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Imaginary part of the optical potential for finite temperature and for preequilibrium processes

M. Abe and S. Yoshida
Ishinomaki Senshu University, Ishinomaki 986-80, Japan

K. Sato
Division of Physics, Tohoku College of Pharmacy, Sendai 981, Japan
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Absorptive potentials for preequilibrium reactions are studied in connection with the absorptive potentials at finite temperature. As a temperature fixed state is a mixture of many-particle many-hole states, an absorptive potential at finite temperature is constructed from exciton number fixed absorptive potentials obtained in the previous paper. Our results, which include terms linear in temperature, are compared with those of Morel and Nozières which lack these terms. The absorptive potential at finite temperature is decomposed by dividing the occupation probability function into the particle and hole parts. The resulting potentials are associated with six processes, and by introducing further approximation we obtain the correspondence with previously obtained ones. Using the Fermi liquid approximation the absorptive potentials for each process and both contributions of polarization and correlation are investigated in detail and the origin of the linear terms in temperature is found coming from the Pauli blocking effect. The results are compared with those without the Fermi liquid approximation using a simplified Skyrme interaction. It is found that the Fermi liquid approximation works fairly well when absorption is not restricted to the bound configurations. Furthermore the Pauli correction term is found important and is shown to be easily taken into account by using the thermal formalism.

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I. INTRODUCTION

Optical potentials have been devised to generate an energy averaged total cross section and applied to numerous analyses of experimental data. If we assume that the incident particle is a nucleon (this assumption is kept throughout this paper), the optical potential is the self-energy of the nucleon and is calculated from the expectation value of the mass operator. The absorptive potential is the imaginary part of the optical potential. If the lowest order terms in perturbation series are taken, the absorptive potential corresponds to the absorption process of the nucleon by creating a particle-hole pair [1].

In finite nuclei, surface vibrational modes are very important in evaluating the absorptive potential, which generates the surface term [2,3]. However in this paper these effects will not be considered but put off for future works, as the volume effects are the first to be treated as a continuation of our previous work [4]. In this paper we regard the nucleus as nuclear matter, and the surface effects are taken into account only through the change of the density around the nuclear surface as in our previous paper.

In that paper we calculated the absorptive potential for preequilibrium process by using semiclassical approximation. In preequilibrium theories [5–7], the exciton number, a sum of particle and hole number, is fixed for each stage, and absorptive potentials are needed to calculate formation and decay rates of the compound system with fixed exciton number. For this we consider three types of processes. First is creation of a particle-hole pair in which the exciton number is increased by 2. Second is the process in which the exciton number does not change. Particle-particle, particle-hole scatterings belong to this process. The last is the process in which the exciton number is reduced by 2. Annihilation of a particle-hole pair by the nucleon corresponds to this. For the calculation of the absorptive potential spectator nucleons are included to share the energy. However the Pauli blocking effect by the spectator was not included in the paper, although the effect was very crudely estimated on an average. This effect is expected to be important.

In most preequilibrium theories the compound system is restricted to the space constructed from bound nucleon states, which is called Q space. The absorption is also restricted to the Q space. However in compound reaction theories and in some preequilibrium reaction theories absorption is allowed to all space in which nucleons occupy bound and continuum states, which we call $P+Q$ space. So the absorptive potential is calculated for both Q and $P+Q$ absorption and we found the differences between them are not small except at low energy.

In preequilibrium theories equilibration must be assumed within states of certain exciton number. Although validity of this assumption is questioned [8], at present this assumption cannot be avoided. On the other hand in equilibrium process like the compound process, equilibration must be maintained through all the stages considered. Of course equilibration within each exciton state must be assured. Therefore the absorptive potentials calculated by thermal formulation for a fixed temperature [9–11] can be utilized in our preequilibrium potential. It is known that an equilibrium state with fixed temperature is considered as a mixture of many-particle many-hole states [12]. So it must be possible to decompose the temperature fixed absorptive potential into an exciton fixed absorptive potential for various processes.

To advance the above program we found that the use the of semiclassical approximation [13] is helpful, as it gives an average over the energy and the mass number, as well as it makes the calculation easy and concrete. An absorptive potential at finite temperature has two contributions, polarization and correlation, and each contains three occupation probability functions of nucleons. In the preequilibrium theory particles and holes are distinguished, and the number of particles and the number of holes have an important role. Therefore to divide the occupation probability function into the particle part and the hole part at the Fermi energy is an essential procedure.

By dividing the occupation probability into two partial ones, the absorptive potential at finite temperature is decomposed into six parts, and each is identified with processes that are the same as those encountered in the preequilibrium calculation except one. The calculation of each term is carried out adopting Fermi liquid approximation [14]. With this help each term is calculated analytically, and furthermore the effect of Pauli blocking is studied in detail.

Next the calculations without the Fermi liquid approximation using a simplified Skyrme interaction are performed and the results are compared with those of the Fermi liquid approximation. The Fermi liquid approximation is expected to be valid at low temperature, and differences between the two are examined. In usual circumstances the Fermi liquid approximation is found to be very useful in practical application for $P+Q$ absorption. Throughout the present paper it is assumed for simplicity that the Coulomb interaction is neglected and the target nucleus has equal and even numbers of protons and neutrons $Z = N = A/2$. Therefore the mean potential is assumed to be independent of spin and isospin. In the ground state the nucleus is the $0p$-$0h$ state; consequently, in the excited state, particle number n_p and hole number n_h are equal and the exciton number is $2m = n_p + n_h$. The type of reactions treated in this work is restricted to nucleon-induced reactions, and the absorption potential is always evaluated on the energy shell.

In the next section the absorptive potential for equilibrium process is constructed from the preequilibrium absorptive potentials obtained in the previous paper. In Sec. III the Wigner transform of the absorptive potential at finite temperature is evaluated in the semiclassical approximation. For this purpose the Wigner transform of a product of residual interaction potential is evaluated for the simplified Skyrme interaction and the finite range Gaussian interaction. In Sec. IV the absorptive potential at finite temperature is decomposed into those for various types of processes and the relation with preequilibrium absorptive potentials is established. Section V is devoted to the calculation in Fermi liquid approximation, and analytical results are shown. Numerical calculations with the simplified Skyrme interaction are carried out in Sec. VI without the Fermi liquid approximation. In Sec. VII summary and discussions will be presented. In Appendices detailed evaluation of integrals appearing in the Fermi liquid approximation will be given.

II. ABSORPTIVE POTENTIAL FOR EXCITED NUCLEI IN EQUILIBRIUM

In the previous paper [4] we calculated the absorptive potential for a *mp-mh* state of a target nucleus. In this case four processes contribute. The first is creation of a particle-hole pair, which is called the (a) process, and the exciton number increases by 2. In the approximation adopted in the paper the absorptive potential $w^{(a)}(E_1)$ is only a function of the incident energy E_1, and independent of m and the excitation energy of the target nucleus.

The second (b) and third (c) processes correspond to scattering by a particle and a hole, respectively. In this case the exciton number does not change. The absorptive potentials without the spectator nucleons are denoted by $w^{(b)}(E_1,\epsilon_2)$ and $w^{(c)}(E_1,\epsilon_2)$, where ϵ_2 is the excitation energy of the nucleon to which the incident nucleon interacts. To obtain the absorptive potential for a nucleus in a *mp-mh* state, the absorptive potentials for the elementary processes $w^{(b)}(E_1,\epsilon_2)$ and $w^{(c)}(E_1,\epsilon_2)$ are multiplied by m with $\varepsilon_2 = 0$.

In the fourth process (e), which was called the (d) process in the previous paper, a particle-hole pair is annihilated and the exciton number decreases by 2. The absorptive potential is obtained if the absorptive potential for the elementary process $w^{(e)}(E_1,\epsilon_2)$ is multiplied by $m(m-1)/2$ which is the number of combinations to pick up two holes from a *mp-mh* state.

When a nucleon impinges on a *mp-mh* nucleus, these four processes contribute and the absorptive potential is given by

$$\begin{aligned} W(m,m,E_1,\epsilon) = w^{(a)}(E_1) + m[w^{(b)}(E_1,0) \\ +w^{(c)}(E_1,0)] \\ +\frac{m(m-1)}{2}w^{(e)}(E_1,0), \end{aligned} \tag{2.1}$$

where the excitation energy of the target nucleus is denoted by ϵ, which is independent of m. Using this we calculate the absorptive potential for an excited nucleus in an equilibrium state.

A. Fixed excitation energy

Let us assume that the target nucleus is excited with an excitation energy ϵ and in an equilibrium state, namely the compound state. This state is considered to be a mixture of many-particle many-hole states. As the state is in equilibrium the probability to find a *mp-mh* state is proportional to the partial state density $\omega(m,m,\epsilon)$,

$$P_\epsilon(m) = \omega(m, m, \epsilon) \Big/ \sum_{m'} \omega(m', m', \epsilon). \tag{2.2}$$

Among available formulas of partial state densities the one given by Ericson [15] and Williams [16] is the simplest, by which the partial state densities are calculated for ^{208}Pb and are shown in Fig. 1(A). The shape has a Gaussian-like form and it is approximated by

$$P_\epsilon(m) \approx \sqrt{\frac{2}{\pi \overline{m}}} \exp[-2(m - \overline{m})^2/\overline{m}]. \tag{2.3}$$

In the above formula the mean particle or hole number $\overline{m}$ is given by

$$\overline{m} = \sqrt{g_F \epsilon / 2}, \tag{2.4}$$

where g_F is the single particle state density for neutrons and protons at the Fermi energy E_F. The variance of the particle or hole number is $\overline{m}/4$.

The equilibrium absorptive potential is obtained by averaging the one for *mp-mh* state (2.1) with the weight (2.2) as

$$\begin{aligned} W(E_1, \epsilon) &= \int_0^\infty P_\epsilon(m) W(m, m, E_1, \epsilon) dm \\ &\approx w^{(a)}(E_1) \\ &\quad + \sqrt{\frac{g_F \epsilon}{2}} \left[w^{(b)}(E_1, 0) + w^{(c)}(E_1, 0) \right] \\ &\quad + \frac{1}{2} \frac{g_F \epsilon}{2} w^{(e)}(E_1, 0). \end{aligned} \tag{2.5}$$

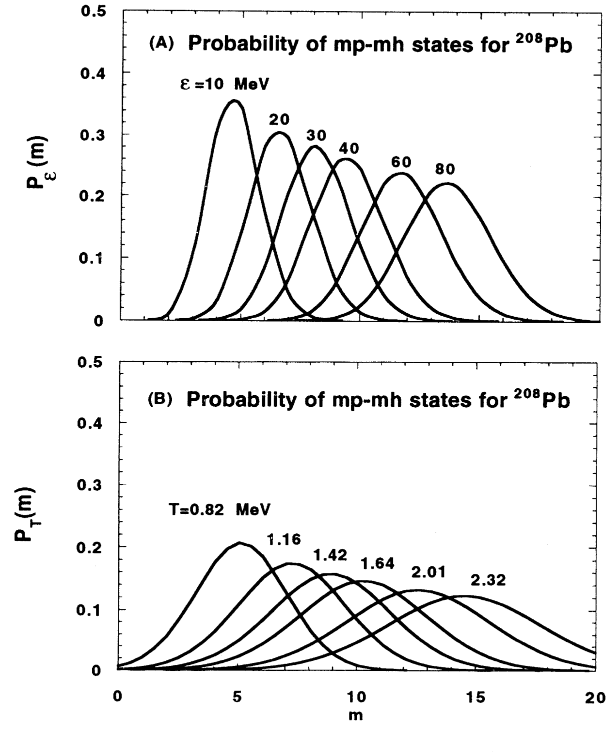

FIG. 1. Probability distribution of the *mp-mh* state in an excited state with excitation energy ϵ for ^{208}Pb [shown in (A)] and with fixed temperature T [shown in (B)]. The values of ϵ shown in (A) correspond to the values of T shown in (B) in that order.

B. Fixed temperature

In the previous subsection the compound nucleus is specified by giving the excitation energy, but it is also possible by assigning the temperature. In a compound nucleus with a given temperature T nucleons are distributed in single particle states according to the Fermi distribution function. From this the distribution function of *mp-mh* states is evaluated as

$$P_T(m) \approx \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(m - \overline{m})^2}{2\sigma^2} \right]. \tag{2.6}$$

The mean value of the particle or hole number $\overline{m}$ and the variance σ^2 are given by

$$\overline{m} = (\ln 2) g_F T, \qquad \sigma^2 = \overline{(m - \overline{m})^2} = \frac{1}{2} g_F T, \tag{2.7}$$

both of which are proportional to the temperature [12]. In Fig. 1(B) this distribution function is illustrated in the case of ^{208}Pb with $g_F = 9.04\ \text{MeV}^{-1}$ [4]. By using the well-known relation for Fermi gas

$$T = \sqrt{\frac{6}{\pi^2} \frac{\epsilon}{g_F}} \tag{2.8}$$

the mean particle or hole number $\overline{m}$ given by (2.7) is expressed in terms of the excitation energy, which does not agree exactly with the one given by (2.4). The width of the distribution for fixed temperature given by Eq. (2.7) is large compared with that for fixed excitation energy given by Eq. (2.3).

After averaging over the probability distribution $P_T(m)$, the equilibrium absorptive potential is expressed by

$$\begin{aligned} W_T(E_1) &\approx w^{(a)}(E_1) \\ &\quad + (\ln 2) g_F T [w^{(b)}(E_1, 0) + w^{(c)}(E_1, 0)] \\ &\quad + \frac{1}{2} [(\ln 2) g_F T]^2 w^{(e)}(E_1, 0), \end{aligned} \tag{2.9}$$

which consists of independent, linear, and quadratic terms in temperature T.

III. ABSORPTIVE POTENTIAL FOR FIXED TEMPERATURE

For fixed temperature the absorptive potential is given by the imaginary part of the self-energy associated with the retarded Green function [3,11]. Using Matsubara formalism it is expressed in the lowest order as

$$\langle 1|W_T|1'\rangle = -\frac{\pi}{2}\sum_{234}\langle 12|V|34\rangle\delta(E_1+E_2-E_3-E_4) \times[n(E_2)\overline{n}(E_3)\overline{n}(E_4)+\overline{n}(E_2)n(E_3)n(E_4)] \times\langle 43|V|21'\rangle\ , \qquad (3.1)$$

where 1,2,3,4 represent the quantum states of participating nucleons. The first term of the square brackets in Eq. (3.1) is often called the polarization contribution and the second term is the correlation contribution. The occupation probability of a hole and of a particle is given respectively by

$$n(E) = \frac{1}{1+e^{(E-\lambda)/T}}, \qquad (3.2a)$$

$$\overline{n}(E) = 1 - n(E) = \frac{1}{1+e^{-(E-\lambda)/T}}, \qquad (3.2b)$$

where λ is the chemical potential.

The absorptive potential for a fixed temperature has been studied by many authors. To our knowledge Morel and Nozières [17] were the first to estimate the polarization term of W_T in an infinite fermion system under the assumption that interaction matrix elements are constant, and obtained the expression

$$W_T(E_1) = -\frac{\pi}{4}\overline{v^2}g_F^3\left[\pi^2T^2+(E_1-E_F)^2\right]\overline{n}(E_1), \quad (3.3)$$

where $\overline{v^2}$ is the average square of the nucleon-nucleon interaction matrix elements. If the correlation contribution, which is proportional to $n(E)$, is added then the factor $\overline{n}(E)$ is dropped because of Eq. (3.2b), and the absorptive potential becomes a function of T^2 and the square of the excitation energy of the incident nucleon, which restores the particle-hole symmetry [3]. This absorptive potential has no linear term in T, and it is not consistent with the results obtained in the previous section. This contradiction will be resolved later.

