Relativistic nuclear Hamiltonians

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Relativistic Hamiltonians are defined as the sum of relativistic one-body kinetic energies, two- and many-body interactions and their boost corrections. We review the calculation of the boost correction of the two-body interaction from commutation relations of the Poincaré group and show that its important terms can be easily understood from classical relativistic mechanics. The boost corrections for scalar- and vector-meson-exchange interactions, obtained from relativistic field theory, are shown to be in agreement with the results of the classical calculation. These boost corrections are also shown to be necessary to reproduce the known results of relativistic mean-field theories. We conclude with comments on the relativistic boost operator for the wave function of a nucleus. Some of the results presented in this article are known. We hope that a better understanding of relativistic Hamiltonians and their relation to relativistic field theory is obtained by putting them together with the new relations.

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I. INTRODUCTION

The concept of an interparticle potential has proved to be extremely useful in the study of nonrelativistic many-particle systems at low energies. The degrees of freedom associated with the fields coupled to the particles, as well as the internal degrees of freedom, if any, of the particles, are eliminated with these potentials so that one can focus on the most important degrees of freedom. *Ab initio* calculations of the interparticle potentials are nontrivial, particularly when the particles are composite like nucleons or rare gas atoms. In practice the potentials are parametrized within a suitable theoretical framework and fitted to observed data. Nonrelativistic Hamiltonians of the type

$$H_{\rm NR} = \sum_i \frac{p_i^2}{2m_i} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \cdots \qquad (1.1)$$

have been used in many contexts. In nuclear physics, for example, the ground and low-energy nuclear states are described by eigenfunctions $\Psi_I(x_1, x_2, \ldots, x_A)$ of $H_{\rm NR}$, where x_i denotes the position $\mathbf{r}_i$, spin $\boldsymbol{\sigma}_i$, and isospin $\boldsymbol{\tau}_i$ of the ith nucleon. Solving the many-body Schrödinger equation

$$H_{\rm NR}\,\Psi_I = E_I \Psi_I \qquad (1.2)$$

is a difficult problem; however, it can now be solved with variational [1] and Green's function [2] Monte Carlo (VMC and GFMC) methods for the ground and some low-energy excited states of up to six nucleons.

Bakamjian and Thomas [3] and Foldy [4] showed many years ago that the concept of potentials can also be useful in describing many-body systems in a relativistically covariant fashion. The relativistic Hamiltonian may be written as

$$H_R = \sum_i \sqrt{m_i^2 + p_i^2} \;+\; \sum_{i<j} [\tilde{v}_{ij} + \delta v_{ij}(\mathbf{P}_{ij})] + \sum_{i<j<k} \left[\tilde{V}_{ijk} + \delta V_{ijk}(\mathbf{P}_{ijk})\right] \;+\; \cdots, \qquad (1.3)$$

where $\tilde{v}_{ij}$ are two-body potentials in the "rest frame" of particles i and j (i.e., the frame in which their total momentum vanishes):

$$\mathbf{P}_{ij} = \mathbf{p}_i + \mathbf{p}_j = 0. \qquad (1.4)$$

Similarly $\tilde{V}_{ijk}$ is the three-body potential in the frame in which

$$\mathbf{P}_{ijk} = \mathbf{p}_i + \mathbf{p}_j + \mathbf{p}_k = 0. \qquad (1.5)$$

The $\delta v_{ij}(\mathbf{P}_{ij})$ and $\delta V_{ijk}(\mathbf{P}_{ijk})$ are called "boost interactions" and depend upon the total momentum of the interacting particles. Obviously,

$$\delta v_{ij}(\mathbf{P}_{ij} = 0) = \delta V_{ijk}(\mathbf{P}_{ijk} = 0) = 0. \qquad (1.6)$$

Only the positive value of $\sqrt{m_i^2 + p_i^2}$ is considered in H_R.

The interaction $\tilde{v}$ is determined by the fields and the internal structure associated with the interacting particles, while $\delta v(\mathbf{P})$ is related to $\tilde{v}$ by relativistic covariance. Krajcik and Foldy [5] formally calculated $\delta v(\mathbf{P})$ to all orders in $P^2/4m^2$, though we will retain only the leading contribution of order $P^2/4m^2$ in this study. An elegant equation for the minimal first order $\delta v(\mathbf{P})$ is found to be [6]

$$\delta v(\mathbf{P}) = -\frac{P^2}{8m^2}\tilde{v} + \frac{1}{8m^2}\,[\,\mathbf{P}\cdot\mathbf{r}\mathbf{P}\cdot\boldsymbol{\nabla},\tilde{v}\,] + \frac{1}{8m^2}\,[\,(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\times\mathbf{P}\cdot\boldsymbol{\nabla},\tilde{v}\,], \qquad (1.7)$$

where the subscripts ij of $\tilde{v}$, $\mathbf{P}$, $\mathbf{r}$ and $\boldsymbol{\nabla}$ have been suppressed for brevity. A brief explanation of this equation is given in Sec. II for completeness.

Present interest in the relativistic Hamiltonian (1.3) stems from the fact that its ground states can be studied with the variational Monte Carlo method. Initially only the ^{3}H and ^{4}He ground states were studied [7], but the methods developed there can also be used to study heavier nuclei like ^{16}O with cluster expansions [8]. Moreover, these methods can also be used to calculate the ground states of ^{2}H, ^{3}H, ^{3}He, ^{4}He, ^{6}He, and ^{6}Li exactly, up to order $P^2/4m^2$, with the Green's function Monte Carlo method [9]. The results obtained in Ref. [7] show that the average value of $P^2/4m^2$ is rather small in nuclei; therefore it is certainly useful to have exact results to this order. It is very likely that higher-order contributions and $\delta V(\mathbf{P}_{ijk})$ contributions are much smaller. For the sake of brevity we will not discuss $\delta V(\mathbf{P}_{ijk})$; a two-pion exchange term in it has been studied by Coon and Friar [10].

