

Structure of Li and Be isotopes studied with antisymmetrized molecular dynamics

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Structure of odd-even and even-even isotopes of Li and Be is studied systematically with antisymmetrized molecular dynamics which is a theoretical method free from any model assumptions such as the existence of clustering. The construction of energy-minimum intrinsic states with definite parity is made by the use of the frictional cooling method. Angular momentum projection is applied to these intrinsic states in order to obtain the eigenstates of angular momentum. It is shown that the clustering structure appears in the nuclei in the $N \approx Z$ region with N and Z standing for the neutron and proton numbers, respectively. The clustering structure changes toward the shell-model-like structure as N increases to around $N \approx 8$. Furthermore appearance of new-type clustering features is suggested in the neutron-richer region with $N > 8$. Energy spectra and other quantities are shown to be reproduced well. Especially the calculated magnetic moments describe the observed data quite well, and the neutron number dependence of the observed magnetic moments is explained as the reflection of the structure change of the isotopes from the clustering structure to the shell-model-like structure. It is indicated that the density dependence of the effective interaction is important.

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I. INTRODUCTION

A lot of information about neutron-rich nuclei has been obtained by recent experiments with radioactive nuclear beams [1-4]. Systematic studies in a single theoretical framework are desired for those newly observed properties of neutron-rich nuclei together with ordinary nuclei. We expect that such a study can check the consistency between the theoretical understanding of unfamiliar features of exotic nuclei and that of ordinary nuclei. However, little systematic theoretical research has been carried out on light nuclei.

Most of theoretical studies for neutron-rich nuclei have treated only one nucleus or a few nuclei at most. For example, the three-body model approaches [5] have been applied to ${}^{11}\text{Li}$ by regarding it as a ${}^9\text{Li}+2n$ system and also applied to ${}^6\text{He}$ by regarding it as an $\alpha+2n$ system. Although these models have given realistic descriptions of the neutron-halo structure [6-8] of these nuclei, they are not necessarily useful for other Li and He isotopes. Even in the study of ${}^{11}\text{Li}$, there remains an important problem of treating ${}^9\text{Li}$ core excitation. The variational shell model [9] has been successfully used to study the system of ${}^{11}\text{Be}$ which is also considered to have the neutron-halo structure. It is not clear, however, whether or not this model is useful for other Be isotopes with remarkable cluster structure as seen in ${}^8\text{Be}$.

There are many other neutron-rich nuclei which have been hardly studied yet. In the neutron-rich side of Li and Be isotopes, the existence of cluster structure should be investigated theoretically, since it is already well known that the clustering structure appears in the ordinary $N \approx Z$ region with N and Z standing for the neutron and proton numbers, respectively, as seen in the $t+\alpha$ cluster structure of ${}^7\text{Li}$ and in the $\alpha+\alpha$ cluster structure of ${}^8\text{Be}$ [10-14]. The recent experimental

developments demand the theoretical researches in the neutron-rich region which can answer the existence of cluster structure as well as the existence of the neutron-halo structure in the exotic neutron-rich region systematically.

For most theoretical frameworks for light nuclei, it is not easy to make systematic researches on the isotopes ranging from ordinary nuclei to neutron-rich nuclei. First of all, the applicability of mean-field approaches is not necessarily assured in light nuclei due to the existence of the cluster structure. In fact it is very difficult for the shell model to describe such light nuclei as ${}^7\text{Li}$ and ${}^8\text{Be}$ which are known to have the well-developed cluster structure. The properties of these ordinary light nuclei with the cluster structure have been studied well with the cluster model. In the model the existence of clusters is assumed *a priori*. Therefore it is difficult to apply the cluster model to the study of exotic nuclei for which we have little information or observed data. If a nucleus has shell-model-like structure instead of cluster structure, the cluster model does not provide a good approach. To obtain a systematic understanding of the isotopes covering from ordinary region to neutron-rich exotic region, it is desirable to use a theoretical framework which can describe both shell-model-like states and clustering states. The method of antisymmetrized molecular dynamics (AMD) [15-27] is the very theoretical framework that satisfies these requirements and enables us to study isotopes of light nuclei systematically in *one framework* without any model assumptions.

AMD is a theory which has been developed intending originally to construct a new microscopic simulation method for the study of heavy-ion reactions. Contrary to other microscopic simulation methods for heavy-ion reactions [28,29], AMD is a quantum-mechanical method which describes the time development of the system wave function. The total wave function of AMD is given as a

Slater determinant of single-particle wave functions represented by Gaussian wave packets. AMD describes the reaction process by calculating the time evolution of the $6A$ variables representing the position and momentum centers of the Gaussian wave packets. The initialization of the collision calculation, namely the construction of the ground-state wave functions for the colliding nuclei, demands us to determine these variables for the ground states. It is achieved by the frictional cooling method which is a kind of variational calculation. The method to construct the minimum-energy wave function by the use of the frictional cooling technique in AMD provides a novel and powerful tool for the nuclear structure study because we have no need at all to rely on model assumptions. According to some preliminary studies [15,16], the AMD wave functions obtained by the frictional cooling method can describe well both the shell-model-like structure and the clustering structure. For instance, the AMD wave function of the ^{16}O ground state has proved to be of the double closed-shell configuration, $(0s)^4(0p)^{12}$, and that of the ^8Be ground state has proved to have a well-developed dumbbell structure of $\alpha + \alpha$. Recently AMD was successfully applied to the study of the structure change along the yrast line of ^{20}Ne [27]. In this study the cranked intrinsic state was constructed for each spin-parity state along the yrast line by the use of the frictional cooling method with constraint on the expectation value of the angular momentum. This AMD approach enabled us, for the first time without introducing any model assumptions, to describe the quite dramatic change of structure along the ^{20}Ne yrast line: namely the $\alpha + ^{16}\text{O}$ clustering structure in low-spin region, the $(sd)^4$ shell-model-like structure around the band terminal 8^+ state, and the appearance of $^{12}\text{C} + 2\alpha$ -like structure in 10^+ and 12^+ states.

The purpose of this paper is to perform systematic study on the structure of Li and Be isotopes with AMD. By virtue of AMD, it is expected that we can trace the structure change with the increase of the neutron number N even if the structure changes so drastically between the cluster structure to the shell-model-like structure. Specific problems we expect to elucidate in our systematic study of these isotopes include the following ones. We expect the answer is obtained for the question whether the clustering structure observed in the $N \approx Z$ region persists or fades out as N increases. Another question to be answered is why the magnetic moment of the exotic nucleus ^{11}Li with neutron halo is close to the Schmidt value while that of the ordinary nucleus ^7Li is fairly different from the Schmidt value.

As for the problem about the formation and dissolution of clusters in neutron-rich nuclei, we have pioneering work by Seya, Kohno, and Nagata [30] on Be and B isotopes. In this work of Ref. [30], the molecular orbital model was adopted which described the system as composed of an α - α core and surrounding neutrons (and proton in B isotopes). It was found that the distance between two α 's changed, with the increase of N , from the large values in ^8Be and ^9B to the very small values in ^{12}Be and ^{13}B which have the magic number $N=8$ for neutrons. Furthermore, as a very interesting result, it

was found that the α - α distance increased again as N increased beyond $N = 8$ toward the neutron dripline. A drawback of this pioneering work is that the existence of an α - α core was postulated from the beginning. In our present approach with AMD, we can check whether this postulation is justified or not. In particular, we expect that we can answer how plausible the existence of the clustering feature is in the neutron dripline region.

This paper is organized as follows. In the next section (Sec. II) we explain the formulation of AMD for our present study of nuclear structure. Namely we explain the AMD wave function, the frictional cooling method, and the angular momentum projection. We also describe the extended AMD wave function improved by a superposition of Slater determinants. In Sec. III, the two-nucleon interaction used in this paper is expressed. The calculated results are given and are compared with data in Sec. IV. Energy spectra and other quantities are shown to be well reproduced. Especially the systematic change of the observed magnetic dipole moments are reproduced very well. It will be shown that the density-dependent interaction is often important to explain the observed data. We will show that a superposition of AMD Slater determinants describes the long tail of the wave function in two cases, the neutron halo structure in ^{11}Be and the outer tail of the relative wave function of α - t clusters in ^7Li . In Sec. V we will discuss the systematic change of the intrinsic state. The calculated results show the well-developed cluster structure in the nuclei with $N \sim Z$ and the shell-model-like structure in the nuclei with the neutron magic number $N = 8$. It means that the shell effects of neutron orbits play an important role in the structure of neutron-rich nuclei. The change of the observed magnetic moment from ^7Li to ^{11}Li is explained mainly by the change of the clustering structure of ^7Li to the shell-model-like structure of ^{11}Li . Possible existence of the clustering feature is seen in the neutron-richer nuclei like ^{14}Be . Finally in Sec. VI we give a summary.

II. FORMULATION OF AMD

The AMD (antisymmetrized molecular dynamics) is a theory which is applicable both to nuclear structure problems and to heavy-ion collision problems. Here we only explain the AMD framework for the sake of nuclear structure study. As for the AMD theory for the sake of nuclear reaction study, the reader is referred to Refs. [18] and [19]. Since the AMD framework for the structure study has recently been given also in Ref. [27], our explanation here is brief, keeping a self-contained style.

A. Wave function of AMD

In AMD the wave function of an A -nucleon system $|\Phi\rangle$ is expressed by a Slater determinant,

$$|\Phi(\mathbf{Z})\rangle = \frac{1}{\sqrt{A!}} \det[\varphi_j(i)], \quad \varphi_j = \phi_{\mathbf{z}_j} \chi_{\alpha_j}, \quad (1)$$

where χ_{α_j} is the spin isospin wave function of j th single-particle state, where α_j indicates the spin isospin, $\alpha_j = p \uparrow, p \downarrow, n \uparrow,$ or $n \downarrow$. The spatial wave function of the j th single-particle state $\phi_{\mathbf{Z}_j}$ is represented by a Gaussian wave packet,

$$\begin{aligned} \langle \mathbf{r} | \phi_{\mathbf{Z}_j} \rangle &= \left(\frac{2\nu}{\pi} \right)^{3/4} \exp \left[-\nu \left(\mathbf{r} - \frac{\mathbf{Z}_j}{\sqrt{\nu}} \right)^2 + \frac{1}{2} \mathbf{Z}_j^2 \right], \\ &\propto \exp \left[-\nu (\mathbf{r} - \mathbf{D}_j)^2 + \frac{i}{\hbar} \mathbf{K}_j \cdot \mathbf{r} \right], \\ \mathbf{Z}_j &= \sqrt{\nu} \mathbf{D}_j + \frac{i}{2\hbar\sqrt{\nu}} \mathbf{K}_j, \end{aligned} \quad (2)$$

whose center is expressed with a complex parameter \mathbf{Z}_j , which ν is the width of the wave packet common to all nucleons. Thus the wave function of the system $|\Phi(\mathbf{Z})\rangle$ is parametrized by complex parameters $\{\mathbf{Z}\} = \{\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A\}$.

The wave function $|\Phi(\mathbf{Z})\rangle$ of Eq. (1) is the same as the wave function of the fermionic molecular dynamics proposed by Feldmeier [31]. Furthermore, $|\Phi(\mathbf{Z})\rangle$ can be regarded as a special case of the Brink-type cluster model wave function [32] where every cluster is composed of a single nucleon. When the parameters of the Brink-type wave function are treated as time-dependent parameters by using the time-dependent variational principle, the Brink-type cluster model is called the time-dependent cluster model (TDCM) [33]. Therefore, $|\Phi(\mathbf{Z})\rangle$ can be also regarded as a special case of the TDCM wave function. The reason we use the name AMD for our present approach to nuclear structure problems is mainly because our approach is characteristic in the point that it is always combined with the frictional cooling method which determines the parameters of the wave function often under given constraints. Other characteristic points of our approach are the use of projection of parity and angular momentum and the frequent use of the superposition of Slater determinants. (In the case of heavy-ion collision problems, the reason why we use the name AMD is mainly because our approach is characteristic in the point that the stochastic two-nucleon collisions are treated by introducing the physical nucleon coordinates.)

