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Quark cluster model study of isospin-2 dibaryons

A. Valcarce,[1] H. Garcilazo,[2] and F. Fernández[1]
[1] *Grupo de Física Nuclear, Universidad de Salamanca, E-37008 Salamanca, Spain*
[2] *Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, Edificio 9, 07738 México D.F., Mexico*
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Based on a quark cluster model for the nonstrange sector that reproduces reasonably well the nucleon-nucleon system and the excitation of the Δ isobar, we generate a nucleon-Δ interaction and present the predictions for the several isospin-2 channels. The only attractive channels are 0^+ and 0^-, but not attractive enough to generate a resonance. If a resonance is artificially generated and is required to have the observed experimental mass, then our model predicts a width that agrees with the experimental result.

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I. INTRODUCTION

The possibility that a resonance may exist in the πNN system with $J^P = 0^-$ has been the subject of great discussion recently [1–3]. A 0^- resonance in the isospin-0 channel was proposed in Ref. [1] in order to explain a peculiar behavior of the pionic double charge exchange cross section in several nuclei from ^{14}C to ^{48}Ca. However, it was pointed out in Ref. [2] that according to the calculations of Ref. [4] a 0^- resonance with isospin zero is not possible and that most likely the resonance must have isospin 2. The motivation of this work is to present the predictions of a quark-model-based interaction about possible resonances in the isospin-2 sector.

While the isospin-1 channel has been extensively studied, the isospin 2 is less well known. However, there have been several works in the literature where the possibility of a bound state or a resonance in the πNN system with isospin 2 has been studied. Gale and Duck [5] studied this problem in the framework of the nonrelativistic Faddeev equations, using a model of a rank-1 separable potential for the π-nucleon interaction in the P_{33} channel without including the important nucleon-nucleon interaction. Their study did not support the existence of πNN bound states, not even when they included relativistic kinematics in the Faddeev equations. They predicted a broad resonance at an energy about 350 MeV above the NN threshold. In subsequent works by Ueda [6] and Kalbermann and Eisenberg [7] the system was investigated using the Heitler-London-Pauli variational method in a nonrelativistic approach. These authors, as in Ref. [5], did not find such bound states. However, the possible existence of a quasibound system was not ruled out.

Lately, there were a series of papers by Garcilazo and Mathelitsch [2,4,8] based on the relativistic Faddeev equations including both π-nucleon and nucleon-nucleon interactions. They used separable potentials for the π-nucleon and nucleon-nucleon subsystems with $J \leq 2$ which were fitted to the phase shifts up to 350 MeV. They found that bound state solutions are possible when a sufficiently short-ranged π-nucleon interaction is assumed. At the same time, they showed that the most likely configuration to have a bound state or a resonance is that with angular momentum and parity quantum numbers $J^P = 2^-$.

There have been other calculations about the same problem. Arenhövel [9] predicted a bound state with the quantum numbers 2^+ and isospin 2, making use of a nucleon-Δ interaction mediated by the exchange of the π and ρ mesons (see also Ref. [10]).

Our aim in this work is to present the predictions based on a quark dynamical model. This model is able to describe correctly the nucleon-nucleon system as well as the deuteron. As it is based only on interactions between quarks its generalization to the nucleon-Δ system is straightforward and it does not mean the inclusion of any new parameter. Therefore, it is our goal to use the nucleon-Δ interaction generated by this quark model to analyze the possibility of the existence of a bound state or a resonance in the isospin-2 sector in all the nucleon-Δ channels with angular momentum $J \leq 2$.

In Sec. II we will briefly describe the quark model. In Sec. III we will review our formalism and we will present and discuss our results. Finally, in Sec. IV we will summarize our conclusions.