A. Semiclassical approximation

As in the previous work [4] we introduce the semiclassical approximation [13], because we are interested in the averaged absorptive potential. Introduction of temperature is also compatible with averaging. The Thomas-Fermi model with the local momentum approximation is adopted for the nucleus which is assumed to be spherically symmetric. If the local wave number at the location $\mathbf{R}$ is denoted by $K(R)$ and the potential by $U(R)$, then the single nucleon energy is given by

$$E = \frac{\hbar^2}{2m_n^*(R)}[K(R)]^2 + U(R) \qquad (3.4)$$

where $m_n^*(R)$ is the effective nucleon mass at the location $\mathbf{R}$. As the real potential $U(R)$ we consider a finite well like the Saxon-Woods type. The probability of finding a nucleon at a location $\mathbf{R}$ and with energy E, which we call the spectral density, is given by

$$g_c(E,\mathbf{R}) = 4\frac{m_n^*(R)K(R)}{2\pi^2\hbar^2}, \qquad (3.5)$$

where the factor 4 is included as this spectral density is for neutrons and protons and spin up and down. The semiclassical state density is obtained as

$$g_c(E) = \int g_c(E,\mathbf{R})d\mathbf{R}. \qquad (3.6)$$

The integral is extended over the classically allowed region. So below the minimum of the potential U, which is denoted by B, the state density vanishes. Above zero energy for neutrons and the Coulomb barrier for protons the state density diverges. The state density given by (3.6) is defined when it has a finite value. When it becomes necessary to use the state density beyond the defined region it must be extrapolated. In nuclear matter the state density may be defined at any energy. Therefore the state density is extrapolated as

$$g_e(E) = g_c(0)\sqrt{\frac{E-B}{-B}}. \qquad (3.7)$$

The above energy dependence is the same as the state density of Fermi gas.

The state density of a single nucleon is conveniently defined as

$$g(E) = \left[g_c(E)\Theta(-E) + g_e(E)\Theta(E)\right]\Theta(E_0-E) \quad (3.8)$$

where E_0 is the upper limit of the energy, and $\Theta(E)$ is the unit step function. For Q space $E_0 = 0$ and for $P+Q$ space $E_0 = \infty$. This extension of the state density to continuum is necessary in calculating the chemical potential when equilibrium reactions are considered.

The chemical potential λ is fixed by the following equation

$$\int_B^{E_0} n(E)g(E)dE = A \qquad (3.9)$$

and we call the chemical potential at zero temperature as the Fermi energy E_F, which is defined to satisfy the relation

$$\int_B^{E_F} g(E)dE = A. \qquad (3.10)$$

In Table I the chemical potentials for ^{208}Pb are shown. For details of the calculation see Sec. VI.

The semiclassical absorptive potential is obtained by taking the Wigner transform of the quantum mechanical one defined by Eq. (3.1) [13]. For this purpose the center of mass $\mathbf{R}_i$ and relative coordinates $\mathbf{x}_i$ of the ith nucleons are defined by

$$\mathbf{r}_i + \mathbf{r}_i' = 2\mathbf{R}_i, \qquad \mathbf{r}_i - \mathbf{r}_i' = \mathbf{x}_i, \qquad (3.11)$$

respectively, where $\mathbf{r}_i$ and $\mathbf{r}_i'$ are the locations of the nucleon, with respect to which a matrix element is defined.

The Wigner transform of the absorptive potential W_T is given by

TABLE I. The chemical potential λ and the number of excited particles or holes $\overline{m}$ given by (4.3) and (4.7) are listed at six temperatures T for ^{208}Pb. In the next rows m_h and m_p are quantities related to the magnitude of the absorptive potential given by (7.1) and (7.2). In the last row those in the Fermi liquid approximation given by (7.3) are listed.

T (MeV)	0	1	2	3	4	5
λ (MeV)	−9.268	−9.337	−9.516	−9.828	−10.256	−10.780
$\overline{m}$	0.00	6.3	12.6	19.1	25.7	32.2
m_h	0.00	6.59	13.70	21.45	29.81	38.67
m_p	0.00	5.96	11.44	16.38	20.87	25.02
$\ln 2 \cdot g_F T$	0.00	6.27	12.54	18.80	25.08	31.35

$$W_T(s_1,E_1,\mathbf{K}_1,\mathbf{R}_1) = \int d\mathbf{x}_1 e^{i\mathbf{K}_1\mathbf{x}_1} \langle s_1\mathbf{r}_1|W_T(E_1)|s_1\mathbf{r}'_1\rangle, \tag{3.12}$$

where s_i stands for spin-isospin quantum numbers for the ith nucleon. However in this paper the absorptive potential is independent of the spin-isospin state, so henceforth the quantum number s_1 will be dropped.

Substituting Eq. (3.1) into Eq. (3.12), the Wigner transform of the absorptive potential is expressed as

$$\begin{aligned} W_T(E_1,\mathbf{K}_1,\mathbf{R}_1) = -\frac{\pi}{2}(2\pi)^{-9} \int & d\mathbf{K}_2 d\mathbf{K}_3 d\mathbf{K}_4 d\mathbf{R}_2 d\mathbf{R}_3 d\mathbf{R}_4 \\ & \times \sum_{s_2,s_3,s_4} \langle s_1\mathbf{K}_1\mathbf{R}_1, s_2\mathbf{K}_2\mathbf{R}_2|VV^\dagger|s_3\mathbf{K}_3\mathbf{R}_3, s_4\mathbf{K}_4\mathbf{R}_4\rangle \\ & \times[n(E_2)\overline{n}(E_3)\overline{n}(E_4) + \overline{n}(E_2)n(E_3)n(E_4)]\delta(E_1+E_2-E_3-E_4), \end{aligned} \tag{3.13}$$

where the Wigner transform of the interaction part is

$$\begin{aligned} \langle|VV^\dagger|\rangle \equiv & \sum_{s_2s_3s_4} \langle s_1\mathbf{K}_1\mathbf{R}_1, s_2\mathbf{K}_2\mathbf{R}_2|VV^\dagger|s_3\mathbf{K}_3\mathbf{R}_3, s_4\mathbf{K}_4\mathbf{R}_4\rangle \\ = & \sum_{s_2s_3s_4} \int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{x}_3 d\mathbf{x}_4 e^{i\mathbf{K}_1\mathbf{x}_1 + i\mathbf{K}_2\mathbf{x}_2 - i\mathbf{K}_3\mathbf{x}_3 - i\mathbf{K}_4\mathbf{x}_4} \\ & \times\langle s_1\mathbf{r}_1, s_2\mathbf{r}_2|V|s_3\mathbf{r}_3, s_4\mathbf{r}_4\rangle\langle s_1\mathbf{r}'_1, s_2\mathbf{r}'_2|V^\dagger|s_3\mathbf{r}'_3, s_4\mathbf{r}'_4\rangle. \end{aligned} \tag{3.14}$$

In the following the Wigner transform of simple interactions is illustrated. First a simplified Skyrme interaction is considered,

$$\begin{aligned} \langle\mathbf{r}_1\mathbf{r}_2|V|\mathbf{r}_3\mathbf{r}_4\rangle = & \delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_2-\mathbf{r}_4)(1-P_\sigma P_\tau) \\ & \times\hat{O}\delta(\mathbf{r}_1-\mathbf{r}_2)V_0(r_1), \end{aligned} \tag{3.15}$$

where the strength of the interaction $V_0(r_1)$ depends on the density which is a function of r_1, and $\hat{O}$ represents its exchange character,

$$\hat{O} = a + a_\sigma P_\sigma + a_\tau P_\tau + a_{\sigma\tau} P_\sigma P_\tau. \tag{3.16}$$

The spin and isospin exchange operators are denoted by P_σ and P_τ respectively, and $a, \ldots, a_{\sigma\tau}$ are parameters expressing the exchange character. The Wigner transform is given by

$$\begin{aligned} \langle|VV^\dagger|\rangle = & \delta(\mathbf{R}_1-\mathbf{R}_3)\delta(\mathbf{R}_2-\mathbf{R}_4)\delta(\mathbf{R}_1-\mathbf{R}_2) \\ & \times J[V_0(R_1)]^2(2\pi)^3\Delta(R_1,q) \end{aligned} \tag{3.17}$$

where

$$J = 6\{(a-a_{\sigma\tau})^2 + (a_\sigma - a_\tau)^2\} \tag{3.18}$$

is a constant associated with the exchange character. The function

$$\begin{aligned} \Delta(R_1,q) = & (2\pi)^{-3} \int d\mathbf{x}_1 e^{-i\mathbf{q}\mathbf{x}_1} \\ & \times V_0\left(\left|\mathbf{R}_1 + \frac{\mathbf{x}_1}{2}\right|\right) V_0\left(\left|\mathbf{R}_1 - \frac{\mathbf{x}_1}{2}\right|\right) \Big/ [V_0(R_1)]^2 \end{aligned} \tag{3.19}$$

depends on the change of the total wave number,

$$\mathbf{q} = \mathbf{K}_1 + \mathbf{K}_2 - \mathbf{K}_3 - \mathbf{K}_4. \tag{3.20}$$

If the strength $V_0(R)$ is proportional to the density and its distribution is approximated by the Gaussian function

$$\rho(r) = \rho_0 e^{-r^2/R_0^2} \tag{3.21}$$

then

$$\Delta(R_1,\mathbf{q}) = \left(\frac{R_0^2}{2\pi}\right)^{\frac{3}{2}} e^{-q^2R_0^2/2}. \tag{3.22}$$

The magnitude of wave number K_i is close to the Fermi wave number $K_F = 1.4\ \mathrm{fm}^{-1}$, while the nuclear radius is R_0=3 $\sim$ 5 fm, so the function $\Delta(R_1, \mathbf{q})$ may be regarded as a delta function of $\mathbf{q}$. In this paper this approximation is adopted as in the previous paper [4],

$$\langle|VV^\dagger|\rangle = \delta(\mathbf{R}_1 - \mathbf{R}_3)\delta(\mathbf{R}_2 - \mathbf{R}_4)\delta(\mathbf{R}_1 - \mathbf{R}_2) \times J[V_0(R_1)]^2(2\pi)^3\delta(\mathbf{q}). \tag{3.23}$$

For a density independent interaction the above result becomes exact provided that $V_0(R_1)$ is a constant.

Next the finite range interaction

$$\langle \mathbf{r}_1\mathbf{r}_2|V|\mathbf{r}_3\mathbf{r}_4\rangle = \delta(\mathbf{r}_1 - \mathbf{r}_3)\delta(\mathbf{r}_2 - \mathbf{r}_4) \times(1 - P_r P_\sigma P_\tau)\hat{O}v(|\mathbf{r}_1 - \mathbf{r}_2|) \tag{3.24}$$

is considered, where P_r is the space exchange operator. Then the Wigner transform is

$$\begin{aligned}\langle|VV^\dagger|\rangle = {}& (2\pi)^3\delta(\mathbf{q}) \int d\mathbf{x} v\left(\left|\mathbf{R}_1 - \mathbf{R}_2 + \frac{\mathbf{x}}{2}\right|\right) v\left(\left|\mathbf{R}_1 - \mathbf{R}_2 - \frac{\mathbf{x}}{2}\right|\right)\\ &\times\Big[\sum_{s_2}\langle s_1s_2|\hat{O}^2|s_1s_2\rangle\Big\{e^{i(\mathbf{K}_1-\mathbf{K}_2)\mathbf{x}}\delta(\mathbf{R}_1 - \mathbf{R}_3)\delta(\mathbf{R}_2 - \mathbf{R}_4)\\ &+e^{i(\mathbf{K}_1-\mathbf{K}_4)\mathbf{x}}\delta(\mathbf{R}_2 - \mathbf{R}_3)\delta(\mathbf{R}_1 - \mathbf{R}_4)\Big\} - 2\sum_{s_2}\langle s_1s_2|P_\sigma P_\tau\hat{O}^2|s_1s_2\rangle\delta(\mathbf{R}_1 + \mathbf{R}_2 - \mathbf{R}_3 - \mathbf{R}_4)2^6\\ &\times\Big\{e^{-i(\mathbf{K}_3-\mathbf{K}_4)(\mathbf{R}_1-\mathbf{R}_2)+i(\mathbf{K}_1-\mathbf{K}_2)(\mathbf{R}_3-\mathbf{R}_4)}\delta(2\mathbf{R}_3 - 2\mathbf{R}_4 - \mathbf{x})\\ &+e^{i(\mathbf{K}_3-\mathbf{K}_4)(\mathbf{R}_1-\mathbf{R}_2)-i(\mathbf{K}_1-\mathbf{K}_2)(\mathbf{R}_3-\mathbf{R}_4)}\delta(2\mathbf{R}_3 - 2\mathbf{R}_4 + \mathbf{x})\Big\}\Big].\end{aligned} \tag{3.25}$$

Sums over spin-isospin give

$$\begin{aligned}J_d &= 2\sum_{s_2}\langle s_1s_2|\hat{O}^2|s_1s_2\rangle\\ &= 8(a^2 + a_\sigma^2 + a_\tau^2 + a_{\sigma\tau}^2) + 8(a + a_{\sigma\tau})(a_\sigma + a_\tau) + 4(aa_{\sigma\tau} + a_\sigma a_\tau),\end{aligned} \tag{3.26}$$

$$J_e = 2\sum_{s_2}\langle s_1s_2|P_\sigma P_\tau\hat{O}^2|s_1s_2\rangle = 16(aa_{\sigma\tau} + a_\sigma a_\tau) + 8(a + a_{\sigma\tau})(a_\sigma + a_\tau) + 2(a^2 + a_\sigma^2 + a_\tau^2 + a_{\sigma\tau}^2). \tag{3.27}$$

From Eq. (3.18), the following relation is easily shown:

$$J = J_d - J_e. \tag{3.28}$$

For the special case of Gaussian interaction with the strength V_G and the range r_0,

$$v_G(\mathbf{r}) = -V_G e^{-r^2/r_0^2}\ , \tag{3.29}$$

the integral is easily calculated

$$\int d\mathbf{x}e^{i\mathbf{k}\mathbf{x}}v_G\left(\left|\mathbf{R}_1 - \mathbf{R}_2 + \frac{\mathbf{x}}{2}\right|\right) v_G\left(\left|\mathbf{R}_1 - \mathbf{R}_2 - \frac{\mathbf{x}}{2}\right|\right) = [v_G(|\mathbf{R}_1 - \mathbf{R}_2|)]^2(2\pi r_0^2)^{\frac{3}{2}}e^{-r_0^2k^2/2}, \tag{3.30}$$

and

$$\begin{aligned}\int\langle|VV^\dagger|\rangle d\mathbf{R}_2 d\mathbf{R}_3 d\mathbf{R}_4 = {}& (2\pi)^3\delta(\mathbf{q})\int\{v_G(|\mathbf{R}_1 - \mathbf{R}_2|)\}^2 d\mathbf{R}_2(2\pi r_0^2)^{\frac{3}{2}}\\ &\times\left[J_d\frac{1}{2}\left\{e^{-\frac{r_0^2}{2}(\mathbf{K}_1-\mathbf{K}_3)^2} + e^{-\frac{r_0^2}{2}(\mathbf{K}_1-\mathbf{K}_4)^2}\right\} - J_e e^{-\frac{r_0^2}{8}\{(\mathbf{K}_1-\mathbf{K}_2)^2+(\mathbf{K}_3-\mathbf{K}_4)^2\}}\right].\end{aligned} \tag{3.31}$$

By using the relation (3.28) and

$$\int[v_G(|\mathbf{R}_1 - \mathbf{R}_2|)]^2 d\mathbf{R}_2(2\pi r_0^2)^{\frac{3}{2}} = V_G^2(\pi r_0^2)^3 = V_0^2, \tag{3.32}$$

it is easily seen that Eq. (3.31) is equivalent to Eq. (3.23) in the limit of $r_0 \to 0$. The notation V_0 is the strength of the delta interaction whose volume integral is equal to the Gaussian interaction (3.29).

