

Gauge invariant unitary theory for pion photoproduction

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The Ward-Takahashi identities are central to the gauge invariance of the photoproduction amplitude. Here we demonstrate that unitarity and in particular the inclusion of both the πN and $\gamma\pi N$ thresholds on equal footing yields a photoproduction amplitude that satisfies both two-body unitarity and the generalized Ward-Takahashi identities. The final amplitude is a solution of a set of coupled channel integral equations for the reactions $\pi N \rightarrow \pi N$ and $\gamma N \rightarrow \pi N$.

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I. INTRODUCTION

Pion photoproduction has a long history going back to the early 1950's [1]. In recent years the interest in this reaction has been revived as a result of new experimental facilities that allow more careful studies of this reaction over a much wider energy range, and the advent of quantum chromodynamics (QCD) as a fundamental theory of strong interaction, which should yield the internal structure of the nucleon. In particular the study of the reaction $p(\gamma, \pi)N$ is motivated by the following: (i) This is the simplest nuclear system one can examine to understand the mechanism of pion photoproduction. Furthermore, the amplitude for this reaction can be used as input into the calculation of pion photoproduction off heavier nuclei [2,3], with the expectation of gaining information about nuclear structure. (ii) There is recent interest in chiral symmetry [4] and its possible violation in this reaction [5,6], which have raised questions regarding the role of unitarity in this reaction [7,8], and the use of pion photoproduction to test chiral models of QCD, e.g., chiral perturbation theory [9]. (iii) The new facilities at CEBAF, and other facilities, which will open the way to examine the structure of the nucleon and the resonances observed in πN scattering and pion photoproduction. These results could shed light on the need to introduce quark-gluon degrees of freedom, and, in particular, determine the energy at which these new degrees of freedom become more important than the traditional meson-baryon degrees of freedom. In addition, this data could be used to test models of QCD.

To examine the structure of resonances observed in πN scattering using pion photoproduction, we need to include the important thresholds for all possible reactions at that resonance energy. To achieve this we may need to include in our formulation those unitarity cuts, and therefore thresholds, important for that resonance [10]. On the other hand, the incident photon will interact with the electromagnetic (e.m.) charge and current distribution in the target, and to that extent it is essential to satisfy charge conservation or U(1) gauge invariance. The fact that the proton has an internal structure, which could be described in terms of quarks at short distances

and mesons at large distances, suggests that these degrees of freedom should be taken into consideration in determining the e.m. charge and current distribution. The meson degrees of freedom, which include nucleon dressing and the distortion of the final outgoing pion wave, are part of the mechanism needed to include unitarity and therefore the proper thresholds. Thus, under ideal conditions one should satisfy both unitarity and U(1) gauge invariance in a consistent manner. In addition, the quark dynamics should be included in the determination of the charge and current distribution even though they do not contribute directly to any thresholds, since the quarks are confined.

Historically, some early calculations of pion photoproduction [11–13] were made to satisfy unitarity by imposing the Watson theorem [14]. This basically consisted of representing the final distortion in the πN channel by the on-shell pion-nucleon amplitude or πN phase shift. In this approach, which has recently been used to extract $E2/M1$ $\Delta - N$ transition [15], the cross section for pion photoproduction consisted of two parts, the Born amplitude, which included the interaction of the photon with the nucleon and meson currents (see Fig. 1), and the on-shell πN amplitude, which was included to satisfy two-body unitarity. The two ingredients were considered to be independent. Thus, for the Born amplitude, the photon coupled to a nucleon with an internal structure represented by an on-mass-shell form factor extracted from elastic electron-proton scattering, while the pion coupling to a finite nucleon (see Fig. 1) involved a πNN form factor, which was constrained by the U(1) gauge invariance to be the same as the e.m. form factor [7]. Here we should note that there is no consistency between the πNN form factor used and the e.m. form factor other than the overall gauge invariance of the Born amplitude. On the other hand, the πN amplitude, which also has the πNN form factor, was determined solely by the πN scattering data. In other words there was no consistency between the Born amplitude for pion photoproduction and the πN amplitude that determined the dynamics of pion-nucleon scattering. More recently [7], extensions of this procedure have been developed with considerable success, that extend the Watson theorem by

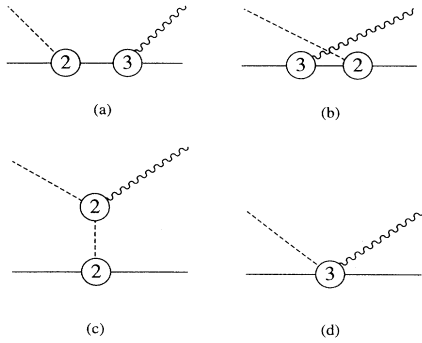


FIG. 1. The Born amplitude for pion photoproduction. The number in the vertices is the irreducibility of the vertex.

employing the off-shell πN amplitude generated by either a separable potential [7] or a chiral Lagrangian [16] to determine the final distortion of the outgoing pion. Even in these calculations no attempt has been made to have any consistency between the πN dynamics and the Born amplitude for pion photoproduction. The present investigation is an attempt at a reformulation of pion photoproduction that satisfies both unitarity and $U(1)$ gauge invariance. The πN dynamics required for unitarity will determine the charge and current distribution. In this way we maintain consistency between the pion dynamics and the requirement of e.m. gauge invariance.

To include gauge invariance and the dynamics that determine the underlying structure of the nucleon, we need to gauge the Lagrangian for the underlying strong interaction dynamics. Ideally this means we need to start from the QCD Lagrangian and include the e.m. interaction. However, at this stage we have no practical procedure for projecting meson-baryon degrees of freedom without resorting to models of QCD. For example, if we assume that the cloudy bag model (CBM) [17,18] determines the dynamics of the quarks and pions, then we should first gauge the CBM Lagrangian [2] to include the e.m. interaction. To substitute the quark degrees of freedom by the baryonic degrees of freedom, we need to take the second step of implementing the procedure adopted in the CBM by effectively integrating out the quark degrees of freedom. This two-step process of getting an effective meson-baryon Lagrangian that includes the electromagnetic and meson interaction consistently, is illustrated in Fig. 2. Unfortunately, to get the correct current for the coupling of the photon to the baryon, we need to construct, in the CBM, a state that is an eigenstate of the total four-momentum that can be boosted from one inertial frame to another [19]. This, in practice, is not possible in a bag model because the bag boundary condition cannot be simply boosted. However, this procedure could be followed in other chiral soliton models [20] with some difficulty, or we could use a model such as the Nambu–Jona-Lasinio (NJL) Lagrangian [21–23] or the global color model [24,25], which can, in principle, give a translationally invariant state [26]. The resultant Lagrangian would have gauge invariance, and we could then proceed to implement unitarity, taking as a starting

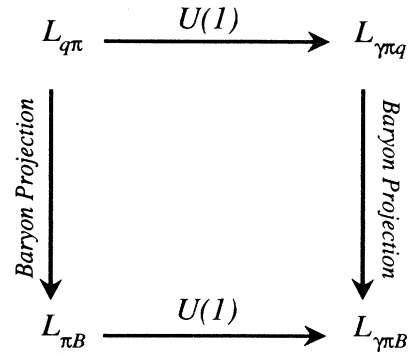


FIG. 2. Two possible scenarios for constructing a gauge invariant Lagrangian for mesons and baryons from a QCD model.

point the Lagrangian in the space of baryons, mesons, and photons. The effect of quarks would be to have form factors for all vertices, but now we would have a self-consistency between the different form factors in the Lagrangian. In particular, the form factor associated with the nucleon e.m. current and the form factor in the πNN vertex would be consistent. Furthermore, this πNN form factor goes into the dynamics that generates the πN interaction. Since the quarks are confined, and therefore do not contribute to unitarity, an alternative procedure that would satisfy gauge invariance and unitarity, see Fig. 2, would be to integrate the quark degrees of freedom in favor of the baryon degrees of freedom. In this case, we would have form factors associated with the meson-baryon Lagrangian, and these form factors are related to the underlying quark structure of the nucleon. A procedure that introduces the e.m. coupling would be to gauge the resultant Lagrangian with form factors (e.g., Ohta's procedure [27,28]). Such a procedure must result in the vertices with photons satisfying their associated Ward-Takahashi identities [29,30] guaranteeing their gauge invariance. At this stage we should point out that gauging the meson-baryon Lagrangian with form factors based on a QCD model may not give the same e.m. currents as the gauging of the underlying QCD model. The currents based on the two methods could differ by a conserved current and to that extent one should ideally derive the currents at the QCD level. The contribution to the nucleon propagator and πNN form factor from meson dressing and meson-nucleon scattering will then be included explicitly, while maintaining both unitarity and gauge invariance.

In Sec. II we introduce a πN Lagrangian in which the nucleon and pion have structure and the πNN vertex has a form factor. At this stage we may assume that this internal structure is due to quark degrees of freedom. This Lagrangian can, in principle, be gauged to give propagators and vertices that satisfy the corresponding Ward-Takahashi identities. Furthermore, this Lagrangian, in lowest order, gives a photoproduction amplitude that is gauge invariant. The actual details of constructing this Lagrangian will not be considered at the present time, as they will depend on the QCD model used. This sets

the stage for the next step of including the contribution to unitarity from the pionic degrees of freedom. This will be detailed in Sec. III, where we derive a set of coupled integral equations for both πN elastic scattering and $\pi + N \leftarrow \gamma + N$. Since the threshold energies for πN scattering and $\gamma\pi N$ are the same, we include both of these thresholds in our coupled channel formulation. To establish the gauge invariance of the photo-production amplitude resulting from the coupled channel approach, we proceed in Sec. IV to make use of the Ward-Takahashi [29,30] identities and the procedure proposed by Kazes [31] to derive the amplitude for this reaction from the one-pion irreducible πNN three-point function derived in Sec. III. We then show in Sec. V that the amplitude resulting from the solution of the coupled channel is, in fact, identical to that derived from the application of the Ward-Takahashi identities. In this way we establish that the solution of the coupled channel problem satisfies both unitarity and gauge invariance. Finally in Sec. VI we present some concluding remarks.

