QCD sum rules, scattering length, and vector mesons in the nuclear medium

Tetsuo Hatsuda,[1,2] Su Houng Lee,[2,3] and Hiroyuki Shiomi[1]
[1]*Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305, Japan*
[2]*Institute for Nuclear Theory, NK-12, University of Washington, Seattle, Washington 98195*
[3]*Department of Physics, Yonsei University, Seoul 120-749, Korea*
(Received 8 May 1995)

Mass shift of vector mesons in the nuclear medium is studied in two different approaches: the in-medium QCD sum-rule approach and the scattering-length approach. The latter is shown to be inapplicable to extract the hadron masses in the medium. Further elaboration of the in-medium QCD sum rules is also discussed.

PACS number(s): 24.85.+p, 12.38.Lg, 21.65.+f

I. INTRODUCTION

The question of changes in vector meson properties in the nuclear medium is of interest in relation to the physics of nuclear matter [1] and relativistic heavy ion collisions [2]. In particular, if there are spectral changes of vector mesons in the medium, they could be observed directly through the lepton-pair spectrum in γ-A, p-A, and A-A reactions [3]. The problem has been studied in effective hadronic models and QCD sum rules (QSR) generalized to finite baryon density. In QSR, it was shown that the vector meson mass drops to about 10–20 % of its vacuum value in nuclear matter density [4]. This is consistent with the idea of the Brown-Rho scaling [5] and also with Walecka model calculations including the vacuum polarization [6].

Recently, it was claimed in Ref. [7] that the vector meson mass should increase slightly on the basis of an analysis of the meson-baryon scattering length in QCD sum rules. Since both Refs. [4] and [7] are based on the Fermi-gas approximation for nuclear matter, it is desirable to clarify the origin of the difference. In this work, we will first give a correct account of the Fermi-gas approximation of the QSR in the medium and show the dangers of relating mass shifts with the scattering length. Secondly, we will show that the determination of the scattering length in QSR is itself rather difficult because of the lack of information on the higher dimensional condensates. Thirdly, the vector meson mass is shown to decrease in the nuclear medium in a correct application of QSR.

This paper is organized as follows. In Sec. II, we will summarize the results of Ref. [4] and make new remarks which are not given in [4] but are relevant for the later discussions. In Sec. III, the essential ingredients of the scattering length approach [7] are summarized. Section IV is devoted to critical investigation of the scattering length approach and its comparison to the in-medium QSR approach. In Sec. V, further elaboration of Ref. [4] is discussed.

II. QSR IN THE NUCLEAR MEDIUM

Let us start with the retarded current correlation function in nuclear matter,

$$\Pi^R_{\mu\nu}(\omega,\mathbf{q};n_B)=i\int d^4x e^{iqx}\langle \mathrm{R}J_\mu(x)J_\nu(0)\rangle_n, \tag{1}$$

where $q^\mu\equiv(\omega,\mathbf{q})$ and $\mathrm{R}J_\mu(x)J_\nu(0)\equiv\theta(x^0)[J_\mu(x),J_\nu(0)]$ with the source currents J_μ defined as $J_\mu=\frac{1}{2}(\bar{u}\gamma_\mu u\mp\bar{d}\gamma_\mu d)$ [$-(+)$ is for the $\rho^0(\omega)$ meson]. n_B denotes the baryon number density and $\langle\ \rangle_n$ is the expectation value in the ground state of nuclear matter.

Although there are two independent invariants in the medium (transverse and longitudinal polarization), they coincide in the limit $\mathbf{q}\rightarrow\mathbf{0}$ and reduce to $\Pi^R_{\mu\mu}/(-3\omega^2)\equiv\Pi^R$. Π^R satisfies the following dispersion relation:

$$\mathrm{Re}\Pi^R(\omega^2)=\frac{1}{\pi}\,\mathrm{P}\int_0^\infty du^2\frac{\mathrm{Im}\Pi^R(u)}{u^2-\omega^2}+(\text{subtraction}). \tag{2}$$

In QSR, the spectral density $\mathrm{Im}\Pi^R$ is modeled with several phenomenological parameters, while $\mathrm{Re}\Pi^R$ is calculated using the operator product expansion (OPE). The phenomenological parameters are then extracted by matching the left and right hand sides of (2) in the asymptotic region $\omega^2\rightarrow-\infty$.

Although the nuclear matter ground state has a complicated structure, the Fermi-gas approximation taking into account the Pauli principle among the nucleons is a good starting point [8]. In this approximation, $\Pi^R_{\mu\nu}$ reads

$$\Pi^R_{\mu\nu}(\omega,\mathbf{q};n_B)=\Pi^R_{\mu\nu}(\omega,\mathbf{q};0)+\gamma\int^{p_F}\frac{d^3p}{(2\pi)^3 2E_N}T^R_{\mu\nu}(\omega,\mathbf{q}|\mathbf{p}), \tag{3}$$

with

$$T^R_{\mu\nu}(\omega,\mathbf{q}|\mathbf{p})=i\int d^4x e^{iqx}\langle N(\mathbf{p})|\mathrm{R}J_\mu(x)J_\nu(0)|N(\mathbf{p})\rangle. \tag{4}$$

Here γ is a degeneracy factor (γ=4 in nuclear matter) and $|N(\mathbf{p})\rangle$ is the nucleon state with four-momentum $p^\mu=(E_N,\mathbf{p})$ in the covariant normalization $\langle N(\mathbf{p})|N(\mathbf{p}')\rangle=(2\pi)^3 2E_N\delta^3(\mathbf{p}-\mathbf{p}')$. The spin average for the nucleon state is assumed in (4). $T^R_{\mu\nu}(\omega,\mathbf{q}|\mathbf{p})$ is nothing but the spin-averaged forward scattering amplitude of the external current

J_μ and the nucleon. In nuclear matter, $\mathbf{p}$ is integrated out in the range $0<|\mathbf{p}|<p_F$ (p_F is the Fermi momentum).

The OPE for $\mathrm{Re}\Pi^R(\omega^2)$, which is the same as the OPE for the causal (Feynman) correlation $\Pi^F(\omega^2)$, has the general form at $\omega^2 \equiv -Q^2 \to -\infty$

$$\mathrm{Re}\Pi^R(\omega^2\to-\infty)=\sum_i \frac{1}{Q^{2i}}a_i(Q^2;\mu^2)\langle \mathcal{O}_i(\mu^2)\rangle_n , \quad (5)$$

where μ is the renormalization point of the local operators, which separates the hard scale $|\omega|$ and soft scales such as Λ_{QCD} and p_F. The Wilson coefficients do not depend on the medium effect. Only the expectation values $\langle \mathcal{O}_i(\mu^2)\rangle_n$, which are obtained within the Fermi-gas approximation, have the p_F dependence.

