Nuclear mesonic Cherenkov-like radiation from high energy nucleons

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Generalized Cherenkov-like effects based on four fundamental interactions are investigated and classified. Two general Cherenkov-like coherence conditions are found as two natural limiting extremes of the same spontaneous particle emission process in nuclear media. Nuclear pionic and nuclear kaonic Cherenkov-like radiation (NPICR and NKCR) produced by high-energy nucleons passing through a nuclear medium are systematically studied using a quantum theoretical approach. The pion and kaon refractive indices, the pion and kaon phase velocities as well as the NPICR and NKCR thresholds are calculated via the Foldy-Lax multiple scattering formalism having as input the experimental data for the elementary pion-nucleon and kaon-nucleon forward scattering amplitudes. The differential cross sections, the angle-energy correlations, as well as some other essential signatures of coherent pion and kaon production via Cherenkov mechanism with high-energy proton beams are presented. Compressed nuclear media with densities up to five times normal nuclear matter density are taken into account.

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I. INTRODUCTION

One important direction in nuclear physics is the study of the influence of the nuclear medium on particles and fields [1–5]. Our investigation concerns specific medium effects such as the spontaneous particle (γ, π, etc.)-*emission as Cherenkov-like radiation* (see Refs. [3,6–9]). The *electromagnetic* Cherenkov effect is a well known collective phenomenon in normal dielectrics which is directly connected with the modification of the phase velocity of photons by the medium. Recently [6], we have extended this concept to nuclear media where *nuclear gamma Cherenkov radiation* (NGCR) should be possible to be emitted from charged particles moving through nuclei with a velocity larger than the phase velocity of photons in the nuclear medium. Using the Foldy-Lax multiple scattering theory [10] and the experimental gamma-nucleon cross sections [11] combined with the dispersion relations predictions, the refractive index of the photons in nuclear media has been calculated [6]. The existence of two nuclear gamma Cherenkov bands, localized in the energy range (140–310) MeV (CB1) and (50–500) GeV (CB2) is predicted. Also, detailed astrophysical implications of the above NGCR prediction are presented in Ref. [8]. Moreover, in the last time there is a revival of interest in the extension of this idea to the investigation of mesonic Cherenkov-like effects in nuclear [7], as well as in hadronic media [9]. For a consistent systematic treatment of the classical and quantum theories of these new possible nuclear effects we refer to our earlier investigations (see Refs. [7,9]) and to the references quoted therein. Other medium effects such as Δ-*hole* collective excitations of the nuclear medium via doorway states are considered in Ref. [14].

In this paper, generalized Cherenkov-like effects based on all four fundamental interactions are investigated and classified. Two general Cherenkov-like coherence conditions are found as two natural limiting extremes of the same spontaneous particle emission process in nuclear media. Moreover, using dispersion relations (DR) for the pion-nucleon forward scattering amplitudes [12] as well as of the kaon-nucleon forward scattering amplitudes [13], we extend our previous results [7] on pion emission as *nuclear pionic Cherenkov-like radiation* (NPICR) to *nuclear kaonic Cherenkov-like radiation* (NKCR) up to pion and kaon energies in the TeV region. Special attention will be dedicated to essential signatures of the NMCR mesons, which can be used by the experimentators to identify such coherent emissions. In Sec. II, an overview on *generalized Cherenkov-like effects* as well as a classification of coherent Cherenkov-like effects are presented. The coherence condition as well as the thresholds for the generalized Cherenkov-like effects are discussed in Sec. III. In Sec. IV, using the quantum theoretical approach [7,9], we give the essential formulas for the *nuclear pseudoscalar meson Cherenkov-like radiation* (NMCR). Numerical results for the pionic and kaonic refractive indices $n^2(\omega)$, for the pion and kaon phase velocities $v_{\rm ph}(\omega)$, for the quantum NPICR- and NKCR-Cherenkov thresholds, as well as for the differential cross sections are presented in Sec. V. Section VI is reserved to the conclusions and an outlook.