II. THE GAUGE INVARIANT LAGRANGIAN

Our main motivation in the present investigation is to set up a unitary and gauge invariant formulation of pion photoproduction. Since the quarks and gluons are confined, they do not contribute to unitarity, and we can include this substructure by introducing form factors into our Lagrangian. In other words we can consider a Lagrangian for the system of nucleons and pions as

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\pi + \mathcal{L}_{\pi NN} . \quad (2.1)$$

The Lagrangians for the baryon and meson with internal structure are taken to be [27]

$$\mathcal{L}_N = \int dx' dx \bar{\psi}(x') [i\gamma \cdot \partial_x r(x' - x) - mh(x' - x)] \psi(x), \quad (2.2a)$$

$$\mathcal{L}_\pi = \int dx dx' \phi(x') (\partial_x^2 - \mu^2) f_\pi(x' - x) \phi(x), \quad (2.2b)$$

where the form factors r , h , and f_π may be determined from the underlying quark-gluon structure; e.g., we could write the corresponding propagators for the nucleon and pion as

$$S_0(p) = [\not{p} - m - \Sigma(p)]^{-1}, \quad (2.3a)$$

$$\Delta(q) = [q^2 - \mu^2 - \Pi(q^2)]^{-1}, \quad (2.3b)$$

where $\Sigma(p)$ and $\Pi(q^2)$ could be written in terms of their quark structure, as illustrated in Fig. 3. It should be pointed out at this stage that the QCD model used in evaluating the diagrams in Fig. 3 should not give rise to unitarity cuts, since the quarks are confined. The mass shifts in both the baryon and the meson propagators can then be written in terms of the functions in the



FIG. 3. Examples of diagrams that could contribute to the baryon (a) and meson (b) dressing in a quark model.

Lagrangian; e.g., by taking $\Sigma(p) = \not{p}A(p^2) + mB(p^2)$ we can write the Fourier transform of the functions r and h in terms of $A(p^2)$ and $B(p^2)$ as $r(p^2) = 1 - A(p^2)$ and $h(p^2) = 1 + B(p^2)$. Thus, the nucleon propagator is given as

$$S_0^{-1}(p) = r(p^2) \not{p} - m h(p^2). \quad (2.4)$$

The conditions that this propagator have a simple pole at the nucleon mass m and that the residue at this pole be one, impose the following conditions on the functions $h(p^2)$ and $r(p^2)$:

$$r(m^2) = h(m^2) \quad (2.5a)$$

and

$$r(m^2) + 2m^2 \{r'(m^2) - h'(m^2)\} = 1 \quad (2.5b)$$

with

$$r'(m^2) = \left. \frac{d}{dp^2} r(p^2) \right|_{p^2=m^2}, \quad (2.5c)$$

$$h'(m^2) = \left. \frac{d}{dp^2} h(p^2) \right|_{p^2=m^2}. \quad (2.5d)$$

In a similar manner we can determine $f_\pi(q^2)$ in terms of $\Pi(q^2)$ with the requirement that the pion propagator have a pole at $q^2 = \mu^2$ with unit residue. In this way the underlying quark-gluon degrees of freedom have been included, and the gauging of these propagators will give us the coupling of the photon to the nucleon and the pion that have internal structure. Here we should point out that any pionic dressing of the nucleon will shift the mass m , and to that extent the nucleon mass in Eq. (2.5) may need to be considered as the bare nucleon mass. This will also apply to the pion mass, if it is to receive any further dressing.

The interaction terms in the Lagrangian in Eq. (2.1) can be written as

$$\mathcal{L}_{\pi NN} = \int dx' dx dy \bar{\psi}(x') \Lambda_5(x', x; y) \tau_i \phi_i(y) \psi(x). \quad (2.6)$$

Here again the vertex function Λ_5 could be determined from an underlying QCD model; e.g., the CBM gives the πNN form factor in terms of the bag radius and the wave function of the quark in the bag.

At this stage the form factors introduced above could be attributed to the internal structure of the pion and nucleon. Given a model for this internal structure one could employ gauge invariance to generate the currents

associated with this structure, and this current would satisfy the Ward-Takahashi identities [32]. On the other hand, one can parametrize these form factors and then use minimal coupling to generate the electromagnetic current. This procedure, followed by Ohta [27,28], give rise to currents that satisfy current conservation but are not unique in that one can always add a conserved current to that derived from the minimal substitution procedure as demonstrated by Bos *et al.* [33]. Since we will need some form of cutoff or form factor to include pionic dressing and therefore unitarity, we will assume that the form factors introduced in the Lagrangian in Eq. (2.1) can be gauged and the resultant currents will satisfy the Ward-Takahashi [29,30] identities.

The behavior of the nucleon propagator, given by Eq. (2.4), under gauging, can now be written to first order in the e.m. field A as

$$S_0^{-1}(p) \rightarrow S_0^{-1}(p) - \Gamma_\mu(k, p', p) A^\mu, \quad (2.7)$$

where k is the photon momentum and $\Gamma_\mu(k, p', p)$ is the resulting electromagnetic current for the nucleon. In the event that $r(p^2) = h(p^2) = 1$, i.e., the nucleon has no internal structure, then $\Gamma_\mu = \frac{e}{2}(1 + \tau_3)\gamma_\mu$, which is the standard Dirac current. We now assume that the proper gauging of the Lagrangian in Eq. (2.2a) should give a current that satisfies the Ward-Takahashi identity [29,30,34,35], i.e.,

$$(p' - p)^\mu \Gamma_\mu(k, p', p) = \frac{e}{2}(1 + \tau_3) \{S_0^{-1}(p') - S_0^{-1}(p)\}. \quad (2.8)$$

In this way the gauging of the nucleon propagator is given by

$$S_0(p) \rightarrow S_0(p) + S_0(p') \Gamma_\mu(k, p', p) S_0(p) A^\mu. \quad (2.9)$$

The actual details involved in calculating $\Gamma_\mu(k, p', p)$ will depend on the model for the internal structure of the nucleon and will not be considered at this stage.

In a similar manner, the gauging of the pion propagator

$$\Delta^{-1}(q) = q^2 - m_\pi^2 - \Pi(q^2), \quad (2.10)$$

can also be considered to first order in the e.m. field to be

$$\Delta^{-1}(q) \rightarrow \Delta^{-1}(q) - \Gamma_\mu^\pi(k, q', q) A^\mu, \quad (2.11)$$

where k is the photon momentum, and $\Gamma_\mu^\pi(k, q', q)$ is the pion electromagnetic current, which is assumed to satisfy the Ward-Takahashi identity

$$(q' - q)^\mu \Gamma_\mu^\pi(k, q', q) = Q_\pi \{ \Delta^{-1}(q') - \Delta^{-1}(q) \}. \quad (2.12)$$

Here Q_π is the pion charge operator. In the limit of a structureless pion, $\Pi(q'^2) = \Pi(q^2) = 0$, $\Gamma_\mu^\pi(k, q', q)$ reduces to the standard current for a point pion, i.e.,

$$\Gamma_\mu^\pi(k, q', q) = Q_\pi (q' + q)_\mu. \quad (2.13)$$

In analogy with Eq. (2.9), we can determine the behavior of the pion propagator $\Delta(q)$ under gauge transformation

to be

$$\Delta(q) \rightarrow \Delta(q) + \Delta(q') \Gamma_\mu^\pi(k, q', q) \Delta(q) A^\mu. \quad (2.14)$$

Having established the gauging of the nucleon and pion propagators, we turn our attention to the gauging of the interaction Lagrangian and, in particular, the pion production vertex. The procedure of gauging the πNN vertex generates the contact term for $\pi N \leftarrow \gamma N$ given by

$$\Lambda_5^\dagger(q, p', p) \tau_i \rightarrow \Lambda_5^\dagger(q, p', p) \tau_i + \Gamma_\mu^{\text{CT}i}(k, q, p', p) A^\mu, \quad (2.15)$$

where $\Gamma_\mu^{\text{CT}i}(k, q, p', p)$ is the $\pi N \leftarrow \gamma N$ vertex. The resultant Ward-Takahashi identity for the contact term is then given by

$$\begin{aligned} k^\mu \Gamma_\mu^{\text{CT}i}(k, q, p', p) &= e_N \tau_i \Lambda_5^\dagger(q, p' - k, p) \\ &\quad - \tau_i e_N \Lambda_5^\dagger(q, p', p + k) \\ &\quad - i e \epsilon_{3ij} \tau_j \Lambda_5^\dagger(q - k, p', p), \end{aligned} \quad (2.16)$$

as indicated by the results of Kazes [31] and Naus, Koch, and Friar [35].

In the present investigation the form factors that are present will play the role of introducing cutoffs that maintain gauge invariance and will allow us to incorporate the pionic dressing of the nucleon without having to resort to other renormalization procedures. From a practical point of view, the gauging procedure will allow the introduction of QCD-based parameters that provide consistency between the different form factors. Although the above Lagrangian is not the most general we can envisage, the procedures followed and the general conclusions can be extended to more general forms for the meson-baryon Lagrangian. In particular, we could have included a πN interaction that under gauging would give us a term for the process $\pi N \leftrightarrow \gamma \pi N$. The detailed form of such an interaction will depend on the model considered; e.g., in some chiral Lagrangians such an interaction would arise from ρ and σ exchange [36].

