Reaction cross sections of intermediate energy α particles within a relativistic optical model

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The suggestion made recently by H. Abele *et al.* that relativistic effects in the interaction of α particles with various nuclei at intermediate energies may remove the existing discrepancy between the theoretical predictions and the experimental data for the reaction cross section is investigated. We use a relativistic model based on the Kemmer-Duffin-Petiau equation and find that relativistic effects do not lead to a reduction in the reaction cross section within the present approach. Alternative explanations are discussed.

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Reaction cross section data have been receiving much interest recently [1–3] from both the theoretical and experimental viewpoints for several reasons, one of which being that these data help to discriminate between different potentials that describe the scattering data equally well. Reaction cross section data for incident protons have been studied extensively in both nonrelativistic and relativistic frameworks. For heavy ions, reaction cross sections [3,4] have also received much attention and the microscopic basis behind the theory reviewed in a recent report [3]. On the other hand, reaction cross section data for α particles have been scarce until the recent measurements [2] where the data for targets from ^{12}C to ^{208}Pb were presented together with optical model predictions. The standard optical model calculations with Woods-Saxon potentials predict values for the reaction cross sections that are on average 10% in excess of the data, while they give reasonable fits to the scattering data [2]. Furthermore, recent optical model calculations for the scattering of α particles from several nuclei have been reported at various energies between 80 and 200 MeV where the real part of the optical potential was obtained by a double folding procedure [1]. These calculations also predicted the angular distributions quite well but gave consistently higher values for the reaction cross sections. It was then suggested that this discrepancy might be lifted if one uses a relativistic model for these calculations [1,2].

In this paper, we report on the results of such an investigation. Relativistic descriptions of heavy ion systems (generally based on solving the Dirac-Bethe-Goldstone equation as in [4], for example) require more laborious calculations than the optical model ones. The comparisons of the data with relativistic model calculations for α particles have been scarce mainly for this reason as it is not easy to develop a simple relativistic wave equation for α scattering. We have developed a simple relativistic optical model [5] for the scattering of α particles based on the Kemmer-Duffin-Petiau (KDP) equation [6]. The KDP equation has been used relatively recently for the description of meson-nucleus scattering [7] and deuteron-nucleus scattering [8]. Relativistic effects for other systems are such that the reaction section is decreased or increased depending on the particular system. For proton scattering, on one hand, relativistic calculations within the Dirac-based optical potential yielded values [9] of σ_R lower than the nonrelativistic folding model ones, thus improving the agreement with the experimental data. On the other hand, for the ^{12}C-^{12}C heavy ion system, for example [4], the nonrelativistic predictions are lower than the data and the relativistic treatment has led to a better agreement with the data. In consequence, a relativistic model for α scattering would not necessarily lead to the desired effect, i.e., a reduction in σ_R in comparison to the nonrelativistic treatment.

In the present calculations, we use the KDP equation as the valid wave equation for α particles and adopt a folding model procedure. This model is described in detail in a recent paper [5] and is the first description of α scattering within the relativistic KDP optical model. Here, we summarize briefly the main steps of the calculations. First, we develop a Schrödinger equation equivalent to the original KDP equation in the same way that a Schrödinger equivalent equation is derived from the Dirac equation. The resulting effective complex potential is given as

$$V_{\rm eff} = U_s + U_v + \frac{E_\alpha}{m_\alpha} U_v + \frac{(U_s^2 - U_v^2)}{2m_\alpha} + U_{\rm Darwin} \,. \quad (1)$$

This potential consists of terms involving a scalar (U_s) and a vector (U_v) potential together with an energy-dependent term and a Darwin term embodying some nonlocal effects. A Coulomb term is also included as a vectorlike potential and the wave equation has the usual relativistic kinematics. The scalar and vector potentials are obtained by folding the Dirac scalar and vector potentials deduced from fits to the proton scattering data at a quarter of the incident α energy. Recoil corrections are also included [5].