The local operators $\mathcal{O}_i(\mu^2)$ in the vector meson sum rule are essentially the same as those in the lepton-nucleon deep inelastic scattering (DIS) and can be characterized by their canonical dimension (d) and the twist (τ=dimension spin). They are given in [9] up to dimension-6 operators and we will not recapitulate them here. Since we are taking $\mathbf{q}\to\mathbf{0}$, Eq. (5) is an asymptotic series in $1/\omega^2$ or equivalently an expansion with respect to d.

In Ref. [4], the leading density dependence of all the operators up to $d=6$ except for relatively small twist-4 spin-2 operators is taken into account. In Sec. IV, we will discuss the effect of the twist-4 operators and Fermi momentum correction to the result of [4].

In the vacuum QSR, the spectral function [i.e., $\mathrm{Im}\Pi^R$ in Eq. (2)] is modeled with a resonance pole and the continuum. In the medium, we have to add additional singularities below the lowest resonance pole within the Fermi-gas approximation, which is called the Landau damping contribution [10]. For $\mathbf{q}\to\mathbf{0}$, it is calculable *exactly* and behaves like a pole at $\omega^2=0$ (see Appendix A for the proof). In total, the hadronic spectral function has the form

$$\begin{aligned}8\pi\,\mathrm{Im}\Pi^R(u>0^-)&=\delta(u^2)\rho_{sc}+F\delta(u^2-m_V^2)\\&\quad+\left(1+\frac{\alpha_s}{\pi}\right)\theta(u^2-S_0)\\&\equiv\rho_{\mathrm{had}}(u^2),\end{aligned} \quad (6)$$

with $\rho_{sc}=2\pi^2 n_B/\sqrt{p_F^2+m_N^2}\simeq 2\pi^2 n_B/m_N$. m_V, F, and S_0 are the three phenomenological parameters to be determined by the sum rules.

Matching the OPE side and the phenomenological side via the dispersion relation in the asymptotic region $\omega^2\to-\infty$, we can relate the resonance parameters to the density-dependent condensates. There are two major procedures for this matching, namely, the finite energy sum rules (FESR) [11] and the Borel sum rules (BSR) [12] which are summarized as the following forms:

$$\int_0^\infty ds\,W(s)[\rho_{\mathrm{had}}(s)-\rho_{\mathrm{OPE}}(s)]=0, \quad (7)$$

$$W(s)=\begin{cases} s^n\theta(S_0-s) & \text{(FESR)}\\ e^{-s/M^2} & \text{(BSR)}.\end{cases}$$

Here the spectral function $\rho_{\mathrm{had}}(s)$ stands for Eq. (6). $\rho_{\mathrm{OPE}}(s)$ is a hypothetical imaginary part of Π^R which, through the dispersion relation (2), reproduces Eq. (5). For more details on (7) and the explicit form of ρ_{OPE}, see Sec. 2 of Ref. [9].

FESR and BSR for $\Pi^R(\omega^2)$

First, for the qualitative argument, let us write down the FESR for the $\rho(\omega)$ meson in the chiral limit. This can be easily obtained by taking the first three moments $n=0,1,2$ in (7):

$$F-S_0\left(1+\frac{\alpha_s}{\pi}\right)=-2\pi^2 m_N^{-1}n_B \quad (n=0),$$

$$Fm_V^2-\frac{S_0^2}{2}\left(1+\frac{\alpha_s}{\pi}\right)=-\mathcal{O}_4-2\pi^2A_1^{u+d}m_N n_B\equiv-\tilde{\mathcal{O}}_4 \quad (n=1),$$

$$Fm_V^4-\frac{S_0^3}{3}\left(1+\frac{\alpha_s}{\pi}\right)=-\mathcal{O}_6-\frac{10}{3}\pi^2A_3^{u+d}m_N^3n_B\equiv-\tilde{\mathcal{O}}_6 \quad (n=2), \quad (8)$$

where $\mathcal{O}_4$ [$\mathcal{O}_6$] is

$$(\pi^2/3)\langle(\alpha_s/\pi)G^2\rangle_n \; [(896/81)\pi^3\langle\alpha_s(\bar{q}q)^2\rangle_n]$$

taken up to linear in n_B. Using the three relations above, we can determine the three phenomenological parameters F, S_0, and m_V or equivalently the changes from the vacuum values δF, δS_0, and δm_V. Important density dependence comes from $A_1^{u+d}n_B$ and the four-quark condensate $\mathcal{O}_6$, which can be shown by solving Eq. (8) numerically.

Although both FESR and BSR give the same qualitative result, the BSR is more reliable for the quantitative estimate since it is rather insensitive to the assumption on the continuum. The $\rho(\omega)$ meson mass in the BSR is given as

$$\frac{m_V^2}{M^2}=\frac{(1+\alpha_s/\pi)[1-e^{-S_0/M^2}(1+S_0/M^2)]-(1/M^4)\tilde{\mathcal{O}}_4+(1/M^6)\tilde{\mathcal{O}}_6}{(1+\alpha_s/\pi)(1-e^{-S_0/M^2})+(1/M^4)\tilde{\mathcal{O}}_4-(1/2M^6)\tilde{\mathcal{O}}_6-\rho_{sc}}. \quad (9)$$

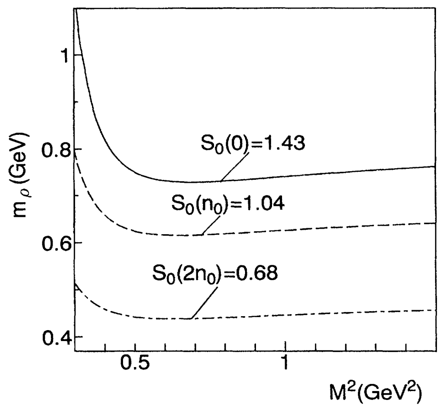

FIG. 1. Borel curve for $m_V(M^2)$ using Π^R. Solid, dashed, and dash-dotted lines correspond to $n_B/n_0=0$, 1.0, and 2.0, respectively. $S_0(n_B)$ determined by the Borel stability method at each density is also shown in GeV^2 units.