III. UNITARITY

Having established the form of the Lagrangian, and therefore the corresponding Hamiltonian, we now proceed to include the pionic contribution to the currents and the amplitude for pion photoproduction. Since the pionic dressing contributes to both the thresholds for πN scattering and modifies the e.m. currents, we need to include pionic contributions, while preserving two-body unitarity and gauge invariance. This can be achieved by deriving coupled integral equations for pion-nucleon elastic scattering and pion photoproduction that include the πN , γN , as well as the $\gamma \pi N$ thresholds. Although this latter threshold is not required for two-body unitarity to be satisfied, the fact that it is at the same energy as the πN threshold suggests that we need to include it for consistency. However, it turns out that the inclusion of the $\gamma \pi N$ threshold is essential if we are to preserve gauge

invariance at the operator level for the pion photoproduction amplitude.

The method employed for the derivation of these coupled integral equations is based on the classification of the Feynman diagrams that contribute to a given amplitude, in perturbation theory, according to their irreducibility. In other words, we take all Feynman diagrams that contribute to a given amplitude and classify these diagrams into classes according to their irreducibility. Then with the help of Taylor's [37,38] last-cut lemma, we can write the diagrams that belong to a given class, and of a given irreducibility, in terms of amplitudes for the same process or a related process. This procedure generates coupled integral equations for all related reactions. This method has been used to derive unitary equations for the $NN - \pi NN$ system [39–43], the $\pi N - \pi\pi N$ system [44,45], and the $\gamma N - \pi N$ system [2,46].

Although Taylor's [37] original classification scheme suffers from double counting problems when applied to covariant perturbation theory, these problems have been recently overcome [47,48] at the cost of imposing irreducibility constraints on subamplitudes in channels other than the s channel, and the requirement that certain subtractions be included in the final equations. Since this double counting does not arise for two-body unitarity in πN scattering and pion photoproduction [47], we have chosen to use the simplified version of the classification of diagrams that is often used in time-ordered perturbation theory, and has been applied to this reaction previously [2,46].

Since the last-cut lemma plays a central role in our derivation of integral equations for pion photoproduction, we will very briefly state the basic definition of a cut and the last-cut lemma as applied in time-ordered perturbation theory. We define a k cut as an arc that separates the initial state from the final state in a given diagram, and cuts k particle lines with at least one line being an internal line. An amplitude is r particle irreducible if all diagrams that contribute to this amplitude will not admit any k cut with $k \leq r$. With these two definitions, we can introduce the last-cut lemma, which states that for a given amplitude that is $(r - 1)$ particle irreducible, there is a unique way of obtaining an r -particle cut closest to the final (initial) state for all diagrams that contribute to the amplitude. By virtue of this lemma, we can expose one-, two- and three-particle intermediate states and the corresponding unitarity cuts and in this way derive equations for the amplitude that satisfy unitarity. From the above statement of the lemma, it is clear that we need to expose the n -particle unitarity cut before the $(n + 1)$ -particle unitarity cut.

With this lemma we can now expose the πN and $\gamma\pi N$ unitarity cuts and derive a set of integral equations governing pion-nucleon scattering and pion photoproduction. However, before we set out to derive integral equations for πN elastic scattering and pion photoproduction, it would be helpful to our discussion in this section if we symbolically write our Lagrangian, derived in Sec. II, as

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_N + \mathcal{L}_\pi + \mathcal{L}_\gamma + \mathcal{L}_{\pi NN} + \mathcal{L}_{\pi N\pi N} + \mathcal{L}_{\gamma NN} \\ & + \mathcal{L}_{\gamma\pi\pi} + \mathcal{L}_{\gamma\pi NN} + \mathcal{L}_{\pi N\gamma\pi N} . \end{aligned} \quad (3.1)$$

The additional terms in this Lagrangian, over those in Eq. (2.1), are the result of gauging the Lagrangian in Eq. (2.1). In addition we have included terms that give rise to $\pi N \leftrightarrow \pi N$, $\mathcal{L}_{\pi N\pi N}$ and $\pi N \leftrightarrow \gamma\pi N$, $\mathcal{L}_{\pi N\gamma\pi N}$. These additional terms are included in the event that we need to include σ and ρ exchange, which are often needed in chiral Lagrangians that describe πN scattering [36]. Here we will not examine the detailed form of such a πN interaction and the associated gauging as the result would then be model dependent. All terms in this Lagrangian will have form factors that are consistent with Ward-Takahashi identities as demonstrated in Sec. II.

A. The πN unitarity cut

Restricting the analysis of the pion nucleon and pion photoproduction amplitudes to lowest order in the electromagnetic coupling e leads to the result that the two amplitudes can be considered separately as there are no radiative corrections to the πN amplitude. The analysis of the πN amplitude under these conditions has been carried out previously [41–45,2] and a summary of the two-body results is presented here. At the two-body level the πN amplitude is given by [2]

$$t^{(0)} = t^{(1)} + \Lambda_5^{(1)\dagger} S \Lambda_5^{(1)}, \quad (3.2)$$

where t is the $\pi - N$ amplitude, $\Lambda_5^{(1)}$ is the $N \leftarrow \pi N$ amplitude, and S is the dressed nucleon propagator. The superscript gives the irreducibility of the amplitudes. The one-particle irreducible $\pi N \leftarrow \pi N$ amplitude $t^{(1)}$ satisfies the two-body equation

$$t^{(1)} = t^{(2)} + t^{(2)} g t^{(1)} = t^{(2)} + t^{(1)} g t^{(2)}, \quad (3.3)$$

with $g = S\Delta$ being the πN propagator. Since we are restricting our analysis to the inclusion of the πN and $\gamma\pi N$ thresholds only, we can choose our πN propagator so that the nucleon propagator does not include pionic dressing, i.e., $g = S_0\Delta$, where S_0 is the "bare" nucleon propagator defined in Sec. II, Eq. (2.3a). However, the substitution $S \rightarrow S_0$, if carried through in the πN propagator g , will require that S_0 have a simple pole at the physical nucleon mass. The input to Eq. (3.3) is the two particle irreducible πN amplitude $t^{(2)}$. Application of the last-cut lemma to this amplitude will expose states with two or more pions and a nucleon, i.e., the $\pi\pi N$ and higher thresholds. If we are not to include these thresholds into our final equation, then we can assume that $t^{(2)}$ is some πN potential whose parameters can be adjusted so that the full πN amplitude $t^{(0)}$ reproduces the experimental phase shifts. On the other hand, if we are to use the Lagrangian defined in Eq. (3.1) for our input, then we need to expose the $\pi\pi N$ intermediate states by the application of the last-cut lemma to $t^{(2)}$. This gives us

$$t^{(2)} = t^{(3)} + \Lambda_5^{(2)} S \Lambda_5^{(2)\dagger}, \quad (3.4)$$

where the second term is the cross diagram. Here again we can make the substitution $S \rightarrow S_0$, with the condition that S_0 has a pole at the physical nucleon mass, if we want to include two-body unitarity only. Since the Lagrangian in Eq. (3.1) does not include any term for two-pion production or absorption, $\pi\pi N \leftrightarrow N$, the three particle irreducible amplitude $t^{(3)}$ is just the πN potential resulting from $\mathcal{L}_{\pi N \pi N}$ and could include σ and ρ exchange. If we were to examine the full $\pi\pi N$ part of the Hilbert space, then the $\pi N \rightarrow N$ amplitude in Eq. (3.4) gets dressed [45] to the extent that $\Lambda_5^{(2)} \rightarrow \Lambda_5^{(1)}$, and the $\pi N \rightarrow N$ amplitude in the crossed digram becomes the same as that in Eq. (3.2) by the pole diagram.

We now turn to the one-particle irreducible $\pi N \rightarrow N$ amplitude $\Lambda_5^{(1)}$. Using the last-cut lemma, we can write

$$\Lambda_5^{(1)} = \Lambda_5^{(2)} + \Lambda_5^{(2)} g t^{(1)} = \Lambda_5^{(2)} + \Lambda_5^{(1)} g t^{(2)}. \quad (3.5)$$

Here again, since the Lagrangian under consideration has no component that gives rise to two-pion production or absorption, i.e., $N \leftrightarrow \pi\pi N$, then the two-particle irreducible $\pi N \leftarrow N$ amplitude $\Lambda_5^{(2)}$ has no intermediate states and is just the $\pi N N$ vertex in the Lagrangian, i.e., $\Lambda_5^{(2)} = \Lambda_5$.

The dressed nucleon propagator in Eq. (3.2) is given by

$$S^{-1} = S_0^{-1} - \Sigma^{(1)}, \quad (3.6)$$

where, with the help of the last-cut lemma, we can write the mass shift due to pionic dressing as

$$\Sigma^{(1)} = \Sigma^{(2)} + \Lambda_5^{(1)} g \Lambda_5^{(2)\dagger} = \Sigma^{(2)} + \Lambda_5^{(2)} g \Lambda_5^{(1)\dagger}. \quad (3.7)$$

Since the Lagrangian has no term for $N \leftrightarrow \pi\pi N$ transition, then $\Sigma^{(2)} = 0$, while the bare nucleon propagator S_0 in Eq. (3.6) is given by

$$S_0^{-1} = \gamma_\mu p^\mu - m_N^{(0)}, \quad (3.8)$$

where $m_N^{(0)}$ is the bare nucleon mass and includes the quark-gluon contribution to the mass. From this point on we assume this bare mass has no momentum dependence as the quark-gluon contribution does not introduce any unitarity thresholds and the mass shift due to quark-gluon structure has no threshold. In this case the dressed propagator S should have a pole at the physical nucleon mass.