Before commencing our pedagogical discussion, it is necessary to categorize the various effects that we will be treating. It is convenient to separate relativistic effects in nuclear physics into three categories: (A) on the interaction $\tilde{v}_{ij}$ of two nucleons in their center of mass (c.m.) frame; (B) on the interaction $\tilde{v}_{ij}+\delta v(\mathbf{P}_{ij})$ of nucleons i and j with a total momentum $\mathbf{P}_{ij}$ in the c.m. frame of the whole nucleus; and (C) on the motion of the nucleus as a whole.

The first effect (A) depends essentially on the nature of the interaction; for example, when $\tilde{v}_{ij}$ is mediated by mesons the relativistic corrections to it depend upon the type, i.e., scalar, vector, etc., of the exchanged meson. Since all realistic models of $\tilde{v}_{ij}$ are obtained by fitting experimental data, they contain relativistic effects in some form. The key question here is how to choose the theoretical form of $\tilde{v}_{ij}$, used to fit the data, such that they are correctly represented.

There is no further model dependence in the second effect (B). The $\delta v(\mathbf{P}_{ij})$ depends only upon $\tilde{v}_{ij}$ and can be obtained from it [5,6]. Many aspects of the relation between $\delta v(\mathbf{P}_{ij})$ and $\tilde{v}_{ij}$ can be understood from classical relativistic mechanics as discussed in Ref. [7] and further elaborated in Sec. III. This allows terms in $\delta v(\mathbf{P}_{ij})$ to be classified as those coming from the relativistic kinematics, Lorentz contraction, Thomas precession, and quantum effects, respectively.

The relativistic effect (C) on the motion of the nucleons as a whole is important because in scattering experiments the struck nucleus recoils and phenomena such as Lorentz contraction, Thomas precession, and retardation ensue and modify the transition matrix elements. The effects (B) and (C) are intimately related [6], and early work [11,12] on relativistic corrections emphasized (C).

Relativistic Hamiltonians are not as widely known as, for example, the relativistic field theory, though many researchers [13,14] have utilized them. In order to study the relation between relativistic Hamiltonians and relativistic field theory we consider interactions between point Dirac particles coupled to scalar and vector fields. The one-meson-exchange scattering of two particles, commonly studied with Feynman's method, is discussed in Sec. IV. The $\delta v(\mathbf{P})$ is necessary to obtain correct scattering amplitudes in frames in which $\mathbf{P}\neq 0$.

In the mean-field limit the problem of infinite matter consisting of Dirac particles coupled to scalar and vector fields has been solved by Serot and Walecka [15]. In Sec. V we study this problem with a relativistic Hamiltonian using the Hartree approximation corresponding to the mean-field limit. This study demonstrates the importance of consistently treating the relativistic effects in $\tilde{v}_{ij}$, $\delta v_{ij}(\mathbf{P}_{ij})$, $\tilde{V}_{ijk}$, etc.

In Sec. VI we discuss relativistic Hamiltonians for nuclei and suggest that they should contain the relativistic correction for the one-pion-exchange contribution to $\tilde{v}_{ij}$ that has been neglected in many realistic models of v_{ij}. The concluding Sec. VII also treats, for the sake of completeness, the motion of the nucleus as a whole.

Many but not all of the results given in this pedagogical article have been published in disparate places; however, no comprehensive discussion of them exists. In view of the recent successes in calculating the behavior of light nuclei from realistic Hamiltonians, such a discussion may be both useful and timely.

II. CALCULATION OF $\delta v(\mathbf{P})$

Equation (1.7) for $\delta v(\mathbf{P})$ has been obtained by Foldy [4] and Friar [6] using general principles of relativistic quantum mechanics as illustrated here. Consider a system of two particles (1 and 2), each with spin $\mathbf{s}$ and mass m. When the momentum and angular momentum generators of the Poincaré group are chosen in the conventional fashion, they are independent of the interaction:

$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2, \qquad (2.1)$$

$$\mathbf{J} = (\mathbf{r}_1\times\mathbf{p}_1) + \mathbf{s}_1 + (\mathbf{r}_2\times\mathbf{p}_2) + \mathbf{s}_2, \qquad (2.2)$$

while the Hamiltonian H and the boost $\mathbf{K}$ will have interaction terms:

$$H = H_0 + H_I, \qquad (2.3)$$

$$\mathbf{K} = \mathbf{K}_0 + \mathbf{K}_I. \qquad (2.4)$$

These generators must obey the commutation relations of the Poincaré group:

$$[P_i,P_j] = [P_i,H] = [J_i,H] = 0, \qquad (2.5)$$

$$[J_i,X_j] = i\epsilon_{ijk}X_k \quad \text{for } \mathbf{X}=\mathbf{J},\mathbf{P},\mathbf{K}, \qquad (2.6)$$

$$[K_i,K_j] = -i\epsilon_{ijk}J_k, \qquad (2.7)$$

$$[K_i,P_j] = iH\delta_{ij}, \qquad (2.8)$$

$$[K_i,H] = iP_i. \qquad (2.9)$$

These relations are obviously satisfied by the H_0 and $\mathbf{K}_0$ of the noninteracting system. The last two commutators in (2.5) require that H_I be translationally and rotationally invariant, while the commutators in (2.6) for $\mathbf{X} = \mathbf{K}$ require that $\mathbf{K}_I$ be a spatial vector. Subtracting the contributions of the noninteracting parts from commutators (2.7)–(2.9) gives

$$[K_{I,i}, K_{0,j}] + [K_{0,i}, K_{I,j}] + [K_{I,i}, K_{I,j}] = 0, \quad (2.10)$$

$$[K_{I,i}, P_j] = iH_I\delta_{ij}, \quad (2.11)$$

$$[\mathbf{K}_0, H_I] = [H_0, \mathbf{K}_I] + [H_I, \mathbf{K}_I]. \quad (2.12)$$