In Fig. 1, the Borel curve (m_V-M^2 relation) is shown for different baryon densities. The continuum threshold S_0 is chosen to make the Borel curve as flat as possible in the Borel window $M^2_{\rm min}<M^2<M^2_{\rm max}$ at given density. We take the density-independent window $M^2_{\rm min}=0.41$ GeV^2 and $M^2_{\rm max}=1.30$ GeV^2 in our analyses. A more general procedure with a density-dependent window (see, e.g., Sec. 4 of Ref. [9]) does not change the results quantitatively.

By making a linear fit using the values at $n=n_0$ and $n=n_B$, we get

$$\frac{m_V(n_B)}{m_V(0)}=1-(0.16\pm 0.06)\frac{n_B}{n_0}, \tag{10}$$

$$\sqrt{\frac{S_0(n_B)}{S_0(0)}}=1-(0.15\pm 0.05)\frac{n_B}{n_0}, \tag{11}$$

$$\frac{F(n_B)}{F(0)}=1-(0.24\pm 0.07)\frac{n_B}{n_0}. \tag{12}$$

These values are slightly different from our previous ones in [4] where the uncertainty discussed in Appendix B is not taken into account. For the decreasing $\rho(\omega)$ mass, the twist-2 and the scalar matrix elements are equally important.

III. MASS SHIFT AND THE SCATTERING LENGTH

In this section, we will first summarize the basic ingredients and assumptions of the scattering length approach [7] [(A)–(C) in the following] in a form appropriate for our discussions. Then critical examination of the approach and its comparison to the in-medium sum rule is given in Secs. III A–III C.

(A) In the scattering length approach, one starts with Eq. (3) and, making a low density approximation for the second term,

$$\gamma\int^{p_F}\frac{d^3p}{(2\pi)^3 2E_N}T^R_{\mu\nu}(\omega,\mathbf{q}|\mathbf{p})\rightarrow n_B\frac{T^R_{\mu\nu}(\omega,\mathbf{q}|\mathbf{0})}{2m_N}. \tag{13}$$

This corresponds to the assumption that all the nucleons in the nuclear medium are at rest ($\mathbf{p}=\mathbf{0}$). If one further takes the kinematics $\mathbf{q}=\mathbf{0}$ and $\omega\simeq m_V$, $T^R\equiv T^R_{\mu\mu}(\omega,\mathbf{q}=0|\mathbf{0})$ is written by the V-N scattering length as

$$T^R\simeq\frac{3Fm_V^2}{8\pi^2}\frac{24\pi(m_N+m_V)a_{VN}}{(\omega^2-m_V^2)^2}+\mathscr{R}(\omega^2), \tag{14}$$

where $a_{VN}=(a_{1/2}+2a_{3/2})/3$ with $a_{1/2}$ and $a_{3/2}$ being the V-N scattering length in the spin-1/2 and spin-3/2 channels, respectively. $\mathscr{R}(\omega^2)$ is the term less singular than the leading double-pole term in the Laurent expansion around $\omega^2=m_V^2$.

By substituting these expressions into Eq. (3) and taking the leading term in Eq. (14), one arrives at the formula

$$\begin{aligned}\frac{1}{3}\Pi^R_{\mu\mu}(\omega\simeq m_V,\mathbf{q}=\mathbf{0};n_B)&\simeq\frac{Fm_V^2}{8\pi^2}\left(\frac{1}{\omega^2-m_V^2}\right.\\&\quad\left.+\frac{12\pi a_{VN}(m_N+m_V)/m_N}{(\omega^2-m_V^2)^2}n_B\right),\\&\propto\frac{1}{\omega^2-(m_V+\delta m_V)^2}\end{aligned} \tag{15}$$

with

$$\delta m_V=6\pi\frac{m_N+m_V}{m_Nm_V}a_{VN}n_B. \tag{16}$$

Hence the positive (negative) scattering length gives an increasing (decreasing) mass in the medium.

(B) To estimate the magnitude and sign of the scattering length a_{VN} in (16), one may use the QSR for T^R satisfying the unsubtracted dispersion relation

$$\mathrm{Re}T^R(\omega^2)=\frac{1}{\pi}\int_0^\infty du^2\frac{\mathrm{Im}T^R(u)}{u^2-\omega^2}. \tag{17}$$

The OPE for T^R is expanded up to $O(1/Q^4)$ as

$$\mathrm{Re}T^R_{\rm OPE}(Q^2)=\frac{1}{8\pi^2}\left(\frac{c_1}{Q^2}-\frac{c_2}{Q^4}\right), \tag{18}$$

where $c_{1,2}/2m_N\equiv d\tilde{\mathscr{O}}_{4,6}/dn_B$ at $n_B=0$. The absence of the logarithmic term in (18) indicates that subtraction is not necessary in (17). Motivated by Eq. (14), the imaginary part is parametrized as

$$\begin{aligned}8\pi\ \mathrm{Im}T^R(u>0)&=b_1\delta'(u^2-m_V(0)^2)+b_2\delta(u^2-m_V(0)^2)\\&\quad+b_3\delta(u^2-S_0(0)),\end{aligned} \tag{19}$$

with three unknowns $b_{1,2,3}$ and known vacuum parameters $m_V(0)$ and $S_0(0)$. [Note that $b_1 = F m_V^2 24\pi(m_N + m_V) a_{VN}$.] The above parametrization is equivalent to taking the following ansatz for the real part:

$$\mathrm{Re}T^R_{\mathrm{had}}(Q^2) = \frac{b_1}{[m_V(0)^2+Q^2]^2} + \frac{b_2}{m_V(0)^2+Q^2} + \frac{b_3}{S_0(0)+Q^2}. \tag{20}$$

By constructing a Borel sum rule using (17)–(19), a positive scattering length $a_{\rho(\omega)} \simeq 0.14$ (0.11) fm is obtained [7] and it is concluded that the mass shift δm_V in Eq. (16) must be positive in [7].