II. THE MODEL OF INTERACTION

The constituent quark model for the nucleon-nucleon (NN) interaction developed during the last decade [11] has been recently extended in a series of papers [12,13] by introducing a scalar meson (the σ-meson) exchange between quarks and not between the nucleons as done previously. In this work, the σ-meson parameters are related to those of the pion exchange and to the constituent quark mass through chiral symmetry requirements. This model has been successfully applied to the description of the NN scattering phase shifts [12] and the static and electromagnetic properties of the deuteron [13]. At the same time it has been used to the description of processes that take place with the excitation of a Δ isobar [14] and to study the pure nucleon-Δ interaction [15].

The model has been introduced in Refs. [12,13]. The ingredients of the quark-quark interaction are the con-

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fining potential (con), the one-gluon exchange (OGE), the one-pion exchange (OPE), and the one-σ exchange (OSE). The explicit form of these interactions is given by (see Refs. [12,13] for details)

$$V_{\rm con}(\vec{r}_{ij}) = -a_c\, \vec{\lambda}_i \cdot \vec{\lambda}_j\, r_{ij}^2 \,, \tag{1}$$

$$V_{\rm OGE}(\vec{r}_{ij}) = \frac{1}{4}\,\alpha_s\, \vec{\lambda}_i \cdot \vec{\lambda}_j \left\{ \frac{1}{r_{ij}} - \frac{\pi}{m_q^2}\left[1 + \frac{2}{3}\vec{\sigma}_i \cdot \vec{\sigma}_j\right] \delta(\vec{r}_{ij}) - \frac{3}{4m_q^2\, r_{ij}^3}\, S_{ij} \right\}, \tag{2}$$

$$V_{\rm OPE}(\vec{r}_{ij}) = \frac{1}{3}\,\alpha_{\rm ch} \frac{\Lambda^2}{\Lambda^2 - m_\pi^2}\, m_\pi \times \left\{ \left[Y(m_\pi\, r_{ij}) - \frac{\Lambda^3}{m_\pi^3}\, Y(\Lambda\, r_{ij}) \right] \vec{\sigma}_i \cdot \vec{\sigma}_j + \left[H(m_\pi\, r_{ij}) - \frac{\Lambda^3}{m_\pi^3}\, H(\Lambda\, r_{ij}) \right] S_{ij} \right\} \vec{\tau}_i \cdot \vec{\tau}_j \,, \tag{3}$$

$$V_{\rm OSE}(\vec{r}_{ij}) = -\alpha_{\rm ch}\, \frac{4\, m_q^2}{m_\pi^2}\, \frac{\Lambda^2}{\Lambda^2 - m_\sigma^2}\, m_\sigma \times \left[Y(m_\sigma\, r_{ij}) - \frac{\Lambda}{m_\sigma}\, Y(\Lambda\, r_{ij}) \right] . \tag{4}$$

The main advantage of this model comes from the fact that it works with a single qq-meson vertex. Therefore its parameters (coupling constants, cutoff masses, ...) are independent of the baryon to which the quarks are coupled, the difference among them being generated by SU(2) scaling. This makes the generalization of the NN interaction to any other nonstrange baryonic system straightforward. Moreover, as explained in Refs. [12,13] the parameters associated with the scalar field are related to those of the pion exchange and the constituent quark mass through $g_{\sigma NN}^2/4\pi = 36\,\alpha_{\rm ch} m_q^2/m_\pi^2$, $\Lambda_\sigma = \Lambda_\pi$, and $m_\sigma^2 = (2m_q)^2 + m_\pi^2$.

Once the quark-quark interaction is chosen, an effective nucleon-Δ potential can be obtained in the Born-Oppenheimer approximation, as the expectation value of the energy of the six-quark system minus the self-energies of the two clusters, which can be computed as the energy of the six-quark system when the two quark clusters do not interact:

$$V_{N\Delta(L\,S\,T)\to N\Delta(L'\,S'\,T)}(R) = \xi_{L\,S\,T}^{L'\,S'\,T}(R) \; - \; \xi_{L\,S\,T}^{L'\,S'\,T}(\infty)\,, \tag{5}$$

