# Algebraic-eikonal approach to medium energy proton scattering from odd-mass nuclei

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We extend the algebraic-eikonal approach to medium energy proton scattering from odd-mass nuclei by combining the eikonal approximation for the scattering with a description of odd-mass nuclei in terms of the interacting boson-fermion model. We derive closed expressions for the transition matrix elements for one of the dynamical symmetries and discuss the interplay between collective and single-particle degrees of freedom in an application to elastic and inelastic proton scattering from <sup>195</sup>Pt.

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#### I. INTRODUCTION

High energy scattering of nucleons from a collective nucleus involves the excitation of a large number of strongly coupled states as well as that of virtual intermediate states. The effects of channel coupling become particularly important for large momentum transfer, where the scattering cannot be treated in the distorted-wave Born approximation (DWBA), but requires calculations to higher order in the channel coupling [1]. The standard approach is that of a coupled channels calculation, which becomes complicated when the number of channels that have to be included is large. An alternative to this approach is provided by the eikonal approximation, which in combination with the interacting boson model (IBM) [2] allows the calculation of the scattering to all orders in closed form for even-even nuclei [1, 3, 4]. This method, also known as the algebraic-eikonal approach, combines the strengths of an algebraic description of the target dynamics with those of the eikonal approximation in the adiabatic limit. In the case of odd-mass nuclei the complications of the channel coupling approach are even greater, because of the interplay between collective and single-particle degrees of freedom and the increase in the number of open channels in the system. On the other hand, the interacting boson-fermion model (IBFM) has provided a tractable model of odd-mass nuclei and its usefulness for the classification and understanding of nuclear structure data has been tested in numerous ways [5]. Among the most interesting aspects of this model is the possibility of the occurrence of boson-fermion dynamical symmetries as well as supersymmetries, the latter of which link the properties of neighboring nuclei.

The purpose of this paper is twofold. First we show that the algebraic-eikonal approach for medium energy proton scattering can be extended to odd-mass nuclei in a simple fashion by describing the target nucleus in terms of the IBFM. Next we consider the particular case of one of the dynamical symmetries of the IBFM, the  $SO(6) \otimes SU(2)$  limit, to derive closed expressions for the transition matrix elements. As an example we discuss the application to elastic and inelastic proton scattering from  $^{195}$ Pt, which is considered to be a paradigm of dynamical boson-fermion symmetry, in terms of energy systematics, electromagnetic decay properties, and single-particle transfer [6]. We consider in particular the interplay between the coupling to collective and single-particle degrees of freedom for the excitation of the lowest negative parity states in  $^{195}$ Pt by medium energy protons.

## **II. EIKONAL APPROXIMATION**

For medium and high energy proton-nucleus scattering the eikonal approach is a good approximation for elastic and inelastic scattering. The Hamiltonian is in general given by

$$H = \frac{\hbar^2 k^2}{2m} + H_t(\xi) + V(\vec{r},\xi) , \qquad (1)$$

where  $H_t$  describes the dynamics of the target nucleus and  $V(\vec{r},\xi)$  represents the proton-nucleus interaction. The projectile coordinate  $\vec{r}$  is measured from the center of mass of the target. The internal coordinates of the target nucleus are collectively denoted by  $\xi$ . If the kinetic energy of the projectile is much larger than the interaction strength, and is also sufficiently large that the projectile wavelength is small compared with the range of variation of the potential, one may use the eikonal approximation to describe the scattering. If, in addition, the projectile energy is large compared with the nuclear excitation energies, one can neglect  $H_t$  (adiabatic limit). Under these approximations the scattering amplitude for scattering a projectile with initial momentum  $\vec{k}$  from an initial state  $|i\rangle$  to final momentum  $\vec{k}'$  and a final state  $|f\rangle$ is given by

$$A_{fi}(\vec{q}) = \frac{k}{2\pi i} \int d^2 b \ e^{i\vec{q}\cdot\vec{b}} \left\langle f|e^{i\chi(\vec{b},\xi)} - 1|i\right\rangle \,, \tag{2}$$

where  $\vec{q} = \vec{k}' - \vec{k}$  is the momentum transfer and  $\chi(\vec{b},\xi)$  is the eikonal phase that the projectile acquires as it goes by the target,