An integral equation for the π - N amplitude, $t^{(0)}$, can now be derived [45]. To achieve this, we first substitute Eqs. (3.3) and (3.5) into (3.2) resulting in

$$t^{(0)} = t^{(2)} + \Lambda_5^{(2)\dagger} S \Lambda_5^{(1)} + t^{(2)} g \left(t^{(1)} + \Lambda_5^{(1)\dagger} S \Lambda_5^{(1)} \right). \quad (3.9)$$

We now make use of the fact that

$$S \Lambda_5^{(1)} = S_0 \Lambda_5^{(0)} = S_0 \Lambda_5^{(2)} \left(1 + g t^{(0)} \right) \quad (3.10)$$

and, noting the definition of $t^{(0)}$, obtain

$$t^{(0)} = v \left(1 + g t^{(0)} \right). \quad (3.11)$$

Here the Born amplitude for πN scattering v is given by

$$v = t^{(2)} + \Lambda_5^{(2)\dagger} S_0 \Lambda_5^{(2)}, \quad (3.12)$$

where, for the present Lagrangian,

$$t^{(2)} = t^{(3)} + \Lambda_5^{(2)} S_0 \Lambda_5^{(2)\dagger} \quad \text{and} \quad \Lambda_5^{(2)} = \Lambda_5, \quad (3.13)$$

where Λ_5 has a form factor that could, in principle, be extracted from a QCD model, and $t^{(3)}$ could be any πN potential we may need to introduce. This Born amplitude, or πN potential, is illustrated in Fig. 4.

We now turn to the amplitude for single-pion photoproduction $\pi + N \leftarrow \gamma + N$. This has been considered in detail previously by Araki and Afnan (AA) [2]. For the sake of completeness we present here a summary of their results. Applying the last-cut lemma to the pion photoproduction amplitude $M^{(0)}$, we first expose the πN unitary cut. Here we will restrict the analysis to first order in the electromagnetic coupling e . We now can divide the diagrams that contribute to this amplitude into two classes: The diagrams that are one-particle irreducible we sum to get the one-particle irreducible amplitude $M^{(1)}$. The rest of the diagrams are one particle reducible and, when summed, give the nucleon pole contribution to the full amplitude. The resultant decomposition for the full amplitude is then given by

$$M^{(0)} = M^{(1)} + \Lambda_5^{(1)\dagger} S \Gamma^{(1)}, \quad (3.14)$$

where $\Gamma^{(1)}$ is the one-particle irreducible $N \leftarrow \gamma N$ amplitude. Following the procedure presented above for the πN amplitude, we employ the last-cut lemma to write an integral equation for the one-particle irreducible photoproduction amplitude $M^{(1)}$. This is given by

$$M^{(1)} = M^{(2)} + t^{(2)} g M^{(1)} = M^{(2)} + t^{(1)} g M^{(2)}, \quad (3.15)$$

where we have retained only those terms linear in e . Finally, applying the last-cut lemma to expose the πN unitarity cut in the $N \leftarrow \gamma N$ amplitude, $\Gamma^{(1)}$, we get

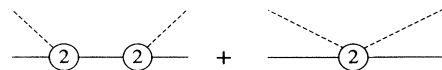


FIG. 4. The Born amplitude for πN scattering.

$$\Gamma^{(1)} = \Gamma^{(2)} + \Lambda_5^{(1)} g M^{(2)} = \Gamma^{(2)} + \Lambda_5^{(2)} g M^{(1)}. \quad (3.16)$$

In this way we have exposed all two-body unitarity cuts in the photoproduction amplitude.

We now write this photoproduction amplitude $M^{(0)}$ as the solution to a two-body integral equation, and in this way define the Born amplitude for the reaction $N\pi \leftarrow N\gamma$. This involves similar steps to those used to derive the equivalent equation for πN elastic scattering, i.e., Eqs. (3.9)–(3.12). This results in the integral equation for $M^{(0)}$ being

$$M^{(0)} = \tilde{v} + v g M^{(0)}, \quad (3.17)$$

with the Born amplitude for pion photoproduction given by

$$\tilde{v} = M^{(2)} + \Lambda_5^{(2)\dagger} S_0 \Gamma^{(2)}, \quad (3.18)$$

where we have made use of the fact that

$$S \Gamma^{(1)} = S_0 \Gamma^{(0)} = S_0 \left(\Gamma^{(2)} + \Lambda_5^{(2)} g M^{(0)} \right). \quad (3.19)$$

At this stage it is premature to link the photoproduction Born amplitude \tilde{v} to the underlying Lagrangian, since the last-cut lemma can be further applied to both $\Gamma^{(2)}$ and $M^{(2)}$ to reveal the contribution of the $\gamma\pi N$ cut. As we will observe, when we expose the $\gamma\pi N$ unitarity cut, \tilde{v} is not the usual Born amplitude for pion photoproduction. It will include additional terms that give rise to the dressing of some of the vertices.

B. The $\gamma\pi N$ cut

To examine the diagrams that contribute to \tilde{v} and in this way determine their physical importance, the last-cut lemma is applied to both $\Gamma^{(2)}$ and $M^{(2)}$ to reveal the $\gamma\pi N$ unitarity cut. The $\gamma\pi N$ branch point occurs at the same energy as the πN branch point but at a lower energy than the start of the $\pi\pi N$ cut as illustrated in Fig. 5. As a result, in this approach, the $\gamma\pi N$ cut will be treated on an equal footing with the πN cut but differently from the $\pi\pi N$ cut, which has been truncated out of our analysis, since we are considering only two-body

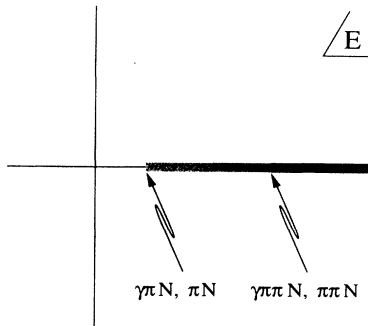


FIG. 5. The energy plane for $\pi + N \leftarrow \gamma + N$ scattering showing the unitarity cuts corresponding to the different thresholds.

unitarity as far as the pion dynamics is concerned. In the approach of AA [2], the $\pi\pi N$ and $\gamma\pi N$ cuts were exposed simultaneously, since their truncation was carried out with respect to the number of particles present in intermediate states in contrast with the present approach of truncation on the basis of the position of the threshold in the energy plane.

To include the $\gamma\pi N$ threshold in our pion photoproduction amplitude, we first consider the diagrams that contribute to the two-particle irreducible $N \leftarrow \gamma N$ amplitude $\Gamma^{(2)}$. These diagrams can be divided into two classes: those that do not have $\gamma\pi N$ intermediate states, the sum of which we denote by $\Gamma^{(3)}$ and those that have $\gamma\pi N$ intermediate states. To the latter we apply the last-cut lemma and thus expose the corresponding threshold. We denote the sum of the diagrams in this class by $\{\Gamma_\pi^{(3)} \tilde{G}^{(2)} \tilde{F}_2^{(2)\dagger}\}_c$, where the subscript c denotes that we include only the connected diagrams in the sum. Here, the subamplitudes $\Gamma_\pi^{(3)}$ for $N \leftarrow \gamma\pi N$ and $\tilde{F}_2^{(2)\dagger}$ for $\gamma\pi N \leftarrow \gamma N$ do not have to be the sum of connected diagrams provided diagrams that contribute to $N \leftarrow \gamma N$ amplitude $\Gamma^{(2)}$ are connected. Finally, $\tilde{G}^{(2)}$ is the product of the propagators for the nucleon, pion, and photon. Since we have a pion in $\tilde{G}^{(2)}$, the nucleon propagator in $\tilde{G}^{(2)}$ need not include any pionic dressing, but the mass in this propagator needs to be the physical nucleon mass. Thus exposing the $\gamma\pi N$ threshold in the $N \leftarrow \gamma N$ amplitude allows us to write this amplitude as

$$\Gamma^{(2)} = \Gamma^{(3)} + \left\{ \Gamma_\pi^{(3)} \tilde{G}^{(2)} \tilde{F}_2^{(2)\dagger} \right\}_c. \quad (3.20)$$

In the absence of a term in the Lagrangian for the $N \leftrightarrow \pi\pi N$ transition, the amplitude $\Gamma^{(3)}$ is nothing more than the nucleon e.m. current, i.e., $\Gamma^{(3)} = \Gamma$. In a similar manner, the absence of a two-pion production or absorption term in the Lagrangian allows us to take the $N \leftarrow \gamma\pi N$ amplitude, $\Gamma_\pi^{(3)}$ to be basically that resulting from the gauging of the πNN vertex, i.e., $\Gamma_\pi^{(3)} = \Gamma_\pi$. Finally, taking into consideration the fact that we are restricting all our results to the inclusion of amplitudes in lowest order in the e.m. coupling, the amplitude $\tilde{F}_2^{(2)\dagger}$ for the process $\gamma\pi N \leftarrow \gamma N$, has to be disconnected and of the form

$$\tilde{F}_2^{(2)\dagger} = D^{-1} \Lambda_5^{(1)\dagger}, \quad (3.21)$$

where D is the photon propagator. This allows us to write the two-particle irreducible $N \leftarrow \gamma N$ amplitude as

$$\Gamma^{(2)} = \Gamma^{(3)} + \Gamma_\pi^{(3)} g \Lambda_5^{(1)\dagger}, \quad (3.22)$$

where for the present Lagrangian we have

$$\Gamma^{(3)} = \langle N | -\mathcal{L}_{\gamma NN} | \gamma N \rangle = \Gamma \quad (3.23)$$

and

$$\Gamma_\pi^{(3)} = \langle N | -\mathcal{L}_{\gamma\pi NN} | \gamma\pi N \rangle = \Gamma_\pi, \quad (3.24)$$

with Γ and Γ_π resulting from the gauging \mathcal{L}_N and $\mathcal{L}_{\pi NN}$ in our basic meson-baryon Lagrangian, respectively.

