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ρ-ω mixing off shell and charge symmetry breaking in the N-N potential

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The role of the off-shell dependence of ρ-ω mixing in the charge symmetry breaking nucleon-nucleon potential is discussed. It is shown that models describing the off-shell dependence of ρ-ω mixing are not sufficient to determine the charge symmetry breaking nucleon-nucleon potential.

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I. INTRODUCTION

Charge symmetry breaking (CSB) has been studied for a long time; see e.g., the reviews [1–7] and especially the references therein. We follow Ref. [7] in summarizing a few main features. Charge independence and charge symmetry breaking are caused by the d-u quark mass difference $m_d - m_u > 0$ and electromagnetic effects. The general goal of this area of research is to find small but observable effects of the breaking of charge independence and charge symmetry. This provides significant insight into strong interaction dynamics since the underlying origin of the breaking is understood. Over the years there has been substantial experimental and theoretical progress. First, we recall the old idea that $m_d - m_u > 0$ along with electromagnetic effects accounts for the observed mass differences between members of hadronic isospin multiplets. This mass difference also leads to the notion that the physical ρ and ω mesons are isospin mixed superpositions of bare states of good isospin. Indeed, substantial effects of $\rho - \omega$ mixing have been observed in the $e^+e^- \to \pi^+\pi^-$ cross section at $q^2 \approx m_\omega^2$ [8,9]. These results allow an extraction of the strong contribution to the ρ-ω mixing matrix element $\langle\rho|H_{\rm str}|\omega\rangle \approx -5200$ MeV^2 [10,7]. Two nucleons may exchange a mixed ρ-ω meson. If one uses $\langle\rho|H_{\rm str}|\omega\rangle \approx -5200$ MeV^2, one obtains a nucleon-nucleon interaction which is consistent with the experimental value $\Delta a_{\rm CSB} = a_{pp}^N - a_{nn}^N = 1.5 \pm 0.5$ fm [11]. Such a force can also consistently account for most of the strong interaction contribution to the ^{3}H-^{3}He binding energy difference [11] and for much of the Nolen-Schiffer anomaly [12]. The TRIUMF (477 MeV [13] and 350 MeV [14]) and IUCF (183 MeV) [15] experiments have compared analyzing powers of $\vec{n}p$ and $n\vec{p}$ scattering and observe charge symmetry breaking at the level expected from π, γ, and ρ-ω exchange effects. The latter effects are important at 183 MeV. Thus ρ-ω mixing seems to describe most of the observed features or charge symmetry breaking in nuclear physics. While it is certainly true that other mechanisms cannot be ruled out, ρ-ω mixing appears to give a consistent description of the bulk of the experimental data.

Recently, this success has been called into question. The momentum dependence of the ρ-ω mixing amplitude has been calculated using several different models [16–19]. While these models are based on quite different physical assumptions, they all share one important quality: The ρ-ω mixing at spacelike momenta in all of these models is quite different from its value at the ω pole—generally of the opposite sign and significantly reduced in magnitude. Indeed it has been shown that for a wide class of models [20] the mixing must go to zero at $q^2 = 0$, implying that amplitude changes sign. Moreover, a QCD sum-rule calculation also apparently gives a similarly large momentum dependence of the coupling [21]. Since the N-N potential probes the spacelike region, this appears to imply that the vector meson exchange part of the charge-symmetry-breaking nucleon-nucleon NN potential is very different from one based on the on-mass-shell mixing. Indeed, NN potentials have been constructed based on these momentum-dependent mixing amplitudes and these are quite different from the ones used in the successful phenomenology [16–19,22].

The purpose of the present paper is to study the general role of the off-shell meson propagator in NN potentials. We find that knowledge of the off-shell meson propagator is not sufficient to determine the potential. In particular, one needs the vertex functions computed from the same theory that supplied the propagator. None of the present treatments of the off-shell propagator deals with the issue of the necessary vertex functions. It is not our intent to compute these functions. Rather, we wish to clarify issues of principle. Accordingly we have included a number of simple illustrative examples. We do show, however, that the CSB induced by the ρ-ω exchange potential can account for the existing data even if the the q^2 dependence is exactly as specified in any of Refs. [16–19,21,22]. This is done by using CSB vertex functions.

We turn to an outline of this paper. In Sec. II we discuss the problem that in hadronic field theoretic models there is never a unique choice for fields, even in a renormalizable theory [23–26]. This means that the propagator and the vertex functions are not unique. We argue generally and with two explicit examples that while the propagator depends on the choice of field variables, the observables do not. Thus, knowledge of the off-shell meson propagator by itself gives no information unless one knows which field is used. One may be able to deduce which definition of the field has been

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used from a complete theory by studying the interactions with the other degrees of freedom in the problem. However, if the theory is incomplete and the interactions of the field with all of the other degrees of freedom are unknown, knowledge of the off-shell propagator by itself is not physically meaningful.

There is an even more serious problem. Modern meson exchange potentials are motivated by field theoretic concerns. However, there is no first principles method for obtaining the "correct" *NN* potential directly from either QCD, or from some hadron field theoretic model, or from any experimentally accessible set of data of hadronic properties. Given this essential difficulty, we believe it is sensible to adopt the general approach used in the construction of meson exchange potentials to the case of charge symmetry breaking. This approach makes the pragmatic assumptions of including the long range features in the meson propagators and the short range features in the vertex functions. This separation is discussed in Sec. III. Such a separation may be questioned, but *a priori* these assumptions should be no worse for the case of CSB potentials than they are for the isospin conserving part of the interaction. Moreover, given the lack of rigor in the construction of potential from the underlying field theory, some assumptions must be made in order to make any connection between ρ-ω mixing and the CSB potential. Given this, it is highly desirable to make sure that the assumptions are consistent with those made elsewhere in the problem.