where

$$\xi_{L\,S\,T}^{L'\,S'\,T}(R) \;=\; \frac{\left\langle \Psi_{N\Delta}^{L'\,S'\,T}(R) \middle| \sum_{i<j=1}^{6} V_{qq}(\mathbf{r}_{ij}) \middle| \Psi_{N\Delta}^{L\,S\,T}(R) \right\rangle}{\sqrt{\langle \Psi_{N\Delta}^{L'\,S'\,T}(R) \mid \Psi_{N\Delta}^{L'\,S'\,T}(R)\rangle}\sqrt{\langle \Psi_{N\Delta}^{L\,S\,T}(R) \mid \Psi_{N\Delta}^{L\,S\,T}(R)\rangle}} \,. \tag{6}$$

The wave function is discussed in detail in Refs. [14,15]. In expression (6) the quark coordinates are integrated out keeping R fixed, the resulting interaction $V_{N\Delta}$ being a function of the nucleon-Δ relative distance. We have neglected the nonlocal terms of the potential; however, for the qualitative analysis we are interested in, the Born-Oppenheimer approximation is known to provide good results [16]. The parameters of the model are summarized in Table I.

III. FORMALISM AND RESULTS

In order to determine the nature (attractive or repulsive) of a given nucleon-Δ ($N\Delta$) channel, we will first calculate the Fredholm determinant of that channel as a function of energy assuming a stable Δ and nonrelativistic kinematics. That means we will use the Lippmann-Schwinger equation

$$T_{ij}(q, q_0) = V_{ij}(q, q_0) + \sum_k \int_0^\infty q'^2 dq' V_{ik}(q, q') \times G_0(E, q') T_{kj}(q', q_0)\,, \tag{7}$$

where the two-body propagator is

$$G_0(E, q) = \frac{1}{E - q^2/2\eta + i\epsilon}\,, \tag{8}$$

with reduced mass

$$\eta = \frac{m_N m_\Delta}{m_N + m_\Delta}\,. \tag{9}$$

The energy and on-shell momentum are related as

$$E = q_0^2/2\eta\,, \tag{10}$$

and we will restrict ourselves to the region $E \leq 0$.

If we replace the integration in Eq. (7) by a numerical quadrature, the integral equations take the form

TABLE I. Quark model parameters.

m_q (MeV)	313
b (fm)	0.5
α_s	0.4
a_c (MeV fm^{-2})	57.96
$\alpha_{\rm ch}$	0.0288
m_σ (fm^{-1})	3.42
m_π (fm^{-1})	0.7
Λ (fm^{-1})	4.2

$$T_{ij}(q_n, q_0) = V_{ij}(q_n, q_0) + \sum_k \sum_m w_m q_m^2 V_{ik}(q_n, q_m) \times G_0(E, q_m) T_{kj}(q_m, q_0) \,, \quad (11)$$

where q_m and w_m are the abscissas and weights of the quadrature (we use a 40-point Gauss quadrature [17]). Equation (11) gives rise to the set of inhomogeneous linear equations

$$\sum_k \sum_m M_{nm}^{ik}(E) T_{kj}(q_m, q_0) = V_{ij}(q_n, q_0) \,, \quad (12)$$

with

$$M_{nm}^{ik}(E) = \delta_{ik}\delta_{nm} - w_m q_m^2 V_{ik}(q_n, q_m) G_0(E, q_m) \,. \quad (13)$$

If a bound state exists at an energy E_B, the determinant of the matrix $M_{nm}^{ik}(E_B)$ (the Fredholm determinant) must vanish, i.e.,

$$\left| M_{nm}^{ik}(E_B) \right| = 0. \quad (14)$$

Even if there is no bound state, the Fredholm determinant is a very useful tool to determine the nature of a given channel. If the Fredholm determinant is larger than 1 that means that channel is repulsive. If the Fredholm determinant is less than 1 that means the channel is attractive. Finally, if the Fredholm determinant passes through zero that means there is a $N\Delta$ bound state at that energy. We show in Fig. 1 the Fredholm determinants of the $N\Delta$ channels with $J \leq 2$ and parity + or − as functions of energy (the channel 2^+ is not shown since it is overwhelmingly repulsive). As seen from this figure, only the channels 0^+ and 0^- are attractive.