To survey the temperature and the incident energy dependence of the absorptive potential W_T, the polarization and correlation contributions are shown in Fig. 2. Adopting the same simplified Skyrme interaction as in Ref. [4] and both $P + Q$ and Q absorptions are assumed. Details of the calculation will be given in Sec. V. First $P + Q$ absorption is examined, which is shown in Fig. 2(A). The curves for the polarization contribution

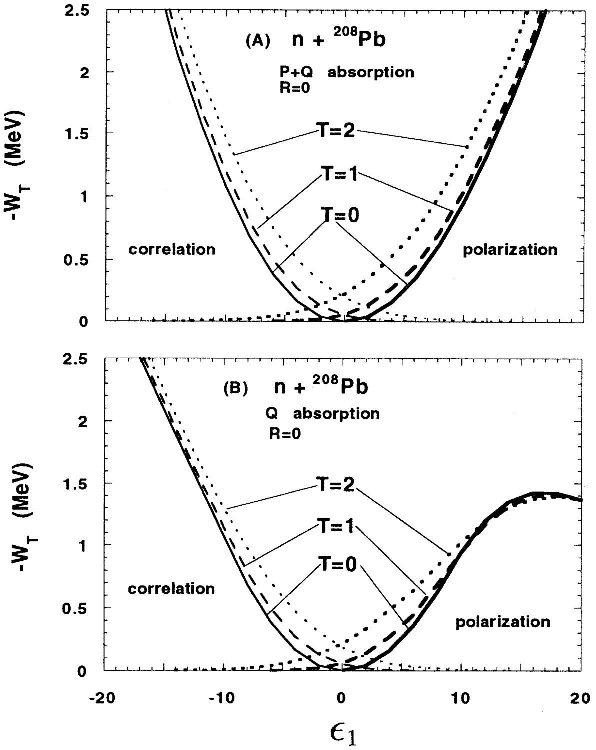

FIG. 2. Absorptive potentials $-W_T$ at temperatures $T = 0, 1$, and 2 MeV and at $R_1 = 0$ are shown both for $P + Q$ absorption ($E_0 = 60$ MeV) in (A) and for Q absorption ($E_0 = 0$ MeV) in (B). As the residual interaction, the simplified Skyrme interaction is adopted, and the numerical calculation is performed without the Fermi liquid approximation.

are well reproduced by the well-known quadratic formula given by (3.3). Below the Fermi energy the curves gradually tend to zero as predicted by $\overline{n}(E_1)$. The correlation contribution is almost the corresponding polarization contribution reflected at the $\epsilon_1 = 0$ line. The case of Q absorption is shown in Fig. 2(B). The correlation contribution for Q absorption is almost the same as the corresponding $P + Q$ absorption. However the polarization contribution is markedly different from those for $P + Q$ absorption. The curves do not increase quadratically as the excitation energy increases. This is because the particle energy is cut off at the energy zero for Q absorption.

IV. DECOMPOSITION OF ABSORPTIVE POTENTIAL W_T INTO PREEQUILIBRIUM POTENTIALS

In this section the correspondence between the absorptive potential at finite temperature given by (3.13) and the preequilibrium absorptive potential whose approximate formulas were intuitively constructed in the previous paper [4] is established.

At zero temperature, states up to the Fermi energy E_F are completely filled, and the relation (3.10) holds. At finite temperature T, nucleons occupying states below the Fermi energy are excited to above the Fermi energy according to the distribution function given by (3.2a), and the average number of excited nucleons and its variance are given approximately by (2.7) at low temperature.

In order to derive the preequilibrium absorptive potential from the fixed temperature one the distribution function $n(E)$ must be decomposed into the particle part and the hole part because nuclear states in the preequilibrium process are specified by the exciton number $2\overline{m}$. The occupation probability $n(E)$ is divided into two partial ones at the Fermi energy E_F

$$n(E) = H(E) + p(E), \tag{4.1}$$

where

$$\begin{aligned} H(E) &= n(E)\Theta(E_F - E), \\ p(E) &= n(E)\Theta(E - E_F) \end{aligned} \tag{4.2}$$

are the hole and particle parts, respectively. The particle number $\overline{m}$ is given by

$$\int_{E_F}^{E_0} p(E)g(E)dE = \overline{m} \tag{4.3}$$

which also gives the relation between the temperature and the average exciton number. By using (3.10) and (4.3) we obtain

$$\int_{B}^{E_F} H(E)g(E)dE = A - \overline{m}. \tag{4.4}$$

For the vacancy probability we divide

$$\overline{n}(E) = 1 - n(E) = h(E) + P(E), \tag{4.5}$$

where

$$\begin{aligned} h(E) &= \overline{n}(E)\Theta(E_F - E), \\ P(E) &= \overline{n}(E)\Theta(E - E_F). \end{aligned} \tag{4.6}$$

By using (3.10) and (4.5) and (4.6) we obtain the hole number

$$\int_{B}^{E_F} h(E)g(E)dE = \overline{m}. \tag{4.7}$$

The divided occupation probabilities (4.1) and the vacancy probabilities (4.5) are put in the absorptive potential given by (3.13), whose last line is expanded in the following form,

$$
\begin{aligned}
&[n(E_2)\overline{n}(E_3)\overline{n}(E_4) + \overline{n}(E_2)n(E_3)n(E_4)]\delta(E_1 + E_2 - E_3 - E_4) \\
&\quad = [H(E_2)P(E_3)P(E_4) + h(E_2)p(E_3)p(E_4)]\delta[E_1 - (-E_2 + E_3 + E_4)] && (a) \\
&\qquad +[p(E_2)P(E_3)P(E_4) + P(E_2)p(E_3)p(E_4)]\delta[(E_1 + E_2) - (E_3 + E_4)] && (b) \\
&\qquad +[2H(E_2)P(E_3)h(E_4) + 2h(E_2)p(E_3)H(E_4)]\delta[(E_1 - E_4) - (E_3 - E_2)] && (c) \\
&\qquad +[2p(E_2)h(E_3)P(E_4) + 2P(E_2)H(E_3)p(E_4)]\delta[(E_1 + E_2 - E_3) - E_4] && (d) \\
&\qquad +[H(E_2)h(E_3)h(E_4) + h(E_2)H(E_3)H(E_4)]\delta[(E_1 - E_3 - E_4) - (-E_2)] && (e) \\
&\qquad +[p(E_2)h(E_3)h(E_4) + P(E_2)H(E_3)H(E_4)]\delta[E_1 - (2E_1 + E_2 - E_3 - E_4)] && (f)
\end{aligned}
\tag{4.8}
$$

where the factor 2 in lines (c) and (d) appears because the interchange of labels 3 and 4 gives the same integral. Now we explain how the right-hand side of the equation is arranged. At each line the first term comes from the polarization contribution $n(E_2)\overline{n}(E_3)\overline{n}(E_4)$ and the second term comes from the correlation contribution $\overline{n}(E_2)n(E_3)n(E_4)$; the latter is obtained from the former by replacing the capital letters H and P by their small letter counterparts. Each line is labeled by $a, b, \ldots, f$ to be identified with each process shown in the upper row of Fig. 3 when the nucleon 1 is a particle and in the lower row when it is a hole.

The polarization contribution for an incident particle is considered first. The hole case will be considered in the last subsection. As the capital letter distributions H and P measure large compared with the small letter distributions, states with capital letter distribution are chosen as intermediate states of nucleons over which a sum is taken. Nucleon states with small letter distribution are assigned to initial states, and it is seen that each line corresponds to each process shown in Fig. 3, although processes up to the intermediate stage are actually drawn there. The arguments of the energy delta function is also rearranged accordingly. The interaction part in Eq. (3.13) will be considered later.

For the correlation contributions states with small letter distribution are assigned to intermediate states, so the correlation contributions are small compared with polarization ones when the nucleon 1 is a particle.

Substituting (4.8) into Eq. (3.13) W_T is decomposed into 12 components as

$$
W_T(E_1, \mathbf{K}_1, \mathbf{R}_1) = \sum_i W^{(i)}(E_1, \mathbf{K}_1, \mathbf{R}_1; T), \tag{4.9}
$$

where

$$
\begin{aligned}
W^{(i)}(E_1, \mathbf{K}_1, \mathbf{R}_1; T) = -\frac{\pi}{2}(2\pi)^{-9} \int & d\mathbf{K}_2 d\mathbf{K}_3 d\mathbf{K}_4 d\mathbf{R}_2 d\mathbf{R}_3 d\mathbf{R}_4 \\
& \times \langle |VV^{\dagger}| \rangle n_2(E_2) n_3(E_3) n_4(E_4) \delta(E_1 + E_2 - E_3 - E_4).
\end{aligned}
\tag{4.10}
$$

In the above equation the superscript (i) specifies the kind of processes (a)–(f) as well as the contribution of the polarization p or the correlation c, and $n_2(E_2)n_3(E_3)n_4(E_4)$ is the corresponding product of three partial occupation probabilities.

In the previous work [4] we introduced intuitively the absorptive potential for preequilibrium processes (a)–(e) for a particle ($E_1 \geq E_F$) shown in Fig. 3. In the fol-

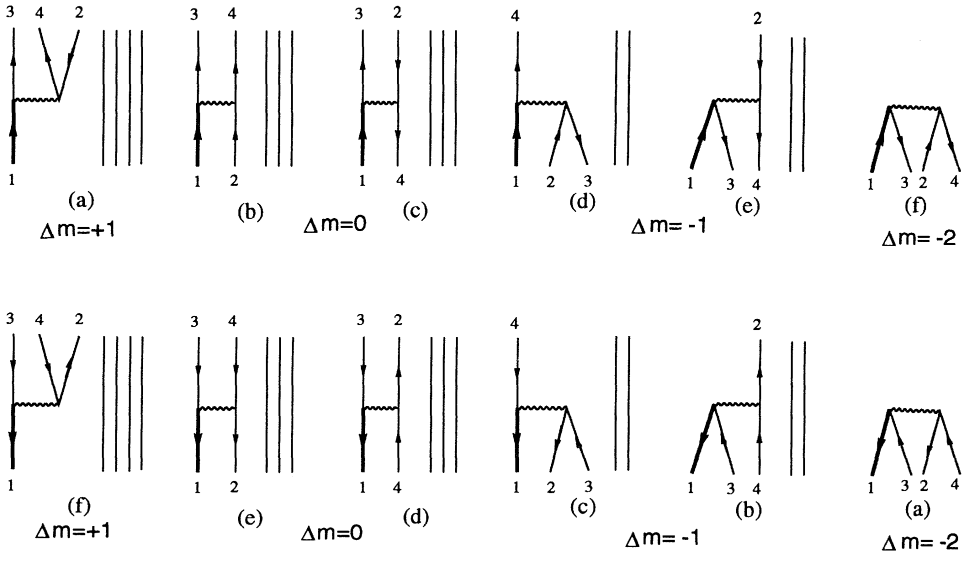

FIG. 3. Diagrams of absorption processes (a)–(f). The upper row shows the case when the incident nucleon is a particle, while the lower row the case of a hole.

lowing the expression (4.10) is written for each process, from which the corresponding preequilibrium absorptive potential is derived, and is shown to reduce to the previous result [4] if further approximation is applied.

A. Calculation of absorptive potential for each process

1. *The process (a)*

This process is a particle-hole pair creation by a particle, so it is similar to the thermal process. By taking the first term of the first line of the right-hand side of (4.8), Eq. (4.10) for this process is given by

$$\begin{aligned} W_p^{(a)}(E_1,\mathbf{K}_1,\mathbf{R}_1;T) = & -\frac{\pi}{2}(2\pi)^{-9}\int d\mathbf{K}_2 d\mathbf{K}_3 d\mathbf{K}_4 d\mathbf{R}_2 d\mathbf{R}_3 d\mathbf{R}_4 \\ & \times\delta[E_1-(-E_2+E_3+E_4)]\langle|VV^{\dagger}|\rangle H(E_2)P(E_3)P(E_4). \end{aligned} \tag{4.11}$$

The above expression reduces to the one we obtained previously if partial occupation probabilities are replaced by those of zero temperature. So our present results take into account the Pauli blocking effects, reduction of the intermediate phase space due to the excitation of nucleons, whose mean number is equal to the particle number given by (4.3).

As the process appeared in the absorptive potential at finite temperature and the process (a) is essentially the same with respect to the interaction, it may not be necessary to explain the interaction. But the matrix element $\langle 2^{-1}34|V|1\rangle$ for the process (a) in Fig. 3 is equal to $\langle 34|V|\overline{2}1\rangle$ by particle-hole conjugation, which appears in (3.14). The overline indicates the time-reversed state. Although 2 instead of $\overline{2}$ appears in (3.14), it does not matter as a sum over 2 is taken there.

Therefore the absorptive potential given by Eq. (4.11) is the preequilibrium one for the process (a) where the target nucleus has the mean particle number $\overline{m}$. The result of numerical calculation of $W_p^{(a)}$ for $n+{}^{208}\mathrm{Pb}$ will be shown in Fig. 6 in Sec. V.

2. *The process (b)*

For this process (4.10) becomes

$$\begin{aligned} W_p^{(b)}(E_1,\mathbf{K}_1,\mathbf{R}_1;T) = & -\frac{\pi}{2}(2\pi)^{-9}\int d\mathbf{K}_2 d\mathbf{K}_3 d\mathbf{K}_4 d\mathbf{R}_2 d\mathbf{R}_3 d\mathbf{R}_4 \\ & \times\delta[(E_1+E_2)-(E_3+E_4)]\langle|VV^{\dagger}|\rangle p(E_2)P(E_3)P(E_4). \end{aligned} \tag{4.12}$$

In this formula the integrals over $\mathbf{K}_3$ and $\mathbf{K}_4$ are for intermediate states, but the integral over $\mathbf{K}_2$ is taken with respect to the initial states, so it is rearranged to fit the process (b). The matrix element of the interaction is just the one that appeared in Eq. (3.14). The integral over the variable $\mathbf{K}_2$ is expressed as an integral over the direction $\hat{\mathbf{K}}_2$ and the magnitude in terms of the energy using (3.5)

$$\int d\mathbf{K}_2 p(E_2) = \frac{(2\pi)^3}{4}\int d\hat{\mathbf{K}}_2 dE_2 p(E_2) g(E_2)\frac{g_c(E_2,\mathbf{R}_2)}{4\pi g(E_2)}, \tag{4.13}$$

where the last term on the right-hand side of the above equation is the probability of the particle 2 with energy E_2 being located at $\mathbf{R}_2$. Then Eq. (4.12) reads

$$\begin{aligned} W_p^{(b)}(E_1,\mathbf{K}_1,\mathbf{R}_1;T) = & -\frac{\pi}{2}(2\pi)^{-6}\int d\hat{\mathbf{K}}_2 dE_2 d\mathbf{R}_2 p(E_2) g(E_2)\frac{g_c(E_2,\mathbf{R}_2)}{4\pi g(E_2)} \\ & \times\int d\mathbf{K}_3 d\mathbf{K}_4\int d\mathbf{R}_3 d\mathbf{R}_4 \delta(E_1+E_2-E_3-E_4)\frac{1}{4}\langle|VV^{\dagger}|\rangle P(E_3)P(E_4). \end{aligned} \tag{4.14}$$

It is convenient to define the elementary absorptive potential for the process (b) by the following expression which appears in Eq. (4.14)

$$w_p^{(b)}(\mathbf{K}_1\mathbf{R}_1,\mathbf{K}_2\mathbf{R}_2;T) = -\frac{\pi}{2}(2\pi)^{-6}\int d\mathbf{K}_3 d\mathbf{K}_4 d\mathbf{R}_3 d\mathbf{R}_4 \delta(E_1+E_2-E_3-E_4)\frac{1}{4}\langle|VV^{\dagger}|\rangle P(E_3)P(E_4), \tag{4.15}$$

although the effect of the target excitation is already included through $P(E_3)$ and $P(E_4)$. The absorptive potential (4.15) is further averaged over the location and the direction of the momentum of particle 2

$$w_p^{(b)}(E_1\mathbf{R}_1, E_2; T) = \int d\hat{\mathbf{K}}_2 d\mathbf{R}_2 \times \frac{g_c(E_2, \mathbf{R}_2)}{4\pi g_c(E_2)} w_p^{(b)}(\mathbf{K}_1\mathbf{R}_1, \mathbf{K}_2\mathbf{R}_2; T). \tag{4.16}$$

Then the absorptive potential (4.12) is given by

$$W_p^{(b)}(E_1\mathbf{R}_1; T) = \int dE_2 p(E_2) \times g(E_2) w_p^{(b)}(E_1\mathbf{R}_1, E_2; T). \tag{4.17}$$

This is the preequilibrium absorptive potential for the process (b) where the target nucleus has 2 $\overline{m}$ excitons. Again our present result takes into account the Pauli blocking effects, and is improved on the previous one in the process (b).