For the study of nuclear structure, the AMD wave function $|\Phi(\mathbf{Z})\rangle$ explained above is projected to the parity eigenstate,

$$|\Phi^\pm(\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A)\rangle = (1 \pm P) |\Phi(\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A)\rangle, \quad (3)$$

with P standing for the parity inversion operator. Since different parity states of a nucleus usually have different structures, this projection is indispensable to discuss the structure of each parity state. Furthermore we often make a linear combination of two AMD Slater determinants for more precise study of the nuclear structure,

$$\begin{aligned} |\Phi^\pm(\mathbf{Z}_1, \dots, \mathbf{Z}_A, \mathbf{Z}'_1, \dots, \mathbf{Z}'_A, C)\rangle \\ = |\Phi^\pm(\mathbf{Z}_1, \dots, \mathbf{Z}_A)\rangle + C |\Phi^\pm(\mathbf{Z}'_1, \dots, \mathbf{Z}'_A)\rangle. \end{aligned} \quad (4)$$

Note that the number of complex parameters of the wave function of Eq. (4) is $6A + 1$ while that of the original AMD is $3A$. In this paper, for the sake of convenience, we call the wave function of Eq. (3) simply the AMD wave function, and the latter one in Eq. (4) the extended AMD (EAMD) wave function.

B. Frictional cooling method

As mentioned in subsection A, the AMD wave function of a system $|\Phi^\pm(\mathbf{Z})\rangle$ is parametrized by the centers of Gaussian wave packets $\{\mathbf{Z}\} = \{\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A\}$. Let us consider constructing the ground-state wave function of the system. First we choose some initial parameters $\{\mathbf{Z}\}$ of all A nucleons rather arbitrarily. The initial wave function $|\Phi^\pm(\mathbf{Z})\rangle$ with this initial choice of $\{\mathbf{Z}\}$ represents in general a highly excited state. We should determine the parameters $\{\mathbf{Z}\}$ which give the minimum energy for the expectation value of the Hamiltonian H

$$\hat{E} = \hat{E}(\mathbf{Z}, \mathbf{Z}^*) \equiv \frac{\langle \Phi^\pm(\mathbf{Z}) | H | \Phi^\pm(\mathbf{Z}) \rangle}{\langle \Phi^\pm(\mathbf{Z}) | \Phi^\pm(\mathbf{Z}) \rangle}. \quad (5)$$

For this purpose we introduce frictional cooling equations for $\{\mathbf{Z}\}$ expressed as

$$\frac{d\mathbf{Z}_k}{dt} = (\lambda + i\mu) \frac{1}{i\hbar} \frac{\partial \hat{E}}{\partial \mathbf{Z}_k^*} \quad \text{and c.c.}, \quad (6)$$

with arbitrary real numbers λ and $\mu < 0$. It is easily proved that the energy decreases with time if the system follows this equation:

$$\begin{aligned} \frac{d\hat{E}}{dt} &= \sum_{i=1}^A \frac{\partial \hat{E}}{\partial \mathbf{Z}_i} \cdot \frac{d\mathbf{Z}_i}{dt} + \frac{\partial \hat{E}}{\partial \mathbf{Z}_i^*} \cdot \frac{d\mathbf{Z}_i^*}{dt} \\ &= \frac{2\mu}{\hbar} \sum_{i=1}^A \frac{\partial \hat{E}}{\partial \mathbf{Z}_i} \cdot \frac{\partial \hat{E}}{\partial \mathbf{Z}_i^*} < 0. \end{aligned} \quad (7)$$

Then the wave function of the minimum-energy state is obtained after sufficient cooling time. This cooling method explained above is called the frictional cooling method. Obviously the frictional cooling method can be used not only in the AMD method but also in the general variational calculation with any kind of wave functions which are parametrized by complex parameters. Therefore just in the same way as AMD, we can obtain the minimum-energy state of the extended AMD wave function described in subsection A.

C. Constrained frictional cooling method

The frictional cooling method can be extended so as to construct the minimum-energy state with a given constraint. The constraint is written by a constraint function as

$$\widehat{W} = \widehat{W}(\mathbf{Z}^*, \mathbf{Z}) = \text{given number}. \quad (8)$$

The constraint function is restricted to be real. An ex-

ample of the constraint is the magnitude of the orbital angular momentum. In this case the constraint function $\widehat{W}(\mathbf{Z}^*, \mathbf{Z})$ is chosen to be

$$\widehat{W}(\mathbf{Z}^*, \mathbf{Z}) = \widehat{\mathbf{L}}(\mathbf{Z}^*, \mathbf{Z}) \cdot \widehat{\mathbf{L}}(\mathbf{Z}^*, \mathbf{Z}), \quad (9)$$

$$\widehat{\mathbf{L}}(\mathbf{Z}^*, \mathbf{Z}) = \frac{\langle \Phi^\pm(\mathbf{Z}) | \sum_{j=1}^A \left(\mathbf{r}_j \times (-i) \frac{\partial}{\partial \mathbf{r}_j} \right) | \Phi^\pm(\mathbf{Z}) \rangle}{\langle \Phi^\pm(\mathbf{Z}) | \Phi^\pm(\mathbf{Z}) \rangle}.$$

First we assume that we have initial coordinates $\{\mathbf{Z}^{\text{int}}\}$ which satisfy the given condition

$$\widehat{W}(\mathbf{Z}^{\text{int}*}, \mathbf{Z}^{\text{int}}) = \text{given number}.$$

$$\begin{aligned} \frac{d}{dt} \widehat{W}(\mathbf{Z}^*, \mathbf{Z}) &= \sum_{j=1}^A \left(\frac{\partial \widehat{W}}{\partial \mathbf{Z}_j} \frac{d\mathbf{Z}_j}{dt} + \frac{\partial \widehat{W}}{\partial \mathbf{Z}_j^*} \frac{d\mathbf{Z}_j^*}{dt} \right) \\ &= (\lambda + i\mu) \frac{1}{i\hbar} \sum_{j=1}^A \left(\frac{\partial \widehat{W}}{\partial \mathbf{Z}_j} \cdot \frac{\partial \widehat{E}}{\partial \mathbf{Z}_j^*} + \eta \frac{\partial \widehat{W}}{\partial \mathbf{Z}_j} \cdot \frac{\partial \widehat{W}}{\partial \mathbf{Z}_j^*} \right) + (\lambda - i\mu) \frac{-1}{i\hbar} \sum_{j=1}^A \left(\frac{\partial \widehat{W}}{\partial \mathbf{Z}_j^*} \cdot \frac{\partial \widehat{E}}{\partial \mathbf{Z}_j} + \eta \frac{\partial \widehat{W}}{\partial \mathbf{Z}_j^*} \cdot \frac{\partial \widehat{W}}{\partial \mathbf{Z}_j} \right). \end{aligned} \quad (12)$$

Hence from Eqs. (11) and (12) η is determined as follows:

$$\eta = -\frac{\mathcal{G}}{\mathcal{F}}, \quad \mathcal{F} = 2 \sum_{j=1}^A \frac{\partial \widehat{W}}{\partial \mathbf{Z}_j} \cdot \frac{\partial \widehat{W}}{\partial \mathbf{Z}_j^*}, \quad (13)$$

$$\begin{aligned} \mathcal{G} &= \sum_{j=1}^A \left[\left(1 - i \frac{\lambda}{\mu} \right) \frac{\partial \widehat{W}}{\partial \mathbf{Z}_j} \cdot \frac{\partial \widehat{E}}{\partial \mathbf{Z}_j^*} \right. \\ &\quad \left. + \left(1 + i \frac{\lambda}{\mu} \right) \frac{\partial \widehat{W}}{\partial \mathbf{Z}_j^*} \cdot \frac{\partial \widehat{E}}{\partial \mathbf{Z}_j} \right]. \end{aligned}$$

Cooling of the total energy is assured for arbitrary λ if μ is negative, because there holds the following relation:

$$\begin{aligned} \frac{d}{dt} \widehat{E} &= \sum_k \left(\frac{\partial \widehat{E}}{\partial \mathbf{Z}_k^*} \frac{d\mathbf{Z}_k^*}{dt} + \frac{\partial \widehat{E}}{\partial \mathbf{Z}_k} \frac{d\mathbf{Z}_k}{dt} \right) \\ &= \sum_k \left[\left(\frac{\hbar}{\mu - i\lambda} \frac{d\mathbf{Z}_k}{dt} - \eta \frac{\partial \widehat{W}}{\partial \mathbf{Z}_k^*} \right) \frac{d\mathbf{Z}_k^*}{dt} \right. \\ &\quad \left. + \left(\frac{\hbar}{\mu + i\lambda} \frac{d\mathbf{Z}_k^*}{dt} - \eta \frac{\partial \widehat{W}}{\partial \mathbf{Z}_k} \right) \frac{d\mathbf{Z}_k}{dt} \right] \\ &= \frac{2\mu\hbar}{\mu^2 + \lambda^2} \sum_k \frac{d\mathbf{Z}_k^*}{dt} \frac{d\mathbf{Z}_k}{dt} - \eta \frac{d}{dt} \widehat{W} \\ &= \frac{2\mu\hbar}{\mu^2 + \lambda^2} \sum_k \frac{d\mathbf{Z}_k^*}{dt} \frac{d\mathbf{Z}_k}{dt} < 0, \end{aligned} \quad (14)$$

with arbitrary real numbers λ and $\mu < 0$. It is easy

We introduce the following frictional cooling equation instead of Eq. (6):

$$\frac{d\mathbf{Z}_k}{dt} = (\lambda + i\mu) \frac{1}{i\hbar} \left(\frac{\partial \widehat{E}}{\partial \mathbf{Z}_k^*} + \eta \frac{\partial \widehat{W}}{\partial \mathbf{Z}_k^*} \right) \text{ and c.c.}, \quad (10)$$

with arbitrary real numbers λ and $\mu < 0$. The multiplier function η is determined by the condition of conserving the value of the constraint function \widehat{W} :

$$\frac{d}{dt} \widehat{W}(\mathbf{Z}^*, \mathbf{Z}) = 0. \quad (11)$$

By using Eq. (10) we have

to extend this constrained frictional cooling method to the general case with many constraints and also to the extended AMD wave function.

The most important feature of the construction of the wave function in AMD by the use of the frictional cooling method with or without the constraint is that the wave function can be obtained without prejudice, i.e., free from any model assumptions such as the existence of clustering. If the resulting wave function proves to have clustering structure, the existence of clustering can be insisted more convincingly than other usual cluster model studies. AMD is a new powerful method for the study of the formation and dissolution of clusters in nuclei.

D. Use of constrained frictional cooling method in the actual calculation of extended AMD

When we calculate the frictional cooling in the framework of the extended AMD (EAMD), we usually encounter the following problem. Let us consider our present EAMD calculation in which we adopt a superposition of two AMD Slater determinants $|\Phi_1^\pm(\mathbf{Z})\rangle$ and $|\Phi_2^\pm(\mathbf{Z}')\rangle$. In many cases, $|\Phi_1^\pm(\mathbf{Z})\rangle$ and $|\Phi_2^\pm(\mathbf{Z}')\rangle$ after the frictional cooling have proved to be the same except for the difference of their spatial orientations. However, as we explain in the next subsection, in our AMD study we always make angular momentum projection from the AMD and EAMD wave functions in order to get eigenstates of the total angular momentum. Therefore the above EAMD result that $|\Phi_1^\pm(\mathbf{Z})\rangle$ and $|\Phi_2^\pm(\mathbf{Z}')\rangle$ are the same except for their spatial orientations does not meet the aim of the EAMD calculation to improve the AMD

calculation.

The method which we adopt in this paper to avoid the above-mentioned difficulty of the EAMD is to impose a constraint to the frictional cooling procedure. The constraint is that the spatial orientations of $|\Phi_1^\pm(\mathbf{Z})\rangle$ and $|\Phi_2^\pm(\mathbf{Z}')\rangle$ should be the same. The spatial orientations of $|\Phi_1^\pm(\mathbf{Z})\rangle$ and $|\Phi_2^\pm(\mathbf{Z}')\rangle$ are defined in the following way. As we will report later in this paper, the AMD results show that in Li isotopes wave-packet centers of two protons with spin-up and -down are located at almost the same spatial points and the wave-packet center of the last proton is located at a different spatial point, while in Be isotopes we have spatially separated two pairs of protons with spin-up and -down and within each pair two protons are located closely to each other. Therefore as the definition of the orientation of the AMD wave function for Li and Be isotopes, we adopt the direction which connects two groups of protons.