Previous calculations of the isospin-2 channels have been based in the three-body formalism of the πNN system [4,5]. In these works [4,5], the possibility of resonances was investigated by calculating the Fredholm determinant of the various channels for energies below the πNN threshold. We show in Table II the comparison of our results for the nature of the six states with those obtained in Refs. [4] and [5]. The difference observed between the predictions of the three-body models is due to the fact that in Ref. [5] the nucleon-nucleon interaction was neglected. As seen from this table, the results of the three-body calculations and our calculation agree with each other in the case of the 0^-, 1^+, and 2^+ channels. The three-body models predict a resonance in the 2^- channel while our result is that this channel is repulsive. Both three-body calculations predict a large attraction in the 0^- channel which is in agreement with our model since we have found that this channel is the most attractive one. In order to determine whether a resonance exists in either the 0^- or 0^+ channels, it is necessary to calculate the $N\Delta$ amplitudes for $E > 0$.

Since we are looking for possible resonances which decay into a pion and two nucleons, only those channels which are attractive or have a $N\Delta$ bound state are good resonance candidates. In order to investigate whether a given attractive channel possesses a resonance, we will calculate Argand diagrams between a stable and an unstable particle for the various $N\Delta$ states, using the formalism of Ref. [10]. In this case, however, we will use relativistic kinematics and will include the width of the Δ. That means that instead of the propagator (8) we will use [10]

$$G_0(S, q) = \frac{2m_\Delta}{s - m_\Delta^2 + i m_\Delta \Gamma_\Delta(s, q)} \,, \quad (15)$$

where S is the invariant mass squared of the system, while s is the invariant mass squared of the πN subsystem (those are the decay products of the Δ) and is given by

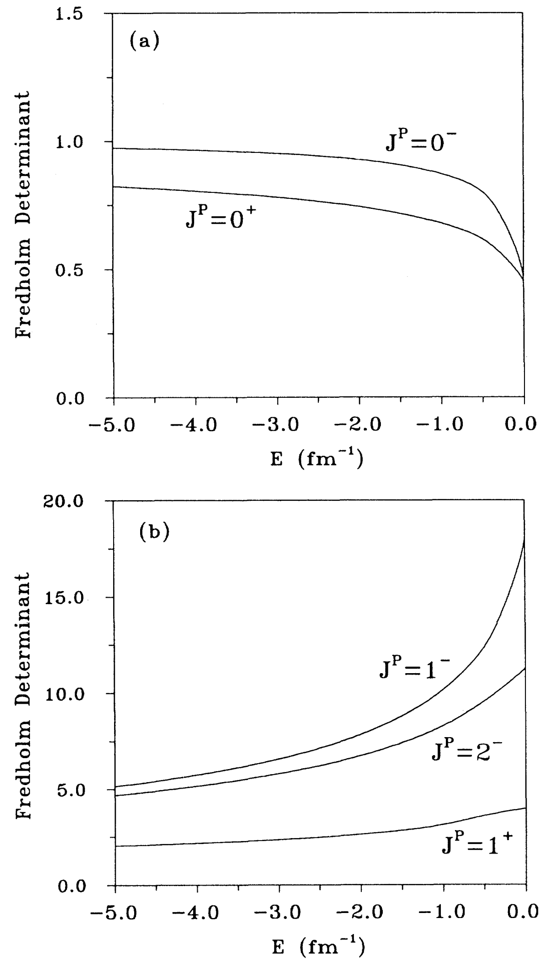

FIG. 1. Fredholm determinant of the nucleon-Δ channels with $J \leq 2$ as a function of the nonrelativistic energy E.

TABLE II. Comparison of the nature of the various $N\Delta$ channels with isospin 2 to those predicted by three-body models.