The factor $p(E_2)g(E_2)$ has a sharp peak at $E_2 = E_F$ while the other factor is a mild function of E_2, so the latter factor may be taken out of the integral with $E_2 = E_F$ and the rest $p(E_2)g(E_2)$ is integrated over E_2, which gives the value $\overline{m}$ by (4.3). Then (4.17) is approximated as

$$W_p^{(b)}(E_1\mathbf{R}_1; T) \approx \overline{m} w_p^{(b)}(E_1\mathbf{R}_1, E_2 = E_F; T). \tag{4.18}$$

Furthermore if the occupation probabilities are replaced by those of zero temperature in the absorptive potential for the elementary process we obtain the absorptive potential for the process (b) of the previous paper [4].

3. The process (c)

The process (c) is treated in the same way as in the process (b), and the absorptive potential (4.10) becomes

$$W_p^{(c)}(E_1, \mathbf{K}_1, \mathbf{R}_1; T) = -\frac{\pi}{2}(2\pi)^{-9} \int d\mathbf{K}_4 d\mathbf{K}_2 d\mathbf{K}_3 d\mathbf{R}_4 d\mathbf{R}_2 d\mathbf{R}_3 \delta[(E_1 - E_4) - (E_3 - E_2)] \langle |VV^\dagger| \rangle 2h(E_4) H(E_2) P(E_3). \tag{4.19}$$

If the absorptive potential for the elementary process (c) is defined by

$$w_p^{(c)}(\mathbf{K}_1\mathbf{R}_1, \mathbf{K}_4\mathbf{R}_4; T) = -\pi(2\pi)^{-6} \int d\mathbf{K}_2 d\mathbf{K}_3 d\mathbf{R}_2 d\mathbf{R}_3 \delta(E_1 + E_2 - E_3 - E_4) \langle |VV^\dagger| \rangle H(E_2) P(E_3) \tag{4.20}$$

and averaged over the location and the direction of the momentum of the hole 4,

$$w_p^{(c)}(E_1\mathbf{R}_1, E_4; T) = \int d\hat{\mathbf{K}}_4 d\mathbf{R}_4 \frac{g_c(E_4, \mathbf{R}_4)}{4\pi g_c(E_4)} w_p^{(c)}(\mathbf{K}_1\mathbf{R}_1, \mathbf{K}_4\mathbf{R}_4; T) \tag{4.21}$$

is obtained. The interaction $\langle 14^{-1}|V|32^{-1}\rangle$ is equal to $\langle 1\bar{2}|V|3\bar{4}\rangle$ which appears in Eq. (3.14). In terms of this we finally obtain the preequilibrium absorptive potential for the process (c) as

$$\begin{aligned} W_p^{(c)}(E_1\mathbf{R}_1; T) &= \int dE_4 h(E_4) g(E_4) w_p^{(c)}(E_1\mathbf{R}_1, E_4; T) \\ &\approx \overline{m} w_p^{(c)}(E_1\mathbf{R}_1, E_4 = E_F; T), \end{aligned} \tag{4.22}$$

where the last line expresses the approximate formula as in the previous process.

4. The process (d)

The absorptive potential for the process (d) is given by

$$\begin{aligned} W_p^{(d)}(E_1, \mathbf{K}_1, \mathbf{R}_1; T) = -\frac{\pi}{2}(2\pi)^{-9} &\int d\mathbf{K}_2 d\mathbf{K}_3 d\mathbf{K}_4 d\mathbf{R}_2 d\mathbf{R}_3 d\mathbf{R}_4 \\ &\times \delta[(E_1 + E_2 - E_3) - E_4)] \langle |VV^\dagger| \rangle 2p(E_2) h(E_3) P(E_4). \end{aligned} \tag{4.23}$$

As in the previous cases the absorptive potential for the elementary process (d) is defined by

$$w_p^{(d)}(\mathbf{K}_1\mathbf{R}_1, \mathbf{K}_2\mathbf{R}_2, \mathbf{K}_3\mathbf{R}_3; T) = -\pi(2\pi)^{-3} \int d\mathbf{K}_4 d\mathbf{R}_4 \delta(E_1 + E_2 - E_3 - E_4) \frac{1}{16} \langle |VV^\dagger| \rangle P(E_4). \tag{4.24}$$

In this case the average must be taken with respect to the initial state of the particle 2 and the hole 3:

$$w_p^{(d)}(E_1\mathbf{R}_1, E_2, E_3; T) = \int d\hat{\mathbf{K}}_2 d\mathbf{R}_2 \frac{g_c(E_2, \mathbf{R}_2)}{4\pi g_c(E_2)} d\hat{\mathbf{K}}_3 d\mathbf{R}_3 \frac{g_c(E_3, \mathbf{R}_3)}{4\pi g_c(E_3)} w_p^{(d)}(\mathbf{K}_1\mathbf{R}_1, \mathbf{K}_2\mathbf{R}_2, \mathbf{K}_3\mathbf{R}_3; T). \tag{4.25}$$

Finally we obtain the absorptive potential for the process (d)

$$W_p^{(d)}(E_1\mathbf{R}_1;T) = \int dE_2 p(E_2)g(E_2)dE_3 h(E_3)g_c(E_3)w_p^{(d)}(E_1\mathbf{R}_1,E_2,E_3;T)$$
$$\approx \overline{m}^2 w_p^{(d)}(E_1\mathbf{R}_1,E_2=E_F,E_3=E_F;T), \tag{4.26}$$

in terms of the absorptive potential for the elementary process (d). As in the previous cases the last line is the approximate formula.

5. *The process (e)*

The absorptive potential for this process is given by

$$W_p^{(e)}(E_1,\mathbf{K}_1,\mathbf{R}_1;T) = -\frac{\pi}{2}(2\pi)^{-9}\int d\mathbf{K}_2 d\mathbf{K}_3 d\mathbf{K}_4 d\mathbf{R}_2 d\mathbf{R}_3 d\mathbf{R}_4 \delta[(E_1-E_3-E_4)+E_2]\langle|VV^\dagger|\rangle H(E_2)h(E_3)h(E_4) \tag{4.27}$$

while the absorptive potential for the elementary process is defined by

$$w_p^{(e)}(\mathbf{K}_1\mathbf{R}_1,\mathbf{K}_3\mathbf{R}_3,\mathbf{K}_4\mathbf{R}_4;T) = -\pi(2\pi)^{-3}\int d\mathbf{K}_2 d\mathbf{R}_2 \delta(E_1+E_2-E_3-E_4)\frac{1}{16}\langle|VV^\dagger|\rangle H(E_2). \tag{4.28}$$

As in the previous processes the absorptive potential for the elementary process is averaged over initial states

$$w_p^{(e)}(E_1\mathbf{R}_1,E_3,E_4;T) = \int d\hat{\mathbf{K}}_3 d\mathbf{R}_3 \frac{g_c(E_3,\mathbf{R}_3)}{4\pi g_c(E_3)} d\hat{\mathbf{K}}_4 d\mathbf{R}_4 \frac{g_c(E_4,\mathbf{R}_4)}{4\pi g_c(E_4)} w_p^{(e)}(\mathbf{K}_1\mathbf{R}_1,\mathbf{K}_3\mathbf{R}_3,\mathbf{K}_4\mathbf{R}_4;T), \tag{4.29}$$

and the absorptive potential is rewritten in terms of (4.29) as

$$W_p^{(e)}(E_1\mathbf{R}_1;T) = \int dE_3 h(E_3)g_c(E_3)dE_4 h(E_4)g_c(E_4)w_p^{(e)}(E_1\mathbf{R}_1,E_3,E_4;T)$$
$$\approx \frac{\overline{m}^2}{2} w_p^{(e)}(E_1\mathbf{R}_1,E_3=E_F,E_4=E_F;T). \tag{4.30}$$

It should be noted that the label of the processes (d) and (e) in the previous paper [4] is interchanged in the present work. In addition, the factor 1/2 in Eq. (2.34) of Ref. [4] should be eliminated.

6. *The process (f)*

The polarization contribution for the process (f) vanishes due to the energy conservation.

B. Correlation contribution and the absorptive potential for a hole

For the correlation contributions the same treatment may be applied, but no correspondence is obtained to the previous work [4] because we did not consider the correlation contribution there.

Now consider the case in which the incident nucleon is a hole. By the time reversal invariance the interchange of all particles and holes does not change the matrix elements of the interaction for the processes (a) through (f). In Eq. (4.9) the main contribution comes from the correlation part in the case of a hole. By this interchange the correlation term of the process (f) becomes the original polarization term (a). In this way correspondence is achieved by reversing the order of the labels. Treatment of a diagram for each process is the same. All particle lines and hole lines are interchanged, and the order of labels is reversed, then the lower row of Fig. 3 is obtained from the upper row. Therefore we need not do any new calculations for a hole.

V. ANALYTIC FORMULA BASED ON THE FERMI LIQUID THEORY

In this section we calculate the absorptive potential $W^{(i)}$ at the nuclear center ($R_1 = 0$) adopting the Fermi liquid approximation [14] which is expected to be valid at low temperatures. By using the resulting analytic expressions for $W^{(i)}$, temperature and energy dependence of each process are examined.

In the Fermi liquid approximation, wave numbers of participating nucleons are supposed to be close to the Fermi wave number K_F, and integrals over the wave number are carried out by introducing the angles θ_2, φ_2, and ϕ. The z axis is taken along $\mathbf{K}_1$, and the polar and azimuthal angles of $\mathbf{K}_2$ are denoted by θ_2 and φ_2. The momentum conservation $\mathbf{K}_1+\mathbf{K}_2=\mathbf{K}_3+\mathbf{K}_4$ and the relation $K_3 \approx K_4$ fix $\mathbf{K}_3$ and $\mathbf{K}_4$ if the angle ϕ between two planes containing $\mathbf{K}_1, \mathbf{K}_2$ and $\mathbf{K}_3, \mathbf{K}_4$ is given. Using these angles the following replacement

$$\delta(\mathbf{K}_1 + \mathbf{K}_2 - \mathbf{K}_3 - \mathbf{K}_4) d\mathbf{K}_2 d\mathbf{K}_3 d\mathbf{K}_4$$

$$\to \left(\frac{m_n^*(R_1)}{\hbar^2}\right)^3 dE_2 dE_3 dE_4 \sin\frac{\theta_2}{2} d\theta_2 d\varphi_2 d\phi \quad (5.1)$$

is applicable. Momentum conservation is no longer necessary to be taken into account, and the calculation becomes easier.

First consider the case of the simplified Skyrme interaction with the approximation (3.23), which contains no angle variables. Therefore the angle integral gives a factor $8\pi^2$. The absorptive potential is given by

$$\begin{aligned} W^{(i)}(E_1, \mathbf{K}_1, R_1; T) = -\frac{\pi}{(2\pi)^4}&\left(\frac{m_n^*(R_1)}{\hbar^2}\right)^3 \\ &\times J[V_0(R_1)]^2 \int dE_2 dE_3 dE_4 \\ &\times \delta(E_1 + E_2 - E_3 - E_4) \\ &\times n_2(E_2) n_3(E_3) n_4(E_4). \quad (5.2)\end{aligned}$$

For the Gaussian interaction, the expression given by Eq. (3.31) contains the quantities $|\mathbf{K}_1 - \mathbf{K}_3|, |\mathbf{K}_1 - \mathbf{K}_4|$, and $|\mathbf{K}_3 - \mathbf{K}_4|$ which depend on the angle variables. These are averaged over angles $\theta_2, \varphi_2, \phi$, which is denoted briefly by Ω,

$$\begin{aligned}\langle e^{-r_0^2(\mathbf{K}_1-\mathbf{K}_3)^2/2}\rangle_\Omega &= \langle e^{-r_0^2(\mathbf{K}_1-\mathbf{K}_4)^2/2}\rangle_\Omega \\ &= \frac{\sqrt{\pi}}{2}\mathrm{erf}\left(\sqrt{2}r_0K_F\right) / \left(\sqrt{2}r_0K_F\right), \quad (5.3)\end{aligned}$$

$$\langle e^{-\frac{r_0^2}{8}\{(\mathbf{K}_1-\mathbf{K}_2)^2+(\mathbf{K}_3-\mathbf{K}_4)^2\}}\rangle_\Omega = \frac{1}{r_0K_F}F(r_0K_F), \quad (5.4)$$

where

$$F(x) = e^{-x^2}\int_0^x e^{t^2} dt \quad (5.5)$$

is Dawson's integral [18]. Then the absorptive potential reads

$$\begin{aligned} W^{(i)}(E_1, R_1; T) = -\frac{\pi}{(2\pi)^4}&\int v_G(|\mathbf{R}_1 - \mathbf{R}_2|)^2 d\mathbf{R}_2 (2\pi r_0^2)^{\frac{3}{2}} \int dE_2 dE_3 dE_4 \\ &\times \left(\frac{m_n^*(R_1)}{\hbar^2}\right)^3 \delta(E_1 + E_2 - E_3 - E_4)\Big[J_d\frac{\sqrt{\pi}}{2}\mathrm{erf}(\sqrt{2}r_0K_F)/(\sqrt{2}r_0K_F) \\ &-J_eF(r_0K_F)/(r_0K_F)\Big] n_2(E_2) n_3(E_3) n_4(E_4). \quad (5.6)\end{aligned}$$

The two functions appeared in the last equation $\frac{\sqrt{\pi}}{2}\mathrm{erf}(\sqrt{2}x)/(\sqrt{2}x)$ and $F(x)/x$ are both decreasing functions of x starting from the value 1 at $x = 0$. However the latter decreases faster than the former. So the effect of the exchange term decreases with increasing the interaction range r_0.

A. Average square of the residual interaction matrix elements and effective state density

To investigate further the relation between the temperature dependent absorptive potential and the exciton-number-fixed absorptive potentials, Eq. (5.2) is rewritten as

$$W^{(i)} = -\pi \overline{v_{\mathrm{eff}}^2} \omega_{\mathrm{eff}}^{(i)}(E_1), \quad (5.7)$$

where $\overline{v_{\mathrm{eff}}^2}$ is the average square of the residual interaction matrix elements given by

$$\overline{v_{\mathrm{eff}}^2} = \frac{J}{(2\pi)^4}\left(\frac{m_n^*}{\hbar^2}\right)^3 \frac{V_0{}^2}{g_F{}^3}. \quad (5.8)$$

The next factor $\omega_{\mathrm{eff}}^{(i)}(E_1)$ is called here the effective state density, which is given by

$$\begin{aligned}\omega_{\mathrm{eff}}^{(i)}(E_1) = \int dE_2 dE_3 dE_4 &\delta(E_1 + E_2 - E_3 - E_4) \\ &\times g_F^3 n_2(E_2) n_3(E_3) n_4(E_4). \quad (5.9)\end{aligned}$$

For the polarization contribution of the process (a) this is the state density of the intermediate states modified by the presence of excited nucleons. For other processes they include the effect of the initial state, but still have a dimension of the state density and Eq. (5.7) has the form of the Fermi golden rule, so we call it the effective state density for convenience. It is also noted that both $\overline{v_{\mathrm{eff}}^2}$ and $\omega_{\mathrm{eff}}^{(i)}(E_1)$ depend on the single particle state density g_F. However these dependences are superficial, and the absorptive potential, the product of the two, is not dependent on g_F.

The relation (5.7) could be used in the general case, but the calculation of $\overline{v_{\mathrm{eff}}^2}$ becomes much more complicated and also depends on the type of processes. On the other hand Herman, Reffo, and Weidenmüller [19] assumed that the factorization (5.7) holds and $\overline{v_{\mathrm{eff}}^2}$ is independent of the process, which is justified under the Fermi liquid approximation.

The integral is expressed in a dimensionless form by introducing the variables

$$\varepsilon_1 = (E_1 - E_F)/T, \tag{5.10a}$$

$$\varepsilon_k = |E_k - E_F|/T \quad (k = 2,3,4), \tag{5.10b}$$

as

$$\omega_{\text{eff}}^{(i)}(\varepsilon_1) = g_F^3 T^2 I^{(i)}(\varepsilon_1). \tag{5.11}$$

The dimensionless integral will be calculated in the next subsection for each process.