The calculations for the angular distributions were done for several nuclei at α energies from 100 to 200 MeV and are reported in [5]. We found that these potentials give a good description of the elastic data and here we focus on one case, namely, the $\alpha + ^{58}$ Ni system at 139 MeV. In Fig. 1, we present the resulting potential together with the phenomenological Woods-Saxon potential of Nolte, Machner, and Bojowald [10] fitted to the elastic scattering data. Figure 2 gives the elastic scattering cross section results compared to the data [11] and to the predictions of the Woods-Saxon potential.

As shown in Fig. 1, the potential obtained has shapes that depart from the usual Woods-Saxon (WS) potentials. The real part is deeper over the entire radial region and follows closely the shapes obtained in folding model calculations using the M3Y interaction based on the G- matrix elements of the NN potential [12]. The imaginary part is surface peaked and is deeper than the WS potential in the surface region. At the center it is markedly shallower. The shape of the absorptive potential is very similar to that obtained for the same system at 172.5 MeV using the Glauber model with first-order noneikonal corrections with a Coulomb term [13]. The agreement with the data (Fig. 2) is reasonable. We stress here that as described in [5] no attempts were made to optimize the fit to the data. It is, however, noteworthy to point out that the effective potential does very well in predicting the positions of the oscillations in the data and reproduces quite well the exponential falloff beyond $\theta = 40°$. The phenomenological Woods-Saxon potential does equally well overall but fails to predict the correct large angle behavior of the data.

In Table I, we give the results obtained for the reaction cross section using the present relativistic potential

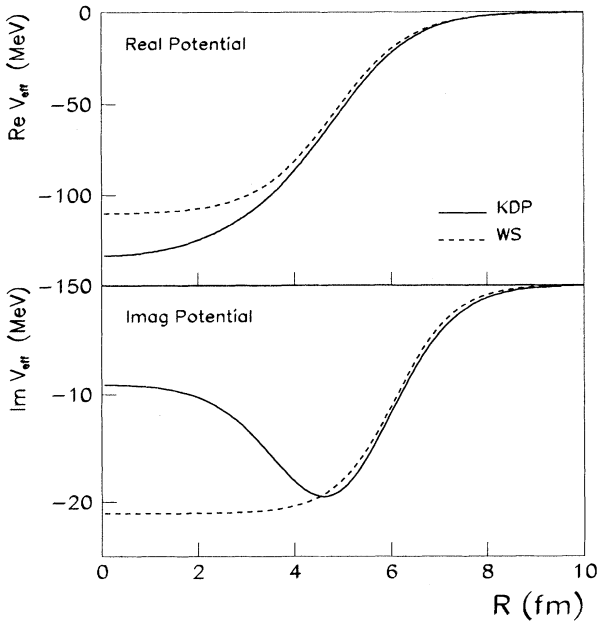

FIG. 1. The effective relativistic potential (solid line) compared with the Woods-Saxon potential (dotted line) for the $\alpha + ^{58}$ Ni system at 139 MeV.

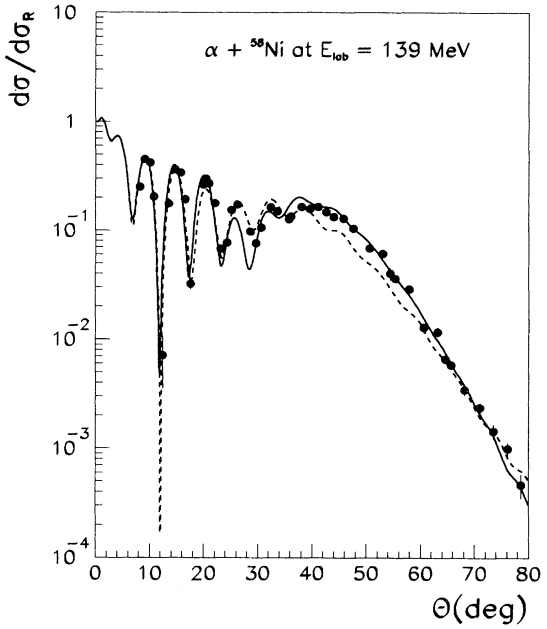

FIG. 2. Angular distributions for $\alpha + ^{58}$ Ni at 139 MeV. The results of the relativistic effective potential (solid line) and the Woods-Saxon potential (dotted line) are compared with the data.

and the Nolte WS potential and compare these with the recent experimental data [2] at different energies. The value at 139 MeV has been obtained by extrapolating between the other experimental values. Apart from the slow decrease of σ_R with energy, both potentials clearly fail to reproduce the data over the entire energy range. We checked that this situation is also valid for other nuclei. It seems therefore that relativistic effects, such as the role of the nuclear-Coulomb cross term as suggested by Abele *et al.* [1], do not lead to a reduction of the reaction cross section and one has to consider other possible explanations.