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$$\chi(\vec{b},\xi) = -\frac{m}{\hbar^2 k} \int_{-\infty}^{\infty} dz \ V(\vec{r},\xi) \ . \tag{3}$$

In the derivation of the scattering amplitude the projectile coordinate is written as  $\vec{r} = \vec{b} + \vec{z}$ , where the impact parameter  $\vec{b}$  is perpendicular to the z axis, which is chosen along  $\hat{z} = (\vec{k} + \vec{k}')/|\vec{k} + \vec{k}'|$ . In the eikonal approximation the scattering amplitude is expressed in terms of an integral over a two-dimensional impact parameter rather than as a sum over partial waves.

For medium energy proton-nucleus scattering from even-even nuclei the eikonal approximation has been applied successfully to elastic and inelastic scattering to forward angles [1, 3, 4]. This procedure can be extended to odd-mass nuclei by considering the coupling of the projectile to both the collective and the single-particle degrees of freedom. If the range of the projectile-nucleus interaction is short compared to the size of the nucleus, the potential can be expressed in terms of the projectilenucleon forward scattering amplitude f [1],

$$V(\vec{r},\xi) = -\frac{2\pi\hbar^2 f}{m} \left\{ \rho(r) + [Q_B(r,\xi) + Q_F(r,\xi)] \cdot Y_2(\hat{r}) \right\}.$$
 (4)

Here  $\rho(r)$  is the nuclear density for the distorting or optical potential, and  $Q_B(r,\xi)$  and  $Q_F(r,\xi)$  denote contributions from the quadrupole coupling of the projectile to the collective (bosonic) and single-particle (fermionic) degrees of freedom of the odd-mass nucleus.

For a strongly absorbing probe the scattering is dominated by peripheral collisions, which allows one to keep only the leading order term in the expansion of the spherical harmonic around  $\theta = \pi/2$ ,

$$Y_{2\mu}(\hat{r}) = Y_{2\mu}(b) + O(z/r) .$$
 (5)

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The calculation of the nuclear matrix elements to all orders in  $\chi$  is a complicated task. The use of algebraic models to describe the nuclear excitations makes such a calculation feasible.

#### III. THE INTERACTING BOSON-FERMION MODEL

In the IBFM the collective and single-particle quadrupole operators are given by

$$Q_{B,\mu}(r,\xi) = \alpha_1(r) \left[ s^{\dagger} \tilde{d} + d^{\dagger} s \right]_{\mu}^{(2)} + \alpha_2(r) \left[ d^{\dagger} \tilde{d} \right]_{\mu}^{(2)} ,$$
$$Q_{F,\mu}(r,\xi) = \sum_{j \le j'} \alpha_{jj'}(r) \left[ a_j^{\dagger} \tilde{a}_{j'} + (-1)^{j-j'} a_{j'}^{\dagger} \tilde{a}_{j} \right]_{\mu}^{(2)} / (1 + \delta_{jj'}) , \qquad (6)$$

with  $\tilde{d}_{\mu} = (-1)^{2-\mu} d_{-\mu}$  and  $\tilde{a}_{j,\mu} = (-1)^{j-\mu} a_{j,-\mu}$ . Since the quadrupole operators are linear in the generators of the symmetry group of the IBFM,  $G = U_B(6) \otimes U_F(m)$ [with  $m = \sum_{j} (2j+1)$ ], the eikonal transition matrix elements can be interpreted as group elements of G. They are thus a generalization of the Wigner  $\mathcal{D}$  matrices for the rotation group. In general, these representation matrix elements can be calculated exactly (albeit numerically) to all orders in the projectile-nucleus coupling strength, either with or without the peripheral approximation. This holds for any collective nucleus, whether spherical, deformed,  $\gamma$  unstable, or an intermediate situation between them. The general result can be expressed in terms of a five-dimensional integral for the collective part [7] and a contribution from the single-particle part, which is easily obtained for a single nucleon. In the peripheral approximation the expression for the scattering amplitude for scattering from an initial state  $|i\rangle = |\alpha, J, M\rangle$  to a final state  $|f\rangle = |\alpha', J', M'\rangle$  reduces to a one-dimensional integral over the impact parameter:

