Saturation properties and incompressibility of nuclear matter: A consistent determination from nuclear masses

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Starting with a two-body effective nucleon-nucleon interaction, it is shown that the infinite nuclear matter model of atomic nuclei is more appropriate than the conventional Bethe-Weizsacker-like mass formulas to extract saturation properties of nuclear matter from nuclear masses. In particular, the saturation density thus obtained agrees with that of electron scattering data and the Hartree-Fock calculations. For the first time using nuclear mass formula, the radius constant r_0=1.138 fm and binding energy per nucleon $a_v = -16.11$ MeV, corresponding to the infinite nuclear matter, are consistently obtained from the same source. An important off-shoot of this study is the determination of nuclear matter incompressibility K_∞ to be 288 ± 28 MeV using the same source of nuclear masses as input.

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I. INTRODUCTION

The binding energy, saturation density, and compression modulus of infinite nuclear matter are fundamental constants of nature. Traditionally, the first two quantities, termed saturation properties, are determined from two different sources, namely, the volume coefficient a_v of the Bethe-Weizsacker- (BW-) like mass formulas and the electron scattering data on heavy nuclei, respectively. Although the Coulomb coefficient $a_C(= 0.6e^2/r_0)$ in BW-like mass formulas specifies the density $\rho = 3/(4\pi r_0^3)$, it is not accepted as the density of nuclear matter. This is because the corresponding radius constant $r_0 \simeq 1.22$ fm obtained [1, 2] in a totally free fit is much higher than the value 1.12–1.13 fm obtained from the electron scattering data [3] on heavy nuclei and Hartree-Fock (HF) calculations [4]. As yet no mass formula fit to nuclear masses has yielded a value of r_0 in this range. This is the so-called "r_0 paradox," which has been a subject of investigation [5] over the years by many. Since the two properties are highly interrelated, the above constrained practice of their determination from two different sources has been a serious drawback in our understanding of nuclear dynamics. Coupled to this, the incompressibility of nuclear matter has posed a serious problem with regard to its determination, both theoretically and experimentally.

In this work, we report our attempt to determine all three properties of nuclear matter using a single model and one kind of experimental data, namely, the nuclear masses, which are abundant in nature and are the best known properties of nuclei. We use the infinite nuclear matter (INM) model [6] based on the generalized Hugenholtz–Van Hove (HVH) theorem [7] of many-body theory, whose success has been well tested through its unique ability to predict masses of nuclei far from stability [6], masses of Na isotopes and other light nuclei, and finally through the 1986–1987 mass predictions [8] of the entire periodic table. In the formulation of the INM model, it was claimed [6] that this model is more suitable than the traditional (BW) ones to extract the properties of nuclear matter, as it is exclusively built in terms of infinite nuclear matter at the ground state. In the present work, we have improved the model and show conclusively, starting from a two-body effective interaction within the energy density formalism, that the saturation properties derived through this improved model are closer to the true properties of nuclear matter than those derived using BW-model-based mass formulas. Then this model is fitted to the experimental masses, which yields a value of r_0 of 1.138 fm, in close agreement with that obtained from the electron scattering data and with the empirical value found through many-body mean-field approaches [9]. Further, using these saturation properties determined from the same set of data on nuclear masses, we arrive at a value of about 288 MeV for the incompressibility K_∞, which is of equal fundamental importance in the realm of nuclear physics and astrophysics.

In Sec. II, the improvements we have made in the INM model are presented. In Sec. III, we show at a microscopic level that the improved INM model is more appropriate than the BW-like model for the extraction of saturation properties of infinite nuclear matter from the nuclear masses. Determination of such properties is presented in Sec. IV. In Sec. V, the value of the nuclear compression modulus is determined from the nuclear masses. Finally, we conclude in Sec. VI.