As discussed in [2], the reaction cross section is very sensitive to the shapes of the potentials used and should thus be included in the fitting procedure simultaneously with the angular distributions. The reaction cross section given as

$$\sigma_R = \frac{\pi}{k^2}\Sigma_l(2l+1)(1-|S_l|^2) \qquad (2)$$

is determined largely by the contributions from high partial waves and is therefore very sensitive to small changes

TABLE I. Reaction cross sections results in mb.

$E_{\rm lab}$(MeV)	KDP	WS	Expt. [2]
74.5	1732	1659	1511
103.2	1726	1648	1468
129.3	1706	1629	1465
139.0	1686	1619	1438
159.7	1675	1604	1381
192.7	1638	1579	1329

in $|S_l|$ for high l values. The angular distributions are not uniquely determined by a set of S-matrix elements and it is thus possible that a simple potential model that yields reasonable values for σ_R while describing adequately the elastic scattering could still exist. We did not investigate this possibility as we have not performed an exact fitting procedure in our calculation. However, we note that, when starting from a different Dirac potential, we obtained a different effective potential that describes the elastic data equally well but which gives a slightly different set of S-matrix elements. In Fig. 3, the $|S|$ of the scattering function is shown for the large l values. The dashed line in this figure represents the results obtained with this second potential denoted as KDP2 at the same laboratory energy of 139 MeV. The difference for the high partial waves leads to a difference of more than 50 mb for σ_R.

One can always invoke that more complex models involving channel coupling, nonlocal effects, etc., could form the basis for an elaborate explanation of the present discrepancy. However, a simple explanation based on a single potential model description would be advantageous. Model-independent fits should be investigated. In this context, the two-step phenomenology advocated in [14] may be very useful. This approach is based on fitting the S-matrix elements directly to the data in the first step and inverting the resulting set of S_l's to determine the optical model potential. Within this method, it would make sense to attempt to constrain the S_l's to yield reasonable values for σ_R. If successful, the inversion part of the calculation will help to gain insight into the important features of the resulting potential. Another simple approach that has been used for other systems is to use different absorptive parts in the optical potential to describe the elastic channel on one hand and the inelastic channels on the other hand (e.g., [15]). Such an approach, together with models where an angular momentum cutoff [15,16] is included, need to be investigated and may help in understanding the origin of the present situation.

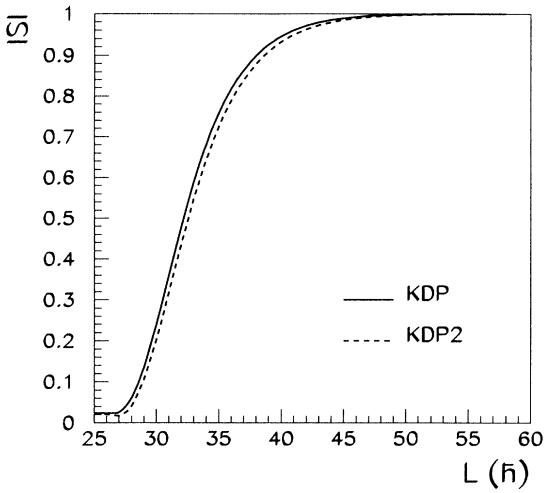

FIG. 3. The $|S_l|$ of the S-matrix elements corresponding to two effective relativistic potentials (see text) for $\alpha + {}^{58}$Ni at 139 MeV.

In conclusion, we have performed relativistic calculations for the scattering of α particles at intermediate energies. Our results indicate that relativistic effects within the present approach do not explain the discrepancy between model calculations and the experimental data of the reaction cross sections. Alternative ideas that may help to resolve the issue are referred to and need to be investigated.

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