# Glueball production from the quark-gluon plasma

Chongshou Gao,[1,3] Xueqian Li,[1,4] and Wei Lu[1,2,3]
[1] Chinese Center of Advanced Science and Technology (World Laboratory), P.O. Box 8730, Beijing 100080, China
[2] Institute of High Energy Physics, P.O. Box 918(4), Beijing 100039, China*
[3] Department of Physics, Peking University, Beijing 100871, China
[4] Department of Physics, Nankai University, Tianjing 300071, China

We suggest that there might be a production of a large number of glueballs in relativistic nucleus-nucleus collisions if the quark-gluon plasma (QGP) phase is realized. The ratio of the number of the produced glueball to that of the neutral pions directly produced in the process of QGP hadronization is estimated phenomenologically. Being a function of the phase transition temperature and chemical potential, it can be as large as a few percents. Considering the fact that the hadronization is very complicated, we anticipate that the abundant population of glueballs from the quark-gluon plasma at least may exert influences on the final-state $K^+/\pi^+$ ratio.

PACS number(s): 24.85.+p, 12.38.Mh, 12.39.Mk, 25.75.+r

The study of quark-gluon plasma (QGP) has attracted much attention from both experimentalists and theoreticians. The reason is that though the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ standard model seems to give a satisfactory description of the physical world, the missing symmetry remains a puzzle [1]. Especially, an important question has not been answered yet: what do a vacuum structure's strong and electroweak interactions possess? To study the vacuum structure, one needs to excite the vacuum to the state of high density and temperature. Hopefully, the Relativistic Heavy Ion Collider (RHIC) under construction can undertake this task and will provide us with a feasible way to investigate the strong vacuum at about 200 A GeV energy scale.

The excited vacuum is just the QGP in which quarks, antiquarks, and gluons are deconfined and frequently colliding with each other. To identify the QGP formation, one must have clear experimental signals. In the past years, it has been suggested that the $J/\Psi$ suppression [2], $K/\pi$ ratio enhancement [3], direct thermal photon [4], and lepton pair production [5] be taken as the signals for the formation of QGP in nucleon-nucleon collisions. In addition, the measurement on the size of the particle-emitting source by the interferometry techniques [6] is also thought to provide direct information on the QGP formation. Because of the complicated backgrounds, however, none of the above signals work individually. To reach reliable conclusions, it is now generally believed that the information from all possible aspects must be synthesized [7].

In this paper, we suggest that an abundant number of glueballs can be formed in relativistic nucleus-nucleus collisions provided the quark-gluon plasma phase is realized. The glueball is predicted by quantum chromodynamics (QCD) and its identification will make a crucial test of QCD. People used to search for them in the low-energy processes such as the $J/\Psi$ and $\Upsilon$ radiative decays and have not succeeded yet due to the low statistics of the accumulated events. Once QGP is formed in heavy-ion collisions, however, we can anticipate a substantial glueball production in its hadronization process. In QGP, there exist a huge amount of thermal gluons. As QGP cools down, they can form color singlet configurations of valence gluons via color interactions, among which the two-gluon glueballs will take up a large portion. Even though the experimental identification of glueballs is very difficult, we can still anticipate that the abundant glueballs in the QGP debris may have some influences on the final-state $K/\pi$ ratio.

Glueballs are, in general, color-singlet configurations $|gg\rangle$ and $|ggg\rangle$. Since the probability for the two-gluon collision is much larger than that for three-gluon collisions, we will consider only the formation of the lowest-energy state of $0^{++}$ glueballs and estimate the ratio of the number of the produced glueballs to that of the neutral pions produced directly during the QGP hadronization process, within a reasonable theoretical framework. The phase transition from the QGP into hadron phase involves nonperturbative aspects of strong intearctions, for which there are no reliable theoretical methods to approach at present. In this work, we make the estimates phenomenologically.

Without getting into any details, we note that the color SU(3) symmetry of strong interaction can give us some indications. The cross section for two constituents forming a color-singlet hadron is proportional to the factor

$$\vec{I}\,(1)\cdot \vec{I}\,(2) = \frac{1}{2}[C_2(1+2) - C_2(1) - C_2(2)], \tag{1}$$

where $\vec{I}$'s are the color SU(3) generators and $C_2$'s the Casimir operators of the SU(3) group. The quark, antiquark, and gluon belong to the **3**, **3***, and **8** representations of the color SU(3) group. Correspondingly,

$$C_2(3) = C_2(3^*) = \tfrac{4}{3},\ C_2(8) = 3. \tag{2}$$

Therefore, for

$$g(k_1) + g(k_2) \to G(k), \tag{3}$$

and

$$q(p_1) + \bar{q}(p_2) \to \pi^0(p), \tag{4}$$

*Mailing address.