II. IMPROVED INM MODEL

We recall here the essential features of the INM model [6] which we have now improved. In this model, the ground-state energy $E^F(A,Z)$ of a nucleus (A,N,Z) with asymmetry β is considered equivalent to the energy E^S of a perfect sphere made up of infinite nuclear matter at a ground-state density with the same asymmetry β plus the residual energy η, called the local energy, which contains all the characteristic contributions like shell, deformation, etc. So

$$E^F(A,Z) = E^S_{\rm INM}(A,Z) + \eta(A,Z), \tag{1}$$

with $E^S_{\rm INM}(A,Z) = E(A,Z) + f(A,Z)$, where

$$f(A,Z) = a_s^I A^{2/3} + a_C^I\{Z^2 - 5[3/(16\pi)]^{2/3}Z^{4/3}\}A^{-1/3} + a_{\rm ss}^I A^{2/3}\beta^2 + a_{\rm cv}^I A^{1/3} - \delta(A,Z) \tag{2}$$

denotes finite-size effects and $E(A,Z)$ is the energy of the infinite part. The superscript I refers to the INM character of the coefficients. Here a_s^I, a_C^I, $a_{\rm ss}^I$, and $a_{\rm cv}^I$ are the surface, Coulomb, surface-symmetry, and curvature coefficients. $\delta(A,Z)$ is the usual pairing term, given as

$$\begin{aligned}\delta(A,Z) &= +\Delta A^{-1/2} \quad \text{for even-even nuclei}\\ &= 0 \quad \text{for odd-}A\text{ nuclei}\\ &= -\Delta A^{-1/2} \quad \text{for odd-odd nuclei.}\end{aligned}$$

Equation (1) now becomes

$$E^F(A,Z) = E(A,Z) + f(A,Z) + \eta(A,Z). \tag{3}$$

Thus, the energy of a finite nucleus is given as a sum of three distinct parts: an infinite part $E(A,Z)$, a finite-size component $f(A,Z)$, and a local energy part $\eta(A,Z)$. The term $E(A,Z)$, being the property of infinite nuclear matter at the ground state, will satisfy the generalized HVH theorem [7]

$$E/A = [(1+\beta)\epsilon_n + (1-\beta)\epsilon_p]/2, \tag{4}$$

where $\epsilon_n = (\partial E/\partial N)_Z$ and $\epsilon_p = (\partial E/\partial Z)_N$ are the neutron and proton Fermi energies, respectively. Using Eq. (3), the INM Fermi energies ϵ_n and ϵ_p can be expressed in terms of their counterparts for finite nuclei as

$$\epsilon_n = \epsilon_n^F - (\partial f/\partial N)\mid_Z - (\partial\eta/\partial N)\mid_Z, \quad \epsilon_p = \epsilon_p^F - (\partial f/\partial Z)\mid_N - (\partial\eta/\partial Z)\mid_N, \tag{5}$$

where $\epsilon_n^F = (\partial E^F/\partial N)_Z$ and $\epsilon_p^F = (\partial E^F/\partial Z)_N$. Using (3) and (5), Eq. (4) is rewritten as

$$E^F/A = [(1+\beta)\epsilon_n^F + (1-\beta)\epsilon_p^F]/2 + S(A,Z), \tag{6}$$

where $S(A,Z) = f/A - (N/A)(\partial f/\partial N)_Z - (Z/A)(\partial f/\partial Z)_N$ is a function of all the finite-size terms a_s^I, a_C^I, $a_{\rm ss}^I$, and $a_{\rm cv}^I$, which are global in nature. As discussed earlier [6], the η terms in Eq. (6) drop out, which plays a crucial role in the success of the INM model, and whose validity has been amply demonstrated [6, 8]. It must be noted that Eq. (6) does not contain the infinite part E as well as the η terms. Thus, through Eq. (6), the decoupling of the finite component f from the infinite one E has been acheived. The coefficients a_s^I, a_C^I, $a_{\rm ss}^I$, and $a_{\rm cv}^I$ can be determined by fitting the $S(A,Z)$ function with the combination of data $E^F/A - [(1+\beta)\epsilon_n^F + (1-\beta)\epsilon_p^F]/2$ obtained from the nuclear masses. We would like to mention here that in the earlier work [6], due to the use of the expressions for Fermi energies, $\epsilon_n^F = E^F(N,Z) - E^F(N-1,Z)$ and $\epsilon_p^F = E^F(N,Z) - E^F(N,Z-1)$, a small contribution $a_a(\beta^2-1)/(A-1)$ survives (of the order of a_a/A) in Eq. (6), whereas in the present work by using the better formulas

$$\begin{aligned}\epsilon_n^F &= \left.\frac{\partial E^F}{\partial N}\right|_Z = \frac{1}{2}\left[E^F(A+1,Z) - E^F(A-1,Z)\right],\\ \epsilon_p^F &= \left.\frac{\partial E^F}{\partial Z}\right|_N = \frac{1}{2}[E^F(A+1,Z+1)\\ &\qquad -E^F(A-1,Z-1)],\end{aligned} \tag{7}$$

the following important improvements are acheived.