$$A_{fi}(\vec{q}) = \frac{k}{i} i^{M-M'} \int_{0}^{\infty} bdb \ J_{M-M'}(qb) \left[ e^{i\chi_{opt}(b)} \sum_{M''} \mathcal{D}_{M'M''}^{(J')}(\hat{q}) \right] \\ \times \langle \alpha', J', M''| e^{i[\chi_{B}(b,\xi) + \chi_{F}(b,\xi)]} |\alpha, J, M'' \rangle \mathcal{D}_{M''M}^{(J)}(-\hat{q}) - \delta_{fi} \right].$$
(7)

The projectile distorted wave is given by

$$\chi_{\rm opt}(b) = \frac{2\pi f}{k} \int_{-\infty}^{\infty} dz \ \rho(r) \ , \tag{8}$$

and the boson and fermion eikonal phases by

$$\chi_B(b,\xi) = g_1(b) \left[ s^{\dagger} \tilde{d} + d^{\dagger} s \right]_0^{(2)} + g_2(b) \left[ d^{\dagger} \tilde{d} \right]_0^{(2)} ,$$
  
$$\chi_F(b,\xi) = \sum_{j \le j'} g_{jj'}(b) \left[ a_j^{\dagger} \tilde{a}_{j'} + (-1)^{j-j'} a_{j'}^{\dagger} \tilde{a}_j \right]_0^{(2)} / (1 + \delta_{jj'}) , \qquad (9)$$

with the eikonal profile functions

$$g_{1}(b) = \frac{2\pi f}{k} \int_{-\infty}^{\infty} dz \, \alpha_{1}(r) \sqrt{5/4\pi} ,$$

$$g_{2}(b) = \frac{2\pi f}{k} \int_{-\infty}^{\infty} dz \, \alpha_{2}(r) \sqrt{5/4\pi} ,$$

$$g_{jj'}(b) = \frac{2\pi f}{k} \int_{-\infty}^{\infty} dz \, \alpha_{jj'}(r) \sqrt{5/4\pi} .$$
(10)

Hence the only representation matrix elements that are needed are those that depend on the z component of the quadrupole operator. Without the peripheral approximation the other components have to be included as well. In the special case of a dynamical symmetry the matrix

elements appearing in Eq. (7) can be derived in closed form. In [3] the results for each of the dynamical symmetries of the IBM for even-even nuclei were analyzed. Here we present the first such study for odd-mass nuclei.

One of the best examples of dynamical symmetries in odd-even nuclei is provided by the low lying negative parity states of <sup>195</sup>Pt, which have been analyzed successfully in terms of the U(6)  $\otimes$  SU(2)  $\supset$  SO(6)  $\otimes$  SU(2) limit of the IBFM [6]. The odd neutron in <sup>195</sup>Pt occupies the  $3p_{1/2}$ ,  $3p_{3/2}$ , and  $2f_{5/2}$  shell model orbits with n = 5, which are treated in a pseudospin coupling scheme as the  $3\bar{s}_{1/2}$ ,  $2\bar{d}_{3/2}$ , and  $2\bar{d}_{5/2}$  pseudo-orbits with  $\bar{n} = n - 1 = 4$ . In this limit the quadrupole operators are

$$\hat{Q}_{B,\mu} = [s^{\dagger}\tilde{d} + d^{\dagger}s]_{\mu}^{(2)} ,$$

$$\hat{Q}_{F,\mu} = -\sqrt{4/5} [a_{1/2}^{\dagger}\tilde{a}_{3/2} - a_{1/2}^{\dagger}\tilde{a}_{3/2}]_{\mu}^{(2)} -\sqrt{6/5} [a_{1/2}^{\dagger}\tilde{a}_{5/2} + a_{1/2}^{\dagger}\tilde{a}_{5/2}]_{\mu}^{(2)} .$$
(11)

Therefore there are only two independent eikonal profile functions, one for the collective part

$$g_1(b) = \epsilon_B(b) ,$$
  

$$g_2(b) = 0 ,$$
(12)

and one for the single-particle part

$$g_{1/2,3/2} = -\sqrt{4/5} \epsilon_F(b) ,$$
  

$$g_{1/2,5/2} = -\sqrt{6/5} \epsilon_F(b) .$$
(13)

The latter values are a consequence of the pseudospin coupling scheme for the single-particle orbits [6].