Channel	Ref. [4]	Ref. [5]	This work
0^+	Repulsive	Attractive	Attractive
0^-	Attractive	Very attractive	Attractive
1^+	Repulsive	Repulsive	Repulsive
1^-	Repulsive	Attractive	Repulsive
2^+	Repulsive	Repulsive	Very repulsive
2^-	Very attractive	Very attractive	Repulsive

$$s = S + m_N^2 - 2\sqrt{S(m_N^2 + q^2)}\,. \tag{16}$$

The width of the Δ is taken to be [10]

$$\Gamma_\Delta(s,q) = \frac{2}{3}\,0.35\,p_0^3\,\frac{\sqrt{m_N^2+q^2}}{m_\pi^2\sqrt{s}}\,, \tag{17}$$

where p_0 is the pion-nucleon relative momentum given by

$$p_0 = \left(\frac{[s-(m_N+m_\pi)^2][s-(m_N-m_\pi)^2]}{4s}\right)^{1/2}. \tag{18}$$

We show in Figs. 2 and 3 the phase shifts of the two attractive channels 0^+ and 0^-. As can be seen from these figures, the attraction in both channels is not strong enough to produce a resonance (it does not reach 90°). As an example of the predictions of our model for other channels, we show in Fig. 4 the Argand diagram corresponding to the $^3P_2$, $^5P_2$, and $^3F_2$ amplitudes of the 2^- channel. The $^5F_2$ amplitude is not shown since it is much smaller. As seen in this figure, the $^5P_2$ wave is strongly repulsive while the $^3P_2$ wave shows a resonancelike behavior. In order to check if this behavior corresponds to a true resonance or it is simply an effect of the channel coupling, we disconnected the $^5P_2$ wave. The resulting Argand diagrams show a weakly attractive behavior, similar to that of the 0^+ and 0^- channels. This shows that indeed the resonancelike behavior observed in Fig. 4 is due simply to channel coupling.

As stated before, the possibility that a resonance may exist in the 0^- channel has been the subject of great discussion recently [1–3]. A 0^- resonance in the isospin-0 channel was proposed in Ref. [1]. However, it was pointed out in Ref. [2] that most likely the resonance must have isospin 2. The proposed resonance [1–3] has a mass of 2065 MeV and a very small width of 0.51 MeV. It is therefore very interesting to investigate whether a model like the one used in this work could predict a resonance with the known features of this resonance, and particularly with such a tiny width.

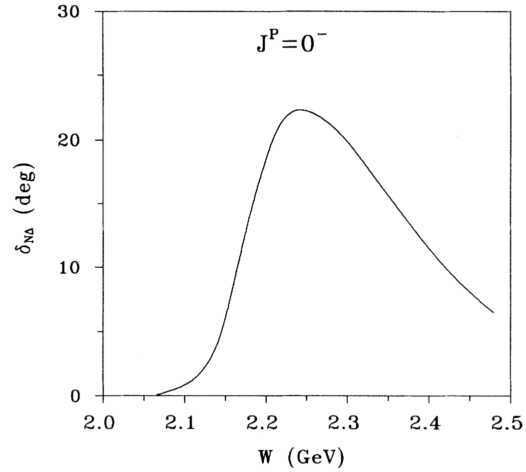

FIG. 3. Phase shift of the 0^- nucleon-Δ channel as a function of the invariant mass $W = S^{1/2}$.

In order to do this, we have artificially varied the mass of the σ meson such as to increase the amount of attraction (the coupling constant of the σ is fixed by chiral symmetry). We found that if $m_\sigma < 375$ MeV a resonance appears in the 0^- channel. We show in Table III the mass and width of this resonance for several values of m_σ. As can be observed from this table, the width of the resonance drops dramatically when its mass approaches the πNN threshold (2017 MeV). This result can be understood from simple angular momentum barrier considerations. If we call q and L the relative momentum and relative orbital angular momentum between a nucleon and the π-nucleon pair, respectively, then since $L = 1$ the width of the resonance will be proportional to $q^{2L+1} = q^3$, so that it will drop very fast as one approaches the πNN threshold since there $q \to 0$.