(i) The decoupling of the infinite part (asymmetry term) from the finite part in Eq. (6) occurs up to an order of a_a/A^2, which can be considered perfect at the numerical level.

(ii) The pairing term δ contained in f effectively drops out in Eq. (6), thereby rendering the determination of other coefficients with greater accuracy due to less correlation.

(iii) The exchange Coulomb term of the standard form $O(Z^{4/3}A^{-1/3})$ [Eq. (2)] exactly cancels in Eq. (6). This cancellation gives rise to a more reliable determination of the INM saturation density through a_C^I.

(iv) The other factors which might affect the determination of the density such as the proton-form factor $[O(Z^2/A)]$ and Nolen-Schiffer anomaly $[O(\beta A)$, referred to as the charge-asymmetry energy] also cancel exactly.

Thus all the finite-size coefficients contained in $S(A,Z)$, which are global in nature, are determined from nuclear masses by a fit to Eq. (6). Now, of the three distinct parts of the energy E^F of a finite nucleus (3), the infinite part E and the local energy part η remain to be determined. The infinite part E must satisfy the generalized HVH theorem (4), whose solution is of the form

$$E = -a_v^I A + a_a^I \beta^2 A, \tag{8}$$

where a_v^I and a_a^I are the global parameters which can be identified as volume and symmetry coefficients corresponding to INM. Using (5) and (8) in the right and left hand sides of Eq. (4), respectively, one obtains

$$-a_v^I + a_a^I\beta^2 = \frac{1}{2}\left[(1+\beta)\epsilon_n^F + (1-\beta)\epsilon_p^F\right] - \left[\frac{N}{A}\frac{\partial f}{\partial N}\bigg|_Z + \frac{Z}{A}\frac{\partial f}{\partial Z}\bigg|_N\right], \tag{9}$$

where the contribution from the local energy part (of the order of η/A) is neglected, which in the limit of large A goes to zero. Since f is known from Eq. (6), the above equation can be used to determine the two parameters a_v^I and a_a^I with the combination of data $\left[(1+\beta)\epsilon_n^F + (1-\beta)\epsilon_p^F\right]/2$ obtained from nuclear masses. Thus all the global parameters are determined essentially in two fits: Equation (6) determines the finite-size coefficients like a_s^I, a_C^I, etc., and Eq. (9) determines the INM coefficients a_v^I and a_a^I. Since the present study is intended for the determination of the properties of nuclear matter, we do not discuss the determination of η and consequently the masses, the details of which can be seen in Refs. [6, 8].

III. IMPROVED INM MODEL VERSUS BW MODEL

In this section, we would like to make a comparative study of the improved INM model and BW model, in regard to their suitability for the determination of saturation properties of infinite nuclear matter from nuclear masses.

As noted in the Introduction, it has not been possible to determine both the energy and density of infinite nuclear matter in the BW model based mass formulas. Further, it has been hoped only that the volume coefficient determined in the BW model corresponds to nuclear matter at the ground state. On the other hand, in the INM model, this fact has been ensured by the explicit use of the HVH theorem, which is valid only at the ground state of infinite nuclear matter. Since in this model the binding energy of a nucleus is written in terms of the properties of INM, it is expected that the INM model is well equipped to extract infinite nuclear matter properties from nuclear masses. We demonstrate this by predicting the *a priori* known INM properties for a given effective interaction.

In this regard, we make use of the extended Thomas-Fermi (ETF) calculation [10] of nuclear binding energies with Skyrme-like forces, which over the years has been firmly established. In such calculations, one obtains the smooth part of the energy corresponding to the liquid-drop nature of nuclei. This smooth part, hereafter referred to as the macroscopic part, describes the energy $E_{\rm INM}^S$ of the INM sphere as defined in Eq. (1). For the purpose of making a comparative study of the INM model and the BW-like model, the appropriate BW mass formula is

$$E_{\rm BW} = -a_v^L A + a_s^L A^{2/3} + a_C^L\left[Z^2 - 5\left(\frac{3}{16\pi}\right)^{2/3} Z^{4/3}\right]A^{-1/3} + a_a^L\beta^2 A + a_{\rm ss}^L A^{2/3}\beta^2 + a_{\rm cv}^L A^{1/3} - \delta(A,Z). \tag{10}$$

In the case of the ETF calculations, the nuclear curvature coefficient comes out to be about 10 MeV as against the BW-like mass formula fit to real nuclei, which gives a value close to zero. For this reason, we have included higher-order terms like curvature and surface-symmetry terms in both the INM and BW models.