II. GENERALIZED CHERENKOV-LIKE EFFECTS

First, we must underline that in order to obtain spontaneous particle production via Cherenkov-like effects in a medium three general conditions are to be fulfilled.

(i) The incident particle source must be coupled to a specific radiation (electromagnetic, mesonic, fermionic-antifermionic, etc.) field (RF).

(ii) The propagation properties of the RF must be modified inside the medium in such a way that the phase velocities of the RF quanta in that medium are smaller than the light velocity in vacuum.

(iii) The particle source must be moving in the medium with a velocity v higher than the phase velocity $v_{\rm ph}$ of the RF in that medium.

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According to these conditions the following four types of Cherenkov-like effects can take place in nature.

I. The strong Cherenkov-like effects: (1) nuclear - (π, K, η, etc.) pseudoscalar mesonic Cherenkov-like radiation (see Refs. [3,7,9]); (2) nuclear - (ρ, K^*, ω, etc.) vector mesonic Cherenkov-like radiation (see Refs. [7,9]); (3) nuclear - ($N, \Lambda, \Sigma, \Xi, \Omega$, etc.) baryonic Cherenkov-like effects (see Refs. [7,9]).

II. Electromagnetic Cherenkov-like effects: (1) the electromagnetic Cherenkov effect in usual (dielectric, crystalline, etc.) media; (2) nuclear gamma Cherenkov radiation (see Refs. [6,8]); (3) nuclear - (e, μ, τ, etc.) leptonic Cherenkov-like effects (accompanied by γ emission).

III. The weak Cherenkov-like effects: (1) nuclear - [$(e, \nu_e), (\mu, \nu_\mu)$, etc.] leptonic Cherenkov-like effects; (2) nuclear weak W-boson emission via Cherenkov-like radiation, etc.

IV. The gravitational Cherenkov-like effects (see Ref. [15]).

Of course, in a nuclear medium all kinds of Cherenkov-like effects can take place since in such a medium all (i)–(iii) above conditions can be fulfilled.

Refractive index and phase velocity. The estimation of the propagation properties of any kind of particle M with total energy ω in a nuclear medium with constituents N can be made in a standard way using the Foldy-Lax formula [10] for the index of refraction n

$$n_M^2(\omega) = 1 + \frac{4\pi\rho}{(\omega^2 - m_M^2)^{1/2}} C(\omega) \bar{f}_{MN\to MN}(\omega), \tag{1}$$

where $\bar{f}_{MN\to MN}(\omega)$ is the average elementary $MN \to MN$ forward scattering amplitude, ρ is the density of the nuclear constituents, while $C(\omega)$ is the coherence factor [10]. Since our investigations are only exploratory, we restrict our discussion to the case of nucleons randomly distributed inside the nuclear medium where $C(\omega)=1$. Therefore, for any kind of particle, γ, (e, μ, τ) leptons, $(\pi, K, \eta$, etc.)-pseudoscalar mesons, $(\rho, \omega, K^*$, etc.)- vector mesons, $(N, \Lambda, \Sigma$, etc.) baryons, we can investigate the possibility of their spontaneous production via a Cherenkov-like effect inside the nuclear medium if their forward elementary scattering amplitudes on nucleons are known from the experimental data or from the available standard amplitude analyses. The lightest mesons for which the elementary scattering amplitudes on nucleons in forward direction are well known from experimental data are the pions [12] and kaons [13].

The phase velocity of any radiated particle M in the medium is defined in the usual way by the relation

$$v_{M\text{ph}}(\omega) = \frac{\omega}{\text{Re}k} = \frac{\omega}{(\omega^2 - m_M^2)^{1/2}} \frac{\sqrt{2}}{\{[(\text{Re}\ n_M^2)^2 + (\text{Im}\ n_M^2)^2]^{1/2} + \text{Re}\ n_M^2\}^{1/2}}, \tag{2}$$

where Rek, ω, and m_M are the magnitude of the momentum, the total energy, and the rest mass of the radiated particle, respectively.