We now turn to the two-particle irreducible $\pi N \leftarrow \gamma N$ amplitude $M^{(2)}$. Here again we divide all diagrams that contribute to this amplitude into two classes: The first is those with $\gamma\pi N$ intermediate states. These diagrams are summed, with the help of the last-cut lemma, to produce the amplitude in which the $\gamma\pi N$ cut is exposed. The sum of all the diagrams in this class is given by $\{\tilde{F}_3^{(2)} \tilde{G}^{(2)} \tilde{F}_2^{(3)\dagger}\}_c$, where $\tilde{F}_3^{(2)}$ is the two-particle irreducible $\pi N \leftarrow \gamma\pi N$ amplitude. Here again the diagrams that contribute to the subamplitude can be disconnected provided the diagrams that are summed to give the full amplitude are the sum of connected diagrams. The second class of diagrams are those with no $\gamma\pi N$ intermediate states. The sum of these diagrams we denote as $M_\pi^{(2)}$. The subscript π indicates that three-particle intermediate states have only pions, i.e., $\pi\pi N$ intermediate states. Thus the two-particle irreducible $\pi N \leftarrow \gamma N$ amplitude can be written as

$$M^{(2)} = M_\pi^{(2)} + \left\{ \tilde{F}_3^{(2)} \tilde{G}^{(2)} \tilde{F}_2^{(3)\dagger} \right\}_c. \quad (3.25)$$

At this stage we could parametrize $M_\pi^{(2)}$ as an energy independent amplitude without affecting the two-body unitarity or the inclusion of the $\gamma\pi N$ threshold of our final amplitude. However, if we want to relate our input to the underlying Lagrangian given in Eq. (3.1), we need to examine the structure of $M_\pi^{(2)}$. In particular, we need to divide the diagrams that contribute to this amplitude into two classes: The first is those with no $\pi\pi N$ intermediate states and that are therefore three-particle irreducible. These we denote by $M^{(3)}$, and for the Lagrangian under consideration we reduce to the contribution of just the basic term in the Lagrangian $\mathcal{L}_{\gamma\pi NN}$. The second class of diagrams has $\pi\pi N$ intermediate states and need to be included if three-particle unitarity is to be included in the final photoproduction amplitude. However, since we are neglecting three-body unitarity, we have a choice of either neglecting the contribution of these diagrams to the $\pi N \leftarrow \gamma N$ amplitude $M_\pi^{(2)}$ or examining the structure of these diagrams with the help of the last-cut lemma in conjunction with the modified version of Taylor's classification of diagrams [47]. Here we should point out that the contribution to $M_\pi^{(2)}$ from diagrams that admit a $(\pi\pi N)$ cut that cut initial, final, and internal lines will be included in $\{\tilde{F}_3^{(2)} \tilde{G}^{(2)} \tilde{F}_2^{(3)\dagger}\}_c$. That leaves diagrams in which the $(\pi\pi N)$ cut can intersect initial and internal lines only, or final and internal lines only. Because we are including the e.m. interaction to first order and have excluded any direct coupling between a $\pi\pi\pi N$ intermediate state and the γN initial state, there are no diagrams in this class. This leaves the diagrams that admit $(\pi\pi N)$ cuts that intersect initial and internal lines only. These will involve the connected $\pi N \leftarrow \pi\pi N$ diagrams, which have the contribution of three-body unitarity. For the present investigation we have chosen to neglect this contribution to three-particle unitarity and have taken $M_\pi^{(2)} = M^{(3)}$, and therefore

$$M^{(2)} = M^{(3)} + \left\{ \tilde{F}_3^{(2)} \tilde{G}^{(2)} \tilde{F}_2^{(3)\dagger} \right\}_c. \quad (3.26)$$

We now turn our attention to the contribution from the $\gamma\pi N$ threshold to $M^{(2)}$, i.e., $\left\{ \tilde{F}_3^{(3)} \tilde{G}^{(2)} \tilde{F}_2^{(2)\dagger} \right\}_c$. Since the e.m. coupling is included to first order in the present analysis, we have that the $\gamma\pi N \leftarrow \gamma N$ amplitude $\tilde{F}_2^{(2)\dagger}$ has to be disconnected and of the form

$$\tilde{F}_2^{(2)\dagger} = \tilde{F}_{2;d}^{(2)\dagger} = D^{-1} \Lambda_5^{(1)\dagger}. \quad (3.27)$$

On the other hand, the three-particle irreducible $\pi N \leftarrow \gamma\pi N$ amplitude $\tilde{F}_3^{(3)}$ can be written in terms of a connected and a disconnected part as

$$\tilde{F}_3^{(3)} = \tilde{F}_{3;d}^{(3)} + \tilde{F}_{3;c}^{(3)}. \quad (3.28)$$

Since the photon is absorbed in this process, the disconnected amplitude $\tilde{F}_{3;d}^{(3)}$ has the form

$$\tilde{F}_{3;d}^{(3)} = S_0^{-1} \Gamma^{\pi(2)} + \Delta^{-1} \Gamma^{(2)}, \quad (3.29)$$

where $\Gamma^{\pi(2)}$ is the two-particle irreducible $\pi \leftarrow \gamma\pi$ amplitude. Since we already have a spectator pion, and to avoid the problem of nonlinearity of our final integral equations, we could take

$$\Gamma^{(2)} \rightarrow \Gamma^{(3)} = \Gamma, \quad (3.30)$$

where Γ is the nucleon e.m. current. Since there is no direct coupling in the Lagrangian between the final πN state and any state with four particles (e.g., $\pi\pi\pi N$ or $\gamma\pi\pi N$), we take $\tilde{F}_{3;c}^{(3)}$ to be the $\pi N \leftarrow \gamma\pi N$ interaction in the Lagrangian, i.e.,

$$\tilde{F}_{3;c}^{(3)} = \langle \pi N | -\mathcal{L}_{\pi N \gamma \pi N} | \gamma \pi N \rangle. \quad (3.31)$$

As with the exposure of the cross diagram in Eq. (3.4), $\tilde{F}_{3;c}^{(3)}$ can also have the last-cut lemma further applied before contact with the underlying Lagrangian is made. At this stage this will be ignored, since we are only interested in the *minimal* requirements of two-body unitarity, and so $t^{(2)}$ will be considered as a background πN potential.

We now make use of Eqs. (3.27) and (3.29) in Eq. (2.12) to write the two-particle irreducible amplitude for $\pi N \leftarrow \gamma N$ as

$$\begin{aligned} M^{(2)} &= M^{(3)} + \tilde{F}_{3;c}^{(3)} g \Lambda_5^{(1)\dagger} + \Gamma^{\pi(2)} \Delta \Lambda_5^{(1)\dagger} + \Gamma^{(2)} S_0 \Lambda_5^{(1)\dagger} \\ &= M^{(3)} + \left[\tilde{F}_{3;c}^{(3)} + \Gamma^{\pi(2)} S_0^{-1} + \Gamma^{(2)} \Delta^{-1} \right] g \Lambda_5^{(1)\dagger}. \end{aligned} \quad (3.32)$$

This basically consists of the "seagull" term $M^{(3)}$, the nucleon emitting a pion with the photon being absorbed on the πN interaction. The details of this $\pi N \leftarrow \gamma\pi N$ amplitude will be determined by the gauging of the πN interaction. The two final terms on the right-hand side of Eq. (3.32) correspond to photon absorption on the pion and the crossed diagrams for pion photoproduction.

We now can write the Born amplitude for pion photoproduction \tilde{v} given in Eq. (3.18) as

$$\begin{aligned} \tilde{v} = M^{(3)} + \left[\tilde{F}_{3;c}^{(3)} + \Gamma^{\pi(2)} S_0^{-1} + \Gamma^{(2)} \Delta^{-1} \right] g \Lambda_5^{(1)\dagger} \\ + \Lambda_5^{(2)\dagger} S_0 \Gamma^{(2)} . \end{aligned} \quad (3.33)$$

For the present, the seagull term is the contact term in the Lagrangian, i.e.,

$$M^{(3)} = \Gamma^{\text{CT}} , \quad (3.34)$$

while the two-particle irreducible pion current

$$\Gamma^{\pi(2)} = \Gamma^\pi . \quad (3.35)$$

For the minimal requirement of two-body unitarity and the inclusion of the πN and $\gamma\pi N$ thresholds, we could replace the reducibility of the $N \leftrightarrow \gamma N$ amplitude $\Gamma^{(2)}$ and the $N \leftrightarrow \pi N$ amplitude $\Lambda_5^{(2)}$ by an amplitude that is of higher reducibility, and which is directly related to the basic terms in our Lagrangian, depending on the diagrams to which these amplitudes contribute. We will come back to this point when we consider the gauge invariance of our overall amplitude.

IV. THE PION PHOTOPRODUCTION AMPLITUDE

Having derived a set of coupled integral equations for the pion photoproduction amplitude, we proceed in this section to derive an expression for this amplitude with the help of the Ward-Takahashi identities [29,30] as employed by Kazes [31]. We then can compare the resultant amplitude, which is gauge invariant, to the unitary am-

plitude derived in the last section.