The macroscopic or the ETF nuclear ground-state energies used here for the comparative study of the INM and BW models are taken from the calculations of Aboussir *et al.* [11]. In their calculation, they used a generalized Skyrme force SkSC4 of the form

$$v_{ij} = t_0[1 + x_0P_\sigma\delta(\vec{r}_{ij})] + t_1(1+x_1P_\sigma)[p_{ij}^2\delta(\vec{r}_{ij})^2 + \text{H.c.}]/2\hbar^2 + t_2(1+x_2P_\sigma)\vec{p}_{ij}\cdot\delta(\vec{r}_{ij})\vec{p}_{ij}/\hbar^2$$
$$+(t_3/6)(1+x_3P_\sigma)[\rho_{qi}(\vec{r}_i) + \rho_{qj}(\vec{r}_j)]^\gamma\delta(\vec{r}_{ij}) + (i/\hbar^2)\; W_0(\vec{\sigma}_i + \vec{\sigma}_j)\cdot\vec{p}_{ij}\times\delta(\vec{r}_{ij})\vec{p}_{ij}.$$

Then, the macroscopic part of the total energy for a given nucleus is calculated using the energy density formalism, i.e., $E = \int \mathcal{E}(\vec{r})d^3r$, where $\mathcal{E} = \tau(\vec{r}) + v(\vec{r})$. The potential energy density v is derived using the two-body force given above. For the kinetic part τ, they use the full fourth-order ETF kinetic functional [10]. It must be noted that realistic nuclear ground-state energies contain shell effects. To incorporate this characteristic feature in a self-consistent way, a Hartree-Fock calculation is performed for the same generalized Skyrme force. Now, using the single-particle states obtained within the HF approximation, the shell corrections can be calculated by directly making use of the Strutinsky procedure. Then, the total energy is given as the sum of the macroscopic part and these shell corrections.

We have made an exhaustive study using those macroscopic energies provided by them for 1492 nuclei. We fitted both the INM formulas given by (6) and (9) and the BW one by (10) to the above macroscopic part of the nuclear masses to determine the corresponding global parameters. The results so obtained are given in Table I. The values obtained directly by Aboussir *et al.* with the SkSC4 force performing nuclear matter and semi-infinite nuclear matter calculations for the various coefficients (hereafter referred to as exact values) are also presented in Table I. It is gratifying to find that the values obtained in the INM fit for the principal coefficients like a_v^I, a_s^I, and a_C^I agree better with the exact values, compared to that of the BW fit. The symmetry coefficient a_β^I agrees reasonably well with the exact value, although it is somewhat inferior to the BW value. Even though the agreement of the higher-order terms like surface-symmetry and curvature in the BW fit agree better, it must be noted that, because of correlations among the coefficients, they significantly affect the principal term like surface and to a lesser extent the other ones also. In case of the INM fit, since the infinite and finite parts are determined in two separate fits, the principal coefficients are not influenced by the higher-order terms. In any case, these two coefficients contribute insignificantly in real nuclei, and are normally ignored. Thus, the saturation properties of nuclear matter, which are *a priori* known for a given effective interaction like SkSC4, are relatively well reproduced by the INM model than the BW-like model. This gives us more confidence in the INM model in extracting real saturation properties from experimental nuclear masses which is done in the next section.

The success of the INM model over that of the BW-like model is essentially due to the following. As also discussed in Ref. [6], the BW-like mass formulas use only the average property of nuclear matter, namely, the average energy per nucleon. However, as demonstrated by Hugenholtz and Van Hove [12], an interacting Fermi system has an additional property, namely, the single-particle property. Such a system has one true single-particle state, i.e., the Fermi state, which has infinite lifetime, while other low-lying ones are metastable. In other words, the lifetime of the single-partcle state approaches infinity in the limit $k \longrightarrow k_F$. This important property is additionally taken into account in the INM model, which is not present in BW-like mass formulas.