Coherence length. Now, we can introduce the idea of the coherence length [16], just as in classical electrodynamics. Therefore, let us consider the motion of a fast particle source with a velocity v in a medium along a trajectory which is close to a straight line. If this particle will emit a radiation M, the phase difference $\Delta\phi$ of the radiated wave at the angle θ between the time moments t and $t + l/v$ will be

$$\Delta\phi = \omega\frac{l}{v} - l\ \text{Re}\ k\ \cos\theta, \tag{3}$$

where l is the path traveled by the particle source in the time interval $(t, t + l/v)$. Then, if we define the coherence length $l(\omega, \theta)$ as the distance in which $\Delta\phi = 1$, we have

$$l(\omega, \theta) = \left(\frac{\omega}{v} - \text{Re}k\ \cos\theta\right)^{-1}. \tag{4}$$

We see that for

$$\cos\theta = \frac{\omega}{v\ \text{Re}\ k} = \frac{v_{\text{ph}}}{v} \tag{5}$$

the coherence length (4) goes to infinity. This case corresponds to Cherenkov-like radiation during uniform motion of a particle source. This approach to an interpretation of the physical essence of Cherenkov-like effects in the case of the electromagnetic radiation is due to Frank [17].

III. KINEMATICS OF THE GENERALIZED CHERENKOV-LIKE EFFECTS

According to classical and quantum theories (see [7] and [9]) the particles M are predicted to be emitted when the coherence condition

$$v_{M\text{ph}}(\omega) \leq v \tag{6}$$

is fulfilled inside the nuclear medium, or equivalently, when the kinetic energy T_N of the incident particle source is higher than the threshold energy T_{thr}, given by

$$T_{\text{thr}}(\omega) = M_1[(1 - v_{1\text{thr}}^2)^{-1/2} - 1], \tag{7}$$

where $v_{1\text{thr}} \equiv v_{M\text{ph}}(\omega)$ for the *classical threshold* $T_{\text{thr}}^C(\omega)$.

In quantum theory the "coherence condition" is obtained as follows. One starts with the Cherenkov-like process schematically described in Fig. 1, where a radiated particle M (e.g., a meson) [with energy ω, momentum $\vec{k}$, rest mass m_M, and refractive index $n_M(\omega)$] is radiated in the medium from an incident particle source B_1 [with energy E_1, momentum $\vec{p}_1$, rest mass M_1, and refractive index $n_1(E_1) = 1$] that itself goes over into a final particle source

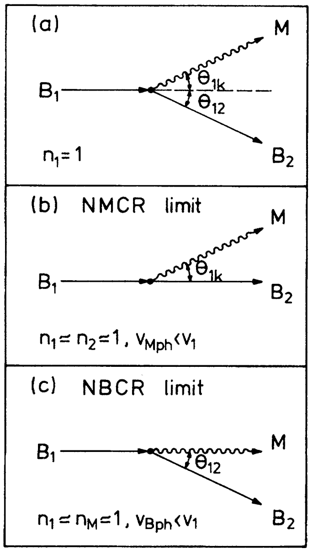

FIG. 1. (a) The nuclear two body decay process $B_1 \to MB_2$. (b) The nuclear mesonic Cherenkov-like radiation (NMCR) limit: $v_{M\text{ph}}(\omega) \le v_1$. (c) The nuclear baryonic Cherenkov-like radiation (NBCR) limit: $v_{B\text{ph}}(E_2) \le v_1$ (see the text).

(e.g., a baryon) B_2 [with energy E_2, momentum $\vec{p}_2$, rest mass M_2, and refractive index $n_2(E_2)$]. The energy-momentum conservation for this transition in the nuclear medium requires

$$E_1 = \omega + E_2, \quad \vec{p}_1 = \text{Re}\vec{k} + \text{Re}\vec{p}_2 . \tag{8}$$