As shown in Table III, when the mass of the σ is taken as $m_\sigma = 234$ MeV the mass of the resonance has the

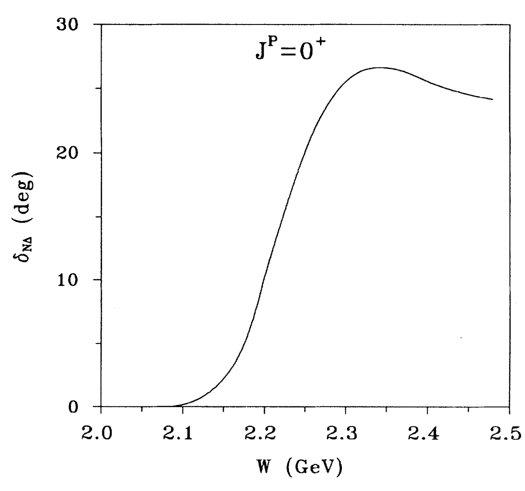

FIG. 2. Phase shift of the 0^+ nucleon-Δ channel as a function of the invariant mass $W = S^{1/2}$.

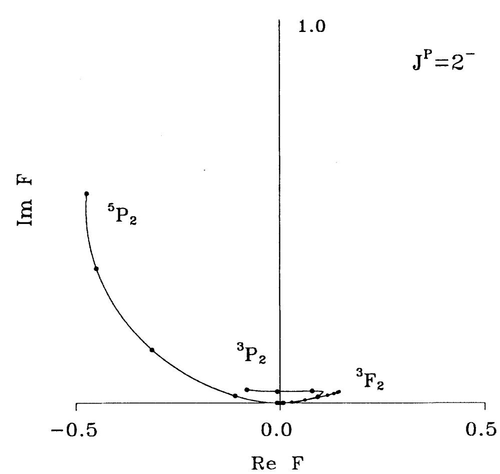

FIG. 4. Argand diagrams of the diagonal amplitudes of the $^3P_2$, $^5P_2$, and $^3F_2$ nucleon-Δ partial waves. The dots correspond to the energies 2.12 – 2.44 GeV.

TABLE III. Dependence of the mass and width of the 0^- resonance on the mass of the σ.

m_σ (MeV)	$M_{\rm res}$ (MeV)	$\Gamma_{\rm res}$ (MeV)
350.0	2170.4	127.6
300.0	2128.7	18.2
250.0	2083.4	2.3
234.0	2064.4	0.6
210.0	2029.5	0.0015

experimental value 2064.4 MeV and a width of 0.6 MeV, which is in very good agreement with the value of the width extracted by Bilger, Clement, and Schepkin [1]. This change in the mass of the σ generates additional attraction in the other channels, giving rise in some cases to resonances. However, all of them lie above the 0^- resonance, which shows that the 0^- channel is the most attractive one. Therefore, the sharp peak seen in the double charge exchange reactions could be justified as a nucleon-Δ resonance in the isospin-2 channel, without resorting to exotic decay channels.

We have generated artificially the additional attraction needed to produce the 0^- resonance by varying the mass of the σ meson. In principle, however, a possible source of this lacking of attraction within the quark cluster model could be the inclusion of the nonlocal terms of the nucleon-Δ interaction and the contribution of the Δ-Δ channels. Both of these effects have been neglected in the present study.

IV. CONCLUSIONS

We have studied the nucleon-Δ system in the isospin-2 channels within the quark cluster model of the baryon-baryon interaction. We have found that this model with standard parameters derived from nucleon-nucleon scattering is unable to generate any resonance with $J \leq 2$. However, we found that the 0^- channel is the most attractive one. We have varied freely the mass of the σ meson in order to generate a resonance in this channel. When the mass of the 0^- resonance reaches the value of 2065 MeV, its width is equal to 0.6 MeV. This very narrow width is in very good agreement with the width extracted in Refs. [1–3].

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