TABLE I. Values obtained for the global parameters [Eqs. (6),(9),(10)] in the infinite nuclear matter (INM) model and Bethe-Weizsacker (BW) mass formula fit using the macroscopic part of nuclear energies (see text). Exact values determined directly using INM and semi-INM calculations are also given. All quantities are in MeV.

Parameters	Exact values	INM	BW
a_v	15.87	15.925	14.769
a_C	0.757	0.7360	0.6945
a_s	17.3	18.10	11.15
a_a	27.0	29.80	25.41
a_{ss}	-16.0	-31.37	-17.77
a_{cv}	11.1	5.06	16.43

IV. DETERMINATION OF NUCLEAR MATTER SATURATION PROPERTIES

Before coming to the actual determination of the various parameters of the INM model, and thereby the saturation properties of nuclear matter, it is essential to assess the relative importance of the possible higher-order terms, which is somewhat different in this model.

The two saturation properties, namely, the density ρ_∞ given by a_C^I and the volume energy a_v^I, are determined in two different fits: Eqs. (6) and (9) respectively. The first fit determines the crucial quantity a_C^I, and hence, it is imperative that we analyze the role of other finite-size effects which may influence the determination of the saturation properties. The finite-size terms which are directly related to the Coulomb effect are the exchange Coulomb, proton-form-factor correction, and charge-asymmetry energy. It may be recalled here that in the INM model the binding energies and Fermi energies are used in the particular combination $E^F/A = [(1+\beta)\epsilon_n^F + (1-\beta)\epsilon_p^F]/2$, in Eq. (6), as dictated by the HVH theorem. As a result, the above stated three effects exactly cancel in Eq. (6), rendering a clean determination of a_C^I and hence the density ρ_∞. This is indeed a very fortunate situation.

The other two higher-order terms which may indirectly affect the value of a_C^I are the curvature $a_{\rm cv}^I$ and surface-symmetry $a_{\rm ss}^I$ coefficients. In real nuclei, the curvature coefficient comes out to be nearly zero, and is normally not included. So we have dropped it. In regard to the surface-symmetry coefficient $a_{\rm ss}^I$, it has been recognized that its values is somewhat difficult to determine from nuclear masses. Even at the theoretical level, the values of $a_{\rm ss}^I$ determined [10] from various effective interactions differ widely. Further, in the modern BW-like mass formulas [13, 14], this coefficient is fixed from considerations other than the ground-state nuclear masses, such as fission barrier heights. Since in the present study we address ourselves to the determination of the properties of INM at the ground state, it is essential that we only use the ground state masses, and not any other property which may drift the system from the ground-state and jeopardize the determination of a_v^I and a_C^I. Therefore, in the present context, it is proper that the important coefficients are treated as free parameters to be fixed by nuclear masses through Eq. (6).

But, fortunately, the surface-symmetry coefficient $a_{\rm ss}^I$ cancels to a major extent ($\sim$ 66%) due to the particular combination of data used in Eq. (6). Although this does not fully cancel like the exchange Coulomb, protron-form-factor correction terms, etc., $a_{\rm ss}^I$ being a second-order term, such cancellation renders it relatively

TABLE II. Values obtained for the global parameters [Eqs. (6),(9)] in the present study using the experimental data [15] on nuclear masses are given for the various sets of data. All quantities are in MeV.

No. of nuclei	a_v^I	a_C^I	a_s^I	a_a^I
1085	16.101	0.7592	19.18	24.65
1191	16.115	0.7596	19.25	24.56
1252	16.112	0.7589	19.23	24.66
1294	16.096	0.7572	19.23	24.32
1371	16.108	0.7593	19.27	24.06

insignificant as compared to a_s^I. At the numerical level, it may be considered to be virtually canceled. Nevertheless, since our main goal is to determine the saturation properties of nuclear matter, which are of fundamental importance, we are anxious to check if any semblance of the survival of the a_{ss}^I term can affect the results.

Hence, we carried out calculations retaining this term as a free parameter in our fit to Eq. (6). It is found that while the values of the other coefficients remain almost unaltered, the value of a_{ss}^I widely varies from -30 MeV to -11 MeV with an accompanied error of about 50–100% as the number of data varies from 1085 to 1371. This fact is also true when one uses presupposed values for a_{ss}^I, while other coefficients are being fitted to the 1371 masses. As the value of a_{ss}^I is varied from -10 MeV to -30 MeV, it was found that $\chi_{\rm rms}$ shows a minimum at $a_{ss}^I \sim -12$ MeV. However, this optimum value of a_{ss}^I fluctuates with the variation of the number of data, resulting in no definite value. These two features are reminiscent of its insignificant presence in Eq. (6). Hence we have omitted this term. The same is true also for the curvature term. Therefore, the optimum representation for the finite-size function $f(A)$ defined in Eq. (1) is $f(A,Z) = a_s^I A^{2/3} + a_C^I\{Z^2 - 5[3/(16\pi)]^{2/3} Z^{4/3}\}A^{-1/3}$.