It is convenient to expand H and $\mathbf{K}$ in powers of $1/m^2$. The expansions for H_0 and $\mathbf{K}_0$ are well known [4]:

$$H_0 = 2m + \frac{1}{2m}\left(p_1^2 + p_2^2\right) + \cdots, \quad (2.13)$$

$$\mathbf{K}_0 = t\mathbf{P} + 2m\mathbf{R} + \frac{1}{2m}\left[\frac{1}{2}\left(\mathbf{r}_1 p_1^2 + p_1^2\mathbf{r}_1 + \mathbf{r}_2 p_2^2 + p_2^2\mathbf{r}_2\right) - \mathbf{s}_1 \times \mathbf{p}_1 - \mathbf{s}_2 \times \mathbf{p}_2\right] + \cdots. \quad (2.14)$$

Note that $t\mathbf{P}$ and $2m\mathbf{R}$ are of the same order and the ellipsis represents terms of order $1/m^3$ or higher. The leading term of H_I, denoted by v, is assumed to be of order $1/m$ since in systems like nuclei the interaction energy and the nonrelativistic kinetic energy are of similar magnitude. Thus

$$H_I = v + \delta v + \cdots, \quad (2.15)$$

where δv is of order v/m^2 or $1/m^3$, and the ellipsis represents terms of order $1/m^5$ or higher. We also assume that the leading term v is independent of $\mathbf{P}$. The commutator (2.11) is then minimally satisfied by taking

$$\mathbf{K}_I = v\mathbf{R} + O\left(\frac{1}{m^3}\right) \text{ and higher terms.} \quad (2.16)$$

The leading terms of Eq. (2.12) are of order $1/m^2$. Retaining only these we get

$$2m\,[\mathbf{R}, \delta v] = \frac{1}{2m}\left[\left(p_1^2 + p_2^2\right), v\mathbf{R}\right] + \frac{1}{4m}[v, (\mathbf{r}_1 p_1^2 + p_1^2\mathbf{r}_1 + \mathbf{r}_2 p_2^2 + p_2^2\mathbf{r}_2 - \boldsymbol{\sigma}_1 \times \mathbf{p}_1 - \boldsymbol{\sigma}_2 \times \mathbf{p}_2)], \quad (2.17)$$

where $\boldsymbol{\sigma} = 2\mathbf{s}$ (i.e., the σ_i are Pauli matrices for spin 1/2 particles). Evaluating the commutators one obtains the basic equation for δv:

$$[\mathbf{R}, \delta v] = -\frac{i}{4m^2}v\mathbf{P} - \frac{1}{4m^2}[\mathbf{r}\mathbf{P}\cdot\mathbf{p}, v] + \frac{1}{16m^2}[(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \times \mathbf{P}, v] + \frac{1}{8m^2}[(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \times \mathbf{p}, v], \quad (2.18)$$

where $\mathbf{p}$ is the relative momentum. This equation cannot determine δv uniquely. We can express

$$\delta v = \delta v' + \delta v(\mathbf{P}), \quad (2.19)$$

where $\delta v'$ commutes with $\mathbf{R}$ (i.e., it is of order $1/m^3$ but independent of P). The $\tilde{v}$ is defined as

$$\tilde{v} = v + \delta v' + \text{all higher-order terms independent of } P, \quad (2.20)$$

and obtained from experiment using theoretical models. Equation (2.18) can be used to determine $\delta v(\mathbf{P})$ from the $\tilde{v}$, and (1.7) provides the simplest solution of (2.18).

It is sufficient to use the v of order $1/m$ in Eq. (1.7) to obtain $\delta v(\mathbf{P})$ up to order $1/m^3$. In some cases this v is a spin-independent function of r, and

$$\delta v(\mathbf{P}) = -\frac{P^2}{8m^2}v(r) + \frac{1}{8m^2}\mathbf{P}\cdot\mathbf{r}\mathbf{P}\cdot\boldsymbol{\nabla}v(r) + \frac{1}{8m^2}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \times \mathbf{P}\cdot\boldsymbol{\nabla}v(r). \quad (2.21)$$

As discussed in the next section, the above three terms of $\delta v(\mathbf{P})$ can be attributed to the relativistic energy-momentum relation, Lorentz contraction, and Thomas precession, respectively.

The boost operator $\mathbf{K}_I$ can have additional terms, denoted by $\mathbf{w}$ in [4,6], which commute with $\mathbf{P}$. These make unitary transformations of the relativistic Hamiltonian, and can be chosen for convenience. The present choice, $\mathbf{w} = \mathbf{0}$, is motivated by the desire to maintain correspondence with classical relativistic mechanics via Eq. (2.21), and is suitable to study energies of nuclear states. When $\mathbf{w} \neq \mathbf{0}$ the $\delta v(\mathbf{P})$ has an additional term $-i[\chi_V, H_0 + v]$ where χ_V depends upon $\mathbf{P}$ and $\mathbf{w}$. The contribution of $\delta v(\mathbf{P})$ to the energy eigenvalue E_I up to order $1/m^3$ is given by $\langle\Psi_I|\delta v(\mathbf{P})|\Psi_I\rangle$, where $|\Psi_I\rangle$ are eigenstates of $(H_0 + v)$. Obviously $[\chi_V, H_0 + v]$ gives zero contribution to E_I for χ_V obtained from any $\mathbf{w}$. When studying reactions, however, special attention must be paid to such terms (see Sec. VII and Ref. 10).

