# Valid QCD sum rules for vector mesons in nuclear matter

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QCD sum rules for vector mesons ( $\rho$ ,  $\omega$ ,  $\phi$ ) in nuclear matter are reexamined with an emphasis on the reliability of various sum rules. Monitoring the continuum contribution and the convergence of the operator product expansion plays a crucial role in determining the validity of a sum rule. The uncertainties arising from less than precise knowledge of the condensate values and other input parameters are analyzed via a Monte Carlo error analysis. Our analysis leaves no doubt that vector-meson masses decrease with increasing density. This resolves the current debate over the behavior of the vector-meson masses and the sum rules to be used in extracting vector meson properties in nuclear matter. We find a ratio of  $\rho$ -meson masses of  $m_{\rho}^{*}/m_{\rho} = 0.78 \pm 0.08$  at nuclear matter saturation density.

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# I. INTRODUCTION

Whether the properties of vector mesons might change significantly with increasing nuclear matter density is of considerable current theoretical interest. This interest is motivated by its relevance to the physics of hot and dense matter and the phase transition of matter from a hadronic phase to a quark-gluon plasma at high density and/or temperature. In particular, the modifications of vector-meson masses in nuclear matter have been studied extensively.

At least three experimentally based studies have been cited as supporting the picture of decreasing vector-meson masses in nuclear matter. These include the quenching of the longitudinal response (relative to the transverse response) in quasielastic electron scattering [1], (e, e'p) reactions [2], and the discrepancy between the total cross section in  $K^+$ -nucleus scattering on <sup>12</sup>C and that predicted from an impulse approximation calculation using  $K^+$ -nucleon scattering amplitudes (extracted from  $K^+$ -D elastic scattering) [3-7]. More direct investigations of vector-meson masses in the nuclear medium have also been proposed. One proposal is to study dileptons as a probe of vector mesons in the dense and hot matter formed during heavy-ion collisions [8]. The dilepton mass spectra should allow one to reconstruct the masses of vector mesons decaying electromagnetically.

Theoretical investigations of vector-meson masses in nuclear matter have used various approaches and models that include the scaling ansatz of Brown and Rho [9], Nambu–Jona-Lasinio model [10], Walecka model [11–17], quark model [18], and the QCD sum-rule approach [19–22]. Previous studies of vector mesons at finite density via the QCD sum-rule approach have been made by Hatsuda and Lee [19], and subsequently by Asakawa and Ko [20,21]. It was found that the vector-meson masses decrease with increasing density. This finding is consistent with the scaling ansatz proposed in Ref. [9], the quark model [18] and predictions ob-

tained from the Walecka model provided the polarizations of the Dirac sea are included [13-17].

However, more recently, another QCD sum-rule analysis has shed considerable doubt on the former conclusions. Reference [22] claims that the previous QCD sum-rule analyses of the in-medium vector-meson masses are incorrect and that the vector-meson masses should in fact increase in the medium. In response, Hatsuda, Lee, and Shiomi [23] have argued that the scattering-length approach used in Ref. [22] is conceptually erroneous. In this paper, we reexamine the QCD sum rules for vector mesons in nuclear matter. Our focus is on the reliability and validity of various sum rules. We will show that careful consideration of the validity of the sum rules used to extract the phenomenological results is crucial to resolving this debate.

Taking the three-momentum to be zero in the rest frame of the nuclear medium, one can only obtain one (direct) sum rule in each vector channel. By taking the derivative of this sum rule with respect to the inverse Borel mass, one may get an infinite series of derivative sum rules. Hatsuda and Lee [19] used the ratio of the first derivative sum rule and the direct sum rule in their analysis while Koike [22] argued that one should use the ratio of the second and first derivative sum rules. We point out that in practical sum-rule applications, the derivative sum rules are much less reliable than the direct sum rule and eventually become useless as the number of derivatives taken increases. The ratio method used by both previous authors does not reveal the validity of each individual sum rule, and hence can lead to erroneous results.

QCD sum rules relate the phenomenological spectral parameters (masses, residues, etc.) to the fundamental properties of QCD. To maintain the predictive power of the sumrule approach, the phenomenological side of the sum rule is typically described by the vector-meson pole of interest plus a model accounting for the contributions of all excited states. By working in a region where the pole dominates the phenomenological side, one can minimize sensitivity to the model and have assurance that it is the spectral parameters of the ground state of interest that are being determined by matching the sum rules. In practice, these considerations effectively set an upper limit in the Borel parameter space,

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beyond which the model for excited states dominates the phenomenological side.

At the same time, the truncated operator product expansion (OPE) must be sufficiently convergent<sup>1</sup> as to accurately describe the true OPE. Since the OPE is an expansion in the inverse squared Borel mass, this consideration sets a lower limit in Borel parameter space, beyond which higher order terms not present in the truncated OPE are significant and important. Monitoring OPE convergence is absolutely crucial to recovering nonperturbative phenomena in the sumrule approach, as it is the lower end of the Borel region where the nonperturbative information of the OPE is most significant. This information must also be accurate and this point will be further illustrated in Sec. III.