The starting point for deriving a gauge invariant pion photoproduction amplitude is the πNN three-point function or Green's function for $\pi N \leftarrow N$, given by

$$\begin{aligned} G(q, p', p) = \int d^4x_1 d^4x_2 d^4x_3 e^{i(p' \cdot x_3 + q \cdot x_1 - p \cdot x_2)} \\ \times \langle 0 | T [\phi(x_1) \psi(x_2) \bar{\psi}(x_3)] | 0 \rangle \\ = S(p') \Delta(q) \Lambda_5^\dagger(q, p', p) \vec{\tau} \cdot \vec{\phi} S(p) , \end{aligned} \quad (4.1)$$

where q is the pion momentum and p' (p) is the momentum of the final (initial) nucleon. Here S and Δ are the nucleon and pion propagators, respectively. In Eq. (4.1) we have taken $\vec{\tau}$ to be the Pauli isospin matrix and $\vec{\phi}$ to be the pion field. To gauge this amplitude and thus generate a gauge invariant pion photoproduction amplitude, we make use of the procedure of coupling the photon to all propagators and vertices. This is equivalent to the substitutions [31]

$$S(p) \rightarrow S(p) + S(p') \Gamma_\mu(k, p', p) S(p) A^\mu, \quad (4.2a)$$

$$\Delta(q) \rightarrow \Delta(q) + \Delta(q') \Gamma_\mu^\pi(k, q', q) \Delta(q) A^\mu, \quad (4.2b)$$

$$\Lambda_5^\dagger(q, p', p) \tau_i \rightarrow \Lambda_5^\dagger(q, p', p) \tau_i + \Gamma_\mu^{\text{CT}i}(k, q, p', p) A^\mu, \quad (4.2c)$$

in the Green's function $G(q, p', p)$. To maintain unitarity we need to include the pionic dressing of both the propagators and vertices. In particular, we will find that the minimal requirement of two-body unitarity will impose a constraint on the level of dressing in both the propagator $S(p)$ and the vertex $\Lambda_5^\dagger(q, p', p)$.

With the above gauging procedure, our πNN Green's function can be gauged to give

$$G(q, p', p) \rightarrow G(q, p', p) + S(p') \Delta(q) M_{\mu; \pi N \leftarrow \gamma N}(q, k, p', p) S(p) A^\mu . \quad (4.3)$$

The resultant pion photoproduction amplitude $M_{\mu; \pi N \leftarrow \gamma N}(q, k, p', p)$ is now given as the sum of four diagrams, i.e.,

$$\begin{aligned} M_{\mu; \pi N \leftarrow \gamma N}^i(q, k, p', p) = \Lambda_5^\dagger(q, p', p+k) \tau_i S(p+k) \Gamma_\mu(k, p+k, p) + \Gamma_\mu(k, p', p'-k) S(p'-k) \Lambda_5^\dagger(q, p'-k, p) \tau_i \\ + \Gamma_\mu^{\pi ij}(k, q, q-k) \Delta(q-k) \Lambda_5^\dagger(q-k, p', p) \tau_j + \Gamma_\mu^{\text{CT}i}(k, q, p', p) . \end{aligned} \quad (4.4)$$

Here q is the pion momentum, k the photon momentum while p' and p are the nucleon momenta in the final and initial states, respectively. The superscripts i and j in $M_{\mu; \pi N \leftarrow \gamma N}^i$ and $\Gamma_\mu^{\pi ij}$ indicate the isospin index, which we have now explicitly included. At this stage the pionic dressing in the $\pi N \leftarrow N$ amplitude Λ_5^\dagger , the nucleon current Γ_μ , the pion current Γ_μ^π , and seagull term Γ_μ^{CT} are included to all orders. These four diagrams are illustrated in Fig. 6 and at this stage give the most general form for the $\pi N \leftarrow \gamma N$ amplitude if we ignore the irreducibility of the subamplitudes in the figure [31,35,49].

Using the Ward-Takahashi identities [29,30] for the nucleon propagator, the pion propagator, and the πNN vertex, we can establish the gauge invariance of the amplitude $M_{\mu; \pi N \leftarrow \gamma N}^i$. In particular we have that $k^\mu M_{\mu; \pi N \leftarrow \gamma N}^i$ reduces to

$$\begin{aligned} k^\mu M_{\mu; \pi N \leftarrow \gamma N}^i(q, k, p', p) = e_N \tau_i S^{-1}(p') S(p'-k) \Lambda_5^\dagger(q, p'-k, p) - \tau_i e_N \Lambda_5^\dagger(q, p', p+k) S(p+k) S^{-1}(p) \\ - ie \epsilon_{3ij} \tau_j \Delta^{-1}(q) \Delta(q-k) \Lambda_5^\dagger(q-k, p', p) , \end{aligned} \quad (4.5)$$

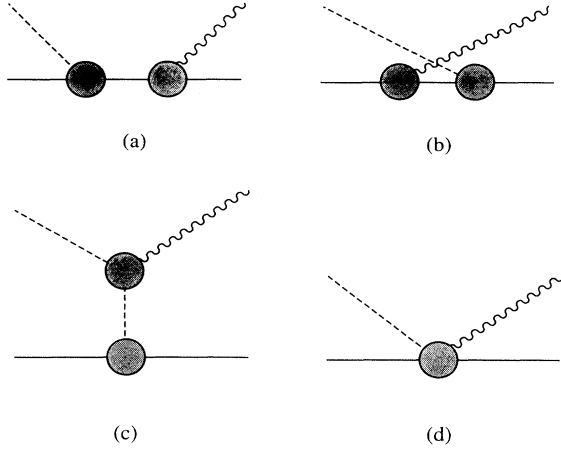


FIG. 6. The four types of diagrams that contribute to the pion photoproduction amplitude. These result from the gauging of the πNN three-point function.

which is in agreement with the results of Kazes [31]. Taking matrix elements of Eq. (4.5) between on-mass-shell initial and final states gives

$$\langle \pi N | k^\mu M_{\mu; \pi N \leftarrow \gamma N}^i(q, k, p', p) | \gamma N \rangle = 0, \quad (4.6)$$

since the inverse propagators are zero on-mass shell. In

this way we have established the fact that $M_{\mu; \pi N \leftarrow \gamma N}^i$ is indeed gauge invariant, and the requirement of current conservation is satisfied.

At this stage the πNN Green's function we have considered has the fully dressed propagators and vertex, and to get these quantities will require a full solution to the underlying field theory. However, to satisfy the minimum requirement of two-body unitarity, we need not include the pionic dressing of the final nucleon propagator, while the initial nucleon propagator and the $\pi N \leftarrow N$ amplitude need only include the minimal pionic dressing to include the πN threshold. In this case the Green's function we need to consider is given by

$$G_1(q, p', p) = S_0(p') \Delta(q) \Lambda_5^{(1)\dagger}(q, p', p) \vec{\tau} \cdot \vec{\phi} S^{(1)}(p). \quad (4.7)$$

Then this Green's function can be gauged to give a corresponding amplitude for pion photoproduction that satisfies the *minimum* requirement of two-body unitarity. Here the final nucleon propagator is gauged by the transformation given in Eq. (2.9), while the gauging of the $\pi N \leftarrow N$ amplitude $\Lambda_5^{(1)\dagger}$ and the dressed nucleon propagator $S^{(1)}$ will be derived using the results of the preceding section. The result of gauging the Green's function in Eq. (4.7) is

$$G_1(q, p', p) \rightarrow G_1(q, p', p) + S_0(p') \Delta(q) \bar{M}_{\mu; \pi N \leftarrow \gamma N}^i(q, k, p', p) S^{(1)}(p) A_\mu, \quad (4.8)$$

where the $\pi N \leftarrow \gamma N$ amplitude $\bar{M}_{\mu; \pi N \leftarrow \gamma N}^i(q, k, p', p)$ is given by

$$\begin{aligned} \bar{M}_{\mu; \pi N \leftarrow \gamma N}^i(q, k, p', p) = & \Lambda_5^{(1)\dagger}(q, p', p+k) \tau_i S^{(1)}(p+k) \Gamma_\mu^{(1)}(k, p+k, p) \\ & + \Gamma_\mu^0(k, p', p'-k) S_0(p'-k) \Lambda_5^{(1)\dagger}(q, p'-k, p) \tau_i \\ & + \Gamma_\mu^{\pi ij}(k, q, q-k) \Delta(q-k) \Lambda_5^{(1)\dagger}(q-k, p', p) \tau_j + \Gamma_\mu^{\text{CT}(1)i}(k, q, p', p). \end{aligned} \quad (4.9)$$

This result differs from that of Eq. (4.4) to the extent that the nucleon in the final state did not require pionic dressing for two-body unitarity to be satisfied, and as a result the corresponding current Γ_μ^0 does not include any pionic corrections. On the other hand the current resulting from gauging the initial nucleon has the necessary pionic corrections to satisfy two-body unitarity. The proof that this amplitude satisfies gauge invariance follows the same procedure considered above, with the difference being the difference in the Ward-Takahashi identities for the bare and dressed nucleon propagators S_0 and S . In this case the gauge invariance of the amplitude $\bar{M}_{\mu; \pi N \leftarrow \gamma N}^i$ is given by

$$\begin{aligned} k^\mu \bar{M}_{\mu; \pi N \leftarrow \gamma N}^i(q, k, p', p) = & e_N \tau_i S_0^{-1}(p') S_0(p'-k) \Lambda_5^{(1)\dagger}(q, p'-k, p) \\ & - \tau_i e_N \Lambda_5^{(1)\dagger}(q, p', p+k) S^{(1)}(p+k) S^{(1)-1}(p) \\ & - i e \epsilon_{3ij} \tau_j \Delta^{-1}(q) \Delta(q-k) \Lambda_5^{(1)\dagger}(q-k, p', p), \end{aligned} \quad (4.10)$$

which on-mass shell gives a conserved current, provided both the bare and dressed nucleon propagators have poles at the physical nucleon mass. Thus we have constructed a gauge invariant amplitude for pion photoproduction starting with the Green's function for the process $\pi N \leftarrow N$ in which the final nucleon propagator has less pionic dressing than the propagator in the initial state. We proceed in the next section to demonstrate that this

gauge invariant amplitude is identical to that resulting from the solution of the coupled equations, and that it satisfies two-body unitarity.