For the occupation probability the notation

$$f(\varepsilon) = \frac{1}{1+e^{\varepsilon}} \tag{5.12}$$

is used, and the partial occupation probabilities are expressed as

$$H(E_k) = 1 - f(\varepsilon_k), \quad P(E_k) = 1 - f(\varepsilon_k),$$
$$p(E_k) = f(\varepsilon_k), \quad h(E_k) = f(\varepsilon_k) \tag{5.13}$$

where ε_k is defined by Eq. (5.10b).

B. Calculation of effective state density

The dimensionless integrals appearing in Eq. (5.11) are calculated in the Fermi liquid approximation. The integral limits of ε_i are 0 and ∞ and the chemical potential λ is set equal to the Fermi energy E_F. In this subsection brief descriptions of calculations and results are presented, while details are given in Appendix A.

The integral for the process (a) in the polarization part is given by

$$I_p^{(a)}(\varepsilon_1) = \int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \delta(\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \times [1 - f(\varepsilon_2)][1 - f(\varepsilon_3)][1 - f(\varepsilon_4)]. \tag{5.14}$$

The integrand is expanded in terms of $f(\varepsilon_i)$ and is expressed as

$$I_p^{(a)}(\varepsilon_1) = I_{000}^{(a)}(\varepsilon_1) - 3I_{100}^{(a)}(\varepsilon_1) + 3I_{110}^{(a)}(\varepsilon_1) - I_{111}^{(a)}(\varepsilon_1). \tag{5.15}$$

The correlation contribution is given by

$$I_c^{(a)}(\varepsilon_1) = \int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \times \delta(\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4) f(\varepsilon_2) f(\varepsilon_3) f(\varepsilon_4) \ ; \tag{5.16}$$

furthermore, it is expressed as

$$I_c^{(a)}(\varepsilon_1) = I_{111}^{(a)}(\varepsilon_1) \ . \tag{5.17}$$

In Eqs. (5.15) and (5.17) the notation

$$I_{k_2 k_3 k_4}^{(a)}(\varepsilon_1) = \int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \times \delta(\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4)(f(\varepsilon_2))^{k_2} \times (f(\varepsilon_3))^{k_3} (f(\varepsilon_4))^{k_4} \tag{5.18}$$

is used, where the indices k_2, k_3, k_4 take either the value 0 or 1. It is easily seen that the integral does not change the value if the order of these indices is changed, so there appears factors 3 in Eq. (5.15)

Similarly the integrals for the process (b) in the polarization contribution

$$I_p^{(b)}(\varepsilon_1) = \int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \times f(\varepsilon_2)[1 - f(\varepsilon_3)][1 - f(\varepsilon_4)] \tag{5.19}$$

is given by

$$I_p^{(b)}(\varepsilon_1) = I_{100}^{(b)}(\varepsilon_1) - 2I_{110}^{(b)}(\varepsilon_1) + I_{111}^{(b)}(\varepsilon_1), \tag{5.20}$$

where $I_{k_2 k_3 k_4}^{(b)}$ is defined by Eq. (5.18) in which the energy delta function is replaced by the one appearing in Eq. (5.19). For the correlation contribution

$$I_c^{(b)}(\varepsilon_1) = \int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \times [1 - f(\varepsilon_2)] f(\varepsilon_3) f(\varepsilon_4) \ , \tag{5.21}$$

we have

$$I_c^{(b)}(\varepsilon_1) = I_{011}^{(b)}(\varepsilon_1) - I_{111}^{(b)}(\varepsilon_1). \tag{5.22}$$

For the process (c)

$$I_p^{(c)}(\varepsilon_1) = 2\int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \delta(\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4) \times [1 - f(\varepsilon_2)][1 - f(\varepsilon_3)] f(\varepsilon_4), \tag{5.23}$$

is obtained from $I_p^{(b)}$ by interchanging ε_2 and ε_4 and multiplying a factor 2,

$$I_p^{(c)}(\varepsilon_1) = 2I_p^{(b)}(\varepsilon_1). \tag{5.24}$$

For correlation contribution,

$$I_c^{(c)}(\varepsilon_1) = 2I_c^{(b)}(\varepsilon_1) \tag{5.25}$$

holds.

The polarization contribution for the process (d) is given by

$$I_p^{(d)}(\varepsilon_1) = 2\int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \delta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 - \varepsilon_4) \times f(\varepsilon_2) f(\varepsilon_3)[1 - f(\varepsilon_4)], \tag{5.26}$$

while the one for the process (e) is

$$I_p^{(e)}(\varepsilon_1) = \int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \delta(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 + \varepsilon_4) \times [1 - f(\varepsilon_2)] f(\varepsilon_3) f(\varepsilon_4). \tag{5.27}$$

As in the case of processes (b) and (c) we have relations

$$I_p^{(d)}(\varepsilon_1) = 2I_p^{(e)}(\varepsilon_1) \tag{5.28}$$

and

$$I_p^{(e)}(\varepsilon_1) = I_{011}^{(e)}(\varepsilon_1) - I_{111}^{(e)}(\varepsilon_1). \tag{5.29}$$

For the correlation contribution

$$I_c^{(e)}(\varepsilon_1) = \int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \delta(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 + \varepsilon_4) \times f(\varepsilon_2)[1 - f(\varepsilon_3)][1 - f(\varepsilon_4)] \tag{5.30}$$

is expressed as

$$I_c^{(e)}(\varepsilon_1) = I_{100}^{(e)}(\varepsilon_1) - 2I_{110}^{(e)}(\varepsilon_1) + I_{111}^{(e)}(\varepsilon_1) \tag{5.31}$$

and the relation

$$I_c^{(d)}(\varepsilon_1) = 2I_c^{(e)}(\varepsilon_1) \tag{5.32}$$

holds.

Finally for the process (f)

$$I_p^{(f)}(\varepsilon_1) = \int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \delta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4) \times f(\varepsilon_2) f(\varepsilon_3) f(\varepsilon_4), \tag{5.33}$$

and

$$I_c^{(f)}(\varepsilon_1) = \int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \delta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4) \times [1 - f(\varepsilon_2)][1 - f(\varepsilon_3)][1 - f(\varepsilon_4)] \tag{5.34}$$

are the polarization and the correlation contributions, respectively. The expansions are given by

$$I_p^{(f)}(\varepsilon_1) = I_{111}^{(f)}(\varepsilon_1), \tag{5.35}$$

$$I_c^{(f)}(\varepsilon_1) = I_{000}^{(f)}(\varepsilon_1) - 3I_{100}^{(f)}(\varepsilon_1) + 3I_{110}^{(f)}(\varepsilon_1) - I_{111}^{(f)}(\varepsilon_1). \tag{5.36}$$

Between polarization and correlation contributions we have relations

$$I_c^{(j)}(\varepsilon_1) = e^{-\varepsilon_1} I_p^{(j)}(\varepsilon_1), \qquad j = a, b, \ldots, f\ , \tag{5.37}$$

which are proved by using the relation $1 - f(\varepsilon) = e^{\varepsilon} f(\varepsilon)$.

Now the integrals $I_{k_1,k_2,k_3}^{(j)}(\varepsilon_1)$ are evaluated assuming $\varepsilon_1 \geq 0$. The integrals are calculated in the order of complexity. The last integral corresponding to $(k_2, k_3, k_4) = (1, 1, 1)$ is expressed in terms of the previously obtained one using the relations (5.37).

The integrals $I_{k_2,k_3,k_4}^{(j)}(\varepsilon_1)$ are expressed in terms of seven independent basic integrals, which result in two constants, two logarithmic functions, and three dilogarithmic functions [18] of variable x defined by

$$x = e^{-\varepsilon_1}. \tag{5.38}$$

The definition [see Eq. (B2)] and main properties of the dilogarithm function are given in Appendix B. The integrals are listed in the first row of Table II, where three functions

$$d_1(x) = \text{dilog}\, x,$$
$$d_2(x) = \text{dilog}\frac{1+x}{2},$$
$$d_3(x) = \text{dilog}(1+x) + \frac{\pi^2}{12} \tag{5.39}$$

are all constructed to vanish at $x = 1$. As the dilogarithm is a monotonically decreasing function of x, the ranges of these three functions are easily fixed as

$$0 \leq d_1(x) \leq d_1(0) = \frac{\pi^2}{6} \approx 1.64,$$
$$0 \leq d_2(x) \leq d_2(0) = \frac{\pi^2}{12} - \frac{1}{2}(\ln 2)^2 \approx 0.58,$$
$$0 \leq d_3(x) \leq d_3(0) = \frac{\pi^2}{12} \approx 0.82 \tag{5.40}$$

for $0 \leq x \leq 1$.

The expansion coefficients of the integral $I_{k_2,k_3,k_4}^{(j)}$ in terms of the seven basic functions are listed in the rows below the first one in Table II. The polarization contribution of each process $I_p^{(j)}(\varepsilon_1)$ and the total polarization contribution $I_p(\varepsilon_1)$ are also listed. The correlation contribution is obtained by multiplying the corresponding

TABLE II. List of integrals appearing in the Fermi liquid theory in terms of two constants and five independent functions.

	$\frac{\pi^2}{12}$	$(\ln 2)^2$	$-\ln 2 \ln x$	$(\ln x)^2$	$d_1(x)$	$d_2(x)$	$d_3(x)$
$I_{000}^{(a)}$				$\frac{1}{2}$			
$-3I_{100}^{(a)}$			-3				3
$3I_{110}^{(a)}$					3	-6	
$-I_{111}^{(a)}$			$\frac{3x}{1+x}$	$\frac{-x/2}{1+x}$	$\frac{-3x}{1+x}$	$\frac{6x}{1+x}$	$\frac{-3x}{1+x}$
$I_p^{(a)}$			$\frac{-3}{1+x}$	$\frac{1/2}{1+x}$	$\frac{3}{1+x}$	$\frac{-6}{1+x}$	$\frac{3}{1+x}$
$3I_{100}^{(b)}$	3		3				
$-6I_{110}^{(b)}$		-3				6	-6
$3I_{111}^{(b)}$	$\frac{-3x}{1+x}$	3	$\frac{-3x}{1+x}$		$\frac{-3}{1+x}$	$\frac{6-6x}{1+x}$	$\frac{6x}{1+x}$
$3I_p^{(b)}$	$\frac{3}{1+x}$		$\frac{3}{1+x}$		$\frac{-3}{1+x}$	$\frac{12}{1+x}$	$\frac{-6}{1+x}$
$3I_{011}^{(e)}$		3					
$-3I_{111}^{(e)}$	$\frac{3}{1+x}$	-3				$\frac{-6}{1+x}$	$\frac{3}{1+x}$
$3I_p^{(e)}$	$\frac{3}{1+x}$					$\frac{-6}{1+x}$	$\frac{3}{1+x}$
I_p	$\frac{6}{1+x}$			$\frac{1/2}{1+x}$			

polarization term by x owing to the relations (5.37).

In order to obtain the integral for the case of a hole $E_1 < E_F$, Table II cannot be used directly, because they are calculated assuming $\varepsilon_1 \geq 0$. Instead the following relations

$$\begin{aligned} I_c^{(a)}(-\varepsilon_1) &= I_p^{(f)}(\varepsilon_1), \\ I_c^{(b)}(-\varepsilon_1) &= I_p^{(e)}(\varepsilon_1), \\ I_c^{(e)}(-\varepsilon_1) &= I_p^{(b)}(\varepsilon_1), \end{aligned} \tag{5.41}$$

should be used. These relations are proved easily by using Eqs. (5.16) and (5.33) and (5.21) and (5.27).

C. Temperature and excitation energy dependence of the effective state density

As we are considering the case of a particle, $\varepsilon_1 \geq 0$, the range of variable x is given by $0 \leq x \leq 1$. We investigate the behavior of the integrals $I^{(j)}_{k_2,k_3,k_4}$ and the absorptive potentials for each process. An interesting limit is the low temperature, but in this case x cannot be expanded in power series of T/ϵ_1, where $\epsilon_1 = E_1 - E_F$. The dilogarithms also cannot be expanded in T/ϵ_1 either. Except for x and $d_1(x) \cdots d_3(x)$ the dependence on ϵ_1 and T may be explicitly shown. Restricting to the $T < \epsilon_1$ case, the functions $d_1(x), d_2(x), d_3(x)$, which cannot be expressed as power series of x, are quantities of order of 1,

$$\begin{aligned} 0.77 \approx d_1(e^{-1}) &\leq d_1(x) \leq d_1(0) \approx 1.64, \\ 0.34 \approx d_2(e^{-1}) &\leq d_2(x) \leq d_2(0) \approx 0.58, \\ 0.41 \approx d_3(e^{-1}) &\leq d_3(x) \leq d_3(0) \approx 0.82 \end{aligned} \tag{5.42}$$

for $0 \leq x \leq e^{-1}$. Because of Eq. (5.11), integrals multiplied by T^2 are considered below.

Let us start from the process (a). The first one $T^2 I^{(a)}_{000}$ is just given by $\epsilon_1^2/2$, which survives in the limit of zero temperature. This absorptive potential has a well-known quadratic dependence on the excitation energy of the incident nucleon.

The second term $T^2 I^{(a)}_{100} \approx (\ln 2)\epsilon_1 T - T^2 d_3(x)$ has a main term proportional to temperature, or the number of particles or holes in the target nucleus. This term is obtained if $f(\varepsilon_2)$ is considered as a function peaked at $\varepsilon_2 = 0$. First evaluating $\int_0^\infty f(\varepsilon_2) d\varepsilon_2 = \ln 2$ the remaining integral is carried out over ε_3 and ε_4 putting $\varepsilon_2 = 0$ which gives ε_1. This term represents the suppression of creating a $2p$-$1h$ state in the intermediate states where particles and holes already exist. The factor 3 that appeared in (5.15) comes from the three sources of the Pauli blocking of two particle and one hole. The remaining term contains higher order terms of order T^2 which comes from the correction to the above approximate evaluation.

The third term $T^2 I^{(a)}_{110}$ has no term proportional to T, and it includes only terms of order T^2 and higher order terms, which are corrections to the second term.

For the process (b) the dominant term $T^2 I^{(b)}_{100} = \ln 2 \cdot \epsilon_1 T + \frac{\pi^2}{12} T^2$ consists of the terms proportional to T and T^2. The former is proportional to the exciton number, or the particle number, and is obtained by exactly the same method as used in evaluating the main term of $T^2 I^{(a)}_{100}$. These two terms have the same magnitude and cancel each other. The next term $T^2 I^{(b)}_{110}$ consists of the term quadratic in T and an additional term with the same order as the main term. This additional term is the Pauli blocking correction and other correction terms to the first term.

TABLE III. List of integrals appearing in the Fermi liquid approximation for $\epsilon_1 \ll T$.

	T^2	$\epsilon_1 T$	ϵ_1^2
$T^2 I^{(a)}_{000}$			$\frac{1}{2}$
$-3T^2 I^{(a)}_{100}$			$-\frac{3}{4}$
$3T^2 I^{(a)}_{110}$			$\frac{3}{8}$
$-T^2 I^{(a)}_{111}$			$-\frac{1}{16}$
$T^2 I^{(a)}_{p}$			$\frac{1}{16}$
$3T^2 I^{(b)}_{100}$	$\frac{\pi^2}{4}$	$3\ln 2$	
$-6T^2 I^{(b)}_{110}$	$-3(\ln 2)^2$	$3(1-2\ln 2)$	$\frac{3}{8}$
$3T^2 I^{(b)}_{111}$	$-\frac{\pi^2}{8} + 3(\ln 2)^2$	$\frac{\pi^2}{16} - \frac{3}{2}(1-\ln 2)$	$\frac{3}{8} - \frac{3}{4}\ln 2$
$3T^2 I^{(b)}_{p}$	$\frac{\pi^2}{8}$	$\frac{\pi^2}{16} + \frac{3}{2}(1-\ln 2)$	$\frac{3}{4}(1-\ln 2)$
$3T^2 I^{(e)}_{011}$	$3(\ln 2)^2$		
$-3T^2 I^{(e)}_{111}$	$\frac{\pi^2}{8} - 3(\ln 2)^2$	$\frac{\pi^2}{16} - \frac{3}{2}(1-\ln 2)$	$\frac{3}{4}\ln 2 - \frac{9}{16}$
$T^2 I^{(e)}_{p}$	$\frac{\pi^2}{8}$	$\frac{\pi^2}{16} - \frac{3}{2}(1-\ln 2)$	$-\frac{9}{16} + \frac{3}{4}\ln 2$
$T^2 I_p$	$\frac{\pi^2}{4}$	$\frac{\pi^2}{8}$	$\frac{1}{4}$

For the process (e) the main contribution $T^2 I^{(e)}_{011}$ is proportional to T^2 representing the product of the particle and hole numbers in the initial state which appeared in Sec. II. The next integral $I^{(e)}_{111}$ is the Pauli and other corrections to the first term.