(C) The above procedure (A)+(B) is equivalent to doing the medium sum rule for $\Pi^R_{\mu\mu}$ but not for Π^R. (Note that $\Pi^R_{\mu\mu} = -3\omega^2\Pi^R$ when $\mathbf{q}=\mathbf{0}$.) This can be seen as follows. The dispersion relation in the medium for $\Pi^R_{\mu\mu}$ reads

$$\mathrm{Re}\Pi^R_{\mu\mu}(\omega^2) = \frac{1}{\pi}\mathrm{P}\int_0^\infty du^2 \frac{\mathrm{Im}[\Pi^R_{\mu\mu}(u)]}{u^2-\omega^2} + (\text{subtraction}). \tag{21}$$

If one adopts Eq. (6) for $\mathrm{Im}\Pi^R$ and uses the relation $\Pi^R_{\mu\mu} = -3\omega^2\Pi^R$, one obtains

$$\left(-\frac{1}{3}\right) 8\pi\, \mathrm{Im}[\Pi^R_{\mu\mu}(u)] = F m_V^2 \delta(u^2 - m_V^2) + \left(1+\frac{\alpha_s}{\pi}\right) u^2 \theta(u^2 - S_0) = u^2 \rho_{\mathrm{had}}(u^2). \tag{22}$$

Since $u^2\delta(u^2)=0$, the Landau damping term in (6) does not arise in (22).

Expansion of the left-hand side (LHS) of (21) in terms of n_B gives [n_B-independent term] + [$n_B\times$ Eq. (18)], while the same expansion of (22) gives [n_B-independent term] + [$n_B\times$Eq. (19)]. The latter is obtained simply by writing $m_V = m_V(0) + \delta m_V$, $S_0 = S_0(0) + \delta S_0$, $F = F(0) + \delta F$, expanding (22) up to linear in δm_V, δF, and δS_0, and doing the following identification:

$$b_1/2m_N = -F m_V^2 \delta m_V^2, \quad b_2/2m_N = m_V^2 \delta F + F\delta m_V^2,$$

$$b_3/2m_N = -S_0 \delta S_0. \tag{23}$$

This means that the sum rule for T^R, Eq. (17), is equivalent to the linear density part of the sum rule for $\Pi^R_{\mu\mu}$, Eq. (21). Assuming that the procedure (A)+(B) is right, it is concluded in [7] that (i) the medium QSR using $\Pi^R_{\mu\mu}$ must give the increasing vector-meson mass, and (ii) the result of the medium sum rule using Π^R in [4] must be wrong.

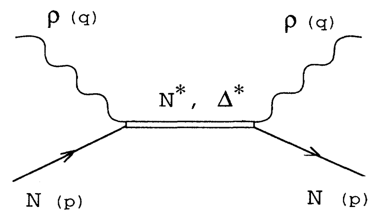

FIG. 2. A schematic illustration of the V-N scattering with s-channel nucleon resonances.

In the following, we will critically examine (A)–(C). The subsection numbers III A, III B, and III C in the following correspond to the statements (A), (B), and (C) in the above, respectively.

A. Mass shift and the scattering length

First of all, let us show that Eq. (13) is an approximation which is not useful around nuclear matter density. In Eq. (13), the motion of nucleons and the Pauli exclusion principle in nuclear matter are completely neglected. Such approximation is valid only when (i) the nucleon density is extremely low, or (ii) $T^R_{\mu\nu}(\omega,\mathbf{q}=\mathbf{0}|\mathbf{p})$ is almost constant as a function of $\mathbf{p}$ in the interval $0<|\mathbf{p}|<p_F$.

Since we are not interested in the case (i), let us concentrate on (ii) and see whether (ii) is plausible or not. At nuclear matter density, the Fermi momentum is sizable, $p_F \simeq 270$ MeV. Thus we should consider, e.g., the ρ-N scattering from $\sqrt{s} = m_\rho + m_N = 1709$ MeV through $\sqrt{s} = [(m_\rho + \sqrt{m_N^2+p_F^2})^2 - p_F^2]^{1/2} = 1726$ MeV. In this interval, there are at least two s-channel resonances $N(1710)$ and $N(1720)$ and also there are two nearby resonances just below the threshold, $N(1700)$ and $\Delta(1700)$ [13]. They are all able to have coupling with ρN. This means that $T^R_{\mu\nu}$ has a rapid variation as a function of $|\mathbf{p}|$ between $|\mathbf{p}|=0$ and $|\mathbf{p}|=p_F$ due to the effect of these s-channel resonances and it is impossible to approximate it by the threshold value (i.e., the V-N scattering length) $T^R(\omega=m_V,\mathbf{q}=\mathbf{0}|\mathbf{p}=\mathbf{0})$. See Fig. 2 for a schematic illustration of the s-channel contributions [14].

What one can expect at best is the approximate linear density formula written in terms of the average of $T^R_{\mu\nu}$ in the region $0<|\mathbf{p}|<p_F$:

$$\gamma\int^{p_F} \frac{d^3p}{(2\pi)^3 2E_N} T^R_{\mu\nu}(\omega,\mathbf{q}=\mathbf{0}|\mathbf{p}) \simeq n_B \left\langle \frac{T^R_{\mu\nu}(\omega,\mathbf{q}=\mathbf{0}|\mathbf{p})}{2E_N} \right\rangle, \tag{24}$$

where $\langle\ \rangle$ stands for the average over the above momentum interval.

It is easy to see what is wrong in (13) in physical terms: the ρ-meson at rest ($\mathbf{q}=\mathbf{0}$) in nuclear matter will suffer the scattering from the nucleons having various three-momenta $\mathbf{p}$ in the interval $0<|\mathbf{p}|<p_F$. The net effect should be the averaged strength of the scattering and not the scattering

length defined at $\mathbf{p}=\mathbf{0}$. This is particularly so when $T^R_{\mu\nu}$ has a rapid $\mathbf{p}$ dependence.

The above point is well known for the nucleon in the nuclear medium. The optical potential for the nucleon at rest in nuclear matter cannot be approximated by the N-N scattering length multiplied by the nuclear density. In fact, the N-N forward scattering amplitude $T_{NN}(p)$ has a huge momentum dependence due to the the deuteron state and the "almost" bound state near the threshold. The relevant quantity for the nucleon optical potential is not the scattering length but the averaged scattering amplitude in the interval $0<|\mathbf{p}|<p_F$. This was recently emphasized in [15] which points out a problem of Ref. [16] where a similar approximation to (13) is used to analyze the nucleon in the nuclear medium.

It is now clear that the mass shift and the scattering length are not related directly at nuclear matter density. In the approach of Ref. [4], neither Eq. (13) nor Eq. (24) is adopted; thus one does not suffer from this problem.

B. Is scattering length calculable in QSR?

Even if one accepts the conclusion of Sec. III A, the V-N scattering length itself is an interesting physical quantity to be calculated in QSR. We will show, however, that this is rather difficult with the present knowledge of the higher dimensional condensate.