In short, one should not expect to extract information on the ground state spectral properties unless the ground state dominates the contributions on the phenomenological side and the OPE is sufficiently convergent. In this paper, we will analyze each individual sum rule with regard to the above criteria. A sum rule with an upper limit in Borel space lower than the lower limit is considered invalid. As a measure of the relative reliability of various sum rules we consider the size of the regime in Borel space where both sides of the sum rules are valid. In addition, the size of continuum contributions throughout the Borel region can also serve as a measure of reliability, with small continuum model contributions being more reliable.

The uncertainties in the OPE are not uniform throughout the Borel regime. These uncertainties arise from an imprecise knowledge of condensate values and other parameters appearing in the OPE. As such, uncertainties in the OPE are larger at the lower end of the Borel region. To estimate these uncertainties we adopt the Monte Carlo error analysis approach recently developed in Ref. [24]. In turn, these uncertainties provide error estimates for the extracted phenomenological spectral properties. This is the first systematic study of uncertainties for in-medium hadronic properties.

In the following we will show in detail how the direct sum rule is valid and the most reliable. The first derivative sum rule suffers from a small Borel region of validity and relatively large continuum model contributions throughout. It is marginal at best and any predictions from this sum rule are unreliable. Higher derivative sum rules are found to be invalid. Both the direct sum rule and the first derivative sum rule lead to the same conclusion that vector-meson masses decrease as the nuclear matter density increases.

This paper is organized as follows: In Sec. II, we sketch the finite-density sum rules for vector mesons and discuss the reliabilities of various sum rules. In Sec. III, the sum rules are analyzed and the sum-rule predictions are presented and discussed. Section IV is devoted to a conclusion.

#### **II. FINITE-DENSITY SUM RULES**

In this section, we briefly review the QCD sum rules for vector mesons in nuclear matter. We focus on some issues raised in these sum rules and refer the reader to the literature [19–21] for more details of the sum rules.

QCD sum rules for vector mesons at finite density study the correlator defined by

$$\Pi_{\mu\nu}(q) \equiv i \int d^4x e^{iq \cdot x} \langle \Psi_0 | T J_\mu(x) J_\nu(0) | \Psi_0 \rangle, \quad (2.1)$$

where  $|\Psi_0\rangle$  is the ground state of nuclear matter, *T* is the covariant time-ordering operator [25], and  $J_{\mu}$  represents any of the three conserved vector currents of QCD:

$$J^{\rho}_{\mu} \equiv \frac{1}{2} (\bar{u} \gamma_{\mu} u - \bar{d} \gamma_{\mu} d), \quad J^{\omega}_{\mu} \equiv \frac{1}{2} (\bar{u} \gamma_{\mu} u + \bar{d} \gamma_{\mu} d),$$
$$J^{\phi}_{\mu} \equiv \bar{s} \gamma_{\mu} s.$$
(2.2)

The nuclear medium is characterized by the rest-frame nucleon density  $\rho_N$  and the four-velocity  $u^{\mu}$ . We assume that the medium is invariant under parity and time reversal. Lorentz covariance and the conservation of the currents imply that the correlator  $\Pi_{\mu\nu}(q)$  can be decomposed into two independent structures multiplying two invariants, corresponding to the transverse and longitudinal polarizations. The medium modifications of these two invariants are in general different. To keep our discussion succinct, we follow the earlier works and take  $\mathbf{q}=0$  in the rest frame of the medium,  $u^{\mu} = \{1, 0\}$ . Since there is no specific spatial direction, the two invariants are related and only the longitudinal part,  $\Pi_L = \Pi_{\mu}^{\mu}/(-3q^2)|_{\mathbf{q}=0}$ , is needed.

All three currents under consideration are neutral currents. This implies that both time orderings in the correlator correspond to the creation or annihilation of the vector meson. Accordingly, the spectral function is necessarily an even function of the energy variable. One can write the invariant function as  $\Pi_L(q_0^2) = \Pi_{\mu}^{\mu}(q_0^2)/(-3q_0^2)$ ,<sup>2</sup> which satisfies the following dispersion relation<sup>3</sup> [19]:

$$\Pi_{L}(Q^{2} \equiv -q_{0}^{2}) = \frac{1}{\pi} \int_{0}^{\infty} ds \, \frac{\mathrm{Im}\Pi_{L}(s)}{s+Q^{2}} + \text{ subtractions.} \quad (2.3)$$

To facilitate our discussion of derivative sum rules, it is useful to derive the following dispersion relation for  $\Pi_L^{(n)}(Q^2) \equiv (Q^2)^n \Pi_L(Q^2)$ 

$$\Pi_{L}^{(n)}(Q^{2}) = \frac{1}{\pi} \int_{0}^{\infty} ds \, \frac{\mathrm{Im}\Pi_{L}^{(n)}(s)}{s + Q^{2}} + \text{ subtractions } (n \ge 1),$$
(2.4)

with  $\text{Im}\Pi_{L}^{(n)}(s) = (-1)^{n} s^{n} \text{Im}\Pi_{L}(s)$ .

<sup>&</sup>lt;sup>1</sup>Here and in the following, "convergence" of the OPE simply means that the highest dimension terms considered in the OPE, with their Wilson coefficients calculated to leading order in perturbation theory, are small relative to the leading terms of the OPE.

<sup>&</sup>lt;sup>2</sup>In the baryon case, the correlation function, considered in the rest frame as a function of  $q_0$ , has both even and odd parts. The reason is that, at finite baryon density, a baryon in medium propagates differently than an antibaryon, yielding a correlation function that is asymmetric in the energy variable (see Refs. [26,27]).