III. THE $\delta v(\mathbf{P})$ IN CLASSICAL RELATIVISTIC MECHANICS

The first two terms of Eq. (2.21) for $\delta v(\mathbf{P})$ were obtained using classical considerations in Ref. [7]. In relativistic classical mechanics two particles at rest a distance $\mathbf{r}_0$ apart have energy

$$E_0 = 2m + \tilde{v}(\mathbf{r}_0) \quad (3.1)$$

in their rest frame by definition of $\tilde{v}$. In a frame in which these particles are moving with momentum $\mathbf{P}$, their energy is given by

$$E_P = 2\left(m^2 + \frac{P^2}{4}\right)^{1/2} + \tilde{v}(\mathbf{r}) + \delta v(\mathbf{P}, \mathbf{r}) \quad (3.2)$$

by definition of $\delta v(\mathbf{P})$. Here $\mathbf{r}$ is the distance in the moving frame,

$$\mathbf{r} = \mathbf{r}_0 - \frac{(\mathbf{P}\cdot\mathbf{r}_0)\,\mathbf{P}}{2E_P^2}, \quad (3.3)$$

due to Lorentz contraction. Now E_P is also given by

$$E_P = \left(E_0^2 + P^2\right)^{1/2}. \tag{3.4}$$

From Eqs. (3.2)–(3.4) we get

$$\delta v(\mathbf{P},\mathbf{r}) = -\frac{P^2}{8m^2}\tilde{v}(\mathbf{r}) + \frac{1}{8m^2}\mathbf{P}\cdot\mathbf{r}\mathbf{P}\cdot\nabla\tilde{v}(\mathbf{r}). \tag{3.5}$$

Its first term is due to the relativistic relation (3.4) between E_P and E_0, and the second due to Lorentz contraction (3.3). These terms are respectively denoted by $\delta v_{\rm RE}(\mathbf{P},\mathbf{r})$ and $\delta v_{\rm LC}(\mathbf{P},\mathbf{r})$ in Ref. [16]. When the interacting particles are spinless (3.5) gives their entire $\delta v(\mathbf{P})$.

When the interacting particles have spin the last term of (2.21) is generated by Thomas precession [17]. The precession of the spin $\mathbf{s}_1$ in the moving frame is given by $-\nabla\tilde{v}(r)\times\mathbf{P}/4m^2$ up to order $1/m^2$. Thus the Thomas precession potential for particle 1 is

$$-\frac{1}{2}\boldsymbol{\sigma}_1\cdot\frac{\nabla\tilde{v}(r)\times\mathbf{P}}{4m^2} = \frac{1}{8m^2}\boldsymbol{\sigma}_1\cdot\mathbf{P}\times\nabla\tilde{v}. \tag{3.6}$$

In the moving frame both particles have the same velocity, but their accelerations are equal and opposite. Thus the Thomas precession potential for the second particle is $-\boldsymbol{\sigma}_2\cdot\mathbf{P}\times\nabla\tilde{v}/8m^2$, giving the total

$$\delta v_{\rm TP}(\mathbf{P},\mathbf{r}) = \frac{1}{8m^2}\left(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2\right)\times\mathbf{P}\cdot\nabla\tilde{v} \tag{3.7}$$

in agreement with the last term of (2.21).

The general $\delta v(\mathbf{P})$ given by Eq. (1.7) has additional terms containing $[(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2),\tilde{v}]$ and $[\mathbf{r},\tilde{v}]$ when $\tilde{v}$ depends upon the spins and the relative momentum. These do not have analogues in classical mechanics, and some of them are discussed in Sec. VI in the context of the one-pion-exchange interaction. They are denoted by $\delta v_{\rm QM}(\mathbf{P},\mathbf{r})$ in Ref. [16]. The contribution of $\delta v_{\rm TP}(\mathbf{P},\mathbf{r})$ to the binding energy of ^{3}H and ^{4}He has been found to be rather small, and that of $\delta v_{\rm QM}(\mathbf{P},\mathbf{r})$ is even smaller [16]. For example, the contributions of $\delta v_{\rm RE}$, $\delta v_{\rm LC}$, $\delta v_{\rm TP}$, and $\delta v_{\rm QM}$ to the energy of the triton are found to be 0.23(2), 0.10(1), 0.016(2), and −0.004(2) MeV, respectively, in Refs. [7] and [16].

IV. MESON-EXCHANGE POTENTIALS

The one-meson-exchange scattering amplitudes, from an initial two-nucleon state $\mathbf{k}_1$, $\mathbf{k}_2$ to final state $\mathbf{k}_1'$, $\mathbf{k}_2'$, depend upon the momentum transfer $\mathbf{q}$,

$$\mathbf{q} = \mathbf{k}_1' - \mathbf{k}_1 = \mathbf{k}_2 - \mathbf{k}_2', \tag{4.1}$$

the relative momenta,

$$\mathbf{p} = \frac{1}{2}\left(\mathbf{k}_1 - \mathbf{k}_2\right), \tag{4.2}$$

$$\mathbf{p}' = \frac{1}{2}\left(\mathbf{k}_1' - \mathbf{k}_2'\right) = \mathbf{p} + \mathbf{q}, \tag{4.3}$$

and the total momentum

$$\mathbf{P} = \mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_1' + \mathbf{k}_2'. \tag{4.4}$$

They can be easily calculated for Dirac particles coupled to a scalar field ϕ of mass μ_S and

$$H_{\rm int} = G_S\bar{\psi}\psi\phi, \tag{4.5}$$

or a vector field V_μ of mass μ_V and

$$H_{\rm int} = G_V\bar{\psi}\gamma^\mu\psi V_\mu \tag{4.6}$$

using well known Feynman diagram rules [18].