The classification scheme and the structure of the wave functions are discussed in detail in [6]. The eigenstates are labeled by  $|[N_1, N_2], (\sigma_1, \sigma_2, \sigma_3), (\tau_1, \tau_2), L, J^P\rangle$ . Here L denotes the pseudo-orbital angular momentum, which is a combination of the core angular momentum R and the pseudo-orbital part of the single-particle angular momenta, l. In this classification scheme the states occur in pseudospin doublets with total angular momentum  $J = L \pm 1/2$  for L > 0 or in singlets with J = 1/2 for L = 0. Some low lying excitations are listed in Table I together with their B(E2) values to the ground state,  $|[N + 1, 0], (N + 1, 0, 0), (0, 0), 0, 1/2^-\rangle$ .

The eikonal transition matrix elements can be obtained by expanding the wave functions of the initial and final states into the direct product of collective (boson) and single-particle (fermion) basis states [6]. The boson part of the transition matrix element can be expressed in terms of Gegenbauer polynomials using similar techniques as for proton scattering from even-even nuclei in the SO(6) limit [3]. The fermion part is easy to evaluate for a single uncoupled nucleon. For elastic transitions we find

$$U_{\rm el}(b) = \frac{4!N!}{(N+2)(N+4)!} \left[ (N+2)\cos\epsilon_F C_N^{(3)}(\cos\epsilon_B) - \left[ (N+4)\cos(\epsilon_B - \epsilon_F) - 2 \right] C_{N-1}^{(3)}(\cos\epsilon_B) \right] .$$
(14)

The matrix elements for transitions to excited states can be derived in a similar way. The matrix element to the first excited pseudospin doublet in the ground state band [N+1,0], (N+1,0,0), (1,0), 2 is given by

$$U_{2_{1}}(\epsilon_{B},\epsilon_{F}) = \frac{4!N!}{(N+2)(N+4)!} \sqrt{\frac{1}{5(N+1)(N+5)}} \left\{ (3N+2)(N+5) \, i \sin \epsilon_{F} \, C_{N}^{(3)}(\cos \epsilon_{B}) + 3(N+4)(N+5) \, i \sin(\epsilon_{B}-\epsilon_{F}) \, C_{N-1}^{(3)}(\cos \epsilon_{B}) + 12(N-1) \, i \sin \epsilon_{B} \cos \epsilon_{F} \, C_{N-1}^{(4)}(\cos \epsilon_{B}) + 12i \sin \epsilon_{B} \, [5-(N+4)\cos(\epsilon_{B}-\epsilon_{F})] \, C_{N-2}^{(4)}(\cos \epsilon_{B}) \right\} .$$

$$(15)$$

The first excited state in <sup>195</sup>Pt belongs to the pseudospin doublet characterized by [N, 1], (N, 1, 0), (1, 0), 2. The

TABLE I. B(E2) values leading to the ground state in <sup>195</sup>Pt, calculated with  $e_B = 0.184$  (e b) and  $e_F = -0.257$  (e b). The number of bosons is N = 6.

