(\mathbf{q})$ of (3.14) in model F, compared to ϵ_σ and μ_σ in model G. We are especially interested in the factored moment expansion in model F. We shall examine the reasons for its success, and under what conditions such an approach may be expected to work for other off-shell assumptions. We argue that a factored moment expansion, like that adopted here, should be accurate in a large class of off-shell form factor models.

In Figs. 5(a) and 5(b), we show the electric and magnetic recoil functions, $r_E(\mathbf{p},\mathbf{q})$ of (3.6) and $r_M(\mathbf{p},\mathbf{q})$ of (3.7), for two values of M^*. In both figures, the solid curve represents the electric term $r_E(\mathbf{0},\mathbf{q})$, which dominates the Coulomb response for a system of Dirac nucleons. The magnetic term $r_M(\mathbf{0},\mathbf{q})=0$, as can be seen from the second equality in (3.7). Thus the Coulomb response for a nucleon at rest is purely electric, as noted in Ref. [3]. The electric function $r_E(\mathbf{0},\mathbf{q})$ decreases from unity at $q=0$ to its limiting value 1/2 as $q\to\infty$. These limits are independent of M^*; however, for smaller M^*, $r_E(\mathbf{0},\mathbf{q})$ decreases more rapidly with q, since it is a monotonic function of q/M^*. This increased suppression is easily accommodated by using an appropriate value for M^*, as discussed for the Dirac case in Fig. 1.

The corrections for Fermi motion to r_E and r_M are also shown in Figs. 5(a) and 5(b), and are denoted by $\delta r_E(\mathbf{q})$ and $\delta r_M(\mathbf{q})$, respectively. These corrections were calculated by averaging $r_E(\mathbf{p},\mathbf{q})$ and $r_M(\mathbf{p},\mathbf{q})$ over the Fermi sphere, taking $p_F = 1.42\ \mathrm{fm}^{-1}$. For a system of Dirac nucleons, δr_E and δr_M enter the response with equal weighting. We noted in Ref. [3] for $M^*/M=1$ that this leads to a nearly exact cancellation over the entire momentum range, and ensures that the lowest-order sum rule, RCSR-I, is accurate to within $\sim 1\%$. Note that this cancellation persists to $\sim 1\%$ accuracy for $M^*/M < 1$, although *both* δr_E and δr_M nearly triple for $M^*/M \sim 1/2$, as in the MFT example.

For nucleons with anomalous magnetic moments, this near cancellation of $\delta r_E(\mathbf{q})$ and $\delta r_M(\mathbf{q})$ no longer occurs, since these functions are multiplied by different factors: ϵ^2_σ and μ^2_σ of (3.8) for model G, or $\epsilon^{*2}_\sigma(\mathbf{q})$ and $\mu^{*2}_\sigma(\mathbf{q})$ of (3.14) for model F. In model G, $\delta r_M(\mathbf{q})$ is multiplied by the constant factor $\mu_p^2+\mu_n^2 \simeq 11.4$, while $\epsilon_p^2+\epsilon_n^2 = 1$. For $M^*/M=1$, this gives a 10–20 % increase in the Coulomb sum $\Sigma(\mathbf{q})$, as was discussed in Ref. [3]. As M^*/M decreases, the enhanced effect of $\delta r_M(\mathbf{q})$ increases further: for $M^*/M=0.556$, the Fermi motion correction increases with q about as quickly as $r_E(\mathbf{0},\mathbf{q})$ decreases, as shown in Fig. 5(b). This leads to the behavior of $\Sigma(\mathbf{q})$ for this value of M^*, shown in Fig. 3(b), and remarked on earlier, i.e., $\Sigma(\mathbf{q}) \simeq 1$. As M^* is decreased to the MFT value, this increase in δr_M effectively triples the importance of higher moments. This accounts for the decreased accuracy of the RCSR at $O(\mathbf{p}^2)$, compared to that applied to free nucleons.