 

the color factor $[\vec{I}(1)\cdot\vec{I}(2)]^2$ is 9 and 16/9, respectively. As a result,

$$\left(\frac{q+\bar{q}\to\pi^0}{g+g\to G}\right)_{\rm color}=\frac{16}{81}. \quad (5)$$

From this simple counting, we see the group structure of color interaction favors the glueball production when QGP hadronizes. In order to evaluate the production rate of particles, however, one has to take into account the dynamical mechanisms.

For process (3), we assume its effective Lagrangian to be

$$\mathcal{L}_{gg\leftrightarrow G}=f_{ggG}F^a_{\mu\nu}F^{a\mu\nu}G, \quad (6)$$

where $F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g_s f^{abc}A^b_\mu A^c_\nu$, $g_s$ is the strong coupling between gluons, and $a=1,...,8$ is the color indices of gluons. Since (6) is the effective Lagrangian, one can be convinced that $g+g\to G$ and its inverse process, i.e., $G\to g+g$, have the same effective coupling $f_{ggG}$. As a reasonable approximation, we simply neglect the gluon self-coupling term in $F^a_{\mu\nu}$. If we assume that the $0^{++}$ glueball decays into two valence gluons and then materializes into the final-state hadrons, the gluon decay width is found to be

$$\Gamma_G=\frac{f^2_{ggG}M^3_G}{2\pi}. \quad (7)$$

If the glueball mass and decay width predicted in models can be taken as inputs, we will be able to determine the effective coupling $f_{ggG}$:

$$f^2_{ggG}=\frac{2\pi\Gamma}{M^3_G}. \quad (8)$$

Then, we can calculate the glueball production rate in unit volume as follows:

$$\frac{\Delta n_G}{\Delta t}=\frac{1}{2}\int\int\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^4k}{(2\pi)^3}\delta(k^2-M^2_G)\theta(E)f_g(k_1)f_g(k_2)(2\pi)^4\delta^4(k_1+k_2-k)|\mathcal{M}_{gg\to\pi}|^2, \quad (9)$$

where 1/2 is a symmetry factor for two identical gluons and $f_g(k)$ is the thermal gluon distributions in the QGP. It is noted that the $\delta(k^2-M^2_G)$ factor sets the on-shell condition for the glueball.

At present, the phase transition between the hadron and QGP phases is generally believed to be a first-order phase transition. Therefore, to the lowest-order approximation one may assume that the local chemical equilibrium is maintained during the QGP hadronization process [8]. At equilibrium, quarks and antiquarks respect the Fermi-Dirac statistics, while gluons obey the Bose-Einstein statistics:

$$f_\pm(p)=\frac{d}{\exp\left(\frac{E-\mu}{T}\right)\pm 1}, \quad (10)$$

where the $\pm$ signs correspond to fermions and bosons, respectively, $d$ is the spin, color, and isospin degeneracy factor ($2\times 8\times 1$ for gluons, $2\times 3\times 2$ for light quarks, $2\times 3\times 1$ for strange quarks, respectively), and $T$ and $\mu$ are the temperature and chemical potential of the QGP, respectively.

Because the glueball is not stable and has a certain width, it is reasonable to use the Breit-Wigner distribution to replace the on-shell delta function in our calculations:

$$\delta(p^2-M^2_G)\Rightarrow\frac{M_G\Gamma_G}{\pi[(p^2-M^2_G)^2+M^2_G\Gamma^2_G]}. \quad (11)$$

In our approximation, the transition amplitude squared for the gluon-gluon fusion into the glueball reads

$$|\mathcal{M}_{ggG}|^2=\frac{1}{4}f^2_{ggG}(k_1\cdot k_2)^2. \quad (12)$$

Combining Eqs. (8) through (12), we obtain

$$\frac{\Delta n_G}{\Delta t}=\frac{4\Gamma_G}{8\pi^3M^2_G}\int\int dE_1dE_2\frac{1}{\exp\left(\frac{E_1}{T}\right)-1}\frac{1}{\exp\left(\frac{E_2}{T}\right)-1}\left[4\Gamma_G E_1E_2+M^2_G\Gamma_G\ln\frac{(M^2_G-4E_1E_2)^2+M^2_G\Gamma^2_G}{M^2_G(M^2_G+\Gamma^2_G)}\right.$$
$$\left.+M_G(M^2_G-\Gamma^2_G)\left(\arctan\frac{M_G}{\Gamma_G}-\arctan\frac{M^2_G-4E_1E_2}{M_G\Gamma_G}\right)\right]. \quad (13)$$