The constraint function $\widehat{W}(\mathbf{Z}^*, \mathbf{Z}'^*, \mathbf{Z}, \mathbf{Z}')$ is given as follows:

$$\widehat{W}(\mathbf{Z}^*, \mathbf{Z}'^*, \mathbf{Z}, \mathbf{Z}') = \frac{\mathbf{Y} \cdot \mathbf{Y}'}{\|\mathbf{Y}\| \cdot \|\mathbf{Y}'\|},$$

$$\mathbf{Y} \equiv \frac{1}{n(G_1)} \sum_{j \in G_1} \text{Re} \mathbf{Z}_j - \frac{1}{n(G_2)} \sum_{j \in G_2} \text{Re} \mathbf{Z}_j, \quad (15)$$

$$\mathbf{Y}' \equiv \frac{1}{n(G_1)} \sum_{j \in G_1} \text{Re} \mathbf{Z}'_j - \frac{1}{n(G_2)} \sum_{j \in G_2} \text{Re} \mathbf{Z}'_j,$$

where G_1 and G_2 stand for two groups of protons explained above, and $n(G_1)$ and $n(G_2)$ are numbers (one or two) of protons contained in the groups G_1 and G_2 , respectively. Here, of course, wave-packet centers $\{\mathbf{Z}_j\}$ and $\{\mathbf{Z}'_j\}$ are those of $|\Phi_1^\pm(\mathbf{Z})\rangle$ and $|\Phi_2^\pm(\mathbf{Z}')\rangle$, respectively. Initial conditions $\{\mathbf{Z}_j^{\text{int}}\}$ and $\{\mathbf{Z}'_j^{\text{int}}\}$ for the procedure of the constrained frictional cooling are chosen so as to satisfy $\widehat{W}(\mathbf{Z}^{\text{int}*}, \mathbf{Z}'^{\text{int}*}, \mathbf{Z}^{\text{int}}, \mathbf{Z}'^{\text{int}}) = 1$, but the protons belonging to the same group G_1 or G_2 need not to be located closely to each other in the initial configurations $\{\mathbf{Z}_j^{\text{int}}\}$ and $\{\mathbf{Z}'_j^{\text{int}}\}$.

E. Projection to total angular momentum eigenstates

The wave function of the system should be a total-angular-momentum eigenstate. It is, however, difficult to cool the total-angular-momentum projected state of the AMD wave function. We regard the minimum-energy state which is obtained with the cooling method described in subsections B and C as the intrinsic state of the system. We project the intrinsic wave function $|\Phi^\pm\rangle$ to the total-angular-momentum eigenstates, and then calculate the expectation values of operators in order to compare with experimental data:

$$\frac{\langle P_{MK}^J \Phi^\pm | \widehat{T}_{q=0}^k | P_{MK}^J \Phi^\pm \rangle}{\langle P_{MK}^J \Phi^\pm | P_{MK}^J \Phi^\pm \rangle} = \frac{\mathcal{T}}{\mathcal{N}},$$

$$\mathcal{T} = (JMk0|JM) \sum_{K'\nu} (JK'k\nu|JK) \times \int d\Omega D_{K'K}^{J*}(\Omega) \langle \Phi^\pm | \widehat{T}_\nu^k R(\Omega) | \Phi^\pm \rangle, \quad (16)$$

$$\mathcal{N} = \int d\Omega D_{KK}^{J*}(\Omega) \langle \Phi^\pm | R(\Omega) | \Phi^\pm \rangle,$$

where P_{MK}^J is a total-angular-momentum projection operator,

$$P_{MK}^J \equiv \int d\Omega D_{MK}^{J*}(\Omega) R(\Omega), \quad (17)$$

with $R(\Omega)$ standing for the rotation operator by Euler angle Ω , and \widehat{T}_q^k is a given tensor operator of rank k . K should be chosen so as to get the minimum expectation value of the Hamiltonian in each system. The K mixing, namely the diagonalization of the Hamiltonian with respect to the K quantum number, is not made in most cases, but in some cases we show results obtained by K -mixing calculation. The spin J of the calculated ground state has been found to be the same as the observed spin value of the ground state for almost every system studied here. In the practical calculation, the three-dimensional integral is evaluated numerically by taking a finite number of mesh points of the Euler angle Ω .

III. INTERACTION

For the effective two-nucleon interaction, we have adopted the Volkov No. 1 force [34] as the central force. The Volkov force contains only Wigner and Majorana components. For some nuclei treated in this paper, we have performed calculations by adding appropriate Bartlett and Heisenberg components to the Volkov force. But the results have proved to be not so much affected by the additional components at least for the quantities studied in this paper. We are now performing more detailed investigations about this point which will be reported elsewhere. As for the two-body spin-orbit force V_{LS} we have adopted the G3RS force [35] expressed as

$$V_{LS} = \{u_I \exp(-\kappa_I \mathbf{r}^2) + u_{II} \exp(-\kappa_{II} \mathbf{r}^2)\} \times P(^3\text{O}) \mathbf{L} \cdot (\mathbf{S}_1 + \mathbf{S}_2),$$

$$\mathbf{L} \equiv \mathbf{r} \times \left(-i \frac{\partial}{\partial \mathbf{r}} \right), \quad u_I = -u_{II} = 900 \text{ MeV}, \quad (18)$$

$$\kappa_I = 5.0 \text{ fm}^{-2}, \quad \kappa_{II} = 2.778 \text{ fm}^{-2},$$

with \mathbf{r} denoting the two-nucleon relative coordinate and with $P(^3\text{O})$ denoting the projection operator onto the triplet odd (^3O) two-nucleon state. Coulomb interaction is approximated by a sum of seven Gaussians following the technique of Ref. [19]. This approximation is valid for

the range of the internucleon distance from 1 to 20 fm, and is precise enough for the study of nuclear structure. In order to study the effect of the density dependent interaction, we have also used the case 3 of the MV1 force

$$V_{DD}^{(2)} = (1 - m - mP_\sigma P_\tau) \left\{ V_A \exp \left[- \left(\frac{r}{r_A} \right)^2 \right] + V_R \exp \left[- \left(\frac{r}{r_R} \right)^2 \right] \right\},$$

$$V_A = -83.34 \text{ MeV}, \quad r_A = 1.60 \text{ fm}, \quad V_R = 104.86 \text{ MeV}, \quad r_R = 0.82 \text{ fm},$$

$$V_{DD}^{(3)} = v^{(3)} \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(\mathbf{r}_1 - \mathbf{r}_3), \quad v^{(3)} = 4000 \text{ MeV fm}^6,$$
(19)

where P_σ and P_τ stand for the spin and isospin exchange operators, respectively. The two-body interaction part $V_{DD}^{(2)}$ is constructed from the Volkov No. 1 force by weakening the strength of its repulsive part from $V_R = 144.86$ MeV to $V_R = 104.86$ MeV. We have compared the results with and without the three-body interaction. The optimum width parameter ν of wave packets is chosen for each parity of the individual system so as to get the minimum energy.

IV. CALCULATED RESULTS AND COMPARISON WITH EXPERIMENTS

The structure of odd-even and even-even isotopes of Li and Be has been studied with AMD and the results are reported in this section. For Be isotopes, studies have been made also with the extended AMD. We have checked the values of the momentum parameters $\{\mathbf{K}_j, j = 1, \dots, A\}$ of the AMD wave function obtained with the frictional cooling method and have found that $\{\mathbf{K}_j\}$ parameters of all the nucleons are small in all the nuclei studied here.

A. Binding energies and energy spectra

Figure 1 shows the binding energies of the ground states of Li and Be isotopes. We have used the Majorana parameter $m = 0.56$ in the case of no three-body force (Volkov force) and $m = 0.576$ with the three-body force (MV1 force). The optimum width parameters ν are shown in Table I. With both of the interactions, the binding energies are qualitatively reproduced for Li isotopes and also for Be isotopes except for ^{11}Be . In Li isotopes the MV1 force gives better fitting to data. The calculated result of ^{11}Be is the binding energy of the lowest $\frac{1}{2}^+$ state. Detailed discussions about the energy and the parity of the ^{11}Be ground state will be given later. The relative difference of the binding energies between neighboring nuclei is important to discuss the neutron halo structure, since the density tail of the neutron halo must be very sensitive to the binding energy of valence neutrons. Hence for the study of the neutron halo structure,

of Ref. [36], which contains a zero-range three-body interaction $V_{DD}^{(3)}$ in addition to the two-body interaction $V_{DD}^{(2)}$:

it is necessary to give careful consideration in choosing the interaction parameters such as the Majorana parameter on which the energies depend rather sensitively.

The calculated energy spectra with and without the three-body interaction are shown and are compared with the observed data in Fig. 2 for Li isotopes and in Fig. 3 for Be isotopes. In the theoretical data with the three-body force, the second J^\pm levels are obtained by diagonalizing the Hamiltonian matrix with respect to the K quantum number in the spin J projected states. The diagonalization of the Hamiltonian gives almost the same energy spectra of the lowest J^\pm states as the spectra obtained without the K mixing. This means that K is approximately a good quantum number in the lowest J^\pm states projected from the AMD wave functions. Comparing the spectra calculated with and without the

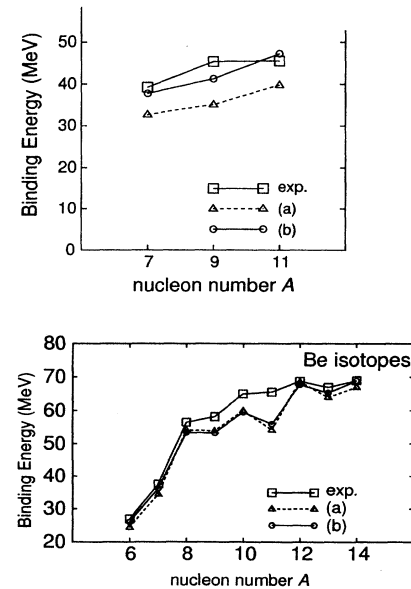


FIG. 1. Binding energies of Li and Be isotopes calculated with (a) the Volkov force No. 1 ($m = 0.56$) and (b) the MV1 force ($m = 0.576$). Experimental data are also shown.

TABLE I. The adopted width parameters ν of Be and Li isotopes in the AMD calculations. They are chosen so as to make the energies minimum.

	$\nu^{(3)} = 0 \text{ MeV fm}^6$	$\nu^{(3)} = 4000 \text{ MeV fm}^6$
	$m = 0.56$ (fm^{-2})	$m = 0.576$ (fm^{-2})
${}^6\text{Be}(+)$	0.215	0.195
${}^7\text{Be}(-)$	0.230	0.200
${}^8\text{Be}(+)$	0.250	0.205
${}^9\text{Be}(-)$	0.245	0.195
${}^9\text{Be}(+)$	0.235	0.200
${}^{10}\text{Be}(+)$	0.230	0.190
${}^{10}\text{Be}(-)$	0.225	0.190
${}^{11}\text{Be}(-)$	0.220	0.180
${}^{11}\text{Be}(+)$	0.220	0.180
${}^{12}\text{Be}(+)$	0.215	0.175
${}^{12}\text{Be}(-)$	0.210	0.180
${}^{13}\text{Be}(+)$	0.205	0.170
${}^{14}\text{Be}(+)$	0.210	0.170
${}^7\text{Li}(-)$	0.230	0.200
${}^9\text{Li}(-)$	0.210	0.180
${}^{11}\text{Li}(-)$	0.195	0.170

three-body force, it is found that the states obtained with the three-body force have larger moment of inertia. The three-body force gives greater effects on the energy difference between normal parity and non-normal parity states and the calculations with the three-body interaction are seen to reproduce the energy spectra better except for ${}^{11}\text{Be}$.