It is worth stressing at the outset that in conventional treatments of meson exchange potentials the off-shell propagator plays no role. This is discussed in Sec. IV where realistic boson exchange charge symmetric potentials are defined to be those that are consistent with the separation discussed in Sec. III. We show that for models with realistic spectral functions the momentum dependence of the meson propagator can be absorbed into that of the vertex function. An example of an unrealistic momentum dependent ω self-energy is presented.

The ideas of Secs. II–IV are applied to the CSB potential caused by ρ-ω exchange in Sec. V. We show that the influence of the momentum dependence of the ρ-ω mixing matrix element can be included by allowing the ρ-nucleon coupling constant to violate charge symmetry. In particular, if the model of Ref. [21] is used, one needs CSB coupling constants that are 0.8% of the standard coupling constants to reproduce the results of a potential obtained without momentum dependence in the ρ-ω mixing matrix element and without CSB in the coupling constants. We summarize the analysis in Sec. VI.

II. FIELD REDEFINITIONS AND OFF-SHELL PROPAGATORS

It has been known for quite some time that value of an off-shell propagator is completely dependent on the choice of field. This is an example of a general theorem proved by Haag [23], Ruelle [24], and Borchers [25] which has been discussed by Coleman, Wess, and Zumino [26]. The off-shell propagators depend on the choice of interpolating fields, whereas all S-matrix elements are independent of this choice. Thus an off-shell propagator, taken in isolation, can have no physical meaning.

To illustrate why this is so, let us consider the simplest possible case, the field corresponding to a stable scalar particle in some nontrivial interacting field theory. The equation of motion for this system may be written as

$$\Box \phi(x) + m^2 \phi(x) = -j(x). \tag{1}$$

This equation of motion is determined from a Lagrangian density $\mathscr{L}(\phi,j)$. Furthermore, let us insist on studying the renormalized field, mass, and current. This means that the correlation function for ϕ will have a pole with residue of unity at the physical mass m:

$$\langle \phi,\vec{p}\,|\,\phi(x)\,|\text{vac}\rangle = e^{ip\cdot x}, \tag{2}$$

which implies that

$$\lim_{q^2\to m^2} (q^2-m^2)\int d^4x e^{iq\cdot x}\langle \text{vac}|T[\phi(x)\phi(0)]|\text{vac}\rangle = i. \tag{3}$$

We are concentrating on the renormalized quantities because un-renormalized properties are not observable and depend on the details of the renormalization procedure. Ultimately we will be interested in the spectral decomposition of the propagator in terms of the physical states of the system and this is directly related to the *renormalized* fields and sources.

It is worth noting that the renormalization conditions put quite stringent constraints on matrix elements of the renormalized source. In particular they imply that source does not connect the vacuum to a one particle state

$$\langle \phi,\vec{p}\,|\,j(0)\,|\text{vac}\rangle = 0. \tag{4}$$

This can be seen simply:

$$\begin{aligned}\langle \phi,\vec{p}\,|\,j(x)\,|\text{vac}\rangle &= \langle \phi,\vec{p}\,|(\Box + m^2)\phi(x)|\text{vac}\rangle \\ &= (-p^2+m^2)\langle \phi,\vec{p}\,|\,\phi(x)\,|\text{vac}\rangle,\end{aligned}$$

where p^2 is the square of the four-momentum of the state which is m^2.

Now we come to the crux of the issue. There is enormous freedom in the choice of field variables, and consequently the Green's functions. In particular, we introduce a new renormalized field and a new source current according to

$$\phi'(x) = \phi(x) + a(x), \tag{5}$$

$$j'(x) = j(x) + (\Box + m^2)a(x), \tag{6}$$

where $a(x)$ is an operator such that $\langle \text{vac}|a|\phi,\vec{p}\rangle = 0$. Thus, for example, $a(x)$ may be a multiple of the renormalized source $j(x)$ or $a(x)$ could have the form $a(x) = (\Box + m^2)b(x)$ where $b(x)$ is an arbitrary renormalized local composite operator. The new field and source satisfy an equation of motion with the same form as the original:

$$(\Box + m^2)\phi'(x) = j'(x). \tag{7}$$

It also satisfies the same renormalization conditions

$$\langle \phi,\vec{p}|\phi'(x)|\text{vac}\rangle = e^{ip\cdot x}, \tag{8}$$

$$\lim_{q^2\to m^2}(q^2-m^2)\int d^4x e^{iq\cdot x}\langle \text{vac}|T[\phi'(x)\phi'(0)]|\text{vac}\rangle = i. \tag{9}$$

The field variable ϕ' is as good a choice for the field variable as the original field ϕ—its equation of motion is of the same form and it satisfies the correct renormalization conditions. It makes no difference to any *physical* amplitude whether one chooses to describe the physics in terms of the field ϕ or ϕ'. Thus, the masses of particles and possible bound states and S-matrix elements for scattering states must be identical with either description. Going from one to the other amounts to nothing more than a change of variables.