Now we turn to the direct $\pi^0$ formation during the QGP hadronization process. The basic effective Lagrangian for $q\bar{q}\to\pi$ reads

$$\mathcal{L}_{q\bar{q}\pi}=f_{q\bar{q}\pi}\bar{q}\gamma_5\frac{\vec{\sigma}}{2}q\cdot\vec{\pi}. \quad (14)$$

However, there is no inverse process $\pi^0\to q\bar{q}$ because of the phase-space constraint. Alternatively, the coupling $g_{q\bar{q}\pi^0}$ can be obtained from the strong coupling $g_{NN\pi}$ through the current algebra method [9],

$$f_{q\bar{q}\pi^0}=\tfrac{3}{5}f_{NN\pi} \quad (15)$$

with $g^2_{NN\pi}/(4\pi)=14.6$ from the nuclear experimental data [10].

The number of the directly produced pions per unit volume and per unit time during the QGP hadronization is

$$\frac{\Delta n_\pi}{\Delta t}=\int\int\frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^4p}{(2\pi)^3}\delta(p^2-M^2_\pi)\theta(E)(2\pi)^4$$
$$\times\delta(p_1+p_2-p)f_q(p_1)f_{\bar{q}}(p_2)|\mathcal{M}_{q\bar{q}\pi^0}|^2, \quad (16)$$

where

$$|\mathcal{M}_{q\bar{q}\pi^0}|^2 = \frac{1}{12} f_{q\bar{q}\pi}^2 (p_1 \cdot p_2 + M_q^2). \tag{17}$$

From Eqs. (15) through (17), we have

$$\frac{\Delta n_\pi}{\Delta t} = \frac{27 f_{NN\pi}^2 M_\pi^2}{400\pi^3} \int\int dE_1 E_2 \times \left( \frac{1}{\exp\left(\frac{E_1-\mu}{T}\right)+1} \right) \left( \frac{1}{\exp\left(\frac{E_2-\mu}{T}\right)+1} \right). \tag{18}$$

With Eqs. (13) and (18), we are able to estimate the ratio of the numbers of the glueballs and that of the neutral pions produced directly during the QGP hadronization:

$$R = \left(\frac{\Delta n_G}{\Delta t}\right) \Big/ \left(\frac{\Delta n_{\pi^0}}{\Delta t}\right). \tag{19}$$

Since the experimental observations of the glueball have not been confirmed so far, we know neither their exact masses nor widths. Among the present glueball candidates, $\eta(1440)$ has the lowest mass [11]. The widths of glueballs are generally estimated to be around 50 MeV [12]. Since we only want to give a rough estimate of the ratio $R$, the inaccurate glueball mass and width will not influence our qualitative conclusion. To be illustrative, we set $M_G$=1440 MeV/$c^2$ and $\Gamma_G = 50$ MeV.

During the QGP hadronization process, as two gluons approach each other, the strong color field would combine them into a glueball. The boundary in the phase diagram is not clear because so far there are neither sufficient data, nor reliable theories to determine the phase-transition condition. But it assures that as the QGP cools down, the system will cross the boundary region and convert itself to the hadron matter. The ratio of the number of the glueballs to that of the pions produced directly during the hadronization depends on along which path QGP matter returns to hadron matter state [1]. That is to say, it depends upon the phase-transition temperature $T$ and density $\mu$.

In Fig. 1, we show the dependence of $R$ on $T$ and $\mu$. If we take the chemical potential $\mu$ to be 100 MeV, $R$ can be as large as 0.50. The transition temperature is generally believed to be around 200 MeV, so $R$ is sizable for a possible range of transition temperature. It is noted that the ratio $R$ is not too small even though the dimensionless effective coupling constant for the gluon-gluon fusion into glueballs, defined as $f'_{ggG} = \sqrt{f_{ggG}^2 M_G^3}$, is much smaller than $f_{q\bar{q}\pi}$. This can be easily understood because both the initial gluon distribution function and the color factor enhance the production of glueballs, as discussed above. Such effects partially compensate the suppression caused by the coupling-constant difference.

Of course, one may argue that there exist other possibilities for the hadronization of thermal gluons. For example, they can directly hadronize into two or more pions. Indeed, these types of processes will decrease our estimated $R$ value. However, such a possibility may be suppressed by the Okubo-Zweig-Iizuka rule [13]. Therefore, we argue that there may be a sizable glueball production from the QGP hadronization compared to that in the $J/\Psi$ and $\Upsilon$ decays.