Model F has qualitatively similar behavior, as we saw in Figs. 4(a) and 4(b). Here, however, $\Sigma(\mathbf{q})$ is modified

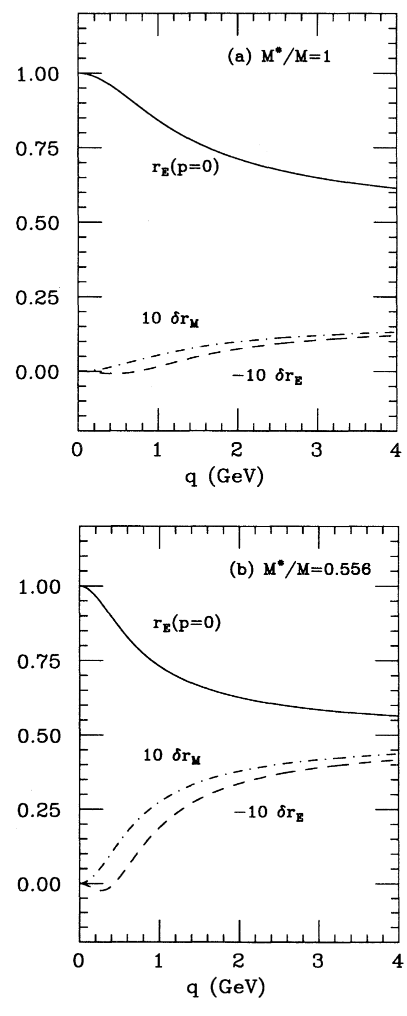

FIG. 5. Electric and magnetic recoil functions, $r_E(\mathbf{p},\mathbf{q})$ and $r_M(\mathbf{p},\mathbf{q})$, in a convenient separation. Results are shown for (a) $M^*/M = 1$ and (b) $M^*/M = 0.556$. The functions $r_E(\mathbf{0},\mathbf{q})$ (solid), $\delta r_E(\mathbf{q})$ (dashed), and $\delta r_M(\mathbf{q})$ (dot-dashed) are shown; $r_M(\mathbf{0},\mathbf{q})=0$. (Note the scale changes for δr_E and δr_M.)

by the effective charges and moments (3.14), which are shown in Fig. 6 (solid curves). For this case $(\mu_p^{*2}+\mu_n^{*2})$ is somewhat smaller than $(\mu_p^2+\mu_n^2)$ and increases with q over the range shown. This explains the smaller values of $\Sigma(\mathbf{q})$ in Figs. 4(a) and 4(b) than in Figs. 3(a) and 3(b), and the upturn with increasing q in Fig. 4(b).

We have seen that the convergence properties of the RCSR in model F are similar to those in model G, in spite of the use of a factored moment expansion that ignores certain Fermi motion effects, which would enter through the effective form factors (3.11). In order to understand this result, we first examine the effective form factors themselves. In Fig. 6, we show these in the form in which they enter the recoil function $r_\sigma(\mathbf{p},\mathbf{q};Q^2)$ in model F, i.e., $[G^*_{E\sigma}/G_{Ep}]^2$ and $[G^*_{M\sigma}/G_{Ep}]^2$. We have used (3.12) and $M^*/M = 0.556$ in their evaluation. The dashed curves show the form factors at $\mathbf{p} = \mathbf{0}$, and the solid curves were obtained by averaging over the Fermi sphere with $p_F = 1.42\ \mathrm{fm}^{-1}$. [Note that the electric form factors tend to their free values in the limit $q \to 0$, while the magnetic form factors tend to their free values in the limit $q \to \infty$. This can be seen from the definitions given in (3.11).] We see that the Fermi motion corrections to the ratios $[G^*/G_{Ep}]^2$ are small compared to the ratios themselves. Thus terms $O(\mathbf{p}^2)$ and higher are small, especially in the magnetic form factors. [Terms $O(\mathbf{p}\cdot\mathbf{q})$ vanish upon angle integration with spherical symmetry, and therefore do not contribute to the results in Fig. 6.]