While the physics clearly does not depend on which field is chosen, the propagator depends strongly on this choice:

$$\int d^4x e^{iq\cdot x}\langle \text{vac}|T[\phi'(x)\phi'(0)]|\text{vac}\rangle$$
$$= \int d^4x e^{iq\cdot x}\langle \text{vac}|T[\phi(x)\phi(0)]\rangle + \int d^4x e^{iq\cdot x}\langle T[\phi(x)a(0)]+T[a(x)\phi(0)]+T[a(x)a(0)]|\text{vac}\rangle. \tag{10}$$

Equation (9), which picks out the pole at $q^2=m^2$, is obtained since by construction a does not connect the vacuum to the one ϕ state. Clearly, this is necessary since the correlation functions for ϕ and ϕ' satisfy the renormalization conditions in Eqs. (3) and (9). Off shell, however, there is no requirement that this term vanish and the two propagators will in general differ. Moreover, since the overall scale of a is arbitrary, it is clear that one can make the difference between the two descriptions arbitrarily large.

Let us make these ideas explicit by considering two examples from a theory in which the current j is a static external source. In this case the energy of the system is given by

$$E = \int d^3r \frac{1}{2} j(\vec{r})\phi(\vec{r}) \tag{11}$$

or

$$E = \int d^3r \frac{1}{2} j(\vec{r})G(\vec{r},\vec{r}\,')j(\vec{r}\,'), \tag{12}$$

where $G(\vec{r},\vec{r}\,')$ is the inverse of the operator ∇^2-m^2. Let us first take $a(x)$ to be a simple function of $\vec{x}$, which is independent of ϕ. Then $\langle \phi,\vec{p}|a(x)|\text{ vac}\rangle=0$ and the renormalization conditions of Eqs. (3,9) are satisfied. One may determine a new Lagrangian density $\mathscr{L}'$ and a new Hamiltonian $\mathscr{H}'$ by starting with the original $\mathscr{L}$ and transforming the variables. Then the new energy E' is given by

$$E' = \int d^3r \frac{1}{2}\{\vec{\nabla}[\phi'(\vec{r})+a(\vec{r})]\cdot[\phi'(\vec{r})+a(\vec{r})]$$
$$+m^2[\phi'(\vec{r})+a(\vec{r})]^2+2j(\vec{r})[\phi'(\vec{r})+a(\vec{r})]\}, \tag{13}$$

and using the equation of motion (7) in the static limit leads to

$$E' = \int d^3r \frac{1}{2} j(\vec{r})[\phi'(\vec{r})+a(\vec{r})]. \tag{14}$$

But Eq. (5) tells us that $E'=E$. Even though the current j' of Eq. (6) is different than j, the energy of the system does not depend on the choice of the function $a(\vec{x})$.

A more interesting example is obtained by letting $\phi=[1+f(\vec{x})]\phi'$ [or $a(\vec{x})=-f(\vec{x})/[1+f(\vec{x})]\,\phi(x)$]. We place the static source j at the origin and choose $f(\vec{x})$ to vanish at large values of $|\vec{x}|$ faster than $e^{-m|\vec{x}|}/|\vec{x}|$. This maintains the original value of the renormalized coupling constant (which is proportional to the asymptotic field) and therefore is the analog of our renormalization for problems with static sources. In this case the equation of motion is

$$D\phi' = -j', \tag{15}$$

where

$$D \equiv (1+f)^2(-\nabla^2+m^2)-2(1+f)\partial_\mu f\partial^\mu-(1+f)\nabla^2 f \tag{16}$$

and

$$j' \equiv j(1+f). \tag{17}$$

Clearly the Green's function G' (the inverse of D) and current j' are both fairly complicated. The use of the new Hamiltonian density $\mathscr{H}'$ gives

$$E' = \int d^3r \frac{1}{2}[(1+f)^2\vec{\nabla}\phi'\cdot\vec{\nabla}\phi'+(\phi')^2\vec{\nabla}f\cdot\vec{\nabla}f$$
$$+2\phi'(1+f)\vec{\nabla}f\cdot\vec{\nabla}\phi'+(m\phi')^2(1+f)^2$$
$$+2j\phi'(1+f)]. \tag{18}$$

Integration by parts and the equation of motion (15) allow one to obtain

$$E'=\frac{1}{2}\int d^3rj(1+f)\phi'=\frac{1}{2}\int d^3rj'(\vec{r})G'(\vec{r},\vec{r}')j'(\vec{r}'), \tag{19}$$

which is just the original energy since $(1+f)\phi'=\phi$.

Thus we have seen two explicit examples in which transformations of field variables change the equation of motion, the Green's functions, and the currents without changing, the physical observable, the energy of the system.

These same arguments of Eqs. (5)–(10) can be used to show that the various n-point vertex functions also depend on the specific choice of field. The generalization of the argument to vector fields rather than scalars and to correlation functions of two different fields uses standard techniques. Again one finds that off-shell propagators and the vertex functions depend explicitly on the choice of field.

It is clear what is going on here. Neither the off-shell propagators nor the vertex functions are directly observable. From a theoretical point of view, the values of these quantities depend explicitly on which arbitrary choice of field one makes. Various combinations of the propagators and the vertex functions correspond to physical quantities and it is only these combinations which can be measured. Choosing a particular field amounts to making a bookkeeping choice—it only determines whether some bit of the physics will be found in the vertex or in the propagator.

The point we wish to stress is that knowledge, however precise, of the off-shell propagator contains no physical information unless one specifies the choice of the quantum field or equivalently unless one has knowledge of how the field couples to the rest of the system—i.e., knowledge of the vertex functions which arise from the same field choice. Thus, a model for the off-shell propagator in the absence of a *consistent* model for the vertex functions is not complete. The models of Refs. [16–19,22] present the mixed ρ-ω propagator off shell, but do not give the necessary simultaneous consistent description of the CSB N-N–vector-meson vertices.