<sup>&</sup>lt;sup>3</sup>Here we have omitted the infinitesimal as we are only concerned with large and spacelike  $q_0^2$ .

For large  $Q^2$ , one can evaluate the correlator by expanding the product of currents according to the operator product expansion (OPE). The result can in general be expressed as

$$\Pi_L(Q^2) = -c_0 \ln(Q^2) + \frac{c_1}{Q^2} + \frac{c_2}{Q^4} + \frac{c_3}{Q^6} + \cdots, \quad (2.5)$$

where we have omitted the polynomials in  $Q^2$ , which vanish under Borel transform. The first term corresponds to the perturbative contribution and the rest are nonperturbative power corrections. The coefficients,  $c_0$ ,  $c_1$ ,  $c_2$ , and  $c_3$ , have been given in literature. For  $\rho$  and  $\omega$  mesons, one has [19,20]

$$c_{0} = \frac{1}{8\pi^{2}} \left( 1 + \frac{\alpha_{s}}{\pi} \right), \quad c_{1} = 0,$$

$$c_{2} = m_{q} \langle \bar{q}q \rangle_{\rho_{N}} + \frac{1}{24} \left\langle \frac{\alpha_{s}}{\pi} G^{2} \right\rangle_{\rho_{N}} + \frac{1}{4} A_{2}^{u+d} M_{N} \rho_{N},$$

$$c_{3} = -\frac{112}{81} \pi \alpha_{s} \langle \bar{q}q \rangle_{\rho_{N}}^{2} - \frac{5}{12} A_{4}^{u+d} M_{N}^{3} \rho_{N}, \quad (2.6)$$

where  $\langle \hat{O} \rangle_{\rho_N} \equiv \langle \Psi_0 | \hat{O} | \Psi_0 \rangle$  denotes the in-medium condensate,  $m_q \equiv \frac{1}{2} (m_u + m_d)$ ,  $\langle \bar{q}q \rangle_{\rho_N} = \langle \bar{u}u \rangle_{\rho_N} = \langle \bar{d}d \rangle_{\rho_N}$ , and  $A_n^q$  is defined as

$$A_n^q \equiv 2 \int_0^1 dx x^{n-1} [q(x,\mu^2) + (-1)^n \bar{q}(x,\mu^2)]. \quad (2.7)$$

Here  $q(x,\mu^2)$  and  $\bar{q}(x,\mu^2)$  are the scale dependent distribution functions for a quark and antiquark (of flavor q) in a nucleon. We follow the standard linear density approximation to the in-medium condensates,  $\langle \hat{O} \rangle_{\rho_N} = \langle \hat{O} \rangle_0 + \langle \hat{O} \rangle_N \rho_N$ , with  $\langle \hat{O} \rangle_0$  the vacuum condensate and  $\langle \hat{O} \rangle_N$  the nucleon matrix element. The corresponding result for the  $\phi$  meson is [21]

$$c_{0} = \frac{1}{4\pi^{2}} \left( 1 + \frac{\alpha_{s}}{\pi} \right), \quad c_{1} = 0,$$

$$c_{2} = 2 m_{s} \langle \bar{s}s \rangle_{\rho_{N}} + \frac{1}{12} \left\langle \frac{\alpha_{s}}{\pi} G^{2} \right\rangle_{\rho_{N}} + A_{2}^{s} M_{N} \rho_{N},$$

$$c_{3} = -\frac{224}{81} \pi \alpha_{s} \langle \bar{s}s \rangle_{\rho_{N}}^{2} - \frac{5}{3} A_{4}^{s} M_{N}^{3} \rho_{N}. \quad (2.8)$$

We adopt the usual pole plus continuum ansatz for the spectral density. This ansatz was recently tested in the lattice QCD investigation of Ref. [28] where it was found to describe nucleon correlation functions very well. The phenomenological spectral density for vector mesons in medium takes the form [19–21]

$$\frac{1}{\pi} \operatorname{Im}\Pi_{L}(s) = \frac{\rho_{sc}}{8\pi^{2}} \,\delta(s) + F_{v}^{*} \,\delta(s - m_{v}^{*2}) + c_{0} \,\theta(s - s_{0}^{*}),$$
(2.9)

where the first term denotes the contribution of the Landau damping,  $m_v^*$  is the vector-meson mass in the medium, and  $s_0^*$  is the continuum threshold. In the calculations to follow,

we use  $m_v$ ,  $F_v$ , and  $s_0$  to denote the corresponding vacuum (zero-density limit) parameters.

Substituting Eqs. (2.5) and (2.9) into the dispersion relation of (2.3) and applying the Borel transform to both sides, one obtains the direct sum rule

$$F_v^* e^{-m_v^{*2}/M^2} = -\frac{\rho_{sc}}{8\pi^2} + c_0 M^2 E_0 + c_1 + \frac{c_2}{M^2} + \frac{c_3}{2!M^4} + \dots + \frac{c_m}{(m-1)!(M^2)^{m-1}} + \dots$$
(2.10)

From the dispersion relation of (2.4), one finds

$$F_{v}^{*}(m_{v}^{*2})^{n}e^{-m_{v}^{*2}/M^{2}}$$

$$= n!c_{0}(M^{2})^{n+1}E_{n}$$

$$+ (-1)^{n}\left[c_{n+1} + \frac{c_{n+2}}{M^{2}} + \dots + \frac{c_{m}}{(m-n-1)!(M^{2})^{m-n-1}} + \dots\right], \qquad (2.11)$$

where we have defined

$$E_{k} \equiv 1 - e^{s_{0}^{*}/M^{2}} \left[ \sum_{i=0}^{k} \frac{1}{(k-i)!} \left( \frac{s_{0}^{*}}{M^{2}} \right)^{k-i} \right].$$
(2.12)

One recognizes that Eq. (2.11) corresponds to the derivative sum rules as they may also be obtained by taking derivatives of Eq. (2.10) with respect to  $1/M^2$ .