For the initial particle source B_1 we assume that the usual mass-shell relation $E_1^2 - \vec{p}_1^{\,2} = M_1^2$ is also valid inside the nuclear medium, while for the radiated particle M as well as for the final particle source B_2 we take "in medium" mass shell relations:

$$(\text{Re}k)^2 = (\omega^2 - m_M^2)[\text{Re}n(\omega)]^2 \tag{9}$$

and

$$(\text{Re}p_2)^2 = (E_2^2 - M_2^2)[\text{Re}n_2(E_2)]^2. \tag{10}$$

For the angles θ_{1k} and θ_{12} [see Fig. 1(a)] we obtain

$$\cos\theta_{1k} = \frac{p_1^2 - (\text{Re}p_2)^2 + (\text{Re}k)^2}{2p_1 \text{Re}k} \tag{11}$$

and

$$\cos\theta_{12} = \frac{p_1^2 + (\text{Re}p_2)^2 - (\text{Re}k)^2}{2p_1 \text{Re}p_2}. \tag{12}$$

Here, we distinguish the following two important limiting cases.

(a) The M-Cherenkov limit [see Fig. 1(b)]: $n_2(E_2)=1$ and $\text{Re}n_M(\omega)>1$ such that $v_{Mph}(\omega)<1$ [see Eq. (2)]. In this NMCR limit we have

$$\cos\theta_{1k} = \frac{v_{M\text{ph}}(\omega)}{v_1} + \frac{\text{Re}k}{2p_1}\left[1 - v_{M\text{ph}}^2(\omega) + \frac{M_2^2 - M_1^2}{(\text{Re}k)^2}\right], \tag{13}$$

i.e.,

$$\cos\theta_{1k} = \frac{v_{M\text{ph}}(\omega)}{v_1} + 0(p_1^{-1}). \tag{14}$$

Now, from the condition that θ_{1k} must be a physical angle ($|\cos\theta_{1k}| \le 1$) we get the *quantum coherence condition*

$$\left|\frac{v_{M\text{ph}}(\omega)}{v_1} + \frac{\text{Re}k}{2p_1}\left[1 - v_{M\text{ph}}^2(\omega) + \frac{M_2^2 - M_1^2}{(\text{Re}k)^2}\right]\right| \le 1 . \tag{15}$$

At high projectile energy, when $E_1 \gg \omega$ or $p_1 \gg \text{Re}k$, the Eqs. (14) and (15) go over into the classical Cherenkov angle, defined by $\cos\theta_{1k} \cong v_{M\text{ph}}(\omega)/v_1$, and into the classical Cherenkov coherence condition $v_{M\text{ph}}(\omega) \le v_1$, respectively.

The *M-quantum threshold* $T_{M\text{thr}}^{Q}(\omega)$ for the kinetic projectile energy $T_1 = E_1 - M_1$, for the particle M produced as Cherenkov-like radiation in the nuclear medium, is given by Eq. (7) where $v_{1\text{thr}}$ now is

$$v_{1\text{thr}} = v_1^M(\omega) \equiv \frac{v_{M\text{ph}}(\omega)}{1+F_M^2} + \frac{F_M}{(1+F_M^2)^{1/2}}\left[1 - \frac{v_{M\text{ph}}^2(\omega)}{1+F_M^2}\right]^{1/2}, \tag{16}$$

with

$$F_M = \frac{\text{Re}k}{2M_1}\left[1 - v_{M\text{ph}}^2(\omega) + \frac{M_2^2 - M_1^2}{(\text{Re}k)^2}\right]. \tag{17}$$

We note that $v_1^M(\omega)$ in Eq. (16) is the solution of Eq. (15) for equality which preserves the classical coherence condition (6).

(b) The B-Cherenkov limit [see Fig. 1(c)]: $n_M(\omega)=1$ and $\text{Re}n_2(E_2)>1$ such that $v_{2ph}(E_2)<1$. In this nuclear baryonic Cherenkov (NBCR) limit we have

$$\cos\theta_{12} = \frac{v_{2\text{ph}}(E_2)}{v_1} + \frac{\text{Re}p_2}{2p_1}\left[1 - v_{2\text{ph}}^2(E_2) + \frac{m_M^2 - M_1^2}{(\text{Re}p_2)^2}\right], \tag{18}$$

i.e.,

$$\cos\theta_{12} = \frac{v_{2\text{ph}}(E_2)}{v_1} + 0(p_1^{-1}), \tag{19}$$

where $v_{2\text{ph}}(E_2)$ is the phase velocity of the particle B_2 defined [see Eq. (2)] as

$$v_{2\text{ph}}(E_2) = \frac{E_2}{\text{Re}p_2} = \frac{E_2}{(E_2^2 - M_2^2)^{1/2}\text{Re}n_2} . \tag{20}$$