To understand the success of the factored moment expansion, consider the second-order sum rule $S_{\mathrm{II}}(\mathbf{q})$. It omits two types of Fermi corrections: (1) $O(\mathbf{p}^2)$ terms which come from a moment expansion of the effective form factors and multiply $r_E(\mathbf{0},\mathbf{q})$ and $r_M(\mathbf{0},\mathbf{q})$, and (2) products of linear terms involving $(\mathbf{p}\cdot\mathbf{q})$, one term from the effective form factors, and one from the recoil factors r_E and r_M. The first of these make at most $\sim 5\%$ correction to the electric contribution, and less than $\sim 1\%$ correction to the magnetic, as can be seen from Fig. 6. These terms can therefore be neglected to a good approximation. Linear terms do not enter the magnetic contribution, as can be seen from the $(\mathbf{p}\times\mathbf{q})^2$ factor in (3.7). Linear terms have been omitted from the electric contribution, and could in principle have been appreciable. In fact, these terms taken together result in less than a 1% error in the RCSR, as can be seen by comparing the dashed and dot-dashed curves for $S_{\mathrm{II}}(\mathbf{q})$ in Figs. 4(a) and 4(b). Thus the success of the factored moment expansion in model F can be attributed to the smallness of Fermi corrections in the effective form factors themselves, and in part to the complete absence of linear terms in the magnetic contribution.

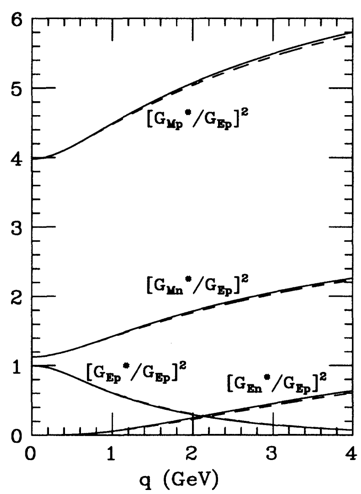

FIG. 6. Effective form factors (squared) in model F, for $M^*/M = 0.556$. Solid curves are obtained by averaging (3.11) with (3.12) over the Fermi sphere, and include Fermi corrections to all orders. Dashed curves are $\epsilon_\sigma^{*2}(\mathbf{q})$ and $\mu_\sigma^{*2}(\mathbf{q})$ of (3.14), as used in the "factored" moment expansion.

We believe that the success of the factored moment expansion in model F may also occur for a much wider variety of off-shell models than we have considered here. The basis for this claim is that Fermi corrections tend to be small in general, representing effects of order $(p_F/M^*)^2$. The main exception is the magnetic correction, which is substantial because of two properties: $r_M(0,\mathbf{q}) = 0$, and $(G^{*2}_{Mp}+G^{*2}_{Mn})/G^2_{Ep} \sim 10$. The magnetic contribution δr_M is small, but is then enhanced by roughly an order of magnitude by the anomalous magnetic moments. No such enhancement occurs for δr_E. Thus in model F the further corrections due to $\mathbf{p}$ dependence in $[G^*_{E\sigma}/G_{Ep}]^2$ and $[G^*_{M\sigma}/G_{Ep}]^2$ are negligible. In other off-shell models, for which $r_\sigma(\mathbf{p},\mathbf{q})$ is expressible in the form (3.9), we expect Fermi motion effects in the corresponding effective form factors also to be small. We therefore propose that a factored moment expansion, like that employed here in model F, may also be accurate for the evaluation of Fermi motion effects in a wide variety of off-shell models. This allows one to evaluate the RCSR using the expansion coefficients given in Appendix B. [We include $\delta r_E(\mathbf{q})$, although a small contribution in nucleon models with anomalous magnetic moments, to ensure the cancellation of Fermi corrections for the Dirac system, and to allow the "consistent" calculation of $S_{\mathrm{II}}(\mathbf{q})$ in Figs. 5(a) and 5(b).]