The results of the AMD calculation without the three-body force give larger value for this energy difference than the observed value in the system of most Li and Be isotopes. On the other hand, in the results with the three-body force the excitation energies of non-normal parity states come down and agree much better with the experimental data. In general non-normal parity states have wider extension of the density distribution and therefore they feel relatively weaker repulsive density-dependent force than normal-parity states. It is the reason why the calculations with the three-body force give smaller ex-

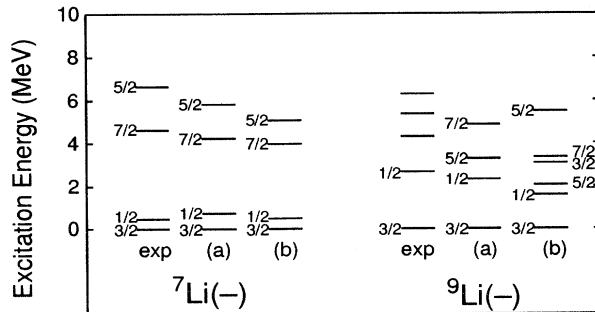


FIG. 2. Energy spectra of ${}^7\text{Li}$ and ${}^9\text{Li}$ calculated with (a) the Volkov force No. 1 ($m = 0.56$) and (b) the MV1 force ($m = 0.576$). Observed spectra are also shown.

citation energies of non-normal parity states than those without the three-body force.

The density-dependent force seems to be important especially for the system of ${}^{11}\text{Be}$. In terms of the shell

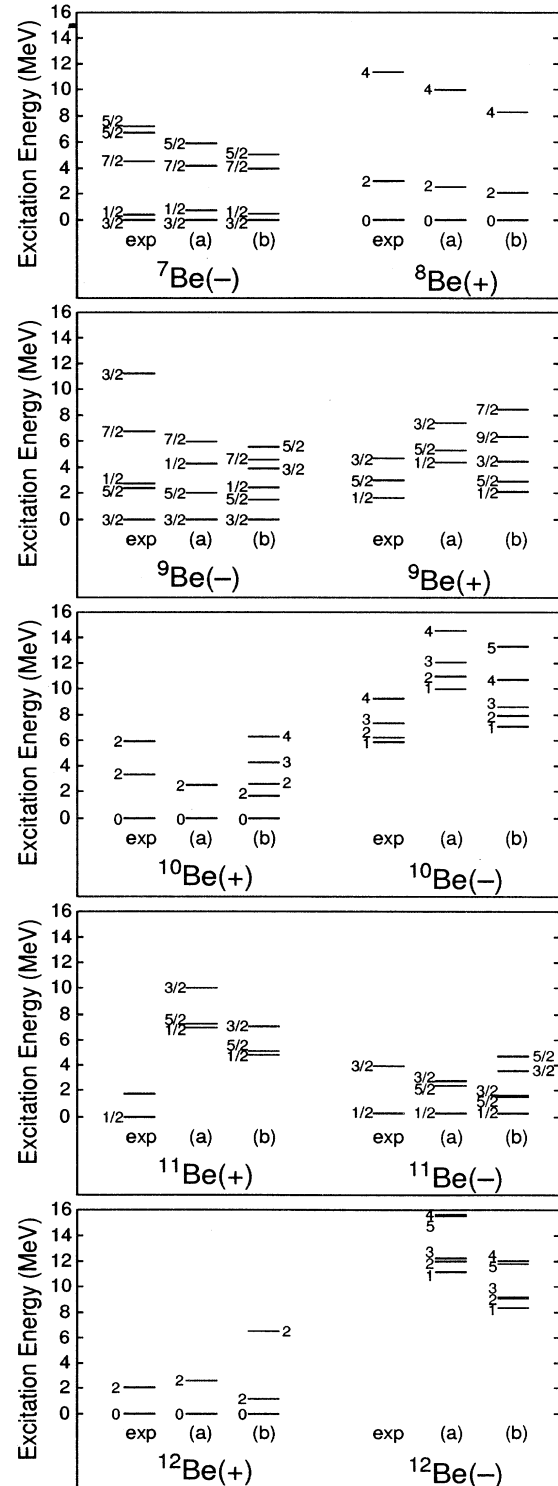


FIG. 3. Energy spectra of Be isotopes. Calculated results with (a) the Volkov force No. 1 ($m = 0.56$) and (b) the MV1 force ($m = 0.576$) are shown and compared with experimental data.

model, the normal parity of the ^{11}Be system should be negative, but it is experimentally known that the ground state is a non-normal positive parity $\frac{1}{2}^+$ state and the lowest $\frac{1}{2}^-$ state is excited by 0.32 MeV. In the AMD results without the three-body force, the lowest $\frac{1}{2}^+$ state is not the ground state but is excited by about 7 MeV from the lowest $\frac{1}{2}^-$. With the three-body force, the lowest $\frac{1}{2}^+$ state has less excitation energy of about 5 MeV. Interaction parameters can be adjusted so as to get better agreement with the experimental value of the energy difference. For example, when we use Majorana parameter $m = 0.65$, the calculated result becomes better and about 3 MeV is obtained as the excitation energy of the lowest $\frac{1}{2}^+$ state. The careful choice of the effective interaction is one of the important points for the reproduction of the non-normal parity ground state.

There are other important problems remaining. Otsuka, Fukunishi, and Sagawa [9] have studied the structure of ^{11}Be with the approach which they call the variational shell model and have obtained the positive parity ground state. They discussed that such a lower energy of the non-normal parity state than that of the normal parity state largely owes to its deformation and to the neutron halo structure. Unfortunately it is not straightforward for the AMD to express the long tail of neutron halo structure because the single-particle wave function of AMD is described with a simple Gaussian wave packet. A superposition of some AMD wave functions is necessary for this problem, and we have tried to make the extended AMD calculation with a superposition of two Slater determinants. In the lowest positive parity state of the EAMD calculation which has a widely distributed density of the neutron, the excitation energy is about 0.5 MeV lower than in the AMD calculation; however, the improvement of the excitation energy is too little to explain the positive parity of the ground state. Our EAMD calculations suggest that further extensions of the AMD wave function should be tried. By supposing that the AMD method describe well the single-particle wave functions of the nucleons other than the last valence neutron (or halo neutron) which is located at a somewhat isolated position from other nucleons, we have adopted the following method in order to see what structure is preferred by the last valence neutron in the lowest positive parity state of ^{11}Be . The single-particle wave functions of the other ten nucleons have been fixed to be the same as the AMD result of ^{11}Be . Only the wave function for the last valence neutron has been described by a superposition of some Gaussian wave packets which have the same width parameter ν but different centers Z_{11} 's. We have determined the coefficient of each Gaussian in the superposition not by cooling but by diagonalizing the matrix elements of the Hamiltonian. This method has been applied to both parity states of ^{11}Be . When the energies are compared before making the angular momentum projection, the excitation energy of the lowest positive parity state is 0.9 MeV smaller than in the AMD calculation. Since the neutron-halo structure is directly concerned with the radius of the nucleus, we will discuss the ^{11}Be problem again in the next subsection.

In the neutron-richer nuclei of Be, some excited levels have been observed. The spin-parity J^π of many states has not been identified yet. The AMD calculations with the density-dependent force suggest that some states of non-normal parity may exist in the rather low excitation energy region in ^{12}Be due to the deformed structure in the intrinsic state with negative parity.

B. Radii of nuclei

Figure 4 shows the comparison of the theoretical values of the radii of Li and Be isotopes with the experimental data. The calculated radius of ^{11}Be is that of the lowest $\frac{1}{2}^+$ state. Dotted lines are the AMD results without the three-body force, dashed lines show results with the three-body force, and solid lines correspond to the EAMD results with the three-body force.

The experimentally observed radii of Be isotopes are seen to be qualitatively reproduced by the calculations of AMD with the three-body repulsive force, while the calculations without the three-body force are too small especially in neutron-rich systems. These results show that the density-dependent repulsive force is important to reproduce the large radii of the experimental data. We consider the reason is as follows: since the three-body repulsive force works weaker in the low density region, the state prefers having the density distribution extended to spatially wider region, which results in the larger radius.

However, even the AMD calculations with the three-body force cannot sufficiently reproduce the large radii of neutron-rich nuclei such as ^{11}Be that extremely deviate from the $A^{1/3}$ law. As similar to the previous discussions of the energy difference between the lowest states with positive and negative parity in ^{11}Be , one of the important points for the problem of the large radius is the careful choice of the effective interaction that is to say the adjustment of interaction parameters. The nuclear radius as well as the binding energy is very sensitive to

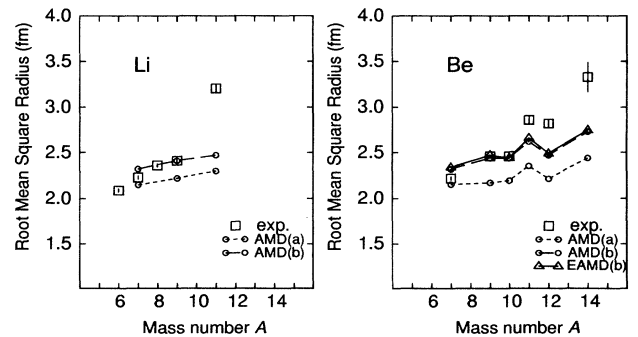


FIG. 4. Root-mean-square radii of Li and Be isotopes. They are calculated with AMD and EAMD using (a) the interaction Volkov force No. 1 ($m = 0.56$) and (b) MV1 force ($m = 0.576$). Squares represent the interaction radii derived from the data of interaction cross sections [6].

the choice of the effective interaction. For example, if we use the Majorana parameter $m = 0.65$ in the AMD calculation with the three-body force, the radius of the $\frac{1}{2}^+$ ^{11}Be state increases up to a large value, 2.78 fm, that is as large as the experimental value. In fact it is generally reasonable to adopt larger m value in a nucleus with larger mass number, and the use of the fixed value of the m parameter for all the isotopes is not necessarily justified. Hence the value $m = 0.65$ may not be so unnatural for ^{11}Be .

The extremely large radius of the ^{11}Be ground state is of course directly related to the neutron halo structure of the state. In the previous subsection, we have reported the calculation in which we have made the superposition of several Gaussians of the last valence neutron. The calculation has been made with the expectation that the long tail of the last valence neutron may reduce the large energy gap between different parity states. The radius obtained by this superposition calculation is 2.33 fm, which is only by 0.05 fm larger than the value obtained by the simple AMD calculation. However, much difference between the AMD calculation and the improved one is found in the neutron density distribution as shown in Fig. 5. The dashed line shows the density distribution of protons and the solid line shows the one of the neutrons in the $\frac{1}{2}^+$ state of ^{11}Be . Contrary to the proton density, the solid line of neutron density has a long tail with low density in the outer region from 5 fm to more than 10 fm. On the other hand, in the simple AMD calculation, the neutron density distributes in almost the same way as the proton density and has no tail. However, when compared with the phenomenologically determined density distribution which provides a good fit to the cross section of the neutron dissociation reaction, the tail described in our improved result is much smaller by about one order in the region around 10 fm. Here we should make a remark that the calculated $\frac{1}{2}^+$ ^{11}Be state still has a higher energy than the $^{10}\text{Be}+n$ threshold energy.

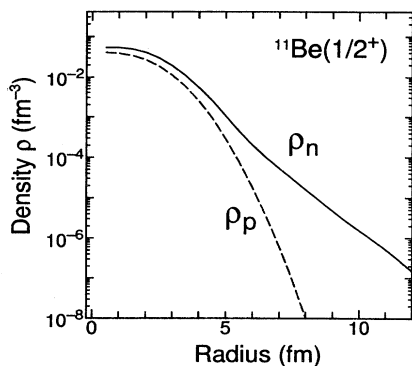


FIG. 5. Density of the positive parity state of ^{11}Be calculated by superposing several Gaussians for the last valence neutron wave function. The solid (dashed) curve shows neutron (proton) density.

Also in the calculations of Li isotopes, the AMD and EAMD have not been able to sufficiently reproduce the large radius of ^{11}Li that deviates largely from the $A^{1/3}$ law. The small binding energy of the neutron in ^{11}Li has been considered to be very important to describe its large radius. It implies the importance of the exact reproduction of the binding energy which is sensitive to the interaction parameters. Besides that, further extensions of the AMD wave function may be necessary for the precise description of the neutron-halo structure.