Now coming to the actual determination of the saturation properties of INM, we use all the nuclear masses with experimental error ≤ 60 keV from the recent mass table of Wapstra and Audi [15]. There are 1371 cases, which have been used in our study. As mentioned earlier the universal parameters in this model are determined in a two-step process. In the first step, we determine the finite-size coefficients a_C^I and a_s^I by making a least-squares fit to Eq. (6) using all 1371 masses. Then these parameters so determined are further used in the second step to obtain the coefficients corresponding to the infinite parts a_v^I and a_β^I, by a fit to Eq. (9) using the same set of data. The $\chi_{\rm rms}$'s obtained for these two fits are 371 keV and 372 keV, respectively, which are substantially lower than the corresponding ones 460 keV and 506 keV obtained in the earlier study [6, 8]. The lowering of $\chi_{\rm rms}$ is almost entirely due to the improvements made in the model, and not as the result of the use of recent masses. To check the goodness and the stability of the parameters obtained in our fits, we have carried out five sets of calculations by varying the number of data randomly considered throughout the mass table, choosing them on the basis of experimental error ranging from 20 keV to 60 keV, and these are presented in Table II. One can clearly see that almost all four parameters are quite stable in spite of widely varying data. Especially remarkable is the stability of the two crucial nuclear parameters, namely, the Coulomb coefficient a_C^I and the volume coefficient a_v^I. It is satisfying to note that the degree of stability of these two important parameters, in which we are specifically interested, is relatively better than in a_s^I and a_β^I. The final values obtained for these two coefficients with a maximum number of data (1371 nuclei), and the corresponding values for r_0 and ρ_∞ are

$$a_v^I = 16.108 \text{ MeV} \quad \text{and} \quad a_C^I = 0.7593 \text{ MeV},$$
$$r_0 = 1.138 \text{ fm} \quad \text{and} \quad \rho_\infty = 0.1620 \text{ fm}^{-3}.$$

We quote no errors for our parameters as they are firmly determined, say, with errors less than 1%. The saturation properties $a_v^I = 18.335$ MeV and $a_C^I = 0.841$ MeV, determined earlier [6], are inaccurate due to the use of the expressions $\epsilon_N^F = E^F(N,Z) - E^F(N-1,Z)$ and $\epsilon_P^F = E^F(N,Z) - E^F(N,Z-1)$ for finite nuclei Fermi energies. It is indeed remarkable that the saturation density $\rho_\infty = 0.162$ fm^{-3} and the corresponding $r_0 = 1.138$ fm found here agree quite well with that obtained from the fit of electron scattering data. This value of r_0 is

TABLE III. Values obtained for the parameters [Eq. (10)] of the BW model using the experimental data [15] with a varying number of higher-order terms in the model (see text). All quantities are in MeV.

No. of parameters	a_v^L	a_s^L	a_a^L	a_C^L	Δ	a_{ss}^L	a_{cv}^L	a_{GC}^L
5	-15.80	18.4	23.0	0.733	11.9			
6	-15.64	18.2	26.6	0.713	11.2	-22.3		
7	-15.48	17.2	26.3	0.707	11.2	-21.9	1.5	
8	-12.66	-10.6	21.7	0.651	10.7	-7.9	102.7	-127.3

TABLE IV. Same as Table II, but using a Bethe-Weizsacker-like mass formula. Values obtained for incompressibility K_∞ using those in Eq. (11) together with the values from Table III for the corresponding set of data are given. All quantities are in MeV.

No. of nuclei	a_v^L	a_C^L	K_∞
1085	15.648	0.7142	291
1191	15.640	0.7134	290
1252	15.651	0.7141	297
1294	15.634	0.7128	303
1371	15.635	0.7131	288

also close to the 1.13 fm obtained in the HF studies, which has been widely accepted in the literature [4]. It may be noted that our value of r_0 is quite similar to the value 1.140±0.005 obtained from the fit of nuclear charge radii [9] extracted from the recent electron scattering data [16]. Thus, the two important ground-state properties, i.e., a_v and ρ_∞, which are interrelated, are consistently determined from one kind of data using a single model.