V. SUMMARY AND CONCLUSIONS

The main results of this paper can be summarized as follows: The effects of nuclear binding in the mean-field approximation enter the Coulomb response function $W_C(\omega,\mathbf{q})$ and the Coulomb sum $\Sigma(\mathbf{q})$ in a way that can be characterized by a reduced effective mass M^*. These effects depend on the behavior of the electromagnetic current for the off-shell kinematics in the nuclear medium. The sensitivity of the Coulomb sum $\Sigma(\mathbf{q})$ to the choice of off-shell model has been discussed previously [10,11]. We consider two illustrative models: F and G. We demonstrate in these models that the RCSR of Ref. [3] can be extended to account for relativistic binding effects, in addition to the purely kinematic effects of recoil and Fermi motion treated in Ref. [3]. The resulting RCSR is no longer model independent, since one must make some assumptions about M^* and the off-shell behavior of the form factors. However, to the extent that the dominant effects of the nuclear medium can be characterized by the simple parameter M^*, the Coulomb sum rule analysis can be applied to data, with the goal of looking for nuclear structure effects beyond the mean field, e.g., two-body correlations. We emphasize that the form of the sum rule is sufficiently general to accommodate a broad class of off-shell form factor models, i.e., those for which the recoil function $r_\sigma(\mathbf{p},\mathbf{q})$ can be expressed in the form (3.9).

The real change in transforming to the M^* basis enters through the nucleons-only approximation, under which the sum rule is derived. Strong potentials in relativistic models mix free $\bar{N}$ components into the initial and final interacting nuclear states, and modify the spacelike response function. Chinn, Picklesimer, and Van Orden [7,8] have isolated the effects of this mixing of $\bar{N}$ components, and show results qualitatively similar to those seen in Fig. 4. (Their form factors are closer to our model F than G.) The transformation to an M^* basis automatically incorporates these potential effects in a convenient representation, although the use of an effective mass independent of position or momentum may only approximate the physical situation.

How might one apply this RCSR to experimental data? First, the Coulomb sum $\Sigma(\mathbf{q})$ must be calculated from the measured Coulomb response function, as in (1.1). Then, one must make some specific assumptions about the off-shell behavior of the current operator, as discussed in Sec. III. This should be cast in the G^* form as in (3.10) and (3.11) for model F [or $G^* \equiv G(Q^2)$ for model G], with the substitution (3.12) to remove (approximately) any residual ω dependence from $G^*_{E\sigma}/G_{Ep}$ and $G^*_{M\sigma}/G_{Ep}$. The choice of model is not restricted to those presented in Secs. III A and III B. With the factored moment expansion, the relativistic recoil function (3.1) takes the form (3.13) and the ratios G^*/G_{Ep} are evaluated at $\mathbf{p}=\mathbf{0}$, as in (3.14). The recoil factor $r_A(\mathbf{q})$ is then evaluated in a moment expansion, e.g., to $O(\mathbf{p}^2)$ as in (4.4), with $\langle \mathbf{p}^2\rangle_\sigma$ fixed by other experimental information, or by a model as in (4.5), or as a free parameter. The expansion coefficients $r_{i\sigma}(\mathbf{q})$ are given in Appendix B, and are functions of the parameter M^*.

The modified Coulomb sum $S_A(\mathbf{q})$ is obtained by forming the ratio (4.2) of the experimentally determined numerator $\Sigma(\mathbf{q})$ and the (model-dependent) recoil denominator $Zr_A(\mathbf{q})$. The expected behavior of $\Sigma(\mathbf{q})$ with increasing q is that it will approach the one-body term $\Sigma^{(1)}(\mathbf{q})$ of (1.3), assuming that both $C(\mathbf{q})$ and $\Sigma^{(2)}_{\text{un}}(\mathbf{q}) \to 0$ in (1.2), as $q\to\infty$. The modified sum $S_A(\mathbf{q})$ will then approach unity *if* the assumed form of the current is correct, *and if* the effective mass M^* is appropriately chosen. Should that be the case for a given set of data, it would be reasonable to assume that the recoil functions have been correctly chosen. The sum rule (1.3) can then be used to investigate the two-nucleon correlation function in the ratio form