Initial state	B(E2) Theor.	$J^P \; ({ m keV})$	$B(E2) (e^2 b^2)$	
			Expt. [8]	Calc.
$[7,0],(7,0,0),(1,0),2,J^P$	$(Ne_B + e_F)^2 \frac{N+5}{5(N+1)}$	$3/2^{-}(211)$	0.240(25)	0.225
		$5/2^{-}(239)$	0.210(23)	0.225
$[6,1],(6,1,0),(1,0),2,J^P$	$(e_B - e_F)^2 \frac{2N(N+3)}{5(N+1)(N+2)}$	$3/2^{-}(99)$	0.085(20)	0.075
		$5/2^{-}(130)$	0.066(10)	0.075
$[6,1],(6,1,0),(1,1),1,J^P$	0	$1/2^{-}$		0
		$3/2^{-}(199)$	0.019(5)	0

transition matrix element to this doublet is given by

$$U_{2_{2}}(\epsilon_{B},\epsilon_{F}) = \frac{4!N!}{(N+4)!} \sqrt{\frac{2(N-1)!}{5(N+3)!}} \left\{ N(N-1) i \sin \epsilon_{F} C_{N}^{(3)}(\cos \epsilon_{B}) + (N+4)(N-5) i \sin(\epsilon_{B}-\epsilon_{F}) C_{N-1}^{(3)}(\cos \epsilon_{B}) - 6(N-1) i \sin \epsilon_{B} \cos \epsilon_{F} C_{N-1}^{(4)}(\cos \epsilon_{B}) - 6i \sin \epsilon_{B} \left[ 5 - (N+4) \cos(\epsilon_{B}-\epsilon_{F}) \right] C_{N-2}^{(4)}(\cos \epsilon_{B}) \right\} .$$
(16)

In the peripheral approximation the states with odd values of the (pseudo-orbital) angular momentum L (such as the pseudospin doublet with L = 1 in Table I) cannot be excited,

$$U_1(b) = 0 . (17)$$

For  $\epsilon_B = \epsilon_F = \epsilon$  the quadrupole operator becomes a generator of the SO(6) group and can therefore connect only states belonging to the same SO(6) representation. Using a recurrence relation for the Gegenbauer polynomials

$$nC_n^{(p)}(x) = 2p\left[xC_{n-1}^{(p+1)}(x) - C_{n-2}^{(p+1)}(x)\right] , \qquad (18)$$

it is easy to show that in this case the above transition matrices reduce to the expressions derived for proton scattering from even-even nuclei with SO(6) symmetry [3]:

$$U_{\rm el}(b) = \frac{3!(N+1)!}{(N+4)!} C_{N+1}^{(2)}(\cos \epsilon) ,$$
  

$$U_{2_1}(b) = \frac{4!N!}{(N+4)!} \sqrt{\frac{5(N+1)}{N+5}} (i\sin \epsilon) C_N^{(3)}(\cos \epsilon) ,$$
  

$$U_{2_2}(b) = 0 .$$
(19)

# IV. APPLICATION TO <sup>195</sup>PT

The mass region of the Pt isotopes is known as a complex transitional region of the nuclear mass table between deformed and  $\gamma$ -unstable nuclei. Nevertheless, some of the best examples of dynamical symmetries of the IBM in even-even nuclei (<sup>194,196</sup>Pt) and of the IBFM in oddmass nuclei (<sup>195</sup>Pt) are found in this mass region. In [9] the even-even Pt nuclei were studied in proton scattering. Here we present the first results of calculations for the scattering of 800 MeV protons from <sup>195</sup>Pt.

The low lying negative parity states of  $^{195}$ Pt show a very small splitting between states belonging to the same pseudospin doublet (too small to be resolved experimentally in proton scattering) (see Table I). Therefore we calculate the differential cross section (DCS) for a given pseudospin doublet which is summed over the contributions of the individual states. Under the assumption of a pseudospin coupling scheme for the single-particle orbits, the angular distributions for the excitation of the individual states of a pseudospin multiplet characterized by L are identical up to a statistical factor

$$\frac{d\sigma(0,1/2 \to L,J|q)}{d\Omega} = \frac{2J+1}{2(2L+1)} \frac{d\sigma(0 \to L|q)}{d\Omega} .$$
 (20)

Hence the summed DCS shows the same dependence

on momentum transfer as the individual contributions. However, if the pseudospin assumption is broken significantly, the predicted narrow oscillations are likely to be washed out.