C. Magnetic moments

The calculated results of magnetic dipole moments are not sensitive to the interaction parameters except for ^{11}Be . Almost the same results are obtained with and without the three-body force in the AMD calculation and also in the EAMD calculation except for the ^{11}Be system. The results of the AMD calculation given in Fig. 6 show very good agreement with the experimental values for even-odd isotopes of Li and Be. It should be emphasized that the AMD method is the first framework which has succeeded in reproducing the magnetic dipole moments systematically for these isotopes as long as we know.

The dependence of the neutron number seen in the experimental values of Li isotopes is expected to carry important information about the nuclear structure, because in terms of the shell model the valence proton in the $0p_{3/2}$ orbital would give dominant effects on the magnetic moments and hence almost the same values of the magnetic moments would be obtained for all these isotopes. We will give detailed discussions about the N dependence in the later section.

Contrary to Li isotopes, in the case of Be isotopes the last valence neutron may contribute mainly, and it is important to make a closer discussion of the neutron orbital. The magnetic dipole moment is measured only for ^9Be , and the result obtained with the AMD agrees well with the data. On the other hand, a little complicated problem exists in the system of ^{11}Be for which the experimental value has not been obtained yet. In the ^{11}Be

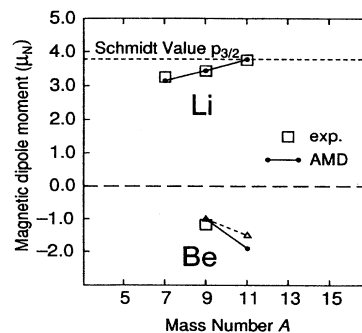


FIG. 6. Magnetic dipole moments. The triangle shows the magnetic moment of another nearly degenerate AMD state. Observed data are shown with squares [37,38].

system, two different states are eventually obtained in the AMD calculation. The two states have almost the same values of the binding energy and the root-mean-square radius, but they have considerably different values of the magnetic dipole moments, $-1.9\mu_N$ and $-1.5\mu_N$. In this situation, it is useful to check the EAMD calculation with two Slater determinants of AMD wave functions. In the EAMD calculation, the value of $-1.9\mu_N$ is obtained. However, the calculation by the use of different interaction parameters m and $u_I = -u_{II}$ of V_{LS} give various results ranging from $-1.5\mu_N$ to $-1.9\mu_N$. We need more careful investigations in order to get a conclusion about the theoretical value for the magnetic moment of ^{11}Be , since there remains the problem to reproduce the observed large radius which is considered to have relation to the orbital of the valence neutron.

D. Electric quadrupole moments and $B(E2)$ values

The electric quadrupole moments for even-odd isotopes of Li and Be calculated with the three-body force are shown in Fig. 7 and are seen to agree well with the experimental data. The AMD results without the three-body force are much smaller in absolute value than the experimental data. It is not surprising that this situation is similar to the case of the radii which have already been discussed in subsection IV B. We can say that the small values of quadrupole moments obtained without the three-body force are mainly due to the problem not in the quadrupole deformation but in the radial density distribution. Although the values with the three-body force seem to be still lightly smaller than the data, it is

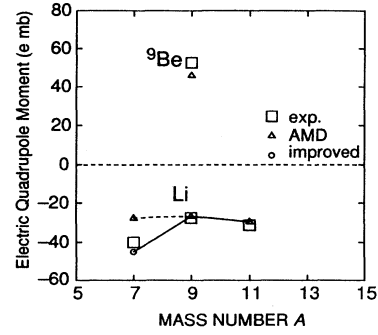


FIG. 7. Electric quadrupole moments calculated with the MV1 force ($m = 0.576$) (triangles). A circle indicates the moment of ^7Li calculated with the improved AMD with the same interaction. They are compared with experimental data (squares) [37,39].

not a serious problem, because about 10% larger values can be obtained just by using $m = 0.60$ as the Majorana parameter instead of $m = 0.576$.

The theoretical value of the quadrupole moment of ^7Li shown in Fig. 7 has been obtained by improving the AMD wave function in the following way. As we discuss later in detail, the AMD wave function of ^7Li has proved to have the cluster structure of $\alpha + t$. However, since the single nucleon wave function of AMD is a Gaussian wave packet, the relative wave function between α and t is also necessarily a Gaussian wave packet of the form

TABLE II. The $E2$ transition strength $B(E2; I_1 \rightarrow I_2)$. The theoretical values are the AMD results calculated with the interaction parameters $m = 0.576$ and $v^{(3)} = 4000 \text{ MeV fm}^6$. The strength $B(E2; \frac{1}{2}^- \rightarrow \frac{3}{2}^-)$ of ^7Li calculated with the improved AMD wave function is shown in parentheses.

	Transition ($I_1 \rightarrow I_2$)	Energy (MeV)	$B(E2)$	
			Exp ($e^2 \text{ fm}^4$)	Theory ($e^2 \text{ fm}^4$)
^7Li	$\frac{1}{2}^- \rightarrow \frac{3}{2}^-$	0.48 \rightarrow 0	16.14	7.51 (18.57)
	$\frac{7}{2}^- \rightarrow \frac{3}{2}^-$	4.63 \rightarrow 0	3.51	4.72
	$\frac{5}{2}^- \rightarrow \frac{3}{2}^-$	6.68 \rightarrow 0		1.05
^9Li	$\frac{1}{2}^- \rightarrow \frac{3}{2}^-$	2.69 \rightarrow 0		7.15
	$\frac{5}{2}^- \rightarrow \frac{3}{2}^-$			0.12
^9Be	$\frac{5}{2}^- \rightarrow \frac{3}{2}^-$	2.43 \rightarrow 0	27.8	18.35
	$\frac{7}{2}^- \rightarrow \frac{3}{2}^-$	6.76 \rightarrow 0	7.24	7.73
^{10}Be	$2^+ \rightarrow 0^+$	3.37 \rightarrow 0	10.49	9.46
	$3^- \rightarrow 1^-$	7.73 \rightarrow 5.96		11.2
^{11}Be	$\frac{3}{2}^- \rightarrow \frac{1}{2}^-$	3.96 \rightarrow 0.32		8.13
	$\frac{5}{2}^- \rightarrow \frac{1}{2}^-$			8.06
^{12}Be	$2^+ \rightarrow 0^+$	2.10 \rightarrow 0		7.75

$$\exp \left[-\frac{4 \times 3}{4 + 3} \nu (\mathbf{X}_\alpha - \mathbf{X}_t - \mathbf{R})^2 \right],$$

$$\mathbf{X}_\alpha \equiv \frac{1}{4} \sum_{j=1}^4 \mathbf{r}_j, \quad \mathbf{X}_t \equiv \frac{1}{3} \sum_{j=5}^7 \mathbf{r}_j, \quad (20)$$

$$\mathbf{R} = \frac{1}{4} \sum_{j=1}^4 \mathbf{Z}_j - \frac{1}{3} \sum_{j=5}^7 \mathbf{Z}_j.$$

When the clustering is well developed, the description of the intercluster relative wave function by a single Gaussian wave function is not sufficient because the relative wave function spreads out toward the outer spatial region resulting in a long tail. The lack of the outer tail part of the intercluster relative wave function may be sensitively reflected in the value of the quadrupole moment. In fact, the AMD calculation with the three-body force has given us a rather small value of the ${}^7\text{Li}$ quadrupole moment compared with the experimental value. Therefore we have improved the intercluster relative wave function of the AMD wave function by superposing several AMD wave functions as described below. The trial AMD wave functions to be superposed have been constructed by adopting the following changes of the original $\{\mathbf{Z}\}$ values:

$$\mathbf{Z}_j \longrightarrow \mathbf{Z}_j + \frac{3}{7} \sqrt{\nu} \mathbf{C} \quad \text{for } j = 1, \dots, 4, \quad (21)$$

$$\mathbf{Z}_j \longrightarrow \mathbf{Z}_j - \frac{4}{7} \sqrt{\nu} \mathbf{C} \quad \text{for } j = 5, \dots, 7.$$

Projection onto the $\frac{3}{2}^-$ state in parity and angular momentum has been applied to the trial AMD wave functions which have different values of the displacement real vector \mathbf{C} . Superposition of these projected states has been made by diagonalizing the total Hamiltonian. The resulting improved AMD wave function has proved to reproduce the electric quadrupole moment well as seen in Fig. 7.

Table II shows the theoretical values of the $E2$ transition strength compared with the observed data. The simple AMD calculations reproduce well the experimental data except for the strength $B(E2; \frac{1}{2}^- \rightarrow \frac{3}{2}^-)$ of ${}^7\text{Li}$. The theoretical value $7.51 e^2 \text{fm}^4$ is much smaller than the observed value $16.14 e^2 \text{fm}^4$. As in the same way as the case of the quadrupole moment, the improved AMD wave function gives the strength of $18.57 e^2 \text{fm}^4$ which is as large as the experimental value. Some of theoretical data predicted by AMD are also shown in the Table II. The AMD calculations predict that the transition strength will be rather large between 3^- and 1^- in ${}^{10}\text{Be}$ and between 2^+ and 0^+ in ${}^{12}\text{Be}$ due to the intrinsic deformation of the proton distribution.

V. STRUCTURE CHANGE BETWEEN CLUSTER STRUCTURE AND SHELL-MODEL-LIKE STRUCTURE

In Sec. III we have reported the theoretical values and compared them with the experimentally observed values.

We have seen that the AMD calculations explain the observed values of various properties qualitatively. Especially in the case of magnetic moments the AMD results reproduce quite well the systematic dependence of the experimental data on the neutron number. It is important to analyze the intrinsic structure of the obtained AMD states in order to understand the fundamental mechanism of such characteristic dependence on the neutron number.

A. Density distribution

The density distributions of the intrinsic states of Li and Be isotopes are shown in the Figs. 8 and 9, respectively. In drawing the figures, the density of each intrinsic state is projected onto an adequate plane by integrating out along the axis perpendicular to the plane. We see here systematic but drastic structure change along the increase of the neutron number. In the results of Li iso-

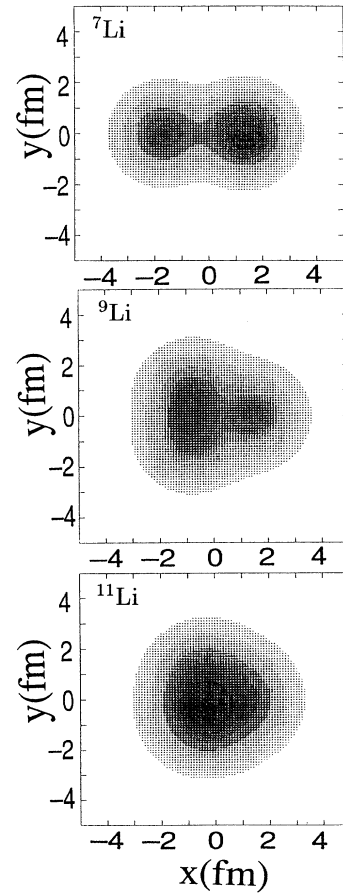


FIG. 8. Matter density distribution of AMD states of Li isotopes. The intrinsic density before parity projection is shown. The density is projected to a x - y plane and integrated along the z axis perpendicular to the plane. Units of x and y axes are in fm.