Hatsuda and Lee used Eq. (2.10) and the first derivative sum rule (n=1) of (2.11) in their calculations [19], while Koike claimed that one should use the first and second (n=2) derivative sum rules [22]. Of course, if one could carry out the OPE to arbitrary accuracy and use a spectral density independent of the model for excited state contributions, the predictions based on Eq. (2.10) and those based on the derivative sum rules should be the same. In practical calculations, however, one has to truncate the OPE and use a simple phenomenological ansatz for the spectral density. Thus it is unrealistic to expect the sum rules to work equally well. The question is, which sum rules give the most reliable predictions. To answer this question, let us compare the *n*th derivative sum rule with the direct sum rule of (2.10). We observe the following.

(1) The perturbative contribution in the derivative sum rule has an extra factor n! relative to the corresponding term in Eq. (2.10), implying that the perturbative contribution is more important in the derivative sum rules than in the direct sum rule, and becomes increasingly important as n increases. Since the perturbative term mainly contributes to the continuum of the spectral density, maintaining dominance of the lowest resonance pole in the sum rule will become increasingly difficult as n increases.

(2) In Eq. (2.10), the term proportional to  $c_m$  is suppressed by a factor of 1/(m-1)!, while it is only suppressed by 1/(m-n-1)! in the derivative sum rule (m>n). This implies that the convergence of the OPE is much slower in the derivative sum rule than in the direct sum rule.

This arises because the convergence of the OPE for  $\Pi_L^{(n)}(Q^2) = (Q^2)^n \Pi_L(Q^2)$  is obviously much slower than that for  $\Pi_L(Q^2)$  for large  $Q^2$ . Consequently, the high order power corrections are more important in the derivative sum rule than in Eq. (2.10), and become more and more important as *n* increases. If one would like to restrict the size of the last term of the OPE to maintain some promise of OPE convergence, the size of the Borel region in which the sum rules are believed to be valid is restricted.

(3) The power corrections proportional to  $c_1, c_2, \ldots, c_n$  do *not* contribute to the *n*th derivative sum rule but do contribute to Eq. (2.10) [29]. If one truncates the OPE, part or all of the nonperturbative information will be lost in the derivative sum rules. It is also worth noting that the leading power corrections are the most desirable terms to have. They do not give rise to a term in the continuum model and they are not the last term in the OPE, whose relative contribution should be restricted to maintain OPE convergence.

In practice, the predictions based on the direct sum rule of (2.10) are more reliable than those from the derivative sum rules, which become less and less reliable as n increases. This can also be demonstrated by analyzing the sum rules numerically.

### **III. SUM-RULE ANALYSIS AND DISCUSSION**

If the sum rules were perfect, one would expect that the two sides of the sum rules overlap for all values of the auxiliary parameter M (Borel mass). As mentioned above, one has to truncate the OPE and use a phenomenological model for the spectral density in practical calculations. Hence, the two sides of the sum rules overlap only in a limited range of M (at best).

All the previous works have used the ratio method of two sum rules. There, one chooses the continuum threshold to make the ratio of the two sum rules as flat as possible as a function of Borel mass (the residue  $F_v^*$  drops out in the ratio). Although it has also been used in various sum-rule calculations in vacuum, we note that the ratio method has certain drawbacks. First, the ratio method does not check the validity of each individual sum rule. It may happen that individual sum rules are not valid while their ratio is flat as function of Borel mass. Second, the ratio method cannot account for the fact that sum rules do not work equally well. The Borel region where a sum rule is valid can vary from one sum rule to another. Finally, the continuum contributions to the sum rules are not monitored in the ratio method. If the continuum contribution is dominant in a sum rule, one should not expect to get any reliable information about the lowest resonance.

### A. Outline of the method

To overcome these shortcomings of the ratio method, we adopt here the optimizing procedure originated in Ref. [30], which has been extensively used in analyzing various vacuum sum rules [31] and finite-density sum rules [26]. In this method, one optimizes the fit of the two sides of each individual sum rule in a fiducial Borel region, which is chosen such that the highest-dimensional condensates contribute no more than  $\sim 10\%$  to the QCD side while the continuum

contribution is less than ~50% of the total phenomenological side (i.e., the sum of the pole and the continuum contribution). The former sets a criterion for the convergence of the OPE while the latter controls the continuum contribution. While the selection of 50% is obvious for pole dominance, the selection of 10% is a reasonably conservative criterion that has not failed in practice.<sup>4</sup> This point is further illustrated in the discussions to follow. The sum rule should be valid in this fiducial Borel region as the pole contribution dominates the phenomenological side and the QCD side is reliable. We then select 51 points in the fiducial region and use a  $\chi^2$  fit to extract the spectral parameters. The reader is referred to Refs. [30,24] for more details of the method.