III. PHILOSOPHY OF MESON EXCHANGE POTENTIALS

The preceding argument that off-shell meson propagators are not sufficient is entirely based on field theoretic considerations. Clearly, this does not help us to to compute observables; it does not address the question of how one can compute CSB (or any other) observables in nuclear physics. One typically constructs a nucleon-nucleon potential and then computes wave functions, hoping that the potentials capture the essential aspects of the underlying field theory. Nevertheless, there is no unambiguous way to construct potentials. Nontrivial assumptions must be made.

Here we will assume that the assumptions underlying phenomenologically successful meson exchange models are reasonable. While one can construct equally successful purely phenomenological models, the meson exchange models make a connection to the spectral properties of the underlying theory. Moreover, the entire question we are investigating—the role of ρ-ω mixing in CSB effects in nuclear physics—can only be addressed in the context of a potential model which employs vector mesons.

There is a definite philosophy underlying the construction of NN potentials from meson exchange. One principal idea is the need for a separation of momentum or length scales. One explicitly includes the exchange of mesons lighter (and hence more long ranged) than some scale separation point. All short ranged effects are either incorporated in phenomenologically determined vertex functions or by some other purely phenomenological means. The physical picture underlying this philosophy is that the nucleon has a three-quark core which cannot be described efficiently in terms of mesons, while at longer distances the nucleon structure is dominated by a meson cloud.

To some extent, the fact that short ranged effects are handled as pure phenomenology is of little importance in most low energy nuclear physics applications. Because of repulsion at short distances, nuclear wave functions have strong short distance correlations which prevent the system from feeling the truly short range part of the potential. Moreover, at very short distances the concept of an NN potential becomes particularly inappropriate. Typically, in meson exchange potentials this scale separation point, which we will call Λ_s, is taken to be of order 1 GeV so that ρ and ω mesons are explicitly included while heavier vector mesons are not. It is worth observing, however, that this does not mean that the short distance physics does not have important long range consequences. In particular, the value of the meson-nucleon coupling constant, determined by short distanced physics, plays an essential role in the potentials at long and intermediate ranges.

We believe that this general approach of treating the short range part of the NN interaction phenomenologically while explicitly including the effects of lighter mesons is reasonable. This general approach ought to be applicable to charge-symmetry-breaking effects.

There is another important assumption which underlies these models. It is assumed that except at short distances the vector part of the potential is dominated by the vector mesons. Thus it is assumed that continuum two-pion vector-isovector and three-pion vector-isoscalar exchange contributions are small—i.e., that the only substantial strength arising from the two pion vector-isovector exchange is sufficiently concentrated at the ρ mass as to be well described by ρ exchange and analogously for three pions and the ω exchange. We note that this assumption can be questioned. In its favor we note that in $e^+e^-\rightarrow$ pions, the ρ and ω peaks do, in fact, completely dominate the low-lying spectral function.

In our discussions we will adopt the Bonn potential [27] strategy of incorporating all short range effects in vertex functions. In such a strategy the scale separation between long and short range is particularly easy to enforce: The phenomenological vertex functions are analytic for $q^2<\Lambda_s^2$ while the propagators are analytic for $q^2>\Lambda_s^2$, where q^2 is the square of the four-momentum.

IV. MOMENTUM-DEPENDENT SELF-ENERGIES IN MESON EXCHANGE POTENTIALS

It is probably useful to discuss an analogous and perhaps somewhat simpler problem before discussing charge symmetry breaking. The ρ-ω mixing matrix element is an off-diagonal mass term. Models which give momentum dependence to this off-diagonal mass can also be expected to give momentum dependence to the analogous diagonal mass terms—i.e., to the vector meson self-energies. It is clearly useful to understand the role of the momentum dependence of the ρ and ω self-energies in the charge-symmetry-preserving potential before taking on the challenge of understanding the momentum dependence of the ρ-ω mixing.

For simplicity we examine the one ω exchange contribution. First consider the traditional meson exchange model description with the scale separation as outlined above. The potential is given by

$$V_\omega(q^2)=\frac{[g^v_\omega(q^2)\gamma^{(1)}_\mu+g^t_\omega(q^2)q^\alpha\sigma^{(1)}_{\alpha\mu}](g^{\mu\nu}-q^\mu q^\nu/m^2_\omega)[g^v_\omega(q^2)\gamma^{(2)}_\nu+g^t_\omega(q^2)\sigma_{\nu\beta}q^\beta]}{q^2-m^2_\omega}, \tag{20}$$

where $g^v_\omega(q^2)$ and $g^t_\omega(q^2)$ are the vector and tensor couplings of the omega to the nucleons. The superscripts 1 and 2 label the nucleon. These couplings are analytic functions of q^2 for $q^2<\Lambda^2_s$ the propagator is clearly analytic for $q^2>\Lambda^2_s$.

In principle, we could consider a more sophisticated model consistent with the philosophy outlined above. For example, one could explicitly include the exchange of three low energy pions (with the quantum numbers of the rho) along with an omega self-energy due to its coupling with the three pion channel and a longer range part of the ω-N vertex due to three-pion exchange. In practice, one expects such effects to be small: In part they serve to simply widen the omega pole by an amount of no practical significance to the potential; other effects of coupling to the three-pion channel are small because they are weakly coupled. In any event, we will stick to the conventional assumptions underlying meson exchange models and neglect such effects. In the remainder of this paper we will ignore such effects.