Next we consider the opposite limit, $\epsilon_1 \ll T$. In this case it is possible to make a series expansion. Using series expansion of $d_i(x)$ and Eq. (5.38), power series expansions of $d_i(x)$'s up to the second order are given by

$$\begin{aligned} d_1(x) &= \varepsilon_1 - \frac{1}{4}\varepsilon_1^2 + \cdots, \\ d_2(x) &= \frac{1}{2}\varepsilon_1 - \frac{3}{16}\varepsilon_1^2 + \cdots, \\ d_3(x) &= \ln 2 \cdot \varepsilon_1 - \frac{1}{4}\varepsilon_1^2 + \cdots . \end{aligned} \tag{5.43}$$

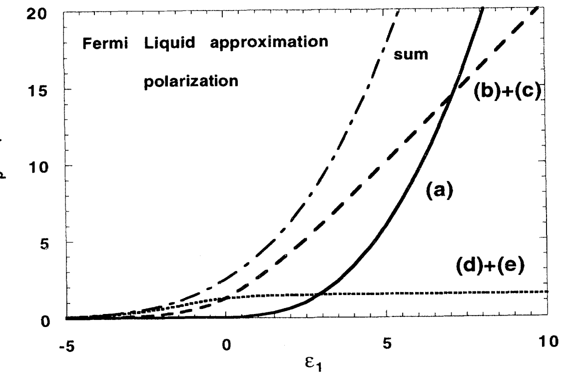

FIG. 4. Integrals $I_p^{(a)}(\varepsilon_1)$, $I_p^{(b)}(\varepsilon_1)+I_p^{(c)}(\varepsilon_1)$, and $I_p^{(d)}(\varepsilon_1)+I_p^{(e)}(\varepsilon_1)$, denoted by (a), (b)+(c), and (d)+(e), respectively, and the total sum are shown as a function of $\varepsilon_1 = (E_1 - E_F)/T$.

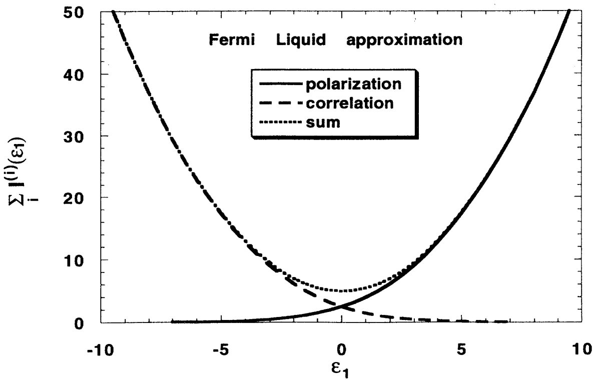

FIG. 5. Sum of all integrals in Fermi liquid approximation $\sum_j I_p^{(j)}(\varepsilon_1)$ is shown by the solid curve, while $\sum_j I_c^{(j)}(\varepsilon_1)$ is shown by the dashed one and the total sum by the dotted curve.

By putting these in Table II we obtain Table III.

The main contribution comes from the processes (b) and (e) and also the correlation and the polarization terms have contributions of the same order. The Pauli blocking effect is large. The $I_{000}^{(a)}$ term is smaller than the magnitude of the next correction term $I_{100}^{(a)}$ and the total contribution of the process (a) is reduced by a factor 8 compared to $I_{000}^{(a)}$. The large Pauli correction arises in other processes, because many nucleons below the Fermi energy jump up.

The absorptive potentials for each process from the polarization contributions are shown in Fig. 4 as a function of ε_1. They are plotted in nondimensional quantities, so they must be converted by appropriate units to obtain the actual values. The absorptive potential for the process (a) starts from $\varepsilon_1 = 0$ and increases roughly quadratically, but the effect of higher order terms is important. The curves for (b)+(c) and (d)+ (e) coincide at $\varepsilon_1 = 0$, and the former increases roughly linearly while the latter keeps almost a constant value. At lower energies the contribution for the $\Delta m = 0$ process is dominant, but at higher energies the contribution for the process $\Delta m = 1$ increases very rapidly and surpasses other contributions. Below $\varepsilon_1 = 0$ the contributions for the process (a) vanishes and others tend to zero by the denominator $1 + x$. The total contributions from the polarization $I_p^{(i)}(\varepsilon_1)$ and also the correlation $I_c^{(i)}(\varepsilon_1)$ are shown in Fig. 5. If one of the former curves is reflected at $\varepsilon_1 = 0$ then the latter curve is obtained. The total contribution has a simple inverted bell shape, which is just given by Eqs. (3.3) where $\overline{n}(E_1)$ is replaced by 1, and it does not vanish at $\varepsilon_1 = 0$. This is because a nucleon at the Fermi level has a finite lifetime if the nucleus is at a finite temperature. The nucleon gets energy from the nucleus and gets excited.

VI. NUMERICAL CALCULATION

In this section absorptive potentials for preequilibrium processes are calculated using a simplified Skyrme interaction. Their temperature as well as incident energy dependence are investigated and compared with those in the Fermi liquid approximation given in the previous section.

If the Wigner transform of the interaction product $VV^\dagger$ given by Eq. (3.23) is put in (4.10) the absorptive potentials are expressed as

$$W^{(i)}(E_1,\mathbf{K}_1,R_1;T) = -\frac{\pi}{2}(2\pi)^{-6}J[V_0(R_1)]^2 \int d\mathbf{K}_2 d\mathbf{K}_3 d\mathbf{K}_4 \delta(\mathbf{K}_1+\mathbf{K}_2-\mathbf{K}_3-\mathbf{K}_4) \times\delta(E_1+E_2-E_3-E_4)n_2(E_2)n_3(E_3)n_4(E_4). \tag{6.1}$$

In this formula energy as well as momentum conservations are observed, so the calculation becomes complicated compared with the Fermi liquid approximation. Furthermore $n_2(E_2)n_3(E_3)n_4(E_4)$ represents the corresponding product of partial occupation probabilities, so this also imposes restriction over the energies, which depends on the process. The directions of $\mathbf{K}_2$ and $\mathbf{K}_3$ are parametrized by

$$\mathbf{K}_1\cdot\mathbf{K}_2 = K_1K_2\cos\theta_2, \quad \mathbf{K}\cdot\mathbf{K}_3 = KK_3\cos\theta_3, \tag{6.2}$$

where

$$\mathbf{K} = \mathbf{K_1} + \mathbf{K_2}. \tag{6.3}$$

After integration over $\mathbf{K_4}$ in Eq. (6.1) we obtain

$$W^{(i)}(E_1,\mathbf{K}_1,R_1;T) = -\frac{\pi}{2}(2\pi)^{-6}J[V_0(R_1)]^2\frac{2m_n^*(R_1)}{\hbar^2}\int d\mathbf{K}_2 d\mathbf{K}_3\frac{1}{2KK_3}\delta\left(\cos\theta_3 - \frac{K_3^2+\mathbf{K}_1\cdot\mathbf{K}_2}{KK_3}\right) \times n_2(E_2)n_3(E_3)n_4(E_1+E_2-E_3). \tag{6.4}$$

Integration over θ_3 is readily performed because of the delta function, in which the condition $\cos^2\theta_3 \leq 1$ imposes the relation

$$\cos^2\theta_2 \le \frac{K_3^2(K_1^2+K_2^2-K_3^2)}{K_1^2K_2^2}. \tag{6.5}$$

From this the integration limits for $\cos\theta_2$ are obtained as

$$\alpha = \min\left(1, \sqrt{\frac{K_3^2(K_1^2+K_2^2-K_3^2)}{K_1^2K_2^2}}\right). \tag{6.6}$$

In Eq. (6.4) θ_2 is contained only in K, so the integration over $\cos\theta_2$ is performed as

$$\int_{-\alpha}^{\alpha}\frac{1}{K}d\cos\theta_2 = \frac{2}{K_2}F(E_1,E_2,E_3), \tag{6.7}$$

where

$$F(E_1,E_2,E_3) = \frac{1}{2K_1}\left(\sqrt{K_1^2+K_2^2+2K_1K_2\alpha} - \sqrt{K_1^2+K_2^2-2K_1K_2\alpha}\right). \tag{6.8}$$

Finally the absorptive potential is obtained as a double integral over energies,

$$W^{(i)}(E_1,\mathbf{K}_1,R_1;T) = -\pi\frac{1}{(2\pi)^4}J[V_0(R_1)]^2\left(\frac{m_n^*(R_1)}{\hbar^2}\right)^3\int_{U(R_1)}^{E_0}dE_2\int_{U(R_1)}^{E_0}dE_3F(E_1,E_2,E_3) \times n_2(E_2)n_3(E_3)n_4(E_1+E_2-E_3). \tag{6.9}$$

It is noted that this formula is applicable both for a particle and for a hole in contrast to our previous work [4].

Numerical calculations are performed for n+^{208}Pb. The interaction is assumed to be the simplified Skyrme interaction of the strength,

$$V_0(R) = t_0 + t_3\rho(R)\ , \tag{6.10}$$

while for the real mean potential the Saxon-Woods type with the effective mass $m_n^*/m_n = 0.7$ is used. Parameters for $V_0(R)$ and the Saxon-Woods potential are the same as in the previous paper [4].

To evaluate the chemical potential λ Eq. (3.9) with (3.8) are used. For the upper limit of the integral in (3.9) $E_0 = 60$ MeV is adopted instead of infinity. The value 60 MeV is much larger than the temperature considered in this work. Only the results of absorptive potentials at the nuclear center ($R_1 = 0$) are shown below, as surface vibrational modes are not taken into account in this paper in contrast with the work by Bortignon *et al.* [3].

First the dependence on the excitation energy of the incident nucleon $\epsilon_1 = E_1 - E_F$ is investigated by fixing the temperature $T = 2$ MeV. Results assuming $P+Q$ absorption are given in Fig. 6. The polarization contribution from each process is shown in Fig. 6(A), whose pattern of the curves is almost the same as the one for Fermi liquid shown in Fig. 4, while the correlation contribution shown in Fig. 6(B) is very similar to the pattern of the curves obtained by inverting the direction of the abscissa of Fig. 6(A). At this low temperature the results of $P+Q$ absorption are not different from the one obtained in the Fermi liquid approximation and also particle-hole symmetry holds in a good accuracy. However we find a difference in detail. The $\Delta m = 0$ curves denoted by (b) and (c)/2 which should coincide in the Fermi liquid approximation gradually split with the excitation energy. The comparison with the Fermi liquid approximation will be discussed later.

Results of calculations for Q absorption are shown in Fig. 7. The pattern of curves for the polarization contribution shown in (A) is noticeably different from those of the $P+Q$ absorption and the Fermi liquid approximation. The particle hole symmetry is hardly observed except for low excitation energy. Of course this is because the continuum part is cut from $P+Q$ space. The absorptive potential for processes (a) and (b) are suppressed greatly, as two particle states are involved in the intermediate states. The curve denoted by (a) has a maximum near $\epsilon_1 = 20$ MeV while the one denoted by (b) has a maximum near $\epsilon_1 = 10$ MeV, and both of them fall off gradually. On the other hand the curves for correlation contribution shown in (B) do not change appreciably from those for $P+Q$ absorption except for the (d) process which shows a plateau below $\epsilon_1 = -10$ MeV and the (b) process which dies off below $\epsilon_1 = -10$ MeV.

Next the temperature dependence of the absorptive potentials is examined and results for $P+Q$ absorption are shown in Fig. 8. For the process (a) three parabola-like curves for three temperatures 1, 3, and 5 MeV are drawn, of which the right-hand part corresponds to the polarization and the left-hand part corresponds to the correlation contributions. As the temperature increases the curvature of the curve decreases. Due to the Pauli blocking effect a particle-hole pair is difficult to create in intermediate states as the temperature increases.

The polarization contributions for the processes (b) and (c) are shown in Fig. 9(A) and the correlation contributions for the processes (d) and (e) are shown in Fig. 9(B). In Fig. 10(A) the polarization contributions for processes (d) and (e) are shown, and in Fig. 10(B) the correlation contributions for processes (b) and (c) are shown. In all cases except processes (a) and (f) the absolute value of the absorptive potential increases with the temperature as seen in the previous paper [4], because the temperature is approximately proportional to the exciton number. Two curves drawn by solid and dotted lines with the same temperatures in one frame, which should coincide if the Fermi liquid approximation is adopted, split with the increase of the temperature, and some of them cross each other.

In Sec. V we discussed the Pauli blocking effects and the linear term in T by decomposing the occupation probability into the zero temperature part and the correction part as shown by Eq. (5.13). The same decomposition is applied to $W_p^{(a)}$ in the present numerical calculation. We consider the following two typical cases.

The first is the case of $T = 1$ MeV and Q absorption. In this case the mean particle number $\overline{m} = 6.3$ and the situation may be realized in a typical multistep compound process. In Fig. 11(A) the absorptive potentials for processes (a) through (e) are shown by full line curves without decomposition. The curve (a0) is the result for $T = 0$ or corresponding to $I_{000}^{(a)}$ in the Fermi liquid approximation, and (a1) corresponds to $-I_{100}^{(a)} - I_{010}^{(a)} - I_{001}^{(a)}$. The figure shows that the sum of curves (a0) and (a1) is approximately equal to the (a) curve. This indicates that (a1) is a dominant Pauli correction term at this low temperature. Furthermore the sum of contributions from (b) and (c) curves is approximately equal to the absolute value of the curve (a1).

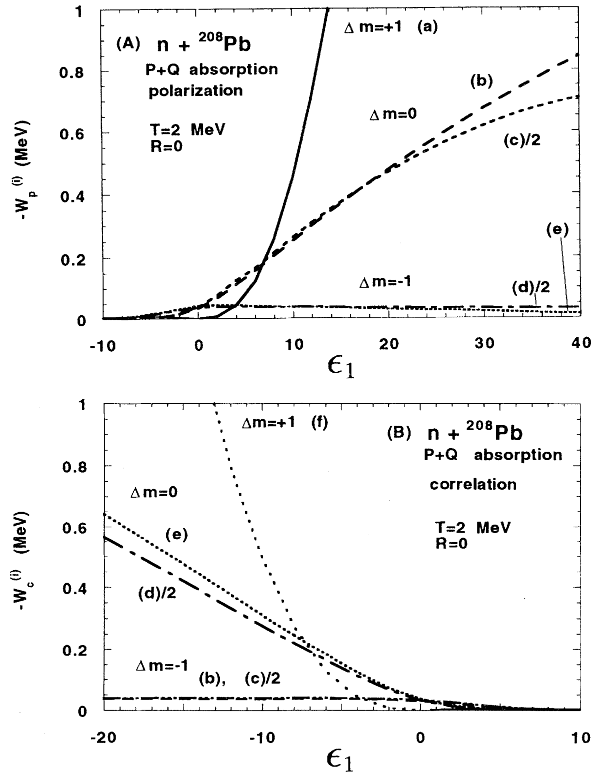

FIG. 6. Polarization contributions $-W_p^{(j)}$ ($j = a - e$) [shown in (A)] and correlation ones $-W_c^{(j)}(j = b - f)$ [shown in (B)] at $T = 2$ MeV for $P + Q$ absorption. The simplified Skyrme interaction is used as the residual interaction and the numerical calculation is performed without the Fermi liquid approximation.