Since QCD sum rules relate the spectral parameters to the properties of QCD, any imprecise knowledge of the condensates and related parameters will give rise to uncertainties in the extracted spectral parameters. These uncertainties have not been analyzed systematically in the previous works. Here we follow Ref. [24] and estimate these uncertainties via a Monte Carlo error analysis. Gaussian distributions for the condensate values and related parameters are generated via Monte Carlo. The distributions are selected to reflect the spread of values assumed in previously published QCD sumrule analyses and uncertainties such as the factorization hypothesis. These distributions provide a distribution for the OPE and thus uncertainty estimates for the QCD side which will be used in the  $\chi^2$  fit. In fitting the sum rules taken from the samples of condensate parameters one learns how these uncertainties are mapped into uncertainties in the extracted spectral parameters.

As in the previous works [19-21], we truncate the OPE at dimension six and keep only the terms considered in the literature. In the linear density approximation, the quark and gluon condensates can be written as

$$m_q \langle \bar{q}q \rangle_{\rho_N} = m_q \langle \bar{q}q \rangle_0 + \frac{\sigma_N}{2} \rho_N, \qquad (3.1)$$

$$m_s \langle \bar{s}s \rangle_{\rho_N} = m_s \langle \bar{s}s \rangle_0 + y \frac{m_s}{m_q} \frac{\sigma_N}{2} \rho_N,$$
 (3.2)

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho_N} = \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0 + \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_N \rho_N,$$
 (3.3)

where  $y \equiv \langle \bar{s}s \rangle_N / \langle \bar{q}q \rangle_N$ . The values of vacuum condensates we use are  $a = -4 \pi^2 \langle \bar{q}q \rangle_0 = 0.62 \pm 0.05$  GeV<sup>3</sup> [24],  $b = 4 \pi^2 \langle (\alpha_s / \pi) G^2 \rangle_0 = 0.4 \pm 0.15$  GeV<sup>4</sup> [24], and  $\langle \bar{s}s \rangle_0 / \langle \bar{q}q \rangle_0 = 0.8 \pm 0.2$  [32,30]. The quark mass  $m_q$  is chosen to satisfy the Gell-Mann–Oakes–Renner relation,  $2 m_q \langle \bar{q}q \rangle_0$  $= -m_\pi^2 f_\pi^2$ , and the strange quark mass is taken to be  $m_s = (26 \pm 2.5) m_q$  [33]. We adopt  $\sigma_N = 45 \pm 7$  MeV [33],  $\langle (\alpha_s / \pi) G^2 \rangle_N = -650 \pm 150$  MeV [27], and  $y = 0.2 \pm 0.1$ [19]. For the moments of the parton distribution functions,  $A_n^{q}$ 's, we quote the values given in Ref. [19] and assign a 20% uncertainty to each value,  $A_2^{u+d} = 0.9 \pm 0.18$ ,

<sup>&</sup>lt;sup>4</sup>Reasonable alternatives to the 10% and 50% criteria are automatically explored in the Monte Carlo error analysis, as the condensate values and the continuum threshold change in each sample.



FIG. 1. Predictions of the direct sum rule for the ratio  $m_{\rho}^*/m_{\rho}$  as a function of the medium density.

 $A_4^{u+d} = 0.12 \pm 0.024$ ,  $A_2^s = 0.05 \pm 0.01$ ,  $A_4^s = 0.002 \pm 0.0004$ . The strong coupling constant is taken to be  $\alpha_s/\pi = 0.117 \pm 0.014$  at 1 GeV scale [24].

The values of both vacuum and in-medium four-quark condensates are not well determined. Early arguments placed the values of vacuum four-quark condensates within 10% of the vacuum factorized values [34]. However, later analyses suggested that factorization underestimates the four-quark condensates significantly [35,36]. Parametrizing the condensate as  $\kappa \langle \bar{q}q \rangle_0^2$ , we will consider values of  $\kappa = 2 \pm 1$  and  $1.0 \le \kappa \le 3.5$  [35,24]. As for the in-medium four-quark condensates, previous authors have adopted the in-medium factorized values (mean field approximation) [19-21]. In the study of finite-density baryon sum rules [26,27], it was found that the in-medium factorized values of certain four-quark condensates led to results in contradiction with experiment. However, it should be pointed out that the four-quark operators appearing in the baryon sum rules are different from those in the vector meson sum rules. Here we parametrize the in-medium four-quark condensates as  $\kappa \langle \bar{q}q \rangle_{\rho_N}^2$ .

The Gaussian distributions for the condensate values and various parameters are generated using the values given above. The error bars in the extracted fit parameters (see below) are given by the standard deviation of the distribution after 100 condensate values generated via Monte Carlo. It is perhaps worth emphasizing that the error bars do not represent the standard error of the mean, which is 10 times smaller for this case. Hence the error bars are representative of the spread of input parameter values. In addition, the uncertainty estimates become insensitive to the number of Monte Carlo samples after about 50 samples. We normalize all finitedensity spectral parameters  $(m_v^*, F_v^*, \text{ and } s_0^*)$  to their corresponding values in vacuum (i.e., zero-density limit). Thus, the error bars in the ratios are dominated by the uncertainties in the density dependent terms of the in-medium condensates since the errors in the vacuum sum rules and finite-density sum rules are correlated.