V. INCOMPRESSIBILITY OF NUCLEAR MATTER

To determine K_∞, we note that INM model determines the binding energy per nucleon a_v and saturation density ρ_∞ at the ground state. The BW-model-based mass formulas give the value of a_v^L at a different density ρ_0 corresponding to their a_c^L, since they do not have any ingredient to ensure that these parameters pertain to nuclear matter at the ground state. Hence, using the values of a_v and densities from the INM model as well as from BW formulas, one can determine K_∞ using the relation

$$a_v^I(\rho_\infty) + (K_\infty/18)(\rho_0/\rho_\infty - 1)^2 = a_v^L(\rho_0), \qquad (11)$$

as shown in Ref. [17].

In order to determine the optimum number of parameters in the BW mass formula given by Eq. (10), we have carried out a least-squares fit with a varying number of parameters, the results of which are presented in Table III. We have used the same 1371 nuclear masses mentioned in our earlier section. It can be seen that the values of the principal five coefficients are not affected when the surface-symmetry a_{ss}^L and curvature a_{cv}^L terms are successively included. Hence, the a_{ss}^L term is well supported and should be retained. The curvature term, in spite of its smallness and relatively large error, can be included as it does not affect the leading terms much. However, the inclusion of the Gauss curvature a_{GC}^L term, the next higher-order term in the model, completely destabilizes the fit by violently disturbing the leading-order coefficients. The surface coefficient has even become negative. This may be due to the very small value [13, 14] of this term a_{GC}^L, which is of the order of 6 MeV.

Quite importantly, the above result is contrary to the common belief that the inclusion of more and more higher-order terms in a liquid-drop model like expansion would result in progressively refined values of the leading-order terms. Therefore, one should be judicious in retaining higher-order terms in such models. In the present study, we consider Eq. (10) having six parameters to be the optimum representation, where we have dropped the curvature term as it comes out to be nearly zero.

With this view, we carried out a least-squares fit to Eq. (10) (without the curvature term) using the same 1371 masses. As in the case of INM model calculations, we have varied the number of data to arrive at stable values of a_v^L and a_C^L, with similar accuracies of second and third decimal places, respectively, since the value of K_∞ is sensitive to these values. The results are given in Table IV. Now, K_∞ is computed using these values in Eq. (11) together with the values of a_v^I and a_C^I from Table II for the corresponding set of data, and are presented in the last column of Table IV. It is remarkable that in spite of the variation of the number of input data ranging from 1085 to 1371, the value of K_∞ comes out to be in between 288 and 305. The average value thus obtained in Table IV is about 294 MeV which is very close to 288 MeV obtained with the maximum number of data used in the fitting procedure, which futher substantiates the stability of our result with respect to the variation of data.

TABLE V. Values of K_∞ obtained with inclusion or noninclusion of higher-order effects like curvature a_{cv}^L, proton-form factor (PFF), and exchange Coulomb. INM4 stands for the INM model mass fit with four parameters, namely, a_v^I, a_s^I, a_a^I, and a_C^I. And BW6 stands for the BW model fit with six parameters; the two additional parameters in this case are the pairing Δ and surface-symmetry a_{ss}^L terms.

	Model set	K_∞
With exc. Coul.	INM4 and BW6	288
	INM4 and BW6+a_{cv}^L	302
	INM4+PFF and BW6 + PFF	326
Without exc. Coul.	INM4 and BW6	303
	INM4 and BW6+a_{cv}^L	309
	INM4+ PFF and BW6 + PFF	330

We then attempt to get an estimate of the error in this value of K_∞ arising out of the limitations of the model, which may be due to the inclusion or noninclusion of higher-order terms like curvature, exchange Coulomb, and proton-form factor. The results of our calculation of K_∞ with inclusion or noninclusion of these three effects are presented in Table V. We have calculated the error χ using the expression $\chi^2 = \frac{1}{N}\Sigma_{i=1}^{N}\left(K_\infty^i - K_\infty^{\mathrm{opt}}\right)^2$, where $K_\infty^{\mathrm{opt}} = 288$ MeV, $N = 5$, and i stands for the five values, other than K_∞^{opt}, tabulated in Table V. The error thus calculated comes out to be 28 MeV.