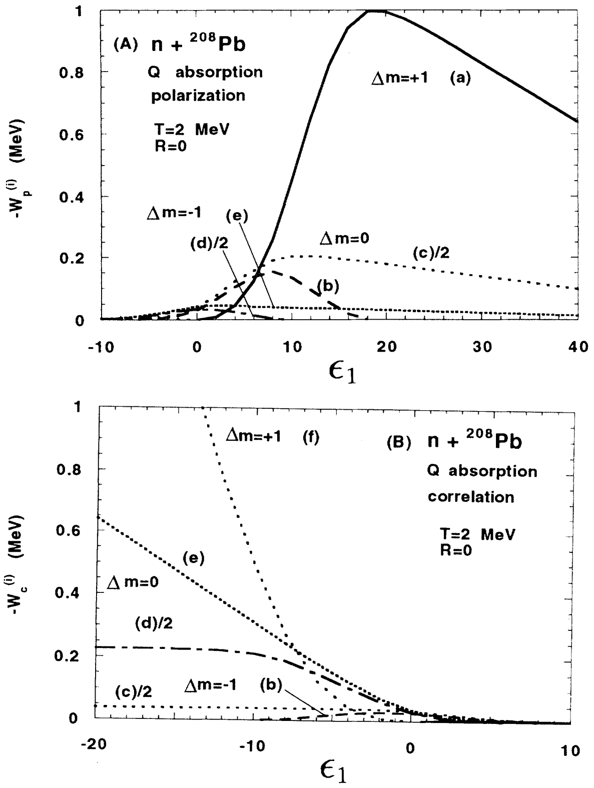

FIG. 7. The same as Fig. 6, but for Q absorption.

Next the case $T = 5$ MeV and $P + Q$ absorption is considered. In this case the mean particle number $\overline{m} = 32$ can possibly be realized in heavy ion reactions. Curves in Fig. 11(B) are results calculated in the same way as in Fig. 11(A) except for the curve (a2). This curve corresponds to $-I_{100}^{(a)} - I_{010}^{(a)} - I_{001}^{(a)} + I_{011}^{(a)} + I_{101}^{(a)} + I_{110}^{(a)}$.

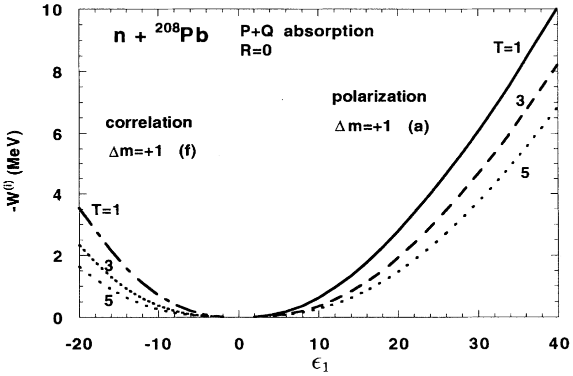

FIG. 8. Polarization contributions $-W_p^{(a)}$ and correlation ones $-W_c^{(f)}$ at temperatures $T = 1, 3$, and 5 MeV in the case of $P + Q$ absorption. Calculations are the same as in Fig. 6.

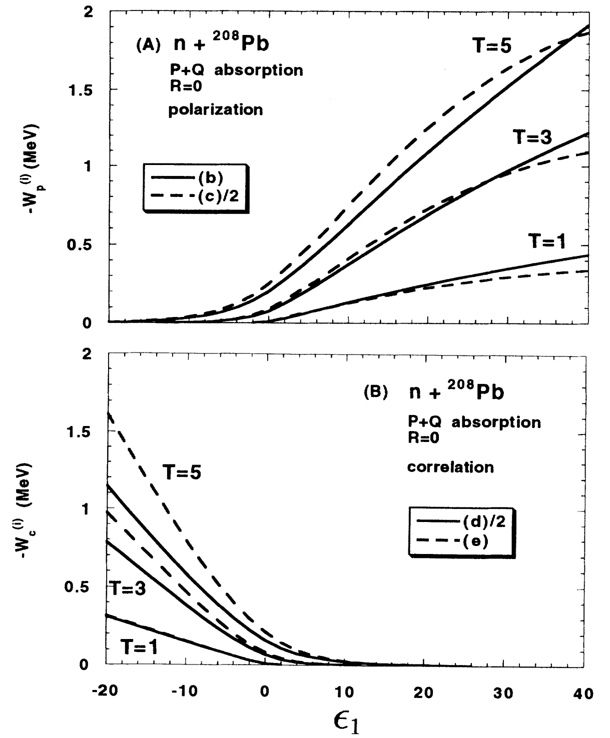

FIG. 9. Polarization contributions $-W_p^{(j)}$ $(j = b, c)$ [shown in (A)] and correlation ones $-W_c^{(j)}(j = d, e)$ [shown in (B)]. Calculations are the same as in Fig. 6.

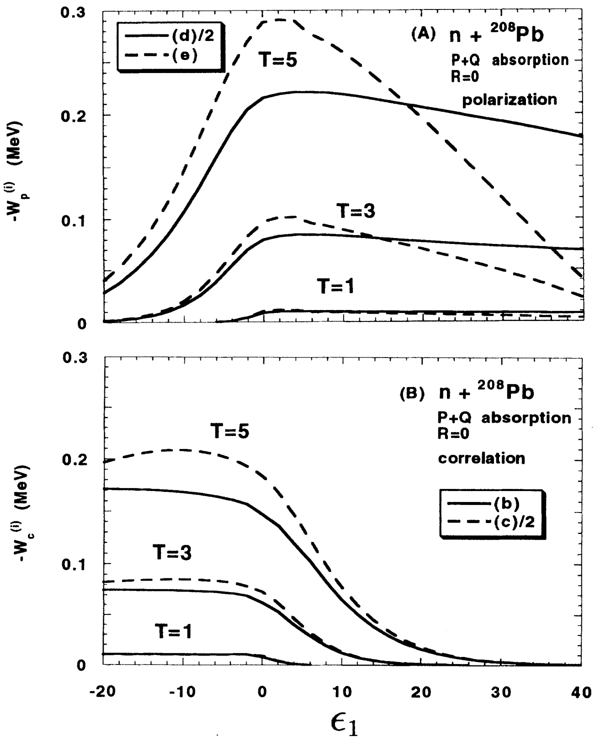

FIG. 10. Polarization contribution $-W_p^{(j)}$ $(j = d, e)$ [shown in (A)] and correlation ones $W_c^{(j)}(j = b, c)$ [shown in (B)]. Calculations are the same as in Fig. 6.

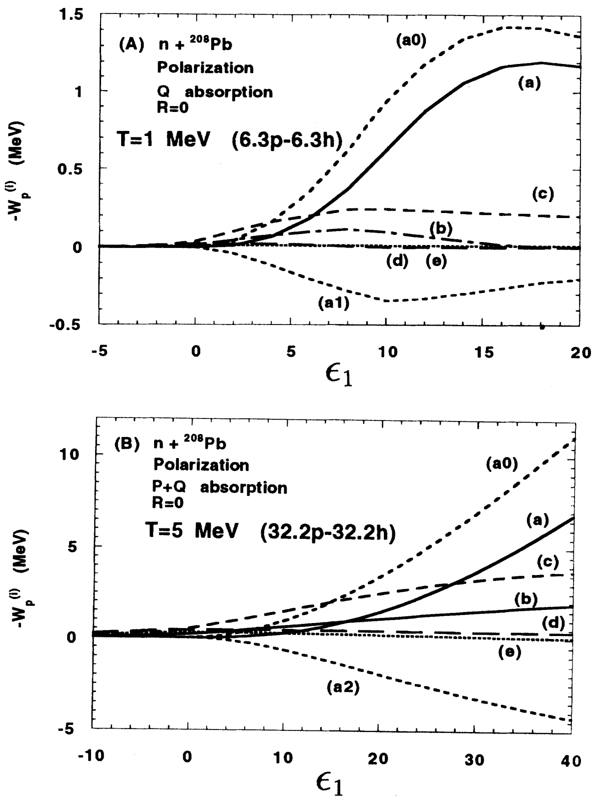

FIG. 11. Polarization contributions $-W_p^{(j)}$ $(j = a - e)$ at $T = 1$ MeV for Q absorption are shown in (A). The dashed curves labeled by (a0) and (a1) represent the $T = 0$ part and the part corresponding to $-I_{100}^{(a)} - I_{010}^{(a)} - I_{001}^{(a)}$ of $-W_p^{(a)}$, respectively. The same absorptive potentials, but at $T = 5$ MeV for $P + Q$ absorption are shown in (B). The (a2) curve corresponds to $-I_{100}^{(a)} - I_{010}^{(a)} - I_{001}^{(a)} + I_{011}^{(a)} + I_{101}^{(a)} + I_{110}^{(a)}$ of $-W_p^{(a)}$.

Among these contributions the same relation observed in Fig. 11(A) (a0)+(a2)=(a) and (b)+(c)$\approx$ (a2) holds. This indicates that at $T = 5$ MeV the second correction terms become important.

VII. SUMMARY AND DISCUSSIONS

In the present paper we considered the absorption process at finite temperature in connection with preequilibrium processes and found a new interrelation between them. The absorptive potential at finite temperature W_T is decomposed into 12 components $W_p^{(j)}$ and $W_c^{(j)}(j = a \cdots f)$. Each component represents the absorptive potential for preequilibrium processes specified with the particle or hole number $\overline{m}$ given by Eq. (4.3) and includes thermal Pauli blocking effects which were considered only approximately in the previous work [4].

Temperature dependence of the absorptive potentials $W^{(i)}$ is one of interesting problems, and the Fermi liquid approximation is introduced to obtain analytic expressions. The results show how each component of W_T depends on temperature, and give a reason why no linear T term appears in W_T. When all components $W_p^{(j)}$ and $W_c^{(j)}$ are summed up, the well-known behavior $W_T \sim (\pi^2 T^2 + \epsilon_1^2)$ appears. Main results obtained through analytic calculation in the Fermi liquid approximation are as follows.

(1) The ϵ_1^2 term comes from the process (a).

(2) Linear T terms consist of the negative contribution from the Pauli blocking effect in the process (a), and the positive contribution from processes (b) and (c). Net contributions to W_T cancel one another to zero, which agrees with the Morel and Nozières result [17]. However T linear terms remain in the absorptive potential for $\Delta m = 1$ and $\Delta m = 0$ processes and give effects on transmission coefficients.

(3) T^2 terms mainly come from processes (b), (c), (d), and (e).

The Fermi liquid approximation is expected to be valid at low temperature, so it is important to undertake calculations without this approximation. For this the simplified Skyrme interaction, which is just a delta interaction in our approximate treatment, is adopted. These calculations are also useful to determine how far the Fermi liquid approximation is valid, especially in the case of Q absorption, where this approximation is supposed to have very limited applicability.

Calculations are performed for ^{208}Pb, and absorptive potentials for processes (a), (b), and (e) are shown in Fig. 12 for $T = 3$ MeV as an example. It is seen that the Fermi liquid approximation is adequate around the Fermi energy E_F while it overestimates the magnitude of the absorptive potential with increasing energy. Although the case of processes (c) and (d) is not shown, a similar tendency is observed.

In the Fermi liquid approximation all participating nucleons are assumed to have momenta very close to the Fermi momentum. Therefore it is expected to be valid in the low temperature limit, and as the temperature increases the Fermi liquid results deviate from those with the delta interaction. Two sources of deviation may be considered. The first is a shift of the chemical potential downward from the Fermi energy. As the temperature increases the curve of the occupation probability moves to the lower energy side by $E_F - \lambda$, which is listed in Table I.

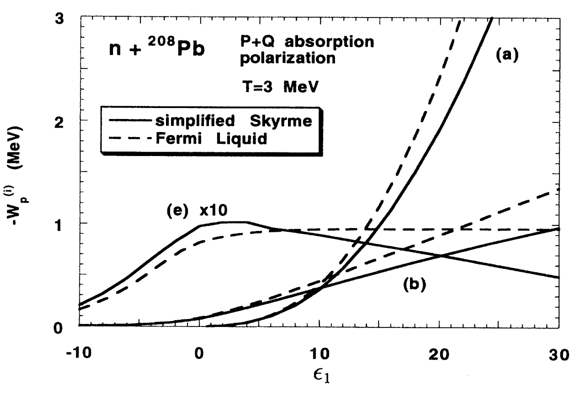

FIG. 12. The absorptive potentials for polarization contribution in the Fermi liquid approximation which are shown by dotted curves are compared with those of the numerical calculations using the simplified Skyrme interaction for (a), (b), and (e) processes. For curves (e) the values multiplied by 10 are shown.

In Sec. IV the absorptive potentials are expressed approximately in terms of the ones for the elementary process and the proportional constant was the mean particle number $\overline{m}$ for processes (b) and (c) and $\overline{m}^2$ for (d) and (e). On the other hand for the absorptive potential calculated with the simplified Skyrme interaction given by (6.9) the quantities

$$m_h = g_F \int_{U(R_1)}^{E_F} h(E)dE \approx g_F T \ln(1 + e^{(E_F - \lambda)/T}) , \tag{7.1}$$

$$m_p = g_F \int_{E_F}^{E_0} p(E)dE \approx g_F T \ln(1 + e^{-(E_F - \lambda)/T}) \tag{7.2}$$

are considered to be related to the magnitude of the absorptive potential. The last expressions on the right-hand side of the equations represent values for $U(R_1) = -\infty$ and $E_0 = \infty$. The constant state density g_F is included just to make these quantities comparable with particle or hole number as given by (4.3) and (4.7), nonetheless they are not the particle or hole number exactly.

In the Fermi liquid approximation

$$\int_0^\infty \frac{g_F}{1 + e^{\epsilon/T}} d\epsilon = (\ln 2) g_F T \tag{7.3}$$

is the corresponding quantity. These values are listed in Table I, and we can estimate how much the magnitude changes in the case of the simplified Skyrme interaction compared with the Fermi liquid approximation. It is noted that this effect is independent of the energy of the incident nucleon, as it is concerned with the initial state of the target nucleus. The partial occupation probabilities $H(E)$ decrease and $P(E)$ increases as the chemical potential decreases. However they appear in the intermediate states and the effect is not too important.

The second source of the deviation comes from neglect of momentum conservation and the upper limit of the excitation energy of particle or hole.

As the momentum conservation imposes restriction on phase space integration, the absorptive potentials with the delta interaction cannot be larger than those in the Fermi liquid approximation provided the same chemical potential is used. This effect is represented as the integral $F(E_1, E_2, E_3)$ given by (6.8), which is unity in the Fermi liquid approximation. This factor is estimated approximately, namely in a linear approximation with respect to the deviation from the Fermi momentum of participating nucleons. For processes (a), (b), and (d), $F(E_1, E_2, E_3) = 1$, while, for (c) and (e), $F(E_1, E_2, E_3) \leq 1$. The upper limit of the excitation energy of particle or hole restricts the integration limit through the partial occupation probabilities.

As seen in Fig. 12 for process (a) the difference between two calculations is small at low excitation energy because the effect of the chemical potential is not present in the initial state and the momentum conservation effect is approximately given by $F = 1$. Reduction at higher excitation energy may be caused by suppression due to the momentum conservation rather than the effect of the chemical potential, by which enhancement in $2p$-$1h$ states is expected. For the process (b) the effect of the chemical potential works to suppress the absorptive potential because $p(E)$ appears in initial states. On the contrary for the process (e) two partial occupation probabilities $h(E)$ appear in the initial states, and the enhancement is expected in agreement with the curves in Fig. 12. At higher energy this tendency is inverted, and this may be due to the momentum conservation and energy limitation of particles.

In the actual situation of a nucleon induced multistep compound (MSC) process, the incident energy is not so large and the typical temperature is estimated to be less than 1 MeV for ^{208}Pb or 2 MeV for ^{93}Nb using Eq. (2.8). So the Fermi liquid approximation is expected to work fairly well.

In the present paper we concentrated on studying the relation between the absorptive potential in thermal equilibrium and those in preequilibrium processes, and on their features at the nuclear center (i.e., in the nuclear matter). Calculations may be extended to the nuclear surface region, but only the density effect may be included as the effect of surface vibrational modes is neglected in this work.