## **B.** Numerical results

Let us start with the sum rules for  $\rho$  and  $\omega$  mesons. We first analyze the direct sum rule of Eq. (2.10). The Landau damping contribution proportional to  $\rho_{sc}$  is very small at the densities considered here [19]. Treating  $\rho_{sc}$  as a search parameter, we find that the direct sum rule predicts a value for  $\rho_{sc}$  in accord with the Fermi-gas approximation,  $\rho_{sc} \approx 2\pi^2 \rho_N / M_N$  [23,19]. In calculating the uncertainty of this parameter we found that there is insufficient information in the sum rules to reliably determine this small contribution. As a result we use the Fermi-gas relation and treat  $m_v^*$ ,  $F_v^*$ , and  $s_0^*$  as search parameters in the results presented below.

The predictions for the ratio  $m_{\rho}^{*}/m_{\rho}$  as a function of the nucleon density is plotted in Fig. 1. One can see that the  $\rho$ -meson mass decreases with increasing density. At nuclear matter saturation density  $\rho_{N} = \rho_{N}^{0} = (110 \text{ MeV})^{3}$ , we find  $m_{\rho}^{*}/m_{\rho} = 0.78 \pm 0.08$ . The residue  $F_{\rho}^{*}$  and the continuum threshold  $s_{0}^{*}$  also decrease as the density increases. The predictions for the ratios  $F_{\rho}^{*}/F_{\rho}$  and  $s_{0}^{*}/s_{0}$  are shown in Figs. 2 and 3, respectively.

In Fig. 4 the left- and right-hand sides of the direct sum rule are plotted as functions of M at the nuclear matter saturation density. The near perfect overlap of the two sides of this sum rule is typical of the quality of fits seen at other densities. The corresponding valid Borel window and the relative contributions of the continuum and the highest order term in the OPE are displayed in Fig. 5 as dashed curves.



FIG. 2. Predictions of the direct sum rule for the ratio  $F_{\rho}^{*}/F_{\rho}$  as a function of the medium density.



FIG. 3. Predictions of the direct sum rule for the ratio  $(s_0^*/s_0)^{1/2}$  as a function of the medium density.



FIG. 4. The left- and right-hand sides of the direct sum rule as functions of the Borel mass M at the nuclear matter saturation density.

One notices that the direct sum rule is valid in a broad Borel regime, where the highest order term contributes less than 10% and the continuum contributes only about 15% at the lower bound and the required 50% at the upper bound. Thus, the pole contribution truly dominates the sum rule in the Borel region of interest, implying that the predictions are reliable. We also find that both lower and upper bounds are functions of the density and decrease as the density increases. The rate of decrease for the upper bound is larger than that for the lower bound, which means that the optimal Borel window shrinks with increasing density.

We proceed now to analyze the first derivative sum rule of Eq. (2.11) (with n=1). It is found that this sum rule is valid in a much smaller Borel regime. The continuum and highest order OPE term contributions are shown in Fig. 5 as dot-dashed curves for  $\rho_N = \rho_N^0$ . It can be seen that the continuum contribution exceeds 33% in the entire Borel window and the relative importance of the highest order term increases as compared to the direct sum rule. Thus, the predictions of the first derivative sum rule are less reliable than those from the



FIG. 5. Relative contributions of the continuum and the highest order OPE terms to the sum rules as functions of the Borel mass at nuclear matter saturation density. The dashed and dot-dashed curves correspond to the direct sum rule and the first derivative sum rule, respectively. Note the relatively broad regime of validity for the direct sum rule.

direct sum rule. At zero density, the first derivative sum rule predicts a very large  $\rho$ -meson mass and the continuum threshold is only about 100 MeV above the pole position. Nevertheless, the first derivative sum rule also predicts that the ratios  $m_{\rho}^{*}/m_{\rho}$ ,  $F_{\rho}^{*}/F_{\rho}$ , and  $s_{0}^{*}/s_{0}$  all decrease as the density increases, which agrees qualitatively with the predictions of the direct sum rule. The reasons for the failure of the first derivative sum rule to reproduce the  $\rho$ -meson mass obtained from the direct sum rule are discussed at length in Ref. [24].

In the second derivative sum rule of Eq. (2.11) (with n=2), the contributions of the quark and gluon condensates drop out and the nonperturbative power correction starts with dimension-six condensates, the last term of the truncated OPE. The perturbative contribution and hence the continuum contribution is multiplied by an extra factor of 2 relative to that for the direct sum rule. Numerical analysis indicates that there is no Borel window where the sum rule is valid. Thus, one cannot get information about ground state vector mesons from this sum rule. This is also true for the third and higher derivative sum rules, where there are no power corrections at the level of the OPE truncation considered here.

All of the results above are for the  $\rho$  and  $\omega$  mesons. A similar analysis can be done for the  $\phi$  meson. Again, we find the same pattern. The direct sum rule gives the most reliable predictions, the first derivative sum rule yields a less reliable result, and the second and higher derivative sum rules are invalid. We find from the direct sum rule  $m_{\phi}^*/m_{\phi} = 0.99 \pm 0.01$  at nuclear matter saturation density. This rate of decrease is much smaller than that for the  $\rho$  and  $\omega$  mesons. This is due to the dominance of  $m_s \langle \bar{s}s \rangle_{\rho_N}$  and its slow change with the medium density.