Let us look at (18) and (20) and compare them at $Q^2\to\infty$, which corresponds to the FESR for T^R. One immediately realizes that only two independent equations can be obtained:

$$b_2+b_3=c_1,$$

$$b_1-m_V(0)^2 b_2-S_0(0)b_3=-c_2, \tag{25}$$

whereas one needs three equations to solve $b_{1,2,3}$. This happens because the OPE is calculated only up to $O(1/Q^4)$ in (18). (25) clearly shows that it is impossible to predict the scattering length a_{VN} (which is proportional to b_1) without extra assumptions [17].

If one tries to make Borel analyses without the above consistency, one does not get stability of the Borel curve, which is actually observed in [7]. Nevertheless, a positive number for b_1 is deduced in [7]: In terms of FESR, this is caused by an extra assumption that the contribution from dimension-8 operators is zero,

$$2m_V(0)^2 b_1-m_V(0)^4 b_2+S_0(0)^2 b_3=0. \tag{26}$$

This assumption together with Eq. (25) yields three equations for three unknown constants. Using the fact that $c_1\sim 0$, $c_2>0$, and $S_0>m_V^2$, one finds a positive value for b_1 as in [7]. However, Eq. (26) has no ground and we do expect a non-negligible contribution from the dimension-8 operators. Since it is technically very hard to estimate dimension-8 condensates precisely at present ($1/Q^6$ terms of T^R), it is dificult to get reliable a_{VN} by using QSR only.

C. Use of QSR for $\Pi^R_{\mu\mu}$

Here, we will demonstrate that the sum rules for $\Pi^R_{\mu\mu}$ do not give reliable predictions for the mass shift. Let us first discuss the FESR for $\Pi^R_{\mu\mu}$:

$$\int_0^{S_0} ds\, s^n[\mathrm{Im}\Pi^R_{\mu\mu}(s)_{\mathrm{had}}-\mathrm{Im}\Pi^R_{\mu\mu}(s)_{\mathrm{OPE}}]=0 \qquad (n=0,1,2,\ldots). \tag{27}$$

One immediately observes that only two relations corresponding to $n=0,1$ are obtained, and they turn out to be equivalent to the second and third relations in Eq. (8). [Note that $n=2$ in (27) cannot give extra condition because the OPE is calculated only up to dimension-6 operators.]

$$Fm_V^2-\frac{S_0^2}{2}\left(1+\frac{\alpha_s}{\pi}\right)=-\tilde{\mathscr{O}}_4 \quad (n=0),$$

$$Fm_V^4-\frac{S_0^3}{3}\left(1+\frac{\alpha_s}{\pi}\right)=-\tilde{\mathscr{O}}_6 \quad (n=1). \tag{28}$$

There are three unknowns F, m_V, and S_0, while only two relations are available. Thus unless one introduces an extra assumption, it is impossible to solve for three *even in the vacuum*. This is similar to the problem which we have discussed in Sec. III B.

One should also note that the missing condition is the duality relation for the spectral density:

$$\int_0^{S_0} ds[\rho_{\mathrm{had}}(s)-\rho_{\mathrm{OPE}}(s)]=0, \tag{29}$$

with ρ_{had} and ρ_{OPE} being the spectral densities for Π^R. This local duality is the cornerstone of vacuum QSR and holds also in the medium since there are no dimension-2 operators in the OPE. If one wants to get a reliable result from FESR, one has either to work out the OPE up to dimension-8 operators (which is a formidable task) or to start with Π^R.

Here one may ask "why not take the $n=-1$ moment in (27) to obtain another relation?" Such a procedure, however, introduces an ambiguity at $s=0$, since one can add any function proportional to $s\,\delta(s)$ to $\mathrm{Im}\Pi^R_{\mu\mu}$ which does not modify the $n=0,1$ sum rules but modifies the $n=-1$ sum rule. If one tries to remove this ambiguity, it is necessary to start with Π^R and to evaluate the Landau damping term in (6) which is exactly calculable as we mentioned in Sec. II (see also Appendix A) [18].

Let us now turn to the BSR for $\Pi^R_{\mu\mu}$

$$\frac{m_V^2}{M^2}=\frac{2[1+(\alpha_s/\pi)][1-e^{-S_0/M^2}(1+S_0/M^2+S_0/2M^2)]-(1/M^6)\tilde{\mathscr{O}}_6}{[1+(\alpha_s/\pi)][1-e^{-S_0/M^2}(1+S_0/M^2)]-(1/M^4)\tilde{\mathscr{O}}_4+(1/M^6)\tilde{\mathscr{O}}_6.} \tag{30}$$

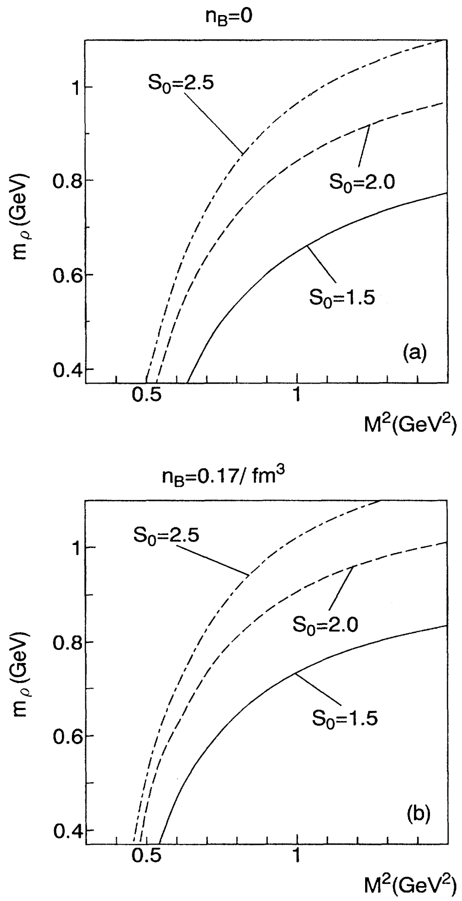

FIG. 3. (a) Borel curves for $m_V(M^2)$ at zero density using $\Pi^R_{\mu\mu}$ with several different values of S_0 in GeV^2 units. (b) Borel curves for $m_V(M^2)$ at nuclear matter density using $\Pi^R_{\mu\mu}$ with several different values of S_0 in GeV^2 units.