The amplitudes for scalar and vector meson exchange are denoted by $v_X(\mathbf{q},\mathbf{p},\mathbf{P})$, $X = S$ and V, respectively. They are expressed as

$$v_X(\mathbf{q},\mathbf{p},\mathbf{P}) = \tilde{v}_X(\mathbf{q},\mathbf{p}) + \delta v_X(\mathbf{P},\mathbf{q},\mathbf{p}) \tag{4.7}$$

to study relations between $\tilde{v}$ and $\delta v(\mathbf{P})$. The $\tilde{v}_X$ independent of m is the familiar Yukawa amplitude denoted by v_X^0:

$$v_S^0(q) = -\frac{G_S^2}{q^2+\mu_S^2}, \tag{4.8}$$

$$v_V^0(q) = \frac{G_V^2}{q^2+\mu_V^2}. \tag{4.9}$$

The $\tilde{v}_X$ containing all terms of order $1/m^2$ is also well known:

$$\tilde{v}_S(\mathbf{q},\mathbf{p}) = v_S^0(q)\left[1 - \frac{(\mathbf{p}+\mathbf{p}')^2}{4m^2} - \frac{i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{q}\times\mathbf{p}}{4m^2}\right], \tag{4.10}$$

$$\tilde{v}_V(\mathbf{q},\mathbf{p}) = v_V^0(q)\left[1 + \frac{(\mathbf{p}+\mathbf{p}')^2}{4m^2} - \frac{q^2}{4m^2} - \frac{\boldsymbol{\sigma}_1\times\mathbf{q}\cdot\boldsymbol{\sigma}_2\times\mathbf{q}}{4m^2} + \frac{3i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{q}\times\mathbf{p}}{4m^2}\right], \tag{4.11}$$

and $\delta v_X(\mathbf{P},\mathbf{q},\mathbf{p})$, up to order $1/m^2$, is given by

$$\delta v_X(\mathbf{P},\mathbf{q}) = v_X^0(q)\left[\frac{(\mathbf{P}\cdot\mathbf{q})^2}{4m^2\left(q^2+\mu_X^2\right)} - \frac{P^2}{4m^2} - \frac{i(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{q}\times\mathbf{P}}{8m^2}\right], \tag{4.12}$$

for both $X = S$ and V.

The first term of the above δv_X comes from the energy

$$\omega^2 = \frac{(\mathbf{P}\cdot\mathbf{q})^2}{4m^2} \tag{4.13}$$

carried by the exchanged meson. Up to order $1/m^2$, the Dirac spinors are given by

$$u(k) = \left(1 - \frac{k^2}{8m^2}\right)\begin{pmatrix}\chi \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{k}}{2m}\chi\end{pmatrix}, \tag{4.14}$$

where χ are Pauli spinors. Their normalizations give a contribution of $-v_X^0(q)P^2/8m^2$ to the δv_X. Only this contribution is considered in the earlier work by Hajduk and Sauer [14], and it accounts for half of the $P^2/4m^2$ term in Eq. (4.12). The other half of the $P^2/4m^2$ term and the last term in δv_X have different origins in the field theories for scalar and vector meson exchange.

The $\delta v_X(\mathbf{P},\mathbf{q})$ can as well be obtained from the $v_X^0(q)$ with the general Eq. (1.7). Since the $v_X^0(q)$ for scalar and vector meson exchange [Eqs. (4.8) and (4.9)] is independent of spins, Eq. (1.7) reduces to the simpler Eq. (2.21). Using that equation for $\delta v_X(\mathbf{P},\mathbf{r})$ we obtain

$$\begin{aligned}\delta v_X(\mathbf{P},\mathbf{q}) &= \int e^{-i\mathbf{q}\cdot\mathbf{r}}\delta v_X(\mathbf{P},\mathbf{r})\, d^3\mathbf{r}\\ &= \frac{1}{8m^2}\int e^{-i\mathbf{q}\cdot\mathbf{r}}[-P^2 + \mathbf{P}\cdot\mathbf{r}\mathbf{P}\cdot\boldsymbol{\nabla}\\ &\quad + (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)\times\mathbf{P}\cdot\boldsymbol{\nabla}]\, v_X^0(r)\, d^3\mathbf{r}. \qquad (4.15)\end{aligned}$$

Integrating the second term by parts gives

$$\begin{aligned}&\int e^{-i\mathbf{q}\cdot\mathbf{r}}\,(\mathbf{P}\cdot\mathbf{r})\,(\mathbf{P}\cdot\boldsymbol{\nabla})\,v_X^0(r)\,d^3\mathbf{r}\\ &\quad = -P^2 v_X^0(q) - \mathbf{P}\cdot\mathbf{q}\,\mathbf{P}\cdot\boldsymbol{\nabla}_q\, v_X^0(q)\\ &\quad = -P^2 v_X^0(q) + 2\,\frac{(\mathbf{P}\cdot\mathbf{q})^2}{q^2+\mu_X^2}\, v_X^0(q)\,. \qquad (4.16)\end{aligned}$$

Thus the first two terms of (4.15) together give the first two terms of (4.12), while the last term of each is in agreement.

In this context Eq. (1.7) appears to be more general. The $\tilde{v}_X$ depends upon the nature of the exchanged meson, but the relation (1.7) between δv_X and $\tilde{v}_X$ is independent of the nature of X. As a matter of fact we expect Eq. (1.7) to be useful to determine the $\mathbf{P}$ dependence of the interaction between two relativistic billiard balls dominated by their structural overlap, rather than boson exchange.

V. RELATIVISTIC MEAN-FIELD THEORY

The problem of extended uniform matter consisting of Dirac particles interacting with scalar and vector fields [Eqs. (4.5) and (4.6)] has been solved by Serot and Walecka [15] in the mean-field limit. The energy density of this matter is given by

$$\begin{aligned}\mathcal{E} &= \frac{G_V^2}{2\mu_V^2}\,\rho^2 + \frac{\mu_S^2}{2G_S^2}\,(m-m^*)^2\\ &\quad + \frac{\gamma}{(2\pi)^3}\int_0^{k_F}(k^2+m^{*2})^{1/2}\,d^3\mathbf{k}, \qquad (5.1)\end{aligned}$$

where ρ is the density, γ is the degeneracy of Dirac particles, and k_F is their Fermi momentum. The effective mass m^* is given by

$$m^* = m - G_S\,\phi_0, \qquad (5.2)$$

where ϕ_0 is the average value of the scalar field. Minimizing $\mathcal{E}$ with respect to variations in ϕ_0 gives the transcendental self-consistency equation

$$m^* = m - \frac{G_S^2}{\mu_S^2}\,\frac{\gamma}{(2\pi)^3}\int_0^{k_F}\frac{m^*}{(k^2+m^{*2})^{1/2}}\,d^3\mathbf{k}\,, \qquad (5.3)$$