Let us now suppose that we had a detailed microscopic model of the ω meson which enables us to calculate a momentum-dependent ω self-energy, $\pi_\omega(q^2)$. As a matter of convention, we will include any effects of mass and wave function renormalizations of the ω in $\pi_\omega(q^2)$. This means that π_ω and its derivative vanishes at $q^2=m^2_\omega$. The omega exchange part of the N-N potential with such a model is given by

$$V_\omega(q^2)=\frac{[\hat{g}^v_\omega(q^2)\gamma^{(1)}_\mu+\hat{g}^t_\omega(q^2)q^\alpha\sigma^{(1)}_{\alpha\mu}](g^{\mu\nu}-q^\mu q^\nu/m^2_\omega)[\hat{g}^v_\omega(q^2)\gamma^{(2)}_\nu+\hat{g}^t_\omega(q^2)\sigma_{\nu\beta}q^\beta]}{q^2-m^2_\omega+\pi_\omega(q^2)}. \tag{21}$$

We have written the couplings as $\hat{g}^{v,t}_\omega$ and rather than $g^{v,t}_\omega$ to make evident the fact that the vertex functions used in the model in Eq. (21) need not be the same as the vertex functions used in the model in Eq. (20): *These vertex functions are phenomenological and depend on how the rest of the problem is treated.*

Given that the vertex functions may differ between the two models, we note that the two models may be identical—i.e., they may be two equivalent ways of representing the same physics. One way for this to occur is if the vertex functions in the two models are related by

$$g^{v,t}_\omega=\hat{g}^{v,t}_\omega\left(\frac{q^2-m^2_\omega}{q^2-m^2_\omega+\pi_\omega(q^2)}\right)^{1/2}. \tag{22}$$

Note that the square root factor is unity for $q^2=m^2_\omega$ due to the renormalization of $\pi_\omega(q^2)$. The result (22) is not surprising in light of the formal analysis of Sec. II. Neither the propagator off shell nor the vertex function are separately meaningful.

Given that vertex functions are fit to some set of data, the only reason the condition in Eq. (22) would not be satisfied would be due to practical and philosophical limitations in the forms used in the fitting of the vertex functions. The practical limitation is that one must take some limited trial form for the phenomenological coupling. To the extent that meson exchange models make sense in the regime where they are used, the trial forms must be rich enough to describe the data with reasonable precision. Thus, apart from the philosophical concerns discussed below, Eq. (22) can be satisfied well enough so that any difference between the potentials of Eqs. (20) and (21) will have a small effect on the physics. The philosophical limitation is that the vertex functions are supposed to only contain effect of a range shorter than Λ_s^{-1}. Longer range effects are to be included by explicit dynamics of the lighter degrees of freedom in the problem.

Thus, the issue of whether the two models are equivalent comes down to whether both $g_{v,t}(q^2)$ and $\hat{g}_{v,t}(q^2)$ can be analytic for $q^2<\Lambda^2_s$ while satisfying Eq. (22). In effect, the question is whether

$$f(q^2)=\left(\frac{q^2-m^2_\omega}{q^2-m^2_\omega+\pi_\omega(q^2)}\right)^{1/2} \tag{23}$$

is analytic for $q^2<\Lambda^2_s$. Nonanalyticity can occur when either the numerator or denominator vanishes or when π_ω is nonanalytic. In fact, we should relax this restriction slightly—the nonanalyticity associated with the ω coupling to three low energy pions which slightly broadens the pole and gives a small nonresonant contribution is, as discussed above, innocuous. In any event, this issue does not arise in the context of the models in Refs. [16–19].

Clearly, the analytic structure of $f(q^2)$ depends in detail on the choice of model. The simplest way to make the physics explicit is to make a spectral representation [28] for the propagator:

$$\frac{1}{q^2-m_\omega^2+\pi_\omega(q^2)}=\int ds\,\frac{\rho(s)}{q^2-s+i\epsilon}+\text{subtraction terms.} \tag{24}$$

Different models will give rise to different spectral functions. However, if the model is realistic, the only substantial spectral strength for $q^2<\Lambda_s^2$ occurs at or near the omega pole. Accordingly any model which gives significant amounts of spectral strength below Λ_s^2 (apart from the ω pole) can be considered as unrealistic in our philosophy. If, however, all of the spectral strength is either at the ω pole or above Λ_s^2, then $f(q^2)$ is analytic for q^2 below Λ_s^2. The apparent nonanalyticity due to the denominator vanishing at $q^2=m_\omega^2$ is precisely canceled by a vanishing numerator. (Recall that all renormalization effects are included in π_ω so that the position of the ω pole does not shift.) In this case, one may redefine the vertex functions according to Eq. (22).