#### **C.** Discussion

The Borel transform plays important roles in making the QCD sum-rule approach viable. It suppresses excited state contributions exponentially on the phenomenological side, thus minimizing the continuum model dependence. It also improves OPE convergence by suppressing the high order power corrections factorially on the QCD side. We observe that taking derivatives of the direct sum rule with respect to  $1/M^2$  is equivalent to a partial reverse of the Borel transform. It is thus not surprising to find that the continuum contribution becomes more important and the convergence of the OPE becomes slower in the derivative sum rules than in the direct sum rule. Since the excited state contributions are modeled roughly by a perturbative evaluation of the correlator starting at an effective threshold, and the higher order OPE terms are not well determined, there are more uncertainties in the derivative sum rules than in the direct sum rule. In fact, a simultaneous fit of both the direct and first derivative sum rules in vacuum reveals that the first derivative sum rule plays a negligible role in determining the fit parameters, when a  $\chi^2$  measure weighted by the OPE uncertainty and relative reliabilities of the sum rules is used [24].

To improve the reliability of the derivative sum rules, one must include more higher order terms in the OPE. However, one usually does not have much control of the values of the higher-dimensional condensates. In addition, the derivative sum rule will always suffer from a factorial enhancement of the terms contributing to the continuum model. Therefore, the direct sum rule will always yield the most reliable results for vector mesons.

Hatsuda and Lee invoked both the direct and the first derivative sum rules [19]. Their results for the ratio  $m_o^*/m_o$ are somewhat larger than those we obtained from the direct sum rule but are somewhat smaller than those from the first derivative sum rule. This discrepancy is obviously attributed to their use of the two sum rules simultaneously without weighing the relative merits of the sum rules. In Ref. [22], Koike adopted both the first and second derivative sum rules. His conclusion of slightly increasing vector-meson masses in the medium depends on the use of the second derivative sum rule, which we have found to be invalid when the OPE is truncated at dimension six. Hatsuda et al. [23] also pointed out some shortcomings of the second derivative sum rule. However, their arguments are based on concerns over the lack of information on the OCD side of the second derivative sum rule and the absence of a "plateau" in the ratio of the second and first derivative sum rules from which a mass is extracted. In this paper, we have extensively explored why these observations come about, and why even Hatsuda et al.'s analysis is less than satisfactory.

The use of the ratio method is mainly driven by the expectation that if the sum rules work well one should see a plateau in the predicted quantities as functions of the Borel mass. The usual interpretation of this criterion is that the ratio of *two* different sum rules, proportional to certain spectral parameter of interest (e.g., mass), should be flat as a function of the Borel mass. Although it is true ideally, this interpretation is potentially problematic in practice. We have seen that the reliabilities and validities of two sum rules are usually different. This feature cannot be revealed in the ratio of the two sum rules. In addition, one can always achieve the flatness of the sum rules are dominated by the continuum. However, one learns little about the lowest pole in this case.

To make contact with the plateau criterion, we propose that if a sum rule works well, one should see a plateau in the plot of an extracted quantity expressed as a function of the Borel mass. For example, from the direct sum rule of (2.10), one can express the vector-meson mass as

$$m_v^* = \left[ M^2 \ln \left( \frac{F_v^*}{\Pi_s(M)} \right) \right]^{1/2},$$
 (3.4)

where  $\Pi_S(M)$  denotes the right-hand side of Eq. (2.10). In Fig. 6, we plot the right-hand side of Eq. (3.4) for the  $\rho$ -meson case at four different densities, with optimized values for  $F_v^*$  and  $s_0^*$ . One indeed sees very flat curves within the region of validity denoted by the error bars. It should be emphasized that (1) this interpretation only involves one sum rule and is thus different from the ratio method previously used in the literature; (2) the value of  $m_v^*$  is only meaningful in the valid region of a sum rule; (3) in this interpretation, the plateau criterion is a true criterion, measuring the quality of the overlap between the two sides of a valid sum rule. For curiosity, we also display the curve outside the validity region for the case of  $\rho_N = \rho_0^N$  in Fig. 6. One notices a devia-



FIG. 6. The  $\rho$ -meson mass as obtained from the right-hand side of Eq. (3.4) as a function of M at various nuclear matter densities. The curves from top down correspond to  $\rho_N = 0$ ,  $\rho_N = 0.5\rho_N^0$ ,  $\rho_N = \rho_N^0$ , and  $\rho_N = 1.5\rho_N^0$ . The error bars denoting the valid regimes are obtained from the relative errors of Fig. 1. Note how the Borel regime shifts and becomes smaller as the density increases. The curve for  $\rho_N = \rho_N^0$  is plotted outside the valid region to demonstrate the importance of carefully selecting a Borel region.

tion from the plateau just outside the valid Borel region. This feature supports our selection of 10% as the criterion for OPE convergence.