which is solved by expanding m^* in powers of k_F. With $\gamma=4$ appropriate for nuclear matter, we obtain

$$\begin{aligned}m^* &= m - \frac{G_S^2\,\rho}{\mu_S^2}\Bigg[1 - \frac{3}{10}\frac{k_F^2}{m^2} + \frac{9}{56}\frac{k_F^4}{m^4} - \frac{5}{48}\frac{k_F^6}{m^6}\\ &\quad + \frac{105}{1408}\frac{k_F^8}{m^8} - \frac{3}{5}\frac{\rho\,k_F^2}{m^3}\frac{G_S^2}{\mu_S^2}\\ &\quad + \frac{144}{175}\frac{\rho\,k_F^4}{m^5}\frac{G_S^2}{\mu_S^2} - \frac{9}{10}\frac{\rho^2\,k_F^2}{\mu_S^4}\left(\frac{G_S^2}{\mu_S^2}\right)^2\Bigg], \qquad (5.4)\end{aligned}$$

up to order k_F^{11}, noting that

$$\rho = \frac{\gamma}{6\pi^2}k_F^3. \qquad (5.5)$$

The energy per particle, given by $\mathcal{E}/\rho$, is obtained as a power series in k_F by substituting the expansion for m^* in Eq. (5.1).

$$\begin{aligned}\mathcal{E}/\rho \;=\; & m + \frac{3}{10}\frac{k_F^2}{m} + \frac{G_V^2\,\rho}{2\mu_V^2} - \frac{G_S^2\,\rho}{2\mu_s^2} + \left[-\frac{3}{56}\frac{k_F^4}{m^3} + \frac{k_F^6}{48\,m^5} - \frac{15}{1408}\frac{k_F^8}{m^7} + \frac{21}{3328}\frac{k_F^{10}}{m^9} + \cdots\right]\\ & + \frac{G_S^2}{\mu_S^2}\,\frac{\rho}{m}\left[\frac{3}{10}\frac{k_F^2}{m} - \frac{36}{175}\frac{k_F^4}{m^3} + \frac{16}{105}\frac{k_F^6}{m^5} - \frac{64}{539}\frac{k_F^8}{m^7} + \cdots\right] + \left(\frac{G_S^2}{\mu_S^2}\,\frac{\rho}{m}\right)^2\left[\frac{3}{10}\frac{k_F^2}{m} - \frac{351}{700}\frac{k_F^4}{m^3} + \cdots\right]\\ & + \left(\frac{G_S^2}{\mu_S^2}\,\frac{\rho}{m}\right)^3\left[\frac{3}{10}\frac{k_F^2}{m} - \cdots\right], \qquad (5.6)\end{aligned}$$

where the ellipsis denotes terms of order k_F^{12} or higher.

We can attempt to obtain this solution starting from a relativistic Hamiltonian appropriate for this system, using the Hartree approximation equivalent to the mean-field approximation. In the Hartree approximation for uniform matter only $q=0$ diagonal interactions contribute ($k_i,k_j \to k_i,k_j$). Therefore the Hamiltonian required for the Hartree approximation is much simpler than that containing the complete $\tilde{v}_X$ and δv_X given by Eqs. (4.10)–(4.12). It is given by

H_R (for Hartree, up to order $1/m^2$)

$$= \sum_i \sqrt{m^2 + k_i^2} + \frac{1}{\Omega} \sum_{i<j} \left\{ -\frac{G_S^2}{\mu_S^2} \left[1 - \frac{(\mathbf{k}_i - \mathbf{k}_j)^2}{4m^2} \right] + \frac{G_V^2}{\mu_V^2} \left[1 + \frac{(\mathbf{k}_i - \mathbf{k}_j)^2}{4m^2} \right] - \frac{(\mathbf{k}_i + \mathbf{k}_j)^2}{4m^2} \left(-\frac{G_S^2}{\mu_S^2} + \frac{G_V^2}{\mu_V^2} \right) \right\}. \quad (5.7)$$

The first two interaction terms come from $\tilde{v}_S$ and $\tilde{v}_V$ [Eqs. (4.10) and (4.11)] and the last from $\delta v_S(\mathbf{P})$ and $\delta v_V(\mathbf{P})$ [Eq. (4.12)]. The factor $1/\Omega$ is from normalization in a box of volume Ω.

The Hartree energy obtained with the complete H_R should be identical to that given by Eq. (5.6) obtained with relativistic mean-field theory. The first row of (5.6) is just the energy obtained in the nonrelativistic limit; it contains the contribution of interactions independent of m in the H_R. The second row gives the relativistic correction to the kinetic energy of a Fermi gas, which is contained in H_R. All the subsequent terms in (5.6) are relativistic corrections to the interaction energy.

There are terms of order $1/m^2$ in the $\tilde{v}_X$ and $\delta v_X(P)$ of H_R. Their Hartree expectation values can be easily obtained by using the average values

$$\overline{(\mathbf{k}_i - \mathbf{k}_j)^2} = \overline{(\mathbf{k}_i + \mathbf{k}_j)^2} = \frac{6}{10} k_F^2. \quad (5.8)$$

We find that the contributions of $1/m^2$ terms of $\tilde{v}_V$ and $\delta v_V(\mathbf{P})$ cancel, while those of $\tilde{v}_S$ and $\delta v_S(\mathbf{P})$ add to give the $1/m^2$ term (first in the third row) in (5.6). The H_R given by (5.7) being valid only up to order $1/m^2$ cannot yield the rest of the terms, of order $1/m^3$ or higher, in Eq. (5.6).