To see how the spectral representation constrains the allowable forms of the self-energies consider the following simple example in which the self-energy has the form

$$\pi_\omega(q^2)=(q^2-m_\omega^2)^2Bq^2. \tag{25}$$

This form is motivated by the renormalization requirements that $\pi_\omega(q^2)$ and its derivative vanish at $q^2=m_\omega^2$. One determines the nucleon-nucleon potential generated by the propagator of Eq. (24) by taking q^2 to be spacelike $q^2=-Q^2<0$; the potential is proportional to the integral

$$\int dQQ\,\frac{\sin(Qr)}{r}\,\frac{1}{-Q^2-m_\omega^2-(Q^2+m_\omega^2)^2BQ^2}\,.$$

One does the contour integration by identifying the poles. There is always a pole at $Q^2=-m_\omega^2$ which is the standard term expected from the exchange of an ω^2 meson. There are other poles at positions determined by the value of B. One finds that if $m_\omega^4>m_\omega^4-4/B\geqslant 0$ there will be poles with Q^2 real and negative. At least one of the poles must be at $|Q^2|<m_\omega^2$, which is unrealistic in our philosophy. If $m_\omega^4-4/B>m_\omega^4$, the poles occur for $Q^2>0$ which are physically unallowable tachyonic excitations. Similarly, if $m_\omega^4-4/B<0$, there are poles off the real axis, which violates the spectral representation and also renders the model for $\pi_\omega(q^2)$ as useless. This analysis demonstrates that a spectral function of the form in Eq. (25) is not viable.

Let us now summarize the effects of the momentum dependence of the ω self-energy on the meson exchange potential. In any realistic model (i.e., any model without unphysical low q^2 spectral strength in the ω propagator) all of the effects of the momentum dependence of the self-energy can be reabsorbed into the momentum dependence of the phenomenological vertex functions. Accordingly, there are no observable physical effects in the *NN* potential induced by such a momentum dependent self-energy. Moreover, including the short range part of the momentum dependence in the propagator of a meson exchange model violates the bookkeeping arrangement in which all of the short range effects are segregated into phenomenological vertices.

V. CHARGE-SYMMETRY BREAKING *NN* POTENTIAL AND THE MOMENTUM DEPENDENCE OF ρ-ω MIXING

The preceding section gives us a paradigm for what happens in the charge-symmetry-breaking part of the potential. We will show, for any realistic model of the momentum dependence of the mixing amplitude, that all of the effects of the momentum dependence can be absorbed into phenomenological short-ranged charge-symmetry-breaking nucleon–vector-meson couplings.

Consider the charge-symmetry-breaking potential arising from vector meson exchange. Let us begin by implementing this according to the philosophy of scale separation discussed in the previous two sections. Assuming that only the meson exchanges we need to consider are the ρ and ω, the charge-symmetry-breaking interaction potential can be written as

$$\begin{aligned}
V^{\rm CSB}_{\omega,\rho}(q^2)=&\frac{[g^v_\omega(q^2)\gamma^{(1)}_\mu+g^t_\omega(q^2)q^\alpha\sigma^{(1)}_{\alpha\mu}][g^{\mu\nu}-q^\mu q^\nu/m_\omega^2][g^{v\rm CSB}_\omega(q^2)\tau_3^{(2)}\gamma_\nu^{(2)}+g^{t\rm CSB}_\omega(q^2)\tau_3^{(2)}\sigma^{(2)}_{\nu\beta}q^\beta]}{q^2-m_\omega^2}\\
&+\frac{[g^v_\omega(q^2)\gamma^{(2)}_\mu+g^t_\omega(q^2)q^\alpha\sigma^{(2)}_{\alpha\mu}][g^{\mu\nu}-q^\mu q^\nu/m_\omega^2][g^{v\rm CSB}_\omega(q^2)\tau_3^{(1)}\gamma_\nu^{(1)}+g^{t\rm CSB}_\omega(q^2)\tau_3^{(1)}\sigma^{(1)}_{\nu\beta}q^\beta]}{q^2-m_\omega^2}\\
&+\frac{[g^v_\rho(q^2)\gamma^{(1)}_\mu\tau_3^{(1)}+g^t_\rho(q^2)q^\alpha\sigma^{(1)}_{\alpha\mu}\tau_3^{(1)}][g^{\mu\nu}-q^\mu q^\nu/m_\rho^2][g^{v\rm CSB}_\rho(q^2)\gamma_\nu^{(2)}+g^{t\rm CSB}_\rho(q^2)\sigma^{(2)}_{\nu\beta}q^\beta]}{q^2-m_\rho^2}\\
&+\frac{[g^v_\rho(q^2)\tau_3^{(2)}\gamma^{(2)}_\mu+g^t_\rho(q^2)q^\alpha\sigma^{(2)}_{\alpha\mu}\tau_3^{(2)}][g^{\mu\nu}-q^\mu q^\nu/m_\rho^2][g^{v\rm CSB}_\rho(q^2)\gamma_\nu^{(1)}+g^{t\rm CSB}_\rho(q^2)\sigma^{(1)}_{\nu\beta}q^\beta]}{q^2-m_\rho^2}\\
&+m_{\rho\omega}^2\frac{[g^v_\omega(q^2)\gamma^{(1)}_\mu+g^t_\omega(q^2)q^\alpha\sigma^{(1)}_{\alpha\mu}][g^{\mu\gamma}-q^\mu q^\gamma/m_\omega^2][g^{\gamma\nu}-q^\gamma q^\nu/m_\rho^2][g^v_\rho(q^2)\tau_3^{(2)}\gamma_\nu^{(2)}+g^t_\rho(q^2)\tau_3^{(2)}\sigma^{(2)}_{\nu\beta}q^\beta]}{(q^2-m_\omega^2)(q^2-m_\rho^2)}\\
&+m_{\rho\omega}^2\frac{[g^v_\omega(q^2)\gamma^{(2)}_\mu+g^t_\omega(q^2)q^\alpha\sigma^{(2)}_{\alpha\mu}][g^{\mu\gamma}-q^\mu q^\gamma/m_\omega^2][g^{\gamma\nu}-q^\gamma q^\nu/m_\rho^2][g^v_\rho(q^2)\tau_3^{(1)}\gamma_\nu^{(1)}+g^t_\rho(q^2)\tau_3^{(1)}\sigma^{(1)}_{\nu\beta}q^\beta]}{(q^2-m_\omega^2)(q^2-m_\rho^2)}\,.
\end{aligned} \tag{26}$$

This form is rather general: In addition to ρ-ω mixing, it explicitly includes possible charge-symmetry-breaking couplings between the vector mesons and the nucleons arising from short distance effects: These couplings are labeled by the superscript CSB. The coefficient $m^2_{\rho\omega}$ is the mixing parameter which in this model is taken to be independent of q^2.