Equation (9) and Eq. (30) in the present paper correspond to Eq. (13) and Eq. (14) in [7], respectively. Since $\tilde{\mathcal{O}}_6$ enters with opposite sign in the RHS of Eq. (9) and Eq. (30) and $\tilde{\mathcal{O}}_6$ decreases in the medium, it is concluded in [7] that m_V decreases in (9) while it increases in (30). However, this conclusion could be completely changed since S_0 in the RHS of these equations is also density dependent. The density dependence of S_0 should be determined from the Borel stability procedure and it is in fact possible if one starts with (9) as shown in Sec. II.

In order to see whether the density dependence of S_0 can be determined in Eq. (30), we show the Borel curves for different values of S_0 in Fig. 3(a) (at zero density) and in Fig. 3(b) (at nuclear matter density). Fig. 3(a) shows that the Borel curve does not have any plateau in the relevant range of M^2 (say, $0.41<M^2<1.30$), which implies that one cannot determine S_0 and hence m_V even in the vacuum. The situation is the same at the nuclear matter density as is shown in Fig. 3(b). Again, one cannot determine $S_0(n_0)$ by the Borel stability method and hence $m_V(n_0)$, which implies that there is no hope of determining the mass shift at finite density. If one sticks to a specific value of S_0 (say, 2.0 GeV^2) and uses it at any density, one finds a fake result $m_V(n_B)/m_V(0)>1$ for given M^2 from Fig. 3, as is found in [7].

The "bad" Borel curves for $\Pi^R_{\mu\mu}$ in Fig. 3 are quite in contrast to the "good" Borel curves for Π^R in Fig. 1. The latter show beautiful stability in the vacuum as well as in the medium, which makes it possible to determine $S_0(n_B)$ at each density and hence $m_V(n_B)$.

The reason for the failure of the BSR for $\Pi^R_{\mu\mu}$ is twofold. First, the higher dimensional operators in the OPE are rather important for $\Pi^R_{\mu\mu}$ sum rules. We have already seen this in (28) where the $n=2$ sum rule can be obtained only when one has dimension-8 operators in the OPE. In the BSR, the lack of information on the dimension-8 operators manifests itself as an instability of the Borel curve at the low M region. Inclusion of the dimension-8 operator would make the Borel curve for $\Pi^R_{\mu\mu}$ flatter. Secondly, the continuum contribution is more important in $\Pi^R_{\mu\mu}$ than in Π^R, since the spectral function is increasing linearly in the former case. This makes the prediction of the resonance parameters less reliable.

Let us summarize here the lessons we learned in subsections III A, III B, and III C. First, the mass shift and the scattering length do not have a direct relation in nuclear matter due to the momentum dependence of the V-N forward scattering amplitude. Secondly, it is hard to predict the V-N scattering length from QSR of the V-N scattering amplitude without the knowledge of the dimension-8 operators in the OPE. Thirdly, sum rules for $\Pi^R_{\mu\mu}$ do not work even in the vacuum without the dimension-8 operators in the OPE, while that for Π^R works very well both in the vacuum and in the medium.

IV. SEVERAL COMMENTS

Full twist-4 calculation

There are three kinds of twist-4 spin-2 operators contributing to the ρ,ω sum rules. At present, their nucleon matrix elements are not known. However, the values of two different combinations in the transverse and longitudinal structure functions of the nucleon have been obtained by two of us [19,20] by analyzing the recent DIS data at CERN and SLAC. Let us further make the following assumption [19]:

$$\frac{\langle \bar{d}\Gamma_\mu \Delta_\nu d\rangle}{\langle \bar{u}\Gamma_\mu \Delta_\nu u\rangle}=\frac{\langle \bar{d}\gamma_\mu D_\nu d\rangle}{\langle \bar{u}\gamma_\mu D_\nu u\rangle}\equiv\beta, \tag{31}$$

where Γ_μ is some gamma matrix and Δ_μ an isospin singlet operator. Then, it is possible to uniquely determine the nucleon matrix element of twist-4 spin-2 operators appearing in the ρ,ω sum rule from combinations of experimental values. It gives the following contribution to Eq. (4):

$$T^{R,\tau=4}_{\mu\mu}(\omega,\mathbf{q}|\mathbf{p})=\frac{-[(q\cdot p)^2-(1/4)m_N^2q^2]}{m_NQ^4}\times\left[-(1+\beta)\left(K_u^1+\frac{1}{4}K_u^2+\frac{5}{8}K_u^g\right)+K_{ud}\pm K_{ud}\right]. \tag{32}$$

The K_u^i $(i=1,2,g)$ are defined in Ref. [19] and $-$ $(+)$ corresponds to the ω (ρ) case. Choosing $\beta=0.476$ as in Ref. [19] the value inside the is square braket A^4 0.40 (0.24) GeV^2 for the ω (ρ) meson.

Now the effect of the twist-4 matrix element can be estimated by making the following substitution for the dimension-6 operators:

$$\left(\mathscr{O}_6+\frac{10}{3}\pi^2 A_3^{u+d} m_N^3 n_B\right)\to\left(\mathscr{O}_6+\frac{10}{3}\pi^2 A_3^{u+d} m_N^3 n_B + 2\pi^2 m_N A^4 n_B\right). \tag{33}$$

The net effect of twist-2 + twist-4 is estimated to be 2.36 (3.29) times larger than the twist-2 effect alone in the $\rho(\omega)$ channel. This could change the slope of the mass shift in (10) from 0.16 to 0.10 (0.075) for the $\rho(\omega)$ meson. Further investigation is necessary, however, to draw definite conclusions, on the magnitude of the twist-4 effect.

Fermi momentum correction

The nonleading p_F corrections modify the leading density-dependent part of the condensates by factors of $B_1(x)=(3/2x^3)(x\sqrt{1+x^2}-\ln[x+\sqrt{1+x^2}])$, $B_2(x)=\sqrt{1+x^2}$, and $B_3(x)=\sqrt{1+x^2}[1+(8/5)x^2]$ for spin-0, spin-2, and spin-4 operators, respectively. Here, $x=p_F/m_N$ and $n_B=\gamma p_F^3/(6\pi^2)$. For nuclear matter (twice nuclear matter) density $x=0.27$ (0.34), one finds that $B_1(0.27)=0.979$, $B_2(0.27)=1.036$, $B_3(0.27)=1.157$, and $B_1(0.34)=0.967$, $B_2(0.34)=1.056$, $B_3(0.34)=1.251$. Thus the effect can be safely neglected at nuclear matter density.