Bortignon *et al.* [3] investigated the absorptive potential for surface vibrations in thermal equilibrium, and found a different temperature dependence from our case. Recently Danielewicz and Schuck [20] have succeeded in obtaining a formula for the absorptive potential for the damping processes of collective motion as well as of a single particle at the same time. If we decompose their absorptive potential into preequilibrium components in line with the present work, it may be possible to get a preequilibrium absorptive potential for collective motion.

In MSC reaction theories the absorptive potentials obtained here are used to calculate the transmission coefficients from which the formation and decay probability of the compound system are obtained. Usually transition from P space to Q space takes place in the first stage of the multistep direct process, but the importance of transition from the second stage was discussed in our previous paper [4]. In such a calculation the absorptive potential obtained here is very useful because they are evaluated more accurately.

In the calculation of level density [21], and also MSC reaction cross section [8,19], the transition matrix elements between exciton states in Q space are assumed to be random matrices and its second moments are important basic quantities. The absorptive potential obtained here is used to evaluate the spreading width semiclassically as given in [22] for particles and holes and the second moments are obtained from them. The calculation becomes much simpler compared with the actual evaluation of matrix elements [21]. These applications are interesting future problems.

ACKNOWLEDGMENTS

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APPENDIX A: INTEGRALS APPEARING IN FERMI LIQUID APPROXIMATION

In this appendix the integrals $I^{(j)}_{k_2k_3k_4}$ appearing in Sec. V are evaluated when the nucleon 1 is a particle, namely $\varepsilon_1 > 0$. The method of the calculation is almost the same for processes (a), (b), and (e), so only the first process is explained in detail.

1. Process (a)

The first integral is defined by

$$I^{(a)}_{000}(\varepsilon_1) = \int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \delta(\varepsilon_1 - \varepsilon_2 - \varepsilon_3 - \varepsilon_4). \tag{A1}$$

Three integral variables $\varepsilon_2, \varepsilon_3, \varepsilon_4$ as well as ε_1 are restricted to positive, and the energy delta function gives rise to the following integral ranges and the integral is readily evaluated as

$$I^{(a)}_{000}(\varepsilon_1) = \int_0^{\varepsilon_1} d\varepsilon_2 \int_0^{\varepsilon_1-\varepsilon_2} d\varepsilon_3 = \frac{1}{2}(\varepsilon_1)^2. \tag{A2}$$

To evaluate the second integral

$$\begin{aligned} I^{(a)}_{100}(\varepsilon_1) &= \int_0^{\varepsilon_1} d\varepsilon_2 \int_0^{\varepsilon_1-\varepsilon_2} d\varepsilon_3 \frac{1}{1+e^{\varepsilon_2}} \\ &= \int_0^{\varepsilon_1} d\varepsilon_2 \frac{1}{1+e^{\varepsilon_2}} (\varepsilon_1 - \varepsilon_2) \ , \end{aligned} \tag{A3}$$

the integral variable ε_2 is transformed to the variable $t = e^{-\varepsilon_2}$ and

$$I^{(a)}_{100}(\varepsilon_1) = -\int_1^x dt \frac{\ln(\frac{t}{x})}{1+t} \tag{A4}$$

is obtained. The upper limit of the integral x is given by Eq. (5.38). After one integration by parts the integral is expressed in terms of the dilogarithm defined by (B2),

$$I^{(a)}_{100}(\varepsilon_1) = -\ln 2 \ln x - \operatorname{dilog}(1+x) - \frac{\pi^2}{12}. \tag{A5}$$

In the third integral

$$I^{(a)}_{110}(\varepsilon_1) = \int_0^{\varepsilon_1} d\varepsilon_2 \int_0^{\varepsilon_1-\varepsilon_2} d\varepsilon_3 \frac{1}{1+e^{\varepsilon_2}} \frac{1}{1+e^{\varepsilon_3}}\,, \tag{A6}$$

the integral over ε_3 is evaluated using formula (B1) as

$$\begin{aligned} I^{(a)}_{110}(\varepsilon_1) &= \int_0^{\varepsilon_1} d\varepsilon_2 \frac{1}{1+e^{\varepsilon_2}} \ln \frac{2e^{\varepsilon_1-\varepsilon_2}}{1+e^{\varepsilon_1-\varepsilon_2}} \\ &= -\int_1^{x} dt \frac{1}{1+t} \ln \frac{2t}{x+t}, \end{aligned} \tag{A7}$$

where $t = e^{-\varepsilon_2}$. This integral is integrated by parts and the variable is again changed to $s = 1 + t$, and

$$\begin{aligned} I^{(a)}_{110}(\varepsilon_1) &= \ln 2 \ln \frac{2}{1+x} \\ &\quad + \int_2^{1+x} ds \left(\frac{\ln s}{s-1} - \frac{\ln s}{x+s-1} \right) \end{aligned} \tag{A8}$$

is obtained. If further change of variable $u = s/(1-x)$ is applied to the last term of the right-hand side of the equation, both integrals become dilogarithms, and the result is given by

$$\begin{aligned} I^{(a)}_{110}(\varepsilon_1) &= \ln 2 \ln \frac{2}{1+x} - \text{dilog}(1+x) \\ &\quad -\frac{\pi^2}{12} \ln(1-x) \ln \frac{2x}{1+x} \\ &\quad +\text{dilog}\frac{1+x}{1-x} - \text{dilog}\frac{2}{1-x}. \end{aligned} \tag{A9}$$

Using formulas (B7) and (B8) and also the relation (B10) it is simplified as

$$I^{(a)}_{110}(\varepsilon_1) = \text{dilog}\, x - 2\text{dilog}\frac{1+x}{2}. \tag{A10}$$

Using Eqs. (5.37), (5.15), and (5.17) the integral $I^{(a)}_{111}(\varepsilon_1)$ is expressed in terms of previously obtained integrals

$$\begin{aligned} I^{(a)}_{111}(\varepsilon_1) &= \frac{x}{1+x} \left[I^{(a)}_{000}(\varepsilon_1) - 3I^{(a)}_{100}(\varepsilon_1) + 3I^{(a)}_{110}(\varepsilon_1) \right] \\ &= \frac{x}{1+x} \left[3 \ln 2 \ln x + \frac{1}{2}(\ln x)^2 + 3\text{dilog}\, x - 6\, \text{dilog}\frac{1+x}{2} + 3\left\{ \text{dilog}(1+x) + \frac{\pi^2}{12} \right\} \right]. \end{aligned} \tag{A11}$$

2. Process (b)

The integral in the process (b) is defined as shown by the simplest case

$$\begin{aligned} I^{(b)}_{100}(\varepsilon_1) &= \int_0^{\infty} d\varepsilon_2 \int_0^{\infty} d\varepsilon_3 \int_0^{\infty} d\varepsilon_4 \\ &\quad \times\delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4) \frac{1}{1+e^{\varepsilon_2}} \\ &= \int_0^{\infty} d\varepsilon_2 \int_0^{\varepsilon_1+\varepsilon_2} d\varepsilon_3 \frac{1}{1+e^{\varepsilon_2}}. \end{aligned} \tag{A12}$$

The integral is calculated quite in the same procedure as in the case of $I^{(a)}_{100}$ and the result is given by

$$I^{(b)}_{100}(\varepsilon_1) = -\ln 2 \ln x + \frac{\pi^2}{12}. \tag{A13}$$

The second integral is calculated as

$$\begin{aligned} I^{(b)}_{110}(\varepsilon) &= \int_0^{\infty} d\varepsilon_2 \int_0^{\varepsilon_1+\varepsilon_2} d\varepsilon_3 \frac{1}{1+e^{\varepsilon_2}} \frac{1}{1+e^{\varepsilon_3}} \\ &= \int_0^{1} dt \frac{1}{1+t} \ln\left(\frac{2}{1+xt} \right) \end{aligned} \tag{A14}$$

and the final result is given by

$$\begin{aligned} I^{(b)}_{110} &= \frac{1}{2}(\ln 2)^2 - \text{dilog}\frac{1+x}{2} \\ &\quad +\text{dilog}(1+x) + \frac{\pi^2}{12}. \end{aligned} \tag{A15}$$

The next integral

$$\begin{aligned} I^{(b)}_{011}(\varepsilon_1) &= \int_0^{\infty} d\varepsilon_2 \int_0^{\varepsilon_1+\varepsilon_2} d\varepsilon_3 \frac{1}{1+e^{\varepsilon_3}} \frac{1}{1+e^{\varepsilon_1+\varepsilon_2-\varepsilon_3}} \\ &= \int_0^{x} dt \frac{1}{1-t} \left[2 \ln \frac{1+t}{2} - \ln t \right] \end{aligned} \tag{A16}$$

is obtained as

$$I^{(b)}_{011}(\varepsilon_1) = 2\text{dilog}\frac{1+x}{2} - \text{dilog}\, x + (\ln 2)^2. \tag{A17}$$

The final integral is

$$\begin{aligned} I^{(b)}_{111}(\varepsilon_1) &= \frac{1}{1+x} \left[-xI^{(b)}_{100}(\varepsilon_1) + 2xI^{(b)}_{110}(\varepsilon_1) + I^{(b)}_{011}(\varepsilon_1) \right] \\ &= \frac{1}{1+x} \left[(1+x)(\ln 2)^2 + x \ln 2 \ln x - \frac{\pi^2}{12} x \right. \\ &\quad +2x \left\{ \text{dilog}(1+x) + \frac{\pi^2}{12} \right\} \\ &\quad \left. -\text{dilog}\, x + 2(1-x)\text{dilog}\frac{1+x}{2} \right]. \end{aligned} \tag{A18}$$

3. Process (e)

The first integral

$$I_{100}^{(e)}(\varepsilon_1) = \int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \times\delta(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 + \varepsilon_4)\frac{1}{1+e^{\varepsilon_2}} = \int_{\varepsilon_1}^\infty d\varepsilon_2 \int_0^{\varepsilon_2-\varepsilon_1} d\varepsilon_3 \frac{1}{1+e^{\varepsilon_2}} \tag{A19}$$

gives the result

$$I_{100}^{(e)}(\varepsilon_1) = -\mathrm{dilog}(1+x). \tag{A20}$$

The next integral

$$I_{110}^{(e)}(\varepsilon_1) = \int_{\varepsilon_1}^\infty d\varepsilon_2 \int_0^{\varepsilon_2-\varepsilon_1} d\varepsilon_3 \frac{1}{1+e^{\varepsilon_2}}\frac{1}{1+e^{\varepsilon_3}} = \int_0^1 dt \frac{1}{t+\frac{1}{x}} \ln\frac{2}{1+t} \tag{A21}$$

becomes

$$I_{110}^{(e)}(\varepsilon_1) = \frac{1}{2}(\ln 2)^2 - \frac{\pi^2}{12} +\mathrm{dilog}\frac{1+x}{2} - \mathrm{dilog}(1+x). \tag{A22}$$

The integral

$$I_{011}^{(e)}(\varepsilon_1) = \int_0^\infty d\varepsilon_2 \int_0^\infty d\varepsilon_3 \int_0^\infty d\varepsilon_4 \times\delta(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 + \varepsilon_4)\frac{1}{1+e^{\varepsilon_3}}\frac{1}{1+e^{\varepsilon_4}} \tag{A23}$$

is easier to integrate first over ε_2 to yield

$$I_{011}^{(e)}(\varepsilon_1) = \int_0^\infty d\varepsilon_3 \frac{1}{1+e^{\varepsilon_3}} \int_0^\infty d\varepsilon_4 \frac{1}{1+e^{\varepsilon_4}} = (\ln 2)^2. \tag{A24}$$

The last integral is given by

$$I_{111}^{(e)}(\varepsilon_1) = \frac{1}{1+x}[-I_{100}^{(e)}(\varepsilon_1) + 2I_{110}^{(e)}(\varepsilon_1) + xI_{011}^{(e)}(\varepsilon_1)] = \frac{1}{1+x}\left[(1+x)(\ln 2)^2 - \frac{\pi^2}{12} +2\ \mathrm{dilog}\frac{1+x}{2} - \mathrm{dilog}(1+x) - \frac{\pi^2}{12}\right]. \tag{A25}$$

4. Process (f)

$$I_p^{(f)}(\varepsilon_1) = I_c^{(f)}(\varepsilon_1) = 0\ . \tag{A26}$$

APPENDIX B: INTEGRAL FORMULAS

The following integral is useful in our calculation:

$$\int_0^a \frac{1}{1+be^{\pm\varepsilon}} d\varepsilon = \pm \ln\frac{(1+b)e^{\pm a}}{1+be^{\pm a}}. \tag{B1}$$

For the integrals appearing in Sec. V and Appendix A the dilogarithms defined by

$$\mathrm{dilog}\ x = \int_1^x \frac{\ln t}{1-t} dt \tag{B2}$$

are necessary [18]. This is expanded in power series around $x = 1$ as

$$\mathrm{dilog}\ x = \sum_{n=1}^\infty \frac{(1-x)^n}{n^2}, \tag{B3}$$

whose convergence region is $0 \leq x \leq 2$. The relations

$$\mathrm{dilog}\frac{1}{x} = -\mathrm{dilog}\ x - \frac{1}{2}(\ln x)^2 \tag{B4}$$

and

$$\mathrm{dilog}(1-x) = \frac{\pi^2}{6} - \ln x \ln(1-x) - \mathrm{dilog} x \tag{B5}$$

are useful, and the following specific values are often used:

$$\mathrm{dilog}\ 0 = \frac{\pi^2}{6}, \quad \mathrm{dilog}\ 1 = 0, \quad \mathrm{dilog}\ 2 = -\frac{\pi^2}{12}. \tag{B6}$$

It is convenient to express the results in terms of the following four dilogarithms: dilog x, dilog $\frac{1+x}{2}$, dilog $\frac{2x}{1+x}$, and $\mathrm{dilog}(1+x) + \frac{\pi^2}{12}$ which vanish at $x = 1$. Other dilogarithms appearing in the integrals are also expressed by these four functions:

$$\mathrm{dilog}\frac{2}{1-x} = -\frac{\pi^2}{6} + \ln\frac{1-x}{2}\ln\frac{1+x}{2} +\mathrm{dilog}\frac{1+x}{2} - \frac{1}{2}\left(\ln\frac{1-x}{2}\right)^2, \tag{B7}$$

$$\mathrm{dilog}\frac{1+x}{1-x} = -\frac{\pi^2}{6} + \ln\frac{1-x}{1+x}\ln\frac{2x}{1+x} +\mathrm{dilog}\frac{2x}{1+x} - \frac{1}{2}\left(\ln\frac{1-x}{2}\right)^2, \tag{B8}$$

$$\mathrm{dilog}\frac{1}{1-x} = -\frac{\pi^2}{6} + \ln(1-x)\ln x +\mathrm{dilog}\ x - \frac{1}{2}[\ln(1-x)]^2. \tag{B9}$$

The four functions are not independent and the following relation holds:

$$\mathrm{dilog}\ x - \mathrm{dilog}\frac{1+x}{2} - \mathrm{dilog}\frac{2x}{1+x} + \mathrm{dilog}(1+x) + \frac{\pi^2}{12} = \frac{1}{2}\left(\ln\frac{1+x}{2}\right)^2 - \ln x \ln(1+x). \tag{B10}$$

By changing variables the left-hand side of the equation is expressed in terms of integrals whose integral range is between 1 and x,

$$\begin{aligned}\int_1^x \frac{\ln t}{1-t}dt - \int_1^x \frac{\ln[(1+t)/2]}{1-t}dt \\ - \int_1^x \left(\frac{1}{1-t}+\frac{1}{1+t}\right)\ln\frac{2t}{1+t}dt \\ - \ln x\ln(1+x) + \int_1^x \frac{\ln t}{1+t}dt. \qquad \text{(B11)}\end{aligned}$$

The integrands containing $1/(1-t)$ cancel each other and the remaining integral is reduced to the right-hand side of the equation. Therefore the three functions $d_1(x) = \mathrm{dilog}(x)$, $d_2(x) = \mathrm{dilog}(\frac{1+x}{2})$, $d_3(x) = \mathrm{dilog}(1+x) + \frac{\pi^2}{12}$ are used as basis functions, and together with logarithms all integrals are expanded.

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