It should be noted that the general form of Eq. (26) is consistent with the general philosophy of meson exchange used here. In particular all short ranged effects are merely parametrized, while the long ranged effects are treated dynamically in terms of the mesons. For this reason we must include the ρ-ω mixing explicitly rather than including all of the effects in terms of the charge-symmetry-breaking couplings.

The couplings $g^{v,t}_{\omega,\rho}(q^2)$ are presumed to have been determined in fits to the charge-symmetry-conserving interactions. In principle, the coupling constants $g^{v,tCSB}_{\omega,\rho}(q^2)$ must be determined phenomenologically from experimental data on charge symmetry breaking. In fact, in the treatments of CSB in Refs. [11,12,10,7] these couplings were all taken to be zero. In that work, model assumptions and existing nucleon-nucleon and pion-nucleon scattering data were used to make *a priori* arguments that these couplings should be small and hence could be neglected. See, for example, Refs. [5] and [7] which reviews the charge dependence of the couplings. The neglect of charge dependence in the meson-nucleon coupling constants is not invalidated by present data. In particular, descriptions of all known CSB effects do not require the inclusion of such terms. Had the data required the inclusion of such terms, they could have been included without violating the spirit of a meson exchange potential model.

Now suppose that we had a detailed model for the structure of the vector mesons in which the ρ-ω mixing amplitude has a nontrivial momentum dependence. The form for the CSB potential is very similar to the form above:

$$\begin{aligned}
V^{\rm CSB}_{\omega,\rho}(q^2) = {} & \frac{[g^v_\omega(q^2)\gamma^{(1)}_\mu + g^t_\omega(q^2)q^\alpha\sigma^{(1)}_{\alpha\mu}][g^{\mu\nu}-q^\mu q^\nu/m^2_\omega][\hat{g}^{v\rm CSB}_\omega(q^2)\tau^{(2)}_3\gamma^{(2)}_\nu + \hat{g}^{t\rm CSB}_\omega(q^2)\tau^{(2)}_3\sigma^{(2)}_{\nu\beta}q^\beta]}{q^2-m^2_\omega} \\
& + \frac{[g^v_\omega(q^2)\gamma^{(2)}_\mu + g^t_\omega(q^2)q^\alpha\sigma^{(2)}_{\alpha\mu}][g^{\mu\nu}-q^\mu q^\nu/m^2_\omega][\hat{g}^{v\rm CSB}_\omega(q^2)\tau^{(1)}_3\gamma^{(1)}_\nu + \hat{g}^{t\rm CSB}_\omega(q^2)\tau^{(1)}_3\sigma^{(1)}_{\nu\beta}q^\beta]}{q^2-m^2_\omega} \\
& + \frac{[g^v_\rho(q^2)\gamma^{(1)}_\mu\tau^{(1)}_3 + g^t_\rho(q^2)q^\alpha\sigma^{(1)}_{\alpha\mu}\tau^{(1)}_3][g^{\mu\nu}-q^\mu q^\nu/m^2_\rho][\hat{g}^{v\rm CSB}_\rho(q^2)\gamma^{(2)}_\nu + \hat{g}^{t\rm CSB}_\rho(q^2)\sigma^{(2)}_{\nu\beta}q^\beta]}{q^2-m^2_\rho} \\
& + \frac{[g^v_\rho(q^2)\tau^{(2)}_3\gamma^{(2)}_\mu + g^t_\rho(q^2)q^\alpha\sigma^{(2)}_{\alpha\mu}\tau^{(2)}_3][g^{\mu\nu}-q^\mu q^\nu/m^2_\rho][\hat{g}^{v\rm CSB}_\rho(q^2)\gamma^{(1)}_\nu + \hat{g}^{t\rm CSB}_\rho(q^2)\sigma^{(1)}_{\nu\beta}q^\beta]}{q^2-m^2_\rho} \\
& + m^2_{\rho\omega}(q^2)\frac{[g^v_\omega(q^2)\gamma^{(1)}_\mu + g^t_\omega(q^2)q^\alpha\sigma^{(1)}_{\alpha\mu}][g^{\mu\gamma}-q^\mu q^\gamma/m^2_\omega][g^{\gamma\nu}-q^\gamma q^\nu/m^2_\rho][g^v_\rho(q^2)\tau^{(2)}_3\gamma^{(2)}_\nu + g^t_\rho(q^2)\tau^{(2)}_3\sigma^{(2)}_{\nu\beta}q^\beta]}{(q^2-m^2_\omega)(q^2-m^2_\rho)} \\
& + m^2_{\rho\omega}(q^2)\frac{[g^v_\omega(q^2)\gamma^{(2)}_\mu + g^t_\omega(q^2)q^\alpha\sigma^{(2)}_{\alpha\mu}][g^{\mu\gamma}-q^\mu q^\gamma/m^2_\omega][g^{\gamma\nu}-q^\gamma q^\nu/m^2_\rho][g^v_\rho(q^2)\tau^{(1)}_3\gamma^{(1)}_\nu + g^t_\rho(q^2)\tau^{(1)}_3\sigma^{(1)}_{\nu\beta}q^\beta]}{(q^2-m^2_\omega)(q^2-m^2_\rho)} .
\end{aligned} \tag{27}$$

We have labeled the CSB coupling as $\hat{g}^{v,t\rm CSB}_{\omega,\rho}$ rather than $g^{v,t\rm CSB}_{\omega,\rho}$ to make explicit the fact the CSB couplings in Eq. (27) may be different from the CSB couplings in Eq. (26).