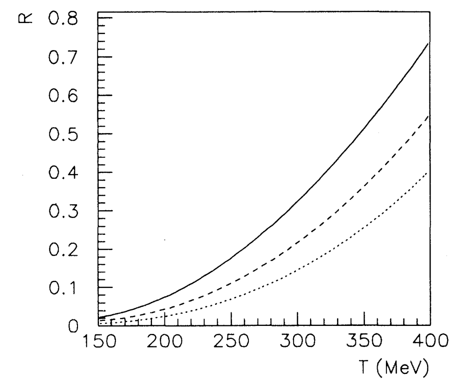


FIG. 1. The dependence of the ratio $R$ on the transition temperature $T$ and the chemical potential $\mu$ of light quarks during the quark-gluon plasma freezing process. The glueball mass and width are set to be 1440 and 50 MeV, respectively. The solid, dashed, and dotted, lines correspond to $\mu$ = 100, 200, and 300 MeV, respectively.

Although we anticipate a relatively large production rate for the glueballs from QGP hadronization, their experimental identification is still not an easy job. As to what we can do for observing glueballs at RHIC, we have no good ideas at the moment. Experimentally, what one detects directly is mostly pions and kaons. Since the glueball is not a stable particle, its identification has to be carried out from measuring its decay products. As we have argued, the glueballs possibly produced at RHIC are dominantly $0^{++}$ glueball with $I$=0, whose main decay channels are the two-meson mode. If the $G$ is not too light, it will mainly decay into two pseudoscalar mesons via the following channels:

$$\pi^+\pi^-, \pi^0\pi^0, K^+K^-, K_SK_S, K_LK_L, \eta\eta, \eta\eta', \eta'\eta'.$$

Because the $0^{++}$ glueball is a flavor singlet, the branching ratios of the above channels are constrained by the SU(3) flavor symmetry. If we ignore the effects of the flavor SU(3) symmetry breaking and the phase-space difference, the ratios of squared transition matrix elements for the corresponding channels read

$$2:1:2:1:1:1:0:1.$$

The decay products of the glueball could be submerged in a large quantity of directly produced mesons; therefore, the intention to identify the glueballs by seeking resonance peaks in the invariant mass spectra of pion pairs may be difficult, unless the detecting technique has achieved great progress.

Our results' arguments bring about a new factor that influences the $K/\pi$ ratio in the final state. The reason is that the rich production of glueballs in the QGP freezing-out process will change the $K/\pi$ ratio that one could expect in the context in which there is no direct glueball formation. The observed pions in the QGP debris can be produced either from $q\bar{q}$ collisions during the QGP hadronization or from the decay of other final-state particles, especially from the glueballs. So far, it is generally accepted that the secondary final-state collisions $\pi\pi \to K\bar{K}$ are the main factor that prevents people from taking the $K/\pi$ ratio as a reliable QGP signal. If the glueballs are plentifully formed in the course of QGP hadronization, it will exert influences on the $K/\pi$ ratio. A quantitative conclusion in this aspect bears more study.

This work was partly supported by the National Natural Science Foundation of China (NSFC) and the Doctoral Programme Foundation of Institution of Higher Education.

[1] T. D. Lee and G. C. Wick, Phys. Rev. D **9**, 2291 (1974).
[2] T. Matsui and H. Satz, Phys. Lett. B **178**, 416 (1986); A. Shor, Phys. Rev. Lett. **54**, 1122 (1985).
[3] J. Rafelski and B. Muller, Phys. Rev. Lett. **48**, 1066 (1982); P. Koch, B. Muller, and J. Rafelski, Phys. Rep. **142**, 167 (1986).
[4] E. V. Shuryak, Sov. J. Nucl. Phys. **28**, 408 (1978); K. Kajantie and H. I. Miettinen, Z. Phys. C **9**, 341 (1981).
[5] P. V. Ruuskanen, Nucl. Phys. **A544**, 169c (1992).
[6] S. Pratt, Phys. Rev. D **33**, 1314 (1986).
[7] B. Muller, Nucl. Phys. **A544**, 95c (1992).
[8] T. Matsui, B. Svetitsky, and L. McLerran, Phys. Rev. D **34**, 783 (1986).
[9] M. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1478 (1958).
[10] A. J. Bearden, Phys. Rev. Lett. **4**, 240 (1960).
[11] Particle Data Group, Phys. Rev. D **50**, 1343 (1994).
[12] M. Shifman, Z. Phys. C **9**, 347 (1981).
[13] S. Okubo, Phys. Lett. **5**, 165 (1963); G. Zweig, CERN preprints, Th-401, 412 (1964); J. Iizuka, Suppl. Progr. Theor. Phys. **37-38**, 21 (1966).