Possible new structure in $\mathrm{Im}\Pi^R$

It is possible that the density-dependent change of the OPE side is balanced by some new structure appearing in the spectral density below the resonance mass. In the QCD sum rule approach, it has to be included by hand before matching it to the OPE. Such a possibility has been examined by Asakawa and Ko [21] by redoing the medium QCD sum rules for the vector meson including other complex structure of the spectral density in the nuclear medium induced by the π,Δ,N,ρ dynamics. They found that even in that case the vector meson mass has to decrease in order to be consistent with the OPE side.

ϕ meson sum rule

The formalism for calculating the change of the ϕ meson sum rule is the same as that for the ρ and ω. However, in the ϕ case, one must include the effect of the strange quark mass in the OPE and this will introduce some basic difference [4]. In the ρ,ω sum rule, the density dependence in the OPE is dominated by $(d,\tau)=(4,2)$ and $(d,\tau)=(6,0)$ operators. However, in the case of the ϕ meson, the dimension-4 strange quark condensate $\langle m_s\bar{s}s\rangle$ is not suppressed by either $1/4\pi^2$ or the light current quark mass and consequently dominates the OPE. In the medium, the change of this condensate, which comes from the K-N sigma term, dominates the small changes in other condensates and introduces a nonnegligible mass reduction by 3–5 % [4]. It is amusing to compare this result with that of an effective model calculation [22] in which the K-N sigma term also induces a small reduction of the ϕ mass.

V. SUMMARY

To estimate the mass shift of vector mesons in the medium, we have carried out a detailed comparison between the approach based on the modification of the vacuum QSR [4] and that based on the scattering length [7]. We found the following in our analyses.

(i) The mass shift and the scattering length do not have a direct relation in nuclear matter due to the momentum dependence of the V-N forward scattering amplitude.

(ii) Sum rules for the V-N scattering amplitude do not give a reliable prediction for the V-N scattering length without knowing dimension-8 condensates in the OPE or without an extra assumption.

(iii) The sum rule for $\Pi^R_{\mu\mu}$, which is sensitive to the continuum ansatz and the higher dimensional operators in the OPE, does not show Borel stability both in the vacuum and in the medium. This is in contrast to the sum rule for Π^R which shows good stability and predicts the decreasing vector-meson mass in the nuclear medium.

ACKNOWLEDGMENTS

The authors thank Dr. Y. Koike for stimulating discussions to clarify the issue, although he does not share our arguments. T.H. thanks Dr. K. Tanaka for useful comments and Professor M. Ichimura for discussions and useful information on nucleons in nuclear matter. T.H. and S.H.L. thank the Institute for Nuclear Theory at the University of Washington for its hospitality and the Department of Energy for partial support during the completion of this work. The work of T.H. and H.S. was supported in part by the Grants-in-Aids of the Japanese Ministry of Education (No. 06102004). The work of S.H.L. was supported by the Basic Science Research Institute program of the Korean Ministry of Education through Grant No. BSRI-94-2425 and by KOSEF through the CTP at Seoul National University.

APPENDIX A

Here, we derive the Landau damping term ρ_{sc}. In the Fermi-gas approximation, the spectral density has two types of contribution: the annihilation term, which is nonzero above the two particle threshold $\omega^2>\mathbf{q}^2+4m_N^2$, and the scattering term, which is nonzero in the spacelike region $\omega^2<\mathbf{q}^2$. We are interested in the second term. For finite $\mathbf{q}$, the spectral density, contributing to the longitudinal polarization, can be obtained by looking at the 00 component of the imaginary part of Eq. (1):

$$\rho_l^s(\omega,\mathbf{q})=\frac{\mathrm{Im}\Pi_{00}}{\mathbf{q}^2}\xrightarrow{\mathbf{q}\to 0}\frac{\mathrm{Im}\Pi_{\mu\mu}}{-3\omega^2}. \tag{A1}$$

Looking at the spectral representation, it is easy to identify the following scattering contribution:

$$\rho_l^s(\omega,\mathbf{q})=\frac{(2\pi)^4}{4\mathbf{q}^2}\int\frac{d^3k_1}{(2\pi)^3E_1}\frac{d^3k_2}{(2\pi)^3E_2}|\langle N(\mathbf{k}_1)|J_0|N(\mathbf{k}_2)\rangle|^2 \times\delta(\omega-E_1+E_2)\delta^3(\mathbf{q}-\mathbf{k}_1+\mathbf{k}_2) \times[n_F(E_2)-n_F(E_1)]. \quad (A2)$$

Here, $E_i=\sqrt{\mathbf{k}_i^2+m_N^2}$ $(i=1,2)$, and $n_F(E_i)=\theta(\sqrt{k_F^2+m_N^2}-E_i)$, where k_F is the Fermi momentum.

In general, the nucleon expectation of the isospin current has two form factors:

$$\langle N(\mathbf{k}_1)|J_\mu^a|N(\mathbf{k}_2)\rangle=\bar{u}(\mathbf{k}_1)\frac{\tau^a}{2}[F_1(q)\gamma_\mu +F_2(q)i\sigma_{\mu\nu}q^\nu]u(\mathbf{k}_2). \quad (A3)$$

Substituting this into Eq. (A2), one obtains

$$\rho_l^s(\omega,\mathbf{q})=\frac{\gamma}{256\pi^2}\int_v^\infty dx[(1-x^2)F_1^2(q)+x^2(\omega^2-\mathbf{q}^2)F_2^2(q)] \times\left[2n_F\left(\frac{|\mathbf{q}|x+\omega}{2}\right)-2n_F\left(\frac{|\mathbf{q}|x-\omega}{2}\right)\right]\theta(\mathbf{q}^2-\omega^2), \quad (A4)$$

where $v=[1-4m^2/(\omega^2-\mathbf{q}^2)]^{1/2}$.

Now, we want to take the limit $|\mathbf{q}|\to 0$. In this limit, the constraint $0<\omega^2<\mathbf{q}^2$ also forces ω to approach zero. Consequently, the contribution proportional to F_2 vanishes, because it is multiplied by either $\mathbf{q}^2$ or ω^2. As for the other term proportional to $F_1(q)$, the integral becomes increasingly large as $|\mathbf{q}|\to 0$ such that the integrated quantity of $\rho_l^s(\omega,\mathbf{q})$ within the phase space for ω remains finite.