The question we wish to address is whether the model in Eq. (27) is equivalent to the model in Eq. (26). The issue comes down to whether the effects of the momentum dependence of the mixing can be entirely absorbed into differences between $\hat{g}^{v,t\rm CSB}_{\omega,\rho}$ and $g^{v,t\rm CSB}_{\omega,\rho}$ without introducing any unnaturally long range effects into the CSB couplings. We shall show that this can be done.

The ρ-ω mixing is measured rather accurately at the pole at $q^2=m^2_\omega$. Accordingly, it is sensible to express

$$m^2_{\rho\omega}(q^2) = m^2_{\rho\omega} + \delta m^2_{\rho\omega}(q^2), \tag{28}$$

with $\delta m^2_{\rho\omega}(m^2_\omega)=0$. Thus, the expression

$$\frac{\delta m^2_{\rho\omega}(q^2)}{(q^2-m^2_\rho)(q^2-m^2_\omega)}$$

has no ω pole. All effects with this term are indistinguishable from terms arising due ρ exchange with a CSB vertex. In particular, if

$$\hat{g}^{v,t\rm CSB}_\omega = g^{v,t\rm CSB}_\omega , \tag{29}$$

$$\hat{g}^{v,t\rm CSB}_\rho = g^{v,t\rm CSB}_\rho - \frac{\delta m^2_{\rho\omega}(q^2)}{q^2-m^2_\omega} g^{v,t}_\rho , \tag{30}$$

then the potential in Eq. (27) is identical with the one of Eq. (26). This result can also be obtained from Feynman diagrams. Let an ω be emitted from a nucleon and then be converted via $m^2_{\rho\omega}(q^2)$ into a ρ. One can draw a box which includes the strong vertex and $m^2_{\rho\omega}(q^2)$. This box is the charge-dependent ρ-nucleon coupling constant. Alternatively one can regard the $m^2_{\rho\omega}(q^2)$ as part of the propagator. Either way, the result is the same.

We can do a specific calculation. For example, suppose $\delta m^2_{\rho\omega}(q^2)=(q^2-m^2_\omega)m^2_{\rho\omega}/m^2_\omega$. This is a good approximation to the $m^2_{\rho\omega}(q^2)$ obtained in the sum rule work of Ref. [21]. Then the difference between $\hat{g}^{v,t\mathrm{CSB}}_\rho$ and $g^{v,t\mathrm{CSB}}_\rho$ is a simple constant $\approx -0.008 g^{v,t}_\rho$; if $\hat{g}^{v,t\mathrm{CSB}}_\rho$ were chosen as the negative of that constant, one would obtain the standard form of the ρ-ω mixing contribution to the NN potential. See also Ref. [29].

Moreover, for any reasonable model of the momentum dependence of the mixing, Eq. (30) can be satisfied without introducing unnaturally long ranged effects into the meson-nucleon vertex functions. The issues are completely analogous to the ones raised in connection with the ω exchange potential discussed in the previous section. First, it should be noted that there is no ω pole singularity on the right hand side of Eq. (30)—it is eliminated because $\delta m^2_{\rho\omega}$ vanishes at the ω pole. Thus, the only source of long range contamination of the couplings is in $\delta m^2_{\rho\omega}(q^2)$ itself. Note that by construction $\delta m^2_{\rho\omega}(q^2)$ cannot have a singularity associated with either the ρ or the ω. Moreover, we know that the only substantial strength in the vector channels at $q^2<\Lambda^2_s$ is through the ρ and ω mesons. Thus, any model which yields long range effects in $\delta m^2_{\rho\omega}(q^2)$ must be regarded as unrealistic according to our philosophy.

VI. SUMMARY

We are working in the framework of boson exchange potentials. This means that in realistic boson exchange models long range effects are included via boson exchanges and that short range effects are included in the vertex functions. For any such realistic model of the momentum dependence of the ρ-ω mixing parameter, there are no effects in the CSB breaking potential which cannot be absorbed into a redefinition of a short ranged CSB ρ-N vertex. Thus, a model which provides knowledge of the momentum dependence of the mixing parameter alone, without simultaneously giving a self-consistent model for the short ranged CSB vector-meson–nucleon couplings, gives no information about the CSB N-N potential.

The work of Refs. [16–19] found major differences between the CSB potentials based on the on-shell ρ-ω mixing and models with a large momentum dependence. Our boson exchange model view is that this is because the short ranged CSB vector meson-nucleon coupling is assumed to be zero—just as in the models based on the on-shell ρ-ω mixing. However, there is no reason *a priori* that this assumption is true for the models under discussion. Indeed, there is *a posteriori* evidence that the assumption may be wrong: The models based on the on-shell ρ-ω mixing and negligible $g^{v,t\mathrm{CSB}}_{\rho\omega}$ reproduce the available data with reasonable accuracy.

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