In Fig. 1 we show the DCS for the scattering of 800 MeV protons from <sup>195</sup>Pt which, as mentioned before, is the best known example of an odd-mass nucleus with  $SO(6) \otimes SU(2)$  symmetry. The three curves represent elastic scattering and the excitation of two low lying pseudospin doublets with L = 2 (see Table I). In the calculations we assume a Woods-Saxon form for the nuclear density with a nuclear radius of  $1.2A^{1/3} = 6.96$  fm and a diffusivity of 0.75 fm, normalized to the total number of nucleons A = 195. For the collective transition density we use the derivative of the nuclear density (Tassie form) and for the single-particle transition density a product of radial wave functions for the pseudo-oscillator orbits. We note that in the pseudospin coupling scheme there is a single transition density for the fermion quadrupole operator of Eq. (11). The transition densities are normalized to the B(E2) values in Table I. The forward proton-nucleon scattering amplitude is  $f = ik\sigma/4\pi$ , in which the isospin averaged proton-nucleon cross section was taken as  $\sigma = 46(1 + 0.38i)$  mb [3].

The DCS's of Fig. 1 incorporate the effects of the interplay of the coupling to the collective and the singleparticle degrees of freedom in the target nucleus. In Figs. 2-4 we gauge these effects by comparing the results of the full calculation (solid lines) with those of a calculation in which the coupling to the single-particle degrees

FIG. 1. Differential cross sections in mb/sr for elastic scattering (solid line) and the excitation of the pseudo-orbital doublets with [7,0], (7,0,0), (1,0), L = 2, J (dashed line) and [6,1], (6,1,0), (1,0), L = 2, J (dotted line) in <sup>195</sup>Pt by 800 MeV protons.





FIG. 2. Elastic differential cross section in mb/sr calculated with and without the coupling to the single-particle degrees of freedom (solid and dashed lines, respectively).

of freedom is turned off (dashed lines). The elastic scattering (L = 0) is completely determined by the collective part. Whereas the excitation of the L = 2 doublet which belongs to the ground band [7,0], (7,0,0) is still largely determined by the collective part, for the excitation of the L = 2 doublet with [6,1], (6,1,0) we predict a large contribution from the single-particle part as well. Note that the scale is logarithmic, so there is almost a factor of 2 difference between the two curves in Fig. 4.

### V. SUMMARY AND CONCLUSIONS

In this paper, we have presented an extension of the algebraic-eikonal approach to medium and high energy proton scattering from odd-mass nuclei described by the



FIG. 3. Differential cross section in mb/sr for the excitation of the pseudo-orbital doublet with [7,0], (7,0,0), (1,0), L = 2, J calculated with and without the coupling to the single-particle degrees of freedom (solid and dashed lines, respectively).



FIG. 4. Differential cross section in mb/sr for the excitation of the pseudo-orbital doublet with [6,1], (6,1,0), (1,0), L = 2, J calculated with and without the coupling to the single-particle degrees of freedom (solid and dashed lines, respectively).

interacting boson-fermion model. The algebraic structure of the IBFM makes it possible to calculate the eikonal transition matrix elements exactly to all orders in the projectile-nucleus coupling strength. This holds for any type of collective nucleus, whether spherical, deformed,  $\gamma$  unstable, or any intermediate situation between them.

In the special case of a dynamical symmetry all transition matrix elements of interest can be obtained in closed form. We have discussed in particular an application to the negative parity states in <sup>195</sup>Pt, which are described in terms of the  $SO(6) \otimes SU(2)$  limit of the IBFM. The analytic expressions for the eikonal transition matrix elements allow the study of various effects in a straightforward way. As an example, we showed that, whereas the excitation of the states belonging to the [N+1,0], (N+1,0,0) ground band is largely dominated by coupling to the collective degrees of freedom, the excitation of the [N, 1], (N, 1, 0) sideband depends sensitively on the interplay between the coupling to the singleparticle and collective degrees of freedom. It would be of interest to experimentally test the pseudospin symmetry assumption discussed in this report through proton scattering from <sup>195</sup>Pt.

Finally, we remark that the formalism presented here can be used to derive closed expressions for the transition matrix elements for other dynamical symmetries of the IBFM, and can be extended to odd-odd nuclei as well.

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