The recent BW-model-based mass formulas usually use a presupposed value of r_0 determined from other considerations. The one which treats r_0 as an adjustable parameter and more or less looks similar to Eq. (10) is by Myers and Swiatecki [1], where r_0 is determined by using the data on both the nuclear masses and fission barriers. Using their values of a_v and the density, and the present values of INM, we obtain K_∞ to be about 299 MeV. Hence, we would like to firmly state that, if one allows r_0 as a free parameter in the fit to nuclear masses, one would invariably arrive at a value of about 288 ± 28 MeV for K_∞.

VI. CONCLUSIONS

In conclusion, we have improved the INM model by using better Fermi energies for the neutron and proton, which has resulted in a cleaner decoupling of the finite-size effects and the INM part of the ground-state energies of nuclei. Unlike in the BW-like mass formulas, the Coulomb-related higher-order terms such as exchange Coulomb, proton-form-factor correction, and charge-asymmetry energy cancel exactly, rendering an accurate determination of the most important quantity, namely, the saturation density. More importantly, we have demonstrated at a fundamental level, starting with an effective two-body interaction, the appropriateness of the INM model over that of the BW-like models to determine the ground-state properties of INM.

The saturation density ρ_∞ and binding energy per nucleon a_v of nuclear matter, the two highly interrelated quantities, are extracted consistently for the first time from a single source, i.e., nuclear masses, through a mass formula. It is particularly satisfying to find that the radius constant corresponding to ρ_∞ determined here agrees quite well with that obtained from electron scattering data and Hartree-Fock calculations. These have been possible because of taking into account additionally the single-particle property of the interacting Fermi system through the use of the generalized HVH theorem in the INM model. Thus, the r_0 anomaly is resolved here satisfactorily.

An important offshoot of this study is the determination of the value of nuclear matter incompressibility starting from nuclear masses, which are the best measured and most abundant data in nuclear physics. The value so obtained for K_∞ is 288±28 MeV. We finally comment that inclusion of the surface-symmetry term a_{ss}^I leads to fluctuation of the value of K_∞ to a larger side of the above value.

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[1] M.D. Myers and W. Swiatecki, Nucl. Phys. **A81**, 1 (1966).
[2] P.A. Seeger and W.M. Howard, Nucl. Phys. **A238**, 491 (1975).
[3] A. deShalit and H. Feshbach, *Nuclear Structure, Theoretical Nuclear Physics*, Vol. I (Wiley, New York, 1974).
[4] J. Decharge and D. Gogny, Phys. Rev. C **21**, 1568 (1980).
[5] J.M. Pearson, Phys. Lett. **91B**, 325 (1980); Nucl. Phys. **A376**, 501 (1982); F. Tondeur, J.M. Pearson, and M. Farine, *ibid.* **A394**, 462 (1983).
[6] L. Satpathy, J. Phys. G **13**, 761 (1987).
[7] L. Satpathy and R. Nayak, Phys. Rev. Lett. **51**, 1243 (1983).
[8] L. Satpathy and R.C. Nayak, At. Data Nucl. Data Tables **39**, 241 (1988).
[9] F. Buchinger, J.E. Crawford, A.K. Dutta, J.M. Pearson, and F. Tondeur, Phys. Rev. C **49**, 1402 (1994).
[10] M. Brack, C. Guet, and H.B. Hakansson, Phys. Rep. **123**, 275 (1985).
[11] Y. Aboussir, J.M. Pearson, A.K. Dutta, and F. Tondeur, Nucl. Phys. **A549**, 155 (1992).
[12] N.H. Hugenholtz and W. Van Hove, Physica (Utrecht) **24**, 363 (1958).
[13] P. Moller and J.R. Nix, At. Data Nucl. Data Tables **39**, 213 (1988).
[14] P. Moller, W.D. Myers, W.J. Swiatecki, and J. Treiner, At. Data Nucl. Data Tables **39**, 225 (1988).
[15] A.H. Wapstra and G. Audi, Nucl. Phys. **A565**, 1 (1993).
[16] H. de Vries, C.W. de Jager, and C. de Vries, At. Data Nucl. Data Tables **36**, 495 (1987).
[17] L. Satpathy, J. Phys. G **17**, 645 (1991).