Now, from the condition that θ_{12} must be a physical angle ($|\cos\theta_{12}| \le 1$) we get the *quantum coherence condition*

$$\left|\frac{v_{2\text{ph}}(E_2)}{v_1} + \frac{\text{Re}p_2}{2p_1}\left[1 - v_{2\text{ph}}^2(E_2) + \frac{m_M^2 - M_1^2}{(\text{Re}p_2)^2}\right]\right| \le 1. \tag{21}$$

At high projectile energy, when $E_1 \gg \omega$ or $p_1 \gg \text{Re}p_2$, the Eqs. (18) and (19) go over into the classical Cherenkov-like angle, defined by $\cos\theta_{12} \cong v_{2\text{ph}}(E_2)/v_1$, and into the classical Cherenkov-like coherence condition $v_{2\text{ph}} \le v_1$, respectively.

The *B-quantum threshold* $T^{Q}_{B\mathrm{thr}}(E_2)$ for the kinetic projectile energy $T_1=E_1-M_1$, for the particle B_2 produced in the nuclear medium by Cherenkov-like mechanism, is given by Eq. (7) where $v_{1\mathrm{thr}}$ now is given by

$$v_{1\mathrm{thr}}=v_1^B(E_2)\equiv\frac{v_{2\mathrm{ph}}(E_2)}{1+F_B^2}+\frac{F_B}{(1+F_B^2)^{1/2}}\left[1-\frac{v_{2\mathrm{ph}}^2(\omega)}{1+F_B^2}\right]^{1/2}, \tag{22}$$

with

$$F_B=\frac{\mathrm{Re}p_2}{2M_1}[1-v_{2\mathrm{ph}}^2(\omega)+(M_2^2-M_1^2)/(\mathrm{Re}p_2)^2]. \tag{23}$$

We note that $v_1^B(E_2)$ in Eq. (22) is the solution of Eq. (21) for equality which preserves the coherence condition $v_{2\mathrm{ph}}\leqslant v_1$. The "duality NMCR-NBCR" is schematically presented in Fig. 2. Thus we see that the NMCR as well as NBCR are two natural limiting extremes of the same spontaneous particle emission process (see Fig. 1) in the nuclear medium.

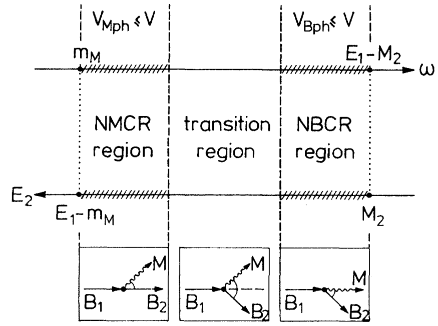

FIG. 2. The "duality NMCR versus NBCR" in the spontaneous particle production in the nuclear medium.

IV. CROSS SECTIONS FOR NUCLEAR MESONIC CHERENKOV-LIKE RADIATION

Let us consider a nuclear mesonic Cherenkov-like process as in Fig. 1, where the incident projectile B_1 is a nucleon (N), the emitted meson (M) is a pion π or a kaon K, and the final particle B_2 is a nucleon. Now, let the incident nucleon be moving with a constant velocity v_1 inside of a nuclear medium (e.g., a ^{208}Pb nucleus). Let $v_{M\mathrm{ph}}(\omega)$ be the phase velocity of the pion or the kaon in this nuclear medium. According to the classical and quantum theories (see Refs. [7,9]) the NMCR mesons are predicted to be emitted when the coherence condition

$$v_{M\mathrm{ph}}(\omega)\leqslant v_1 \tag{24}$$

is fulfilled inside the nuclear medium, or equivalently, when the kinetic energy T_N of the incident nucleon is higher than the threshold energy T_{thr}, given by Eq. (7).