The first term in the fourth row, of order $1/m^3$, is known [19] to be the Hartree contribution of the three-body force V_{ijk}, shown in Fig. 1. It is obtained by eliminating the antiparticle degrees of freedom from H_R. There are two terms of order $1/m^4$ in (5.6). The second term in the third row gives the contribution of the $1/m^4$ parts of $\tilde{v}_S$ and $\delta v_S(\mathbf{P})$, while the first term in the fifth row is the Hartree contribution of the four-body forces. In such cases the relativistic Hamiltonian (1.3), with only two- and three-body forces along with their exact boost corrections, can at most account for all terms up to k_F^{10}. In contrast the nonrelativistic Hamiltonian (1.1) can reproduce terms up to k_F^3. When $G_S^2\rho/\mu_S^2 m$ is of order unity many-body forces give significant contributions, and the usefulness of Hamiltonians like (1.3) diminishes. At low densities the effects of correlations between particles can be important, and these are more easily treated using Hamiltonians.

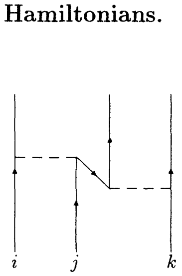

FIG. 1. Z diagram for three-body interaction.

VI. PION-EXCHANGE INTERACTIONS

The one-pion-exchange interaction between two nucleons may be calculated from the pseudovector interaction

$$H_{\text{int}} = -\frac{f}{\mu_\pi} \bar{\psi} \gamma^\mu \gamma_5 \tau_i \psi \partial_\mu \phi_i, \quad (6.1)$$

where ψ is a Dirac field representing nucleons, and ϕ_i denotes the pion field with isospin i. The v_π is calculated using standard techniques of field theory and expressed as a sum of $\tilde{v}_\pi$ and $\delta v_\pi(\mathbf{P})$. Keeping terms up to order $1/m^2$ we obtain

$$\tilde{v}_\pi(\mathbf{q}, \mathbf{p}) = -\frac{f^2}{\mu_\pi^2 (q^2 + \mu_\pi^2)} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \boldsymbol{\sigma}_1 \cdot \mathbf{q} \, \boldsymbol{\sigma}_2 \cdot \mathbf{q} \times \left(1 - \frac{p^2}{m^2} \right), \quad (6.2)$$

$$\delta v_\pi(\mathbf{P}, \mathbf{q}, \mathbf{p}) = -\frac{f^2 \, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{\mu_\pi^2 (q^2 + \mu_\pi^2)} \left\{ \boldsymbol{\sigma}_1 \cdot \mathbf{q} \, \boldsymbol{\sigma}_2 \cdot \mathbf{q} \times \left(\frac{(\mathbf{P} \cdot \mathbf{q})^2}{4m^2 (q^2 + \mu_\pi^2)} - \frac{P^2}{4m^2} \right) - \frac{\mathbf{P} \cdot \mathbf{q}}{8m^2} \left[\boldsymbol{\sigma}_2 \cdot \mathbf{q} \, \boldsymbol{\sigma}_1 \cdot (\mathbf{P} + 2\mathbf{p}) + \boldsymbol{\sigma}_1 \cdot \mathbf{q} \, \boldsymbol{\sigma}_2 \cdot (\mathbf{P} - 2\mathbf{p}) \right] \right\}. \quad (6.3)$$

This result can also be obtained assuming pseudoscalar coupling:

$$H_{\text{int}} = iG \bar{\psi} \gamma_5 \tau_i \psi \phi_i, \quad (6.4)$$

with

$$G^2 = \frac{4m^2 f^2}{\mu_\pi^2}, \quad (6.5)$$

and it can be verified that the δv_π obtained by inserting $\tilde{v}_\pi$ in Eq. (1.7) is identical to that given by (6.3).

The contribution of the $\delta v_\pi(\mathbf{P})$ term containing $\boldsymbol{\sigma}_1 \cdot \mathbf{q} \, \boldsymbol{\sigma}_2 \cdot \mathbf{q}$, to the binding energy of ^{3}H and ^{4}He is calculated in Ref. [7], and those of the rest of the terms of δv_π in [16]. However, the p^2/m^2 term in $\tilde{v}_\pi$ has been neglected in Ref. [7] and almost all other models of v_{NN}. In principle it can be as important as the $P^2/4m^2$ term of δv_π.

We do not as yet have a complete understanding of the nucleon-nucleon interaction. It is generally believed that the long-range part of the interaction is given by one-pion exchange, and this belief is strongly supported by the Nijmegen analysis [20] of the two-nucleon scattering data. The one-pion-exchange interaction is responsible for the quadrupole moment of the deuteron, and it appears to give large contributions to the nuclear binding energy [21]. The interaction at shorter distance probably has comparable contributions from the internal structure of the nucleon, $N\Delta$ box diagrams for example, and from the exchange of heavier mesons. It is convenient to separate the $\tilde{v}_{NN}$ into the one-pion-exchange part and the rest of it:

$$\tilde{v}_{NN} = \tilde{v}_\pi + \tilde{v}_R. \tag{6.6}$$

The short-range cutoff of $\tilde{v}_\pi$ and the entire $\tilde{v}_R$ are primarily determined by fitting the observed nucleon-nucleon scattering data.

The available models, except for Bonn models [22], use only the leading term, independent of m, of the $\tilde{v}_\pi$ [Eq. (6.2)]. Thus the $\tilde{v}_R$ in these models compensates for the neglected p^2/m^2 term of $\tilde{v}_\pi$. However, this compensation cannot be exact since the p^2/m^2 term in $\tilde{v}_\pi$ generates a momentum-dependent tensor force. Such a force is not yet included in other models of $\tilde{v}_{NN}$. Attempts to refit the NN scattering data with the $\tilde{v}_\pi$ correct up to order $1/m^2$ are in progress. These will presumably provide better empirical models of $\tilde{v}_R$, and also be useful to study relativistic effects of order $1/m^2$.

VII. CONCLUSION

Unique identification of relativistic effects is difficult in nuclear physics due to a lack of *ab initio* understanding of nuclear forces from QCD. Several relativistic effects are inadvertently included in the nonrelativistic Hamiltonian (1.1) via the phenomenological interactions v_{ij} and V_{ijk} obtained by fits to observed data. However, nonrelativistic Hamiltonians do not contain several known relativistic effects. The relativistic Hamiltonians given by Eq. (1.3) seem to offer a practical method to include these effects in nuclear many-body theory.