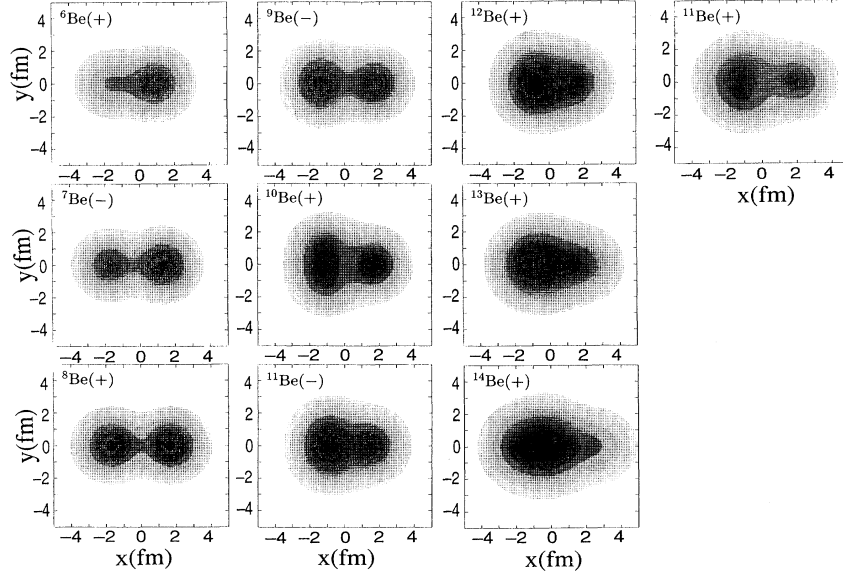


FIG. 9. Matter density distribution of AMD states of Be isotopes. The intrinsic density before parity projection is shown. Density is projected to a x - y plane and integrated along the z axis perpendicular to the plane. Units of x and y axes are in fm.

topes (Fig. 8), it is easily seen that the ${}^7\text{Li}$ system has the largest deformation with the cluster structure. ${}^9\text{Li}$ also has a deformed shape, though the degree of the deformation is not as large as the one in ${}^7\text{Li}$. ${}^{11}\text{Li}$ has an almost spherical state that can be expressed by a shell model wave function. In Fig. 9 for Be isotopes, more interesting features are seen. In the figure the density distributions of only the normal parity states of Be isotopes are shown except the case of ${}^{11}\text{Be}$. Just like the structure change from ${}^7\text{Li}$ to ${}^{11}\text{Li}$, the deformation is most developed in ${}^8\text{Be}$ due to the cluster structure and then gradually decreases toward ${}^{12}\text{Be}$ which has the most spherical shell-model-like structure. What is novel and interesting in Be is the result that the deformation appears again as the neutron number increases in the neutron-richer isotopes than ${}^{12}\text{Be}$. Rather large deformation is seen in the positive parity state of ${}^{11}\text{Be}$ which is known to have the ground state with the anomalous positive parity. It is to be noted that deformation has been considered to be one of the essential mechanisms for the decrease of the excitation energy of the $\frac{1}{2}^+$ level in ${}^{11}\text{Be}$.

B. Clustering aspect

As mentioned in Sec. I, it is well known that in the $N \approx Z$ region of Li and Be isotopes there appear the well-developed cluster structure like the α - α structure of ${}^8\text{Be}$ and the α - t structure of ${}^7\text{Li}$. Many theoretical studies

by the use of the cluster model have been successfully made both for nuclear structure problems and for nuclear reaction problems. However, there have been a very small number of theoretical works which have ascertained the formation of clusters microscopically without assuming the existence of any kinds of clusters. The present AMD theory is a theoretical framework very suitable for the above-mentioned ascertainment.

By checking the spatial centers of Gaussian wave packets given by the values $\{\mathbf{D}\} = \{\text{Re}\mathbf{Z}/\sqrt{\nu}\}$, we have found that the AMD wave functions of ${}^7\text{Li}$ and ${}^8\text{Be}$ have actually the α - t and α - α clustering structure, respectively. Further, the AMD wave function of ${}^9\text{Be}$ has been found to be of the α - α - n structure (or α - ${}^5\text{He}$ structure), which has also been well known for a long time as the structure of ${}^9\text{Be}$. We are interested in how the clustering structure changes as a function of the neutron number N . In Li isotopes, ${}^9\text{Li}$ has the ${}^6\text{He}+t$ clustering, although it is not so well developed as the $\alpha+t$ clustering in the ${}^7\text{Li}$. The heavier nucleus ${}^{11}\text{Li}$ has no clustering structure.

Figure 10 shows the spatial configuration of the center of each Gaussian wave packet for Be isotopes. The values $\{\mathbf{D}\} = \{\text{Re}\mathbf{Z}/\sqrt{\nu}\}$ are projected to an appropriate plane. The squares and circles correspond to the centers of protons and neutrons, respectively. In every Be isotope, four protons are always grouped spatially into two pairs. Each pair is composed of $p \uparrow$ and $p \downarrow$ and is seen in the figure as just two squares located very closely to each other. The neutrons except for the valence neutron in the

even-odd Be isotopes also couple to compose some pairs of $n \uparrow$ and $n \downarrow$. It is shown that many of the Be isotopes have two-center clustering structure with at least one α cluster. One will find the $2p + \alpha$ clustering in the positive parity state of ${}^6\text{Be}$, ${}^3\text{He} + \alpha$ in ${}^7\text{Be}$, $\alpha + \alpha$ in ${}^8\text{Be}$, ${}^5\text{He} + \alpha$ in ${}^9\text{Be}$, ${}^6\text{He} + \alpha$ in ${}^{10}\text{Be}$, and the ${}^7\text{He} + \alpha$ clustering in the negative parity state of ${}^{11}\text{Be}$.

The degree of development of clustering structure is roughly estimated with the relative distances between two pairs of protons which is shown in Fig. 11. The α clustering is most developed in ${}^8\text{Be}$ and it becomes weaker in heavier isotopes as the neutron number increases. No developed clustering structure is found in ${}^{12}\text{Be}$ with the neutron magic number $N = 8$. In ${}^{13}\text{Be}$ and ${}^{14}\text{Be}$ which have neutron number $N > 8$, the clustering structure develops again. In ${}^{13}\text{Be}$, the α is not so normal but is somewhat polarized with the neutrons distributed in the spatially outer region. In ${}^{14}\text{Be}$, the distortion of α is larger since the centers of two valence neutrons are seen in the further outer space. In Ref. [30] Be

isotopes were studied with the molecular orbital model which describes the Be isotope as composed of an α - α core and surrounding neutrons. It was found there that the inter- α distance decreases when going from ${}^8\text{Be}$ to ${}^{12}\text{Be}$ but increases when going from ${}^{12}\text{Be}$ to ${}^{14}\text{Be}$. Since the relative distance between two proton pairs mentioned above is similar to the α - α distance, their results are consistent with ours which have been obtained without any assumption of the existence of clusters.

The calculations of the extended AMD present us with interesting results about clustering features. In EAMD we have adopted the superposition of two parity-projected Slater determinants which we denote as $|\Phi_1^\pm(\mathbf{Z})\rangle$ and $|\Phi_2^\pm(\mathbf{Z}')\rangle$. In most normal parity states of Be isotopes, the main component $|\Phi_1^\pm(\mathbf{Z})\rangle$ has proved to be almost the same as the wave function of the simple AMD calculation. The minor component $|\Phi_2^\pm(\mathbf{Z}')\rangle$ mixes in with only a small amount and improves the description of the state expressed with the major component. By analyzing $\{\mathbf{Z}'\}$ in the minor component $|\Phi_2^\pm(\mathbf{Z}')\rangle$, we

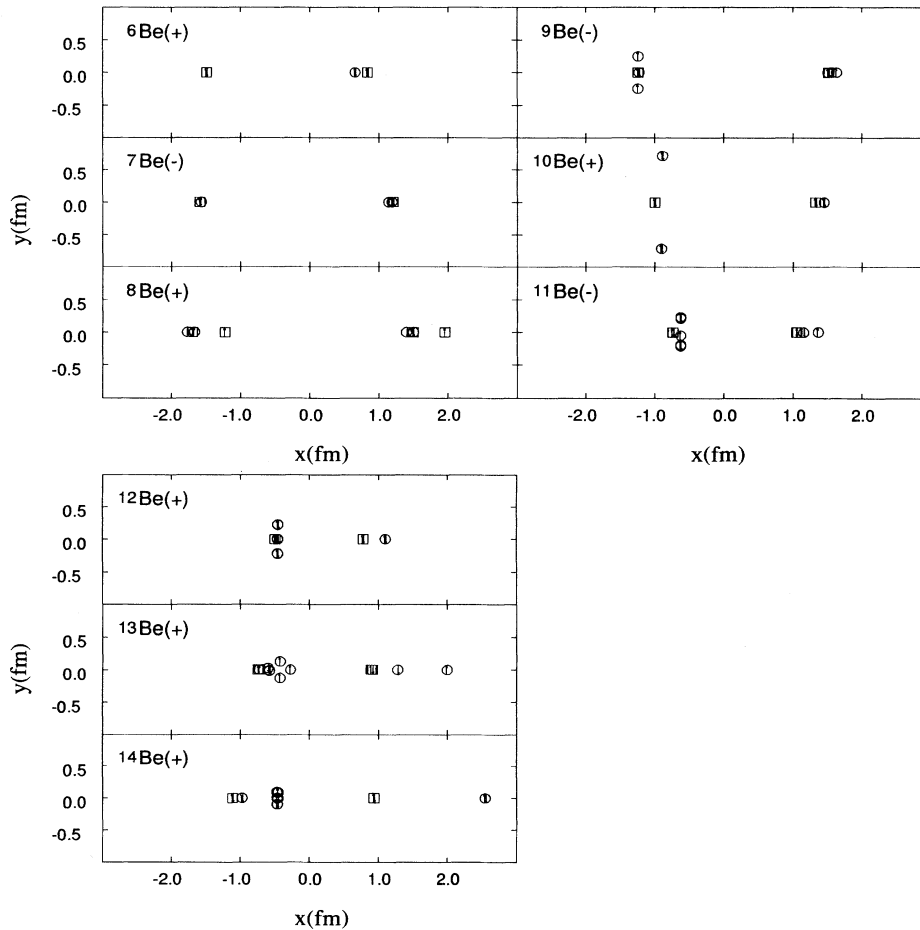


FIG. 10. Spatial configurations of the centers of nucleon wave packets in normal parity states of Be isotopes. $\{\text{Re}\mathbf{Z}/\sqrt{\nu}\}$ in AMD calculations are projected to adequate planes. Circles and squares with up arrow (down arrow) represent the centers of $n \uparrow$ (\downarrow) and $p \uparrow$ (\downarrow), respectively.

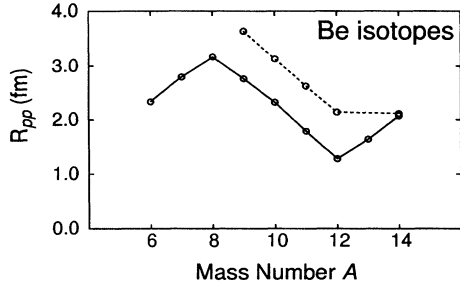


FIG. 11. Mass number A dependence of the relative distance between two proton pairs of normal and non-normal parity intrinsic states in Be isotopes. Solid line is for normal parity states and dashed line for non-normal parity states.

have found that the types of the improvement made by $|\Phi_2^\pm(\mathbf{Z}')\rangle$ can be classified into three types.

The first type is seen in the cases of ${}^6\text{Be}$ and ${}^7\text{Be}$. The state $|\Phi_1^\pm(\mathbf{Z})\rangle$ in ${}^7\text{Be}$ (${}^6\text{Be}$), has the same clustering configuration ${}^3\text{He} + \alpha$ ($2p + \alpha$) but with larger intercluster distance compared to the one in the simple AMD result. On the other hand, $\{\mathbf{Z}'_i\}$ in $|\Phi_2^\pm(\mathbf{Z}')\rangle$ are distributed between the two clusters of $|\Phi_1^\pm(\mathbf{Z})\rangle$ so that the spatial region between the two clusters does not become too low in density. The second type of the improvement is concerned with the relative wave function between the clusters. In ${}^8\text{Be}$ (${}^9\text{Be}$), the state $|\Phi_1^\pm(\mathbf{Z})\rangle$ is found to have the same $\alpha + \alpha$ (${}^5\text{He} + \alpha$) clustering structure as the one obtained by simple AMD. The second component $|\Phi_2^\pm(\mathbf{Z}')\rangle$ has also the same clustering configuration, but the intercluster distance is larger by about 2 fm than the distance in $|\Phi_1^\pm(\mathbf{Z})\rangle$. The total wave function $|\Phi_1^\pm(\mathbf{Z})\rangle + |\Phi_2^\pm(\mathbf{Z}')\rangle$ results in improving the outer tail of the wave function of the relative motion between the clusters. In heavier

TABLE III. Percentage of the normalized component $|\Phi_1\rangle^N \equiv |\Phi_1\rangle/\| |\Phi_1\rangle \|$ contained in the normalized total wave function $|\Phi_1 + \Phi_2\rangle^N \equiv (|\Phi_1\rangle + |\Phi_2\rangle)/\| |\Phi_1\rangle + |\Phi_2\rangle \|$.