$$S_A(\mathbf{q}) \simeq 1 + \tilde{C}_A(\mathbf{q}) + \frac{\Sigma^{(2)}_{\text{un}}(\mathbf{q})}{Zr_A(\mathbf{q})}, \tag{5.1}$$

by looking for deviations from unity at moderate momentum transfers. The two-body correlation function,

$$\tilde{C}_A(\mathbf{q}) \equiv \frac{1}{Z}\frac{C(\mathbf{q})}{r_A(\mathbf{q})}, \tag{5.2}$$

is related to the standard nonrelativistic correlation function. [See Eqs. (5.8) and (5.16) in Ref. [3].] An interesting application of the RCSR is the constraint of the viable off-shell form factor models by analyzing the experimentally determined Coulomb sum $\Sigma(\mathbf{q})$ in different off-shell models and comparing to $S_A(\mathbf{q})\simeq 1$ beyond the expected range of correlations.

A further remark about the analysis of data with the RCSR seems appropriate. It has become customary for the experimental Coulomb response data to be integrated in a modified form of (1.1), as suggested by de Forest [13], in which the proton electric form factor $G_{Ep}(Q^2)$ is replaced by $\tilde{G}_{Ep}(Q^2) \equiv G_{Ep}(Q^2)\sqrt{(1+\tau)/(1+2\tau)}$, with $\tau \equiv Q^2/4M^2$ (see, e.g., Refs. [14,15]). We explained in Ref. [3] why this procedure will not lead to a *non-energy-weighted* sum rule: to obtain a sum rule of the form (1.2) or (1.5), the extra ω dependence should not be introduced into the definition of the Coulomb sum. The kinematic effects included in the recoil factors $r_A(\mathbf{q})$ depend on the three-momentum transfer $\mathbf{q}$, rather than on the invariant Q^2. It is this property which preserves the sum rule in passing from (1.2) to (5.1).

The use of the M^* basis for both initial and final states, with the same value of M^*, implicitly assumes that the nucleus is large enough that the nucleon kinematics in the final state are sensitive to the scalar field. For smaller nuclei, it may be necessary to account for the effects of finite size. This could possibly be accomplished through a rederivation of the RCSR in a Hartree representation based on bound, localized nuclear states. Another issue is the momentum dependence of the mean-field potentials, as discussed in Refs. [16,17], for example. Since the RCSR has been derived in momentum space, such a modification is relatively straightforward. As a first approximation, which is consistent with our conclusion that Fermi motion effects tend to be small, it seems reasonable to use $M^*(0)$ for initial states and $M^*(\mathbf{q})$ for final states. This prescription preserves the basic structure of the RCSR, in that no further dependence on $\mathbf{p}$ is introduced. This would require the initial and final state masses to be treated distinctly, however, and would complicate the form of the coefficients in Appendix B.

We are currently investigating the effects of virtual $N\bar{N}$ pairs (of mass M^*) on the RCSR, as they enter intermediate excited states in the random phase approximation. These were considered previously by Horowitz [18], and appear to be significant. We are also interested in how energy-dependent terms, which can enter in off-shell models where a substitution of the form (3.12) is not appropriate, affect the RCSR.

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APPENDIX A: Γ_μ IN TERMS OF SACHS FORM FACTORS

An alternative form of the current operator (2.7) may be obtained by making a Gordon transformation [19,20] on matrix elements between free plane-wave spinors:

$$\Gamma_\mu = \frac{1}{1+\tau}\left[G_E(Q^2)\ \frac{P_\mu}{M} + G_M(Q^2)\ \frac{r_\mu}{4M^2}\right], \quad \text{(A1)}$$

where $P_\mu \equiv \frac{1}{2}(p+p')_\mu$, and

$$r_\mu \equiv \frac{1}{2}\left[\gamma_\mu(P\cdot\gamma)(q\cdot\gamma) - (q\cdot\gamma)(P\cdot\gamma)\gamma_\mu\right], \quad \text{(A2)}$$

and the F's and G's are related by (2.8). Another form of r_μ may be obtained [21]:

$$r_\mu = 2\gamma_5\epsilon_{\mu\nu\rho\sigma}P^\nu q^\rho \gamma^\sigma. \quad \text{(A3)}$$