By integrating over this region with $|\mathbf{q}|$ finite and then taking the limit, we find

$$\lim_{|\mathbf{q}|\to 0}\int_0^{\mathbf{q}^2}d\omega^2\rho_l^2(\omega,\mathbf{q})=\frac{\gamma}{12}\int\frac{d^3p}{(2\pi)^3 2E}n_F(E)v(3-v) \equiv\rho_{sc}/8\pi, \quad (A5)$$

so that $\rho_l^s(\omega,p)$ effectively becomes a δ function. Thus the final result is that the spectral density reduces to

$$\lim_{|\mathbf{q}|\to 0}\rho_l^s(\omega,\mathbf{q})=\delta(\omega^2)\rho_{sc}/8\pi. \quad (A6)$$

It should be noted here that we did not make any approximation from (A2) through (A5); thus the result is exact. Also note that there arises no ambiguity from the nucleon form factor because $F_1(q=0)=1$.

APPENDIX B

At present, there are no experimental data for the four quark condensate in the medium. Thus we use a simple mean field approximation in nuclear matter [4]:

$$\langle(\bar{q}\gamma_\mu\gamma_5\lambda^a q)^2\rangle_n\simeq-\langle(\bar{q}\gamma_\mu\lambda^a q)^2\rangle_n \simeq\frac{16}{9}[\langle(\bar{q}q)^2\rangle_0+2\langle\bar{q}q\rangle_0 \times\langle N|\bar{q}q|N\rangle n_B B_1(x)]. \quad (B1)$$

As for $\langle(\bar{q}q)^2\rangle_0$ at 1 GeV scale, we will use the canonical value $(-281\text{ MeV})^6$ [12] with $\alpha_s(1\text{ GeV})\simeq 0.36$. This number, which is substantially larger than the current algebra value $\langle\bar{q}q\rangle_0\simeq(-230\text{ MeV})^3$ at 1 GeV, should be considered as an effective one containing nonleading $1/N_c$ contributions. As for $\langle\bar{q}q\rangle_0$ in the second term of (B1), it is not clear whether one should use $(-230\text{ MeV})^3$ or $(-281\text{ MeV})^3$. (Note that the latter number was used in [4].) Taking into account such ambiguity as well as the "experimental" errors of $\Sigma_{\pi N}$ and $\hat{m}$, we adopt $\langle\bar{q}q\rangle_0\langle N|\bar{q}q|N\rangle=(-256\text{ MeV})^3(45/14)(1\pm 0.368)$ as a standard value to be used in Eq. (B1).

[1] G. E. Brown, Nucl. Phys. **A522**, 397 (1991).
[2] *Quark Matter '95*, edited by A. M. Poskanzer *et al.*, Nucl. Phys. **A590**, Nos. 1, 2 (1995).
[3] See the review by T. Hatsuda, "Hadron Structure and the QCD Phase Transition," hep-ph/9502345, 1995.
[4] T. Hatsuda and Su H. Lee, Phys. Rev. C **46**, R34 (1992).
[5] G. E. Brown and M. Rho, Phys. Rev. Lett. **66**, 2720 (1991).
[6] K. Saito, T. Maruyama, and K. Soutome, Phys. Rev. C **40**, 407 (1989); H. Kurasawa and T. Suzuki, Prog. Theor. Phys. **84**, 1030 (1990); H.-C. Jean, J. Piekarewicz, and A. G. Williams, Phys. Rev. C **49**, 1981 (1994); H. Shiomi and T. Hatsuda, Phys. Lett. B **334**, 281 (1994).
[7] Y. Koike, Phys. Rev. C **51**, 1488 (1995).
[8] The seminal applications of the QSR to the nucleon in nuclear matter have been given in the following papers: E. G. Drukarev and E. M. Levin, Prog. Part. Nucl. Phys. **A556**, 467 (1991); T. Hatsuda, H. Hogaasen, and M. Prakash, Phys. Rev. Lett. **66**, 2851 (1991); R. J. Furnstahl, D. K. Griegel, and T. D. Cohen, *ibid.* **67**, 961 (1991).
[9] T. Hatsuda, Y. Koike, and S. H. Lee, Nucl. Phys. **B394**, 221 (1993).
[10] A. I. Bochkarev and M. E. Shaposhnikov, Nucl. Phys. **B268**, 220 (1986).
[11] N. V. Krasnikov, A. A. Pivovarov, and N. N. Tavkhelidze, Z. Phys. C **19**, 301 (1983).
[12] A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979).
[13] Review of Particle Properties, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
[14] There are also t-channel meson exchanges in the process. One of them is the scalar-meson (σ) exchange which gives an energy-independent contribution to the amplitude and is more relevant to the vector-meson mass shift in nuclear matter.
[15] R. J. Furnstahl and T. Hatsuda, Phys. Rev. Lett. **72**, 3128 (1994); R. J. Furnstahl, Phys. Rev. C **50**, 1735 (1994).
[16] Y. Kondo and O. Morimatsu, Report No. INS-933, 1992 (unpublished); Phys. Rev. Lett. **71**, 2855 (1993).
[17] In QCD sum rules, one should always check that the number

of phenomenological parameters to be determined is equal to or smaller than the number of OPE terms, otherwise sum rules are not closed and cannot give reliable predictions. The FESR provide a useful tool to do this consistency check.

[18] For the readers who have not been convinced yet, we give here another argument. The FESR can be constructed by taking a certain number of derivatives of the real part of $\Pi^R_{\mu\nu}(Q^2)$ [11]. If one starts with $\Pi^R(Q^2) \sim \ln Q^2 + a_1/Q^2 + a^2/Q^4 + a_3/Q^6$ and applies $Q^2\partial/\partial Q^2$, one obtains the canonical FESR (8). The local duality relation in (8) is nothing but the statement that $a_1 = 0$. On the other hand, if one starts with $\Pi^R_{\mu\mu}(Q^2) \sim Q^2\Pi^R(Q^2) \sim Q^2 \ln Q^2 + a_1 + a^2/Q^2 + a_3/Q^3$, any derivative with respect to Q^2 will lose the information of a_1. This is equivalent to the missing local duality relation or the ambiguity of $s\delta(s)$ in $\mathrm{Im}\Pi^R_{\mu\mu}(s)$.

[19] S. Choi, T. Hatsuda, Y. Koike, and Su H. Lee, Phys. Lett. B **312**, 351 (1993).

[20] Su H. Lee, Phys. Rev. D **49**, 2242 (1994).

[21] M. Asakawa and C. M. Ko, Phys. Rev. C **48**, 526 (1993).

[22] C. M. Ko, P. Levai, X. J. Qiu, and C. T. Li, Phys. Rev. C **45**, 1400 (1992).