	$ ^N\langle \Phi_1 (\Phi_1 + \Phi_2) \rangle^N ^2$
${}^6\text{Be}(+)$	0.83
${}^7\text{Be}(-)$	0.84
${}^8\text{Be}(+)$	0.83
${}^9\text{Be}(-)$	0.90
${}^{10}\text{Be}(+)$	0.85
${}^{11}\text{Be}(-)$	0.96
${}^{11}\text{Be}(+)$	0.88
${}^{12}\text{Be}(+)$	0.96
${}^{13}\text{Be}(+)$	0.90

Be isotopes, we observe the third type of the improvement by the minor component $|\Phi_2^\pm(\mathbf{Z}')\rangle$. In this case, $|\Phi_1^\pm(\mathbf{Z})\rangle + |\Phi_2^\pm(\mathbf{Z}')\rangle$ represents the mixing of two different channels of clustering. In ${}^{10}\text{Be}$, ${}^{11}\text{Be}$, and ${}^{12}\text{Be}$, the main component $|\Phi_1^\pm(\mathbf{Z})\rangle$ represents the ${}^6\text{He} + \alpha$, ${}^7\text{He} + \alpha$, and ${}^8\text{He} + \alpha$ channels and the minor component $|\Phi_2^\pm(\mathbf{Z}')\rangle$ corresponds to the ${}^5\text{He} + {}^5\text{He}$, ${}^6\text{He} + {}^5\text{He}$, and ${}^6\text{He} + {}^6\text{He}$ channels, respectively. In ${}^{13}\text{Be}$, the main component $|\Phi_1^\pm(\mathbf{Z})\rangle$ has the ${}^8\text{He} + {}^5\text{He}$ configuration with the polarized ${}^5\text{He}$, while the minor component $|\Phi_2^\pm(\mathbf{Z}')\rangle$ has ${}^8\text{He} + \alpha + n$ structure with the valence neutron locating far from the center of the nucleus.

More interesting aspects of clustering are found in the non-normal parity state of ${}^{11}\text{Be}$. Figure 12 shows the spatial configurations of $\{\text{Re}\mathbf{Z}/\sqrt{\nu}\}$ in $|\Phi_1^\pm(\mathbf{Z})\rangle$ and $\{\text{Re}\mathbf{Z}'/\sqrt{\nu}\}$ in $|\Phi_2^\pm(\mathbf{Z}')\rangle$ comparing with the result of simple AMD. The positive parity state obtained with the simple AMD has the ${}^7\text{He} + \alpha$ clustering feature [seen in Fig. 12(a)]. On the other hand, the EAMD calculation gives the state with a mixture of $\alpha + {}^6\text{He} + n$ and ${}^5\text{He} + {}^6\text{He}$ configurations [seen in Figs. 12(b-i) and 12(b-ii), respec-

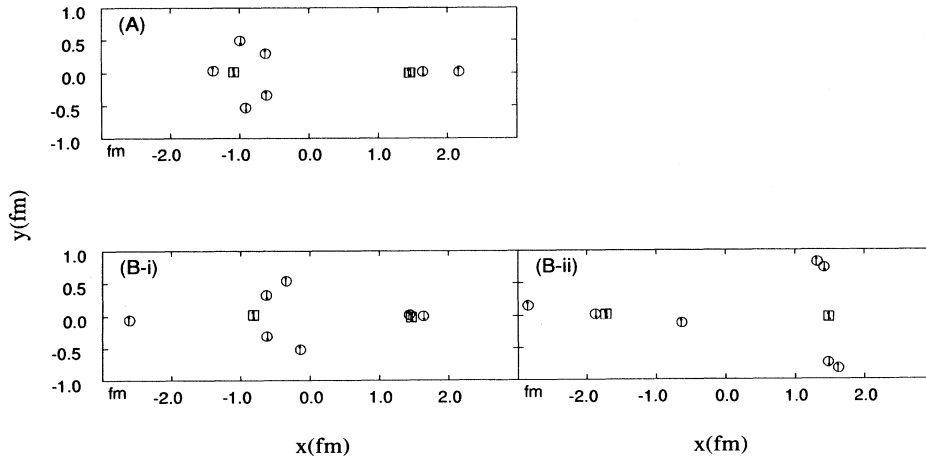


FIG. 12. Spatial configurations for the positive parity state of ${}^{11}\text{Be}$. (a) shows $\{\text{Re}\mathbf{Z}/\sqrt{\nu}\}$ of the AMD wave function. (b) shows $\{\text{Re}\mathbf{Z}/\sqrt{\nu}\}$ in Φ_1 (b-i) and $\{\text{Re}\mathbf{Z}'/\sqrt{\nu}\}$ in Φ_2 (b-ii) obtained with EAMD. Circles and squares with up arrow (down arrow) correspond to the centers of $n \uparrow$ (\downarrow) and $p \uparrow$ (\downarrow), respectively.

tively]. Both configurations in the EAMD calculation differ from the one in the AMD calculation, and they describe the behavior of the valence neutron far from the center better than the simple AMD calculation.

Though the two states $|\Phi_1^\pm\rangle$ and $|\Phi_2^\pm\rangle$ are not orthogonal to each other, in most of the Be isotopes it is found that the second state $|\Phi_2^\pm\rangle$ has a significant component orthogonal to $|\Phi_1^\pm\rangle$. In the total wave function $|\Phi_1^\pm\rangle + |\Phi_2^\pm\rangle$, the orthogonal component to $|\Phi_1^\pm\rangle$ is about 10%. In Table III we show the percentage of the normalized component $|\Phi_1^\pm\rangle^N \equiv |\Phi_1^\pm\rangle / \|\Phi_1^\pm\rangle\|$ contained in the normalized total wave function $|\Phi_1^\pm + \Phi_2^\pm\rangle^N \equiv (|\Phi_1^\pm\rangle + |\Phi_2^\pm\rangle) / \|(|\Phi_1^\pm\rangle + |\Phi_2^\pm\rangle)\|$, $|\Phi_1^\pm\rangle^N \langle \Phi_1^\pm | (\Phi_1^\pm + \Phi_2^\pm)^N |^2$. Considering the above-mentioned facts that the main component $|\Phi_1^\pm\rangle$ is very similar to the wave function of the simple AMD calculation and that the second component $|\Phi_2^\pm\rangle$ mixes in with a small mixing amplitude, it is implied that the AMD study adopting only one parity-projected Slater determinant is usually sufficiently reliable.

C. Indispensable role of parity projection in describing asymmetric cluster structure

We point out here that the parity projection is essential for precise description of the clustering aspects. As shown above, $\alpha + t$ clustering in ${}^7\text{Li}$ is described by the variational calculation by the use of the parity projected AMD wave function. However, when we have adopted the variational calculation without the parity projection we have obtained not the state with the $\alpha + t$ clustering configuration but the one with $t + n + t$ clustering. The AMD wave function without parity projection is given by a single Slater determinant, and its variational calculation is considered to be one of approximated frameworks of the Hartree-Fock method. In any Hartree-Fock-type theory which adopts a single Slater determinant, the description of the $\alpha + t$ clustering can be made only by a parity-violating intrinsic state. But, usually the minimum energy is obtained by a parity-conserving configuration rather than by a parity-violating configuration. This is the reason why we have failed to obtain the asymmetric configuration of $\alpha + t$ but have obtained the symmetric configuration of $t + n + t$ in ${}^7\text{Li}$ in the AMD calculation without parity projection. We have encountered the same situation in the study of ${}^{20}\text{Ne}$ [27]. By the AMD calculation with parity projection, we could confirm the existence of $\alpha + {}^{16}\text{O}$ clustering in ${}^{20}\text{Ne}$. However, when we made the AMD calculation without parity projection, we never obtained the asymmetric (parity-violating) $\alpha + {}^{16}\text{O}$ clustering configuration but a symmetric (parity-conserving) configuration. Therefore we stress here that the parity projection is important for the description of the structure of light nuclei and it is sometimes dangerous to extract a conclusion with the framework in which the functional space is restricted within a single Slater determinant without parity projection.

D. Correlations of the structure change with the observed electromagnetic properties

The drastic change of the intrinsic structure has been discussed in the previous subsections A and B. Our aim

here is to understand the fundamental mechanism how the observed values of the electromagnetic properties reflect the systematic structure change. Furthermore we aim to find what information about the nuclear structure can be extracted from the observed data.

The observed magnetic dipole moment of Li isotopes changes systematically as the neutron number N increases. The shift of the ${}^7\text{Li}$ magnetic moment from the Schmidt value $3.79\mu_N$ of the $0p_{3/2}$ proton orbit has been considered to be closely related with the clustering structure. Below we discuss this point on the basis of our AMD calculations. An important point of the AMD results which we need to remember in the following discussions is that neutrons make no contribution to the calculated magnetic moments. This is because of the property of the calculated AMD wave functions of Li isotopes that neutrons are all paired off, namely every spin-up neutron wave packet always shares the same spatial point with a spin-down neutron wave packet. It implies that the total intrinsic spin of neutrons is zero and the total angular momentum of neutrons is exhausted by the total orbital angular momentum.

In the following discussions, one should recall that the clustering gives two kinds of fundamental effects on the nuclear structure. One is caused by the spatial relative distance between clusters (spatial clustering), and the other is concerned with the angular momentum coupling of nucleons caused by the clustering correlation of nucleons (cluster coupling of angular momenta). A typical example of the latter kind is found in the so-called shell-model cluster which is the cluster appearing in the SU_3 coupling shell-model configuration. According to the Bayman-Bohr theorem [40], the clustering wave function with the minimum spatial separation of clusters is equivalent to the SU_3 -shell-model wave function. As we see below, the effect of clustering on the magnetic moments is not due to the spatial clustering but due to the cluster coupling of angular momenta. In order to extract the effect of the cluster coupling of angular momenta from our AMD wave functions, we have artificially made the intercluster relative distance in the AMD wave function small so as to get the shell-model limit state. In practice we have transformed all the parameters \mathbf{Z}_i as $\{\mathbf{Z}\} \rightarrow \{a\mathbf{Z}\}$ where a is a real constant that is sufficiently small. It should be noted that this transformation does not largely affect the internal wave functions of clusters contained in the AMD wave functions, because nucleons inside each cluster are located closely to each other. The state obtained in the shell-model limit does not have the developed clustering in view of the intercluster relative distance any more, but keeps the angular momentum coupling correlation caused by the clustering structure of the original AMD wave function. Table IV shows some electromagnetic quantities calculated with the angular-momentum-projected states from the shell-model-limit intrinsic states mentioned above, which are compared with the original AMD calculations. In Table IV we have also shown the expectation values of squared total angular momenta of protons $\langle \mathbf{J}_p^2 \rangle$ and neutrons $\langle \mathbf{J}_n^2 \rangle$ and those of squared total orbital angular momentum of protons $\langle \mathbf{L}_p^2 \rangle$. We see that the magnetic dipole moments

TABLE IV. Comparison of various quantities calculated with the AMD wave functions to those calculated with the shell-model-limit wave functions of AMD. The notations $\langle \mathbf{J}_p^2 \rangle$, $\langle \mathbf{J}_n^2 \rangle$, and $\langle \mathbf{L}_p^2 \rangle$ are explained in the text. The experimental data of the magnetic dipole moments μ and the electric quadrupole moments Q are also shown for comparison. The expectation values are calculated with the total angular momentum projected states. The adopted interaction parameters are $m = 0.576$ and $v^{(3)} = 4000 \text{ MeV fm}^6$.