It is technically possible to treat the kinetic energy of nucleons relativistically. The two-body problem can be easily solved in momentum space and realistic models of $\tilde{v}_{NN}$ can be obtained by fitting the scattering data. Faddeev-Yakubovsky [23] and the quantum Monte Carlo methods [1,2,8] can be used with the relativistic kinetic energy operator $\sqrt{m_i^2 + p_i^2}$. It is difficult to expand the square root beyond the nonrelativistic term. The next term, $-p_i^4/8m_i^3$, is attractive, and a Hamiltonian unbounded from below results when p_i^6 and higher terms are neglected.

In contrast it appears to be useful to expand the $\delta v(\mathbf{P}_{ij})$ in powers of $P_{ij}^2/4m^2$ because the total momentum of an interacting pair of nucleons in nuclei is generally much less than m. The lowest order $\delta v(\mathbf{P}_{ij})$ is relatively simple [Eq. (1.7)] and seems to be dominated by the classical terms coming from the relativistic energy-momentum relation and Lorentz contraction.

A relativistic many-body theory of nuclei can also be developed starting from quantum field theory. The relativistic Hamiltonians and quantum field theory imply the same relation between $\tilde{v}_{ij}$ and $\delta v(\mathbf{P}_{ij})$ dictated by the invariance of the Poincaré group. The theory based on hadron fields also provides a theoretical framework to construct models of $\tilde{v}_{ij}$, V_{ijk}, and many-nucleon interactions [10,22,15,24]. This framework is certainly useful, but limited by the relatively small number of fields, such as N, Δ, π, ρ, ω, ..., that can be treated. If the internal structure of nucleons strongly influences the $\tilde{v}_{ij}$ then one would need to treat consistently a large number of hadron fields. In this case it may be advantageous to use relativistic Hamiltonians containing semiphenomenological models of $\tilde{v}_{ij}$ and V_{ijk} having field-theoretic long-range pion-exchange parts and shorter-range phenomenological parts.

Finally we note that in this approach it is rather simple to describe a nucleus moving with a velocity $\mathbf{V}$. Its wave function is given by

$$|\Psi\rangle = e^{i\mathbf{K}\cdot\mathbf{u}}|\Psi_0\rangle, \tag{7.1}$$

where $|\Psi_0\rangle$ describes the nucleus with zero total momentum,

$$\mathbf{u} = \mathbf{V}\tanh^{-1}(|\mathbf{V}|) = \mathbf{V}\left(1 + \frac{1}{3}|\mathbf{V}|^2 + \cdots\right), \tag{7.2}$$

and $\mathbf{K}$ is given by Eqs. (2.4), (2.14), and (2.16). Up to order $|\mathbf{V}|^2$ for a two-nucleon system at time $t = 0$ we obtain

$$\begin{aligned}\mathbf{K}\cdot\mathbf{u} = \mathbf{R}\cdot\mathbf{V}&\left(2m + \frac{p^2}{m} + \tilde{v} + \frac{1}{4m}P^2 + \frac{2m}{3}|\mathbf{V}|^2\right)\\ &-\frac{i}{2m}\mathbf{P}\cdot\mathbf{V} + \frac{1}{2m}(\mathbf{r}\cdot\mathbf{V})(\mathbf{P}\cdot\mathbf{p})\\ &-\frac{1}{2m}\left[\frac{1}{2}(\mathbf{s}_1+\mathbf{s}_2)\times\mathbf{P} + (\mathbf{s}_1-\mathbf{s}_2)\times\mathbf{p}\right]\cdot\mathbf{V}.\end{aligned} \tag{7.3}$$

The $\mathbf{P}$ and $\mathbf{p}$ in the above equation are operators. Using

$$e^{i\mathbf{K}\cdot\mathbf{u}}\,|\Psi_0\rangle = \lim_{n\to\infty}\left(e^{i\mathbf{K}\cdot\mathbf{u}/n}\right)^n|\Psi_0\rangle, \tag{7.4}$$

we obtain

$$\begin{aligned}|\Psi\rangle = &\left[1 + \frac{1}{2}(\mathbf{r}\cdot\mathbf{V})(\mathbf{V}\cdot\boldsymbol{\nabla}) - \frac{1}{2m}(\mathbf{s}_1-\mathbf{s}_2)\times\boldsymbol{\nabla}\cdot\mathbf{V}\right]\\ &\times\left(1 + \frac{1}{4}|\mathbf{V}|^2\right)^2\\ &\times\ \exp\left[i\mathbf{R}\cdot\mathbf{V}\left(2m + \frac{p^2}{m} + \tilde{v} + m|\mathbf{V}|^2\right)\right]|\Psi_0\rangle.\end{aligned} \tag{7.5}$$

Since the energy of the two-nucleon state is given by

$$E = 2m + \frac{p^2}{m} + \tilde{v} + m|\mathbf{V}|^2, \tag{7.6}$$

we can identify $\mathbf{V}E$ as the value (not operator) of the total momentum. The factor $(1 + \mathbf{r}\cdot\mathbf{V}\ \mathbf{V}\cdot\boldsymbol{\nabla}/2)$ in $|\Psi\rangle$ produces the Lorentz contraction of the wave function and the $(\mathbf{s}_1 - \mathbf{s}_2)$ term gives spin rotations. One of the $\left(1 + |\mathbf{V}|^2/4\right)$ factors compensates for the change in normalization due to the Lorentz contraction, while the other represents the covariant normalization $[E/E(P=0)]$ of the boosted wave function [25]. Due to the choice $\mathbf{w}=\mathbf{0}$ made in Sec. II, only kinematical changes occur in the boosted wave function. One can show that for a one-pion-exchange potential (OPEP) in the form of Eq. (6.2) the $\mathbf{w}$ is nonvanishing [see Eq. (A21) of Ref. [10]].

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