For $\mu=0$ we have

$$P_0 = \frac{1}{2}(E_{\mathbf{p}+\mathbf{q}}+E_{\mathbf{p}}),$$
$$r_0 = 2\gamma_5\boldsymbol{\gamma}\cdot(\mathbf{p}\times\mathbf{q}). \quad \text{(A4)}$$

For matrix elements between mean-field spinors, we have for $\mu=0$

$$\Gamma_0 = \frac{1}{1+\tilde{\tau}^*}\left[G_E(Q^2)\ \frac{(E^*_{\mathbf{p}+\mathbf{q}}+E^*_{\mathbf{p}})}{2M^*} + G_M(Q^2)\ \frac{\gamma_5\boldsymbol{\gamma}\cdot(\mathbf{p}\times\mathbf{q})}{2M^{*2}}\right]. \quad \text{(A5)}$$

In model G, we use the on-shell forms of $G_E(Q^2)$ and $G_M(Q^2)$, given in (A5). In model F, we use $G^*_{E\sigma}$ and $G^*_{M\sigma}$, given in (3.11). Expression (A5) makes clear the origin of the functional forms in (3.6) and (3.7).

APPENDIX B: COEFFICIENTS OF FACTORED MOMENT EXPANSION

In this Appendix, we give the coefficients $r_{i\sigma}$ for even powers through $O(\mathbf{p}^4)$ in the moment expansion. These are written as they appear in model G, i.e., in terms of the usual nucleon charge (ϵ_σ) and magnetic moment (μ_σ):

$$r_{0\sigma}(\mathbf{q}) = \epsilon_\sigma^2\ \frac{E^*_{\mathbf{q}}+M^*}{2E^*_{\mathbf{q}}}, \quad \text{(B1)}$$

$$r_{2\sigma}(\mathbf{q}) = \epsilon_\sigma^2\left[\frac{-4E^{*5}_{\mathbf{q}} + 5E^{*4}_{\mathbf{q}}M^* + 2E^{*2}_{\mathbf{q}}M^{*3} - 3M^{*5}}{12E^{*5}_{\mathbf{q}}M^{*2}}\right] + \mu_\sigma^2\left[\frac{E^*_{\mathbf{q}}-M^*}{3E^*_{\mathbf{q}}M^{*2}}\right], \quad \text{(B2)}$$

$$r_{4\sigma}(\mathbf{q}) = \frac{1}{240E^{*9}_{\mathbf{q}}M^{*4}}\ \frac{E^*_{\mathbf{q}}-M^*}{E^*_{\mathbf{q}}+M^*}\left[\epsilon_\sigma^2\left[64E^{*9}_{\mathbf{q}} + 29E^{*8}_{\mathbf{q}}M^* - 6E^{*7}_{\mathbf{q}}M^{*2} + 6E^{*6}_{\mathbf{q}}M^{*3} + 18E^{*5}_{\mathbf{q}}M^{*4} + 24E^{*4}_{\mathbf{q}}M^{*5} + 30E^{*3}_{\mathbf{q}}M^{*6} - 90E^{*2}_{\mathbf{q}}M^{*7} - 210E^*_{\mathbf{q}}M^{*8}\right] + \mu_\sigma^2\left[-64E^{*9}_{\mathbf{q}} - 24E^{*8}_{\mathbf{q}}M^* + 16E^{*7}_{\mathbf{q}}M^{*2} - 16E^{*6}_{\mathbf{q}}M^{*3} - 48E^{*5}_{\mathbf{q}}M^{*4} - 24E^{*4}_{\mathbf{q}}M^{*5}\right]\right]. \quad \text{(B3)}$$

These forms are also applicable to model F, upon making the replacements $\epsilon_\sigma\to\epsilon^*_\sigma(\mathbf{q})$ and $\mu_\sigma\to\mu^*_\sigma(\mathbf{q})$ using (3.14).

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