		μ (μ_N)	Q (e mb)	$\langle \mathbf{J}_p^2 \rangle$	$\langle \mathbf{J}_n^2 \rangle$	$\langle \mathbf{L}_p^2 \rangle$
${}^7\text{Li}$	Exp.	3.27	-40(3)			
	AMD	3.15	-27.6	3.31	2.74	2.17
	shell-model limit	3.14	-15.1	3.12	2.61	2.00
${}^9\text{Li}$	Exp.	3.44	-27(1)			
	AMD	3.52	-26.6	3.54	1.07	2.01
	shell-model limit	3.53	-23.3	3.54	1.02	2.00
${}^{11}\text{Li}$	Exp.	3.76	-31(5)			
	AMD	3.79	-29.4	3.75	0.02	2.00
	shell-model limit	3.79	-29.4	3.75	0.00	2.00

in the shell-model limit are almost the same as the original AMD results and reproduce the experimental data. Furthermore we see that the values of $\langle \mathbf{J}_p^2 \rangle$, $\langle \mathbf{J}_n^2 \rangle$, and $\langle \mathbf{L}_p^2 \rangle$ in the shell-model limit are close to those of the original AMD. These results given in Table IV confirm that the angular momentum coupling of nucleons in the shell-model-limit wave functions is similar to the one in the original AMD wave functions, and that the magnetic moments are not sensitive to the spatial clustering but to the cluster coupling of angular momenta. Below we explain the characters of the angular momentum coupling of nucleons in the shell-model-limit wave functions. These characters are of course different from those of the j - j coupling shell-model wave functions when the original AMD wave functions have clustering structure.

In Li isotopes, the magnetic dipole moment in the shell-model limit is determined by the orbit of only the third valence proton in $0p$ orbitals. In ${}^{11}\text{Li}$ with the closed shell for neutron orbits, $\langle \mathbf{J}_p^2 \rangle$ is $3.75 [= \frac{3}{2}(\frac{3}{2} + 1)]$ and $\langle \mathbf{J}_n^2 \rangle$ is 0 in the shell-model limit. This is because the angular momenta of all neutrons couple totally to 0 and only the $0p_{3/2}$ orbit is allowed for the valence proton in the ground state with total spin $J = 3/2$. In this case the magnetic moment μ is as large as the Schmidt value. In ${}^7\text{Li}$, the magnitude 2.61 of $\langle \mathbf{J}_n^2 \rangle$ implies that the component with the nonzero total angular momentum of neutrons is considerably large. It is to be noted that, as we mentioned before, the total angular momentum of neutrons is equal to the total orbital angular momentum of neutrons, namely $\langle \mathbf{J}_n^2 \rangle = \langle \mathbf{L}_n^2 \rangle$. The magnitude 3.12 of $\langle \mathbf{J}_p^2 \rangle$ which is smaller than 3.75 is due to the mixing of $0p_{3/2}$ with $0p_{1/2}$, and this proton angular momentum couples with the nonzero angular momentum of neutrons so as to compose totally $\frac{3}{2}$ spin. This $0p_{1/2}$ mixing of the proton reduces the μ value from the Schmidt value. ${}^9\text{Li}$ has the medium properties between ${}^{11}\text{Li}$ and ${}^7\text{Li}$. In Ta-

ble IV we see that the $0p_{1/2}$ mixing of proton in ${}^9\text{Li}$ is smaller than in ${}^7\text{Li}$ but is larger than in ${}^{11}\text{Li}$ and also that the total angular momentum of neutrons in ${}^9\text{Li}$ is smaller than in ${}^7\text{Li}$ but is larger than in ${}^{11}\text{Li}$.

In summary, it is concluded that the dependence of the μ moments of Li isotopes on the neutron number N originates from the angular momentum coupling correlation caused by the clustering structure. Our results show that the magnetic dipole moments are not sensitive to the relative distance between clusters and that the observed data give little information about the detail of the intercluster relative motion.

In contrast to magnetic dipole moments, electric quadrupole moments are sensitive to the relative distance between clusters. In the following discussion of the N dependence of the electric quadrupole moments, we try to decompose the calculated Q moments into two components; the first component is due to the spatial clustering and the second component is due to the other properties of the AMD wave function including the cluster coupling of angular momenta. We regard the second components as being given by the Q moments calculated by the shell-model-limit wave functions defined above. They are shown in Table IV together with the Q moments of the original AMD calculations and are -15.1, -23.28, and -29.41 e mb for ${}^7\text{Li}$, ${}^9\text{Li}$, and ${}^{11}\text{Li}$, respectively. These values show that the second component becomes smaller as the neutron number N decreases. Such an N dependence can be explained by the argument similar to that made in the above discussion of μ moments about the mixture of proton's $0p_{1/2}$ and $0p_{3/2}$ orbits. The mixing of the $0p_{1/2}$ proton configuration into the $0p_{3/2}$ proton configuration is larger for the isotope with smaller N . It makes the Q moment smaller because of the fact $\langle 0p_{1/2} | Q^{\text{op}} | 0p_{1/2} \rangle = 0$ with Q^{op} standing for the quadrupole moment operator which is a tensor operator

with rank 2. By subtracting this component from the total Q moments (namely the Q moments of the original AMD calculations), we obtain the first component which is due to the spatial clustering. The first component becomes larger as the neutron number decreases from ^{11}Li to ^7Li . Such a dependence of the first component on the neutron number is consistent with the drastic change of clustering structure. Thus the systematic experimental values are qualitatively explained by the structure change seen in our AMD results. The AMD result of the Q moment of ^7Li given in Table IV is not the value obtained by improving the α - t relative wave function mentioned in Sec. III D but the value obtained by using a single parity-projected AMD Slater determinant. Yet the first component of the Q moment is largest for ^7Li than for ^9Li and ^{11}Li .

Finally we analyze the AMD wave functions by calculating the total number of the oscillator quanta. The state with clustering structure usually contains a large amount of the high-lying shell-model orbits and gives a larger expectation value of the total number of the oscillator quanta than the state with the shell-model-like structure. Our analysis is made separately for the neutron and proton orbits. We introduce the value ΔN_p and ΔN_n which stand for the deviation of the proton and neutron orbits in the AMD wave function from those in the simple shell-model wave function:

$$\Delta N_p \equiv \frac{\langle P_{MK}^J \Phi^\pm | N_p^{\text{op}} | P_{MK}^J \Phi^\pm \rangle}{\langle P_{MK}^J \Phi^\pm | P_{MK}^J \Phi^\pm \rangle} - N_p^{\text{min}},$$

$$\Delta N_n \equiv \frac{\langle P_{MK}^J \Phi^\pm | N_n^{\text{op}} | P_{MK}^J \Phi^\pm \rangle}{\langle P_{MK}^J \Phi^\pm | P_{MK}^J \Phi^\pm \rangle} - N_n^{\text{min}},$$

where N_p^{op} and N_n^{op} are the oscillator quantum number operators and N_p^{min} and N_n^{min} are the minimum values of oscillator quantum numbers given by the simple shell model for protons and neutrons, respectively. The state with shell-model-like structure should have small ΔN values, while the state with well-developed clustering structure should have large values of ΔN . Figure 13 shows ΔN_p and ΔN_n of Be and Li isotopes. In each of Li and Be, ΔN_n is quite large with the neutron number $N = 4$ and it decreases as N increases. ΔN_n has the smallest value 0 in the nuclei with neutron magic number $N = 8$, and it increases again as N increases in the region with $N \geq 8$. Such a neutron-number dependence of ΔN_n directly reflects the shell effect of neutron orbits. We see that the ΔN_p for proton orbits has almost the same dependence on the neutron number N . A very interesting fact is that the shell effect of neutron orbits is reflected on the proton orbits which have a close relation to the electric and magnetic properties. What causes the clustering structure is the feature that the orbits of protons change following the change of neutron orbits due to the mean field made by neutrons. This feature is fundamental for the mechanism of the N dependence of the electromagnetic data.

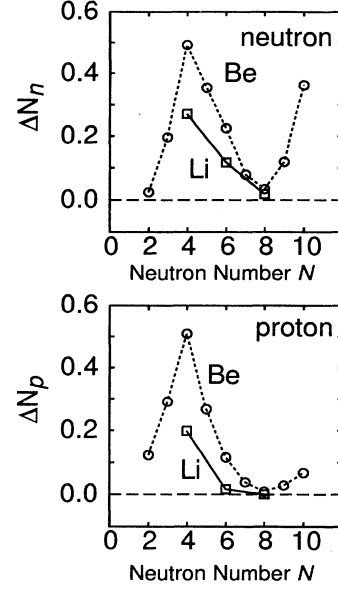


FIG. 13. Deviation of the total number of oscillator quanta of the AMD state from that of the simple shell-model wave function. Deviation for proton and neutron orbits is shown as ΔN_p and ΔN_n which are defined in the text.

VI. SUMMARY

Structure of odd-even and even-even Li and Be isotopes has been studied with the AMD (antisymmetrized molecular dynamics) method. Energy spectra and other quantities have been reproduced well. AMD is the first framework which has succeeded in describing the systematic data of electric and magnetic properties such as magnetic dipole moments and electric quadrupole moments in a wide range of the light isotopes covering both neutron-rich nuclei and ordinary nuclei. In our AMD framework we never need to introduce the effective charge but we use only the bare charge. This is because the AMD wave function automatically describes the drastic change of proton orbits from clustering structure to the shell-model-like structure.

It has been ascertained that the well-developed clustering structure appears in ^7Li and ^8Be without assuming the existence of any kind of clusters. The clustering structure seen in the $N \approx Z$ region in Li and Be isotopes gets weaker as the neutron number N increases, and changes toward the shell-model-like structure in ^{11}Li and ^{12}Be with the neutron magic number $N = 8$. It is suggested that possible clustering aspects appear again in neutron-richer Be isotopes like ^{14}Be . Thus the drastic change of the structure as a function of the neutron number N has been explained in connection with the shell effect of neutron orbits.

N dependence of the experimental data of the electric and magnetic properties has been discussed in relation with the structure change. It has been shown that the N dependence of the observed data for Li isotopes has been

described as the direct reflection of the drastic structure change between clustering structure and shell-model-like structure. Strictly speaking, the N dependence of μ moments has been explained in terms of the cluster coupling of angular momenta, namely the angular momentum coupling correlation of nucleons which is caused by clustering structure. In the case of Q moments which are more sensitive to the nuclear deformation, N dependence has been explained in terms of two effects of clustering structure; one is due to the spatial clustering and the other is due to the cluster coupling of angular momenta.

We have also tried to make an extended AMD calculation with a superposition of two parity-projected Slater determinants for Be isotopes. It has been found that in most cases the first Slater determinant is almost the same as the wave function of the simple AMD calculation and is the major component in the total wave function. The second Slater determinant is not so large a component and is found to give an improvement of the description of the state expressed with the major Slater determinant. Therefore the AMD calculation by the use of the single parity-projected Slater determinant is approved to be sufficient for describing leading properties of nuclear structure. In some nuclei, however, two Slater determinants have been found to describe two channels of different clustering configurations.

In some cases, further improvements of AMD wave functions have been found to be important. For example, the improved AMD calculation which adopts a superposition of several Slater determinants has proved to be necessary for the lowest $J^\pm = \frac{1}{2}^+$ state in ^{11}Be in order to describe the behavior of the valence neutron better than the simple AMD and also in order to give better results about the energy and radius, even though the reproduction is not sufficient yet. We have shown that the calculations by a superposition of several AMD wave

functions reproduce the long tail of the wave functions in two cases, the neutron halo in ^{11}Be and the $\alpha + t$ relative motion of ^7Li .

We have found that the density dependence of the effective interaction plays important roles in getting better agreements with the experimental data of radii, electric quadrupole moments, and low excitation energies of non-normal parity states. These quantities are sensitive to the density distribution. Our results show that the density dependence of the effective interaction is indispensable in obtaining better reproduction of density distribution in a wide range of isotopes.

It has been suggested that many kinds of interesting structure exist in excited states of neutron-rich nuclei. Though some low excited states have been described by angular momentum projection from the intrinsic state obtained with AMD, the detailed structure of excited states are to be investigated by using the constrained frictional cooling method in the AMD approach. Such kind of study has been already made in ^{20}Ne for the investigation of the structure change in yrast states [27].

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