V. IS THE GAUGE INVARIANT AMPLITUDE UNITARY?

In the preceding section we demonstrated how one may construct a gauge invariant amplitude for pion photopro-

duction given the $\pi N \leftarrow N$ Green's function and the transformation of the pion propagator, nucleon propagator, and the $\pi N \leftarrow N$ amplitude under the gauge symmetry. In Sec. II these gauge transformations were given for the different terms in the basic meson-baryon Lagrangian. Here we are going to use the gauging of these basic terms in the Lagrangian to derive the results of gauging the pionic dressed nucleon propagator $S(p)$, the pion propagator $\Delta(q)$, and the one-particle irreducible $\pi N \leftarrow N$ amplitude $\Lambda_5^{(1)\dagger}$. From this point on we will drop the superscript (1) on the dressed nucleon propagator assuming that $S(p)$ includes the necessary pionic dressing to include two-body unitarity. These results, when used in the definition of the three-point function, will give us the amplitude for pion photoproduction with the application of Lehmann-Symanzik-Zimmerman (LSZ) [50] reduction to the resultant Green's function. The amplitude resulting from this gauging procedure is identical to that resulting from the solution of the coupled set of integral equations derived in Sec. III. In this way we establish the gauge invariance of the pion photoproduction amplitude resulting from the solution of the coupled integral equations, which already satisfy unitarity.

Since the nucleon propagator $S_0(p')$ need not include pionic dressing, but might include the quark-gluon structure, we may use the result of Sec. II to write

$$S_0 \xrightarrow{U(1)} S_0 + S_0 \Gamma^{(3)} S_0 . \quad (5.1)$$

If we assume that our bare nucleon has the quark-gluon structure, and this structure does not contribute to unitarity, then the nucleon current is $\Gamma_\mu^{(3)} = \Gamma_\mu$. Note that in the absence of the quark-gluon structure, this current will be the standard Dirac current for a point fermion. In a similar manner, the gauge prescription for the basic input into our coupled integral equations, which are also the terms in the Lagrangian, are given as

$$\Lambda_5^{(2)\dagger} \xrightarrow{U(1)} \Lambda_5^{(2)\dagger} + \Gamma^{\text{CT}(3)} , \quad (5.2)$$

$$\Lambda_5^{(2)} \xrightarrow{U(1)} \Lambda_5^{(2)} + \Gamma_\pi^{(3)} , \quad (5.3)$$

$$t^{(2)} \xrightarrow{U(1)} t^{(2)} + \tilde{F}_{3,c}^{(3)} . \quad (5.4)$$

Here, in addition to the gauging of the πNN vertex, we have included the gauging of the two-particle irreducible $\pi N \leftarrow \pi N$ amplitude. For the pion propagator we take

$$\Delta \xrightarrow{U(1)} \Delta + \Delta \Gamma^{\pi(1)} \Delta . \quad (5.5)$$

In our meson-baryon Lagrangian the only dressing the pion can have is via nucleon antinucleon loops [51]. If we ignore these loop corrections to the pion dressing, the only dressing we could have is the result of the underlying quark-gluon structure. The pion current in this case can be written in terms of the gauge invariant current, i.e., $\Gamma_\mu^{\pi(1)} = \Gamma_\mu^{\pi(2)} = \Gamma_\mu^\pi$.

To gauge the one-particle irreducible πN amplitude $t^{(1)}$, we need to first gauge the πN propagator g . Since only two-body unitarity need be considered, the nucleon propagator in g can be taken to be the bare propagator S_0 , and the πN propagator reduces to $g = S_0 \Delta$. To first

order in the e.m. coupling the gauge transformation of the πN propagator reduces to

$$\begin{aligned} g &\xrightarrow{U(1)} g + \Delta S_0 \Gamma^{(3)} S_0 + S_0 \Delta \Gamma^{\pi(2)} \Delta \\ &= g + g \tilde{F}_{3;d}^{(3)} g . \end{aligned} \quad (5.6)$$

With the help of the two-body equation for $t^{(1)}$, and the transformation properties of the πN amplitude $t^{(2)}$ and g , the gauging of the one particle irreducible πN amplitude $t^{(1)}$ reduces to

$$t^{(1)} \xrightarrow{U(1)} t^{(1)} + \tilde{F}_3 , \quad (5.7)$$

where, to first order in the e.m. coupling,

$$\tilde{F}_3 = \left(t^{(1)} g + 1 \right) \tilde{F}_{3;c}^{(3)} \left(1 + g t^{(1)} \right) + t^{(1)} g \tilde{F}_{3;d}^{(3)} g t^{(1)} . \quad (5.8)$$

With these results in hand, we can now determine how $\Lambda_5^{(1)\dagger}$ behaves under gauging. Given that the one-particle irreducible amplitude for $\pi N \leftarrow N$ is given, using Eq. (3.5), by the relation

$$\Lambda_5^{(1)\dagger} = \left(t^{(1)} g + 1 \right) \Lambda_5^{(2)\dagger} , \quad (5.9)$$

we can make use of Eqs. (5.2), (5.6), and (5.7) to write the gauge transformation for $\Lambda_5^{(1)\dagger}$ as

$$\Lambda_5^{(1)\dagger} \xrightarrow{U(1)} \Lambda_5^{(1)\dagger} + M' , \quad (5.10)$$

where to first order in e.m. coupling the $\pi N \leftarrow \gamma N$ amplitude M' is given by

$$\begin{aligned} M' &= \left(t^{(1)} g + 1 \right) \left(\Gamma^{\text{CT}} + \tilde{F}_{3;c}^{(3)} g \Lambda_5^{(1)\dagger} \right) \\ &\quad + t^{(1)} g \tilde{F}_{3;d}^{(3)} g \Lambda_5^{(1)\dagger} . \end{aligned} \quad (5.11)$$

All that is left to determine is how the nucleon propagator S behaves under gauging when the πN unitarity cut has been exposed. From Eqs. (3.6) and (3.7) we have that

$$\begin{aligned} S &= \left[S_0^{-1} - \Sigma^{(1)} \right]^{-1} \\ &= S_0 + S_0 \Sigma^{(1)} S , \end{aligned} \quad (5.12)$$

where

$$\Sigma^{(1)} = \Lambda_5^{(2)} g \Lambda_5^{(1)\dagger} . \quad (5.13)$$

With the help of Eqs. (5.1), (5.3), (5.6), and (5.10), the dressed nucleon propagator can be gauged to give

$$S^{-1} \xrightarrow{U(1)} S^{-1} - \tilde{\Gamma}^{(1)} , \quad (5.14)$$

where the dressed $N \leftarrow \gamma N$ amplitude $\tilde{\Gamma}^{(1)}$ is given, to first order in the e.m. coupling, by

$$\begin{aligned} \tilde{\Gamma}^{(1)} &= \Gamma^{(3)} + \Gamma_\pi^{(3)} g \Lambda_5^{(1)\dagger} + \Lambda_5^{(2)} g \left[\tilde{F}_{3;d}^{(3)} g \Lambda_5^{(1)\dagger} + M' \right] \\ &= \Gamma^{(2)} + \Lambda^{(1)} g \left[\Gamma^{\text{CT}} + \tilde{F}_3^{(3)} g \Lambda_5^{(1)\dagger} \right] . \end{aligned} \quad (5.15)$$

In writing the second line in Eq. (5.15), we have made use of Eqs. (3.22) and (5.11). If we now compare the dressed nucleon current resulting from the gauging of the dressed nucleon propagator, and make use of Eqs. (3.16) and (3.32), we find that this current is in fact identical to the one-particle irreducible current derived in Sec. III, i.e.,

$$\tilde{\Gamma}^{(1)} = \Gamma^{(1)}. \quad (5.16)$$

Therefore the nucleon propagator S behaves, to first order in the electromagnetic coupling, under gauging as

$$S \xrightarrow{U(1)} S + S \Gamma^{(1)} S. \quad (5.17)$$

Thus exposing the πN unitarity cut in S , and gauging, leads to the same results as exposing the $\gamma\pi N$ unitarity cut, which is what one expects.

To complete the proof of the equivalence of the two formulations of Secs. III and IV, we are required to show that

$$M = M^{(0)}, \quad (5.18)$$

where M results from substituting Eqs. (5.17), (5.10), (5.5), and (5.1) into Eq. (4.7) with the result that gauging the three-point function for the process $\pi N \leftarrow N$ is given by

$$G_1 \xrightarrow{U(1)} G_1 + S_0 \Delta M S, \quad (5.19)$$

and the corresponding pion photoproduction amplitude M is of the form

$$M = \Gamma^{(3)} S_0 \Lambda_5^{(1)\dagger} + \Gamma^{\pi(1)} \Delta \Lambda_5^{(1)\dagger} + \Lambda_5^{(1)\dagger} S \Gamma^{(1)} + M'. \quad (5.20)$$

In writing Eq. (5.19), we have restricted ourselves to first order in the electromagnetic coupling.