In the present analysis as well as the previous works, the linear density approximation has been assumed for the inmedium condensates. For a general operator there is no systematic way to study contributions that are of higher order in the medium density. Model-dependent estimates in Ref. [37] suggest that the linear approximation to  $\langle \bar{q}q \rangle_{\rho_N}$  should be good (higher-order corrections ~20% of the linear term) up to nuclear matter saturation density. In our analyses, we have assigned generous uncertainties to various condensate values and parameters. We expect these will cover the uncertainties arising from the linear density approximation. As the medium density increases, the deviation from the linear density approximation will increase. One then needs more precise knowledge of the density dependence of various condensates in order to have reliable QCD sum-rule predictions.

As in previous works, we have neglected the dimensionsix twist-four operators in (2.6), as the nucleon matrix elements of these operators are unknown. However, estimates may be obtained from deep-inelastic-scattering data provided one is willing to make a few additional assumptions [23]. Taking the estimates given in Ref. [23], we find these additional contributions have little effect on our results. At saturation density, our present in-medium  $\rho$ -meson mass of 0.59 GeV is increased slightly to 0.62 GeV. Taking a 100% uncertainty on the twist-four contributions has no apparent effect on the present uncertainty of 0.11 GeV. At saturation density, the ratio  $m_{\rho*}/m_{\rho}$  is shifted from 0.78 to 0.82 which is small relative to the uncertainty of 0.08. Certainly further study of the twist-four contributions is required. However, we do not expect such contributions to significantly alter the conclusions of this paper.

As a final remark, we comment on the electromagnetic width of the  $\rho$  meson,  $\Gamma(\rho^0 \rightarrow e^+ e^-)$ . In free space, it is given by [38,34]

$$\Gamma(\rho^0 \to e^+ e^-) = \frac{1}{3} \alpha_E^2 m_\rho \left(\frac{4\pi}{g_\rho^2}\right) = \frac{4\pi}{3} \alpha_E^2 \left(\frac{F_\rho}{m_\rho}\right), \quad (3.5)$$

where  $\alpha_E$  is the electromagnetic coupling constant. The modification of this result in medium may be estimated by replacing  $m_\rho$  and  $F_\rho$  with their corresponding values in medium. The ratio of the free space and in-medium widths can be expressed as

$$\frac{\Gamma^{*}(\rho^{0} \to e^{+}e^{-})}{\Gamma(\rho^{0} \to e^{+}e^{-})} = \frac{m_{\rho}}{m_{\rho}^{*}} \frac{F_{\rho}^{*}}{F_{\rho}}.$$
(3.6)

Note that  $m_{\rho}/m_{\rho}^*$  increases while  $F_{\rho}^*/F_{\rho}$  decreases with increasing density. However, the rate of decrease for  $F_{\rho}^*/F_{\rho}$  is larger than the rate of increase for  $m_{\rho}/m_{\rho}^*$ . Consequently, the ratio for the widths is less than 1. At  $\rho_N = \rho_N^0$ , our estimate is  $\Gamma^*/\Gamma = 0.85 \pm 0.10$ . This implies that the  $\rho$ -meson electromagnetic width becomes smaller in nuclear matter. This behavior might be observed in the proposed experiment studying dileptons as a probe of vector mesons in the dense and hot matter [8].

## **IV. CONCLUSION**

In this paper, we have carefully examined the QCD sum rules for vector mesons in nuclear matter. Our primary concern has been on the validity and reliability of various sum rules. We emphasize that the sum rules do not work equally well due to the truncation of the OPE and the use of a model for the phenomenological spectral density. In particular, the derivative sum rules are less reliable than the direct sum rule. This is attributed to (1) the perturbative contribution and hence the continuum contribution become increasingly important in the derivative sum rules; (2) the high order terms in the OPE become increasingly important in the derivative sum rules; (3) part (or all) of the nonperturbative information is lost in the derivative sum rules. We therefore conclude that any predictions based on (or partially on) second or higher derivative sum rules are incorrect given the level of the OPE truncation adopted in the literature. One should avoid using the derivative sum rules in practical applications.

We tested this conclusion numerically by analyzing the sum rules with regard to pole dominance and OPE convergence [30]. A Monte Carlo based error analysis was used to provide reliable uncertainties on our predictions and remove the sensitivity of the results to the input parameters [24]. We found that the direct sum rule satisfies our criteria and leads to reliable predictions. The first derivative sum rule suffers from a small region of validity and large continuum contributions throughout. The second and higher derivative sum rules are invalid.

Our analysis confirms that the QCD sum-rule approach predicts a decrease of vector-meson masses with increasing density, and resolves the debate between Hatsuda *et al.* [23] and Koike [22]. The prediction of a slight increase of vectormeson masses in medium is based on an invalid second derivative sum rule.

We note that all previous authors have used the ratio method in the analysis of the sum rules, which has many drawbacks and may lead to incorrect results. We encourage the community to adopt the approach developed in Ref. [24] which checks the quality of the overlap between two sides of each individual sum rule by monitoring pole dominance and the convergence of the OPE. This approach also allows one to realistically estimate the uncertainties and reveal the predictive ability of QCD sum rules.

The analysis presented here is the most reliable QCD analysis of in-medium vector-meson properties. At nuclear matter saturation density, we predict

$$\frac{m_{\rho}^{*}}{m_{\rho}} = 0.78 \pm 0.08, \quad \frac{\Gamma^{*}(\rho^{0} \to e^{+}e^{-})}{\Gamma(\rho^{0} \to e^{+}e^{-})} = 0.85 \pm 0.10,$$
(4.1)

and look forward to experimental vindication of these results [8].

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