Substituting for M' from Eq. (5.11) and making use of the definition of the two-particle irreducible $\pi N \leftarrow \gamma N$ amplitude $M^{(2)}$ given in Eq. (3.32), we get

$$\begin{aligned} M &= \left(t^{(1)} g + 1 \right) M^{(2)} + \Lambda_5^{(1)\dagger} S \Gamma^{(1)} \\ &= M^{(2)} + t^{(2)} g M^{(1)} + \Lambda_5^{(1)\dagger} S \Gamma^{(1)}. \end{aligned} \quad (5.21)$$

In writing the second line we have made use of Eq. (3.15) to relate the one-particle and two-particle irreducible $\pi N \leftarrow \gamma N$ amplitudes. Making use of the adjoint of Eq. (5.3), we can write M as

$$\begin{aligned} M &= M^{(2)} + \Lambda_5^{(2)\dagger} S \Gamma^{(1)} + t^{(2)} g \left[M^{(1)} + \Lambda_5^{(1)\dagger} S \Gamma^{(1)} \right] \\ &= M^{(2)} + \Lambda_5^{(2)\dagger} S_0 \left[\Gamma^{(2)} + \Lambda_5^{(2)} g M^{(0)} \right] + t^{(2)} g M^{(0)}. \end{aligned} \quad (5.22)$$

The second line of Eq. (5.22) is derived by making use of Eq. (3.19) to write $S \Gamma^{(1)}$ in terms of $M^{(0)}$ and Eq. (3.14) to get the last term on the right-hand side. We now employ the definition of the Born amplitude for pion photoproduction \tilde{v} , Eq. (3.33), and the πN potential v given

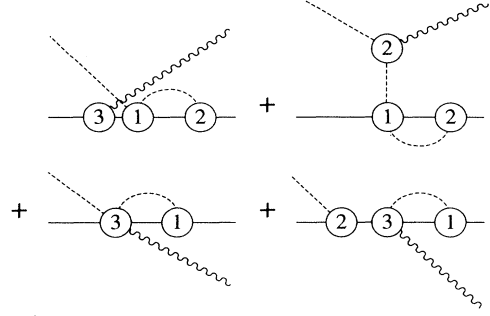


FIG. 7. The non-Born diagrams that contribute to the pion photoproduction amplitude; the number in the circle gives the irreducibility of each amplitude.

in Eq. (3.12) to write Eq. (5.22) as

$$M = \tilde{v} + v g M^{(0)}. \quad (5.23)$$

If we compare this result with Eq. (3.17), we observe that the right-hand sides of both equations are identical. This proves the fact that the amplitude, which is a solution of the coupled equations that include the γN , πN , and $\gamma\pi N$ thresholds, is identical to that derived using the gauging of the $\pi N \leftarrow N$ Green's function with the help of the Ward-Takahashi identities. Thus we have that

$$M = M^{(0)}, \quad (5.24)$$

and in this way we have established the fact that the solution of the coupled integral equation, Eq. (3.17), gives us an amplitude that satisfies two-body unitarity and is gauge invariant.

To establish the connection with previous results in the literature, we iterate Eq. (5.23) with the result

$$\begin{aligned} M^{(0)} &= \tilde{v} + (v + v g v + v g v g v + \dots) \tilde{v} \\ &= \left(t^{(0)} g + 1 \right) \tilde{v} \\ &= \left(t^{(0)} g + 1 \right) \tilde{v}_B + \left(t^{(0)} g + 1 \right) \tilde{v}_R, \end{aligned} \quad (5.25)$$

where \tilde{v}_B is the Born term, illustrated in Fig. 1, and has often been used as a gauge invariant term on its own. The factor of $(t^{(0)} g + 1)$ gives the distortion in the πN channel. The second term, illustrated in Fig. 7, is required to maintain gauge invariance at the operator level for $M^{(0)}$ and results from including the $\gamma\pi N$ channel into our integral equations. This inclusion of the $\gamma\pi N$ channel effectively allows us to couple the photon not only to the initial nucleon in the $\pi N \leftarrow N$ vertex, but also to the nucleon in the final state. In this way we have coupled the nucleon to every propagator and vertex in the three-point Green's function for $\pi N \leftarrow N$.

VI. CONCLUSION

To employ pion photoproduction at medium energies to examine nucleon structure and, in particular, the resonances observed in πN scattering, we need a theory that

incorporates charged current conservation and the conservation of probability, i.e., gauge invariance and unitarity. If in addition we would like to test current models of QCD, based on being able to separate quark-gluon from meson-baryon degrees of freedom, we need to include form factors in which we have consistency between the πNN and γNN form factors resulting from QCD. In the present investigation, we considered a Lagrangian written in terms of the meson-baryon degrees of freedom. This Lagrangian has form factors that, in principle, could be derived from a QCD model. We then gauged this Lagrangian. This gauging procedure could have been carried out for a specific QCD model (see, e.g., Ref. [32]); however, to maintain generality we assumed that the gauging of the πN Lagrangian gives rise to photon vertices that satisfy the Ward-Takahashi identity.

Within the framework of this Lagrangian, which includes not only the meson-baryon degrees of freedom but also the coupling of the photon to these degrees of freedom, we derived a set of coupled integral equations that satisfy two-particle unitarity. This was achieved by exposing the πN , γN , and $\pi\gamma N$ thresholds following the analysis of Taylor [37,38]. To prove that the solution of these coupled equations is also gauge invariant, we employed the Ward-Takahashi identities, derived for the basic terms in the Lagrangian, to gauge the one-particle irreducible three point function for the process $\pi N \leftarrow N$. Since we demanded that our amplitude satisfy *only* two-body unitarity, we assumed that the final nucleon propagator in the $\pi N \leftarrow N$ Green's function did not have any pionic dressing, and the initial nucleon propagator had the minimal dressing to include the πN threshold, while the πNN vertex in the three-point function was one-particle irreducible. This gauging procedure gave us an amplitude that is identical to that resulting from a solution of coupled integral equations. We found it essential that we include the $\gamma\pi N$ threshold into our coupled integral equation.

The need for the inclusion of the $\gamma\pi N$ threshold to satisfy gauge invariance at the operator level, raises some questions regarding the application of Watson's theorem [14] or its off-shell modification [7] to unitarize the gauge invariant Born term represented in Fig. 1, and to maintain gauge invariance in the final result. In fact, when we cast our amplitude into a form of Watson's theorem, i.e., a distorted wave in the πN channel, we found that we had an additional term that has not been included in most calculations in the past. These additional

terms are required in order to preserve gauge invariance of the photoproduction amplitude at the operator level.

The present coupled integral equations for $\pi N \rightarrow \pi N$ and $\gamma N \rightarrow \pi N$ reactions are four-dimensional Bethe-Salpeter [52] equations, and any reduction to a three-dimensional equation, e.g., the Blankenbecler-Sugar reduction [53], will destroy the gauge invariance. This could be overcome by either deriving the Ward-Takahashi identities within the framework of propagators in three dimensions [54] or developing the necessary numerical procedure for solving the Bethe-Salpeter equations. We are presently examining both approaches.

The Δ resonance has not been considered explicitly in the present analysis, since we have not detailed our model for πN scattering. We could include the contribution of the Δ in one of two ways depending on the dynamics used to generate the πN amplitude: (i) In a model where the Δ is taken to be a baryon to be treated on equal footing with the nucleon, we will need to consider the inclusion of the Δ explicitly. In this case we will need to determine the gauging of the Δ propagator to generate the coupling of the photon to the Δ . (ii) If the Δ is considered a πN resonance that is generated dynamically, then the gauging of the πN amplitude detailed above will include the contribution of the Δ to pion photoproduction. For the πN amplitude to satisfy both unitarity and crossing symmetry will require the solution of a nonlinear integral equations, and to that extent we have not considered the inclusion of crossing symmetry in the present analysis. To include higher order πN resonances, we need to include the $\pi\pi N$ threshold as well as the $\gamma\pi\pi N$ threshold into the formulation. At this stage we have developed the formalism to include the $\pi\pi N$ threshold [2,46] with application in the cloudy bag model [55] for πN scattering. We propose to examine the inclusion of the $\gamma\pi\pi N$ threshold, which will allow us to extend the coupled channel approach to energies above the threshold for two-pion production and the rich spectrum of πN resonances.

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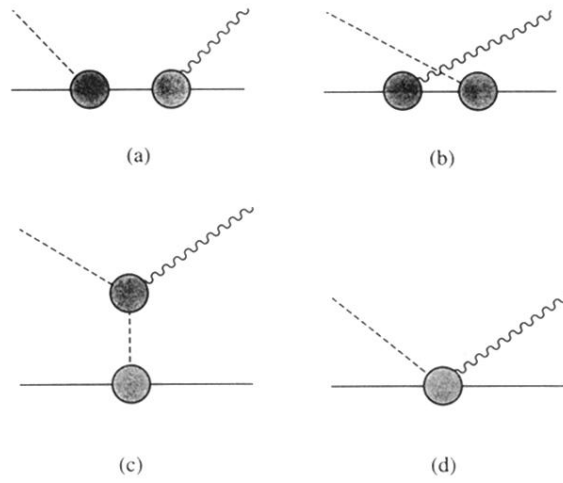


FIG. 6. The four types of diagrams that contribute to the pion photoproduction amplitude. These result from the gauging of the πNN three-point function.