The transition probability (per unit time) for the emission of a pseudoscalar meson via the nuclear mesonic Cherenkov-like (NMCR) mechanism can be obtained via the golden rule in which $\tilde{H}_{\mathrm{fi}}$ are matrix elements of the Hamiltonian describing the interaction between the effective quantized mesonic field and initial and final nucleons inside the nuclear medium. Then, the number of physical mesons emitted per unit time as NMCR into the energy interval $(\omega,\omega+d\omega)$ is given by (see Refs. [7,9] for details)

$$\frac{dN^M}{d\omega}(\omega)=\frac{1}{2\pi v_1}|\tilde{H}_{\mathrm{fi}}|^2\mathrm{Re}k\frac{d\,\mathrm{Re}k}{d\omega}\Theta[1-v_{1\mathrm{thr}}(\omega)/v_1] \quad , \tag{25}$$

where $\Theta(x)$ is the Heavyside step function. The matrix element $|\tilde{H}_{\mathrm{fi}}|^2$ is given by

$$|\tilde{H}_{\mathrm{fi}}|^2=\frac{G_{MNN}^2}{|n_M(\omega)|^2}\frac{1}{2\omega}S(E_1,\omega)F_I=\frac{G_{MNN}^2}{|n_M(\omega)|^2}\frac{\mathrm{Re}k^2-\omega^2+(M_1-M_2)^2}{8\omega E_1(E_1-\omega)}F_I\ , \tag{26}$$

where $G_{MNN}^2/4\pi$ $[G_{\pi NN}^2/4\pi=14.6; G_{KN\Lambda}^2/4\pi=15.6$ (see Ref. [18])] is the usual pseudoscalar (ps) meson-baryon coupling constant and F_I is the isospin factor ($F_I=1$ for $N\rightarrow\pi^0N$, as well as for $N\rightarrow KN$ NKCR channels, and $F_I=2$ for $p\rightarrow n\pi^+$ or $n\rightarrow p\pi^-$ NPICR channels, respectively).

The result (26) was obtained after summing and averaging over initial and final nucleon spin states, respectively. This fact is contained in the spin function: $S(E_1,\omega)=\Sigma_f[\bar{u}_2(\vec{p}_2)\gamma_5u_1(\vec{p}_1)]\cdot[\bar{u}_1(\vec{p}_1)\gamma_5\bar{u}_2(\vec{p}_2)]$, where γ_5 and $u_i(\vec{p}_i)$ are the Dirac matrix and Dirac spinors, respectively.

Using Eqs. (25) and (26) we get

$$\frac{dN^M}{d\omega}(\omega)=\frac{G_{MNN}^2}{4\pi v_1}\frac{F_I}{|n_M(\omega)|^2}\frac{1}{v_{M\mathrm{ph}}(\omega)}\frac{d\,\mathrm{Re}k}{d\omega}\frac{(\mathrm{Re}k)^2[1-v_{M\mathrm{ph}}^2(\omega)]+(M_1-M_2)^2}{4E_1(E_1-\omega)}. \tag{27}$$

Now, the differential cross section $d\sigma/d\omega$ for meson emission as NMCR can be calculated according to the usual formula:

$$\frac{d\sigma}{d\omega}=2\pi\int_0^R b\,db\frac{dN^M}{d\omega}(b,\omega) \tag{28}$$

with

$$\frac{dN^M}{d\omega}(b,\omega)=\int_0^{\tau(b)} dt\frac{dN^M}{d\omega}(\omega), \tag{29}$$

where $\tau(b)$ is the NMCR-*emission time*, i.e., the time in which the incident baryon with impact parameter b is traversing the target nucleus with radius R. An exact estimation of $\tau(b)$ is a very complex and difficult problem. However, a rough estimation of this time is given by formula

$$\tau=\frac{2[R^2-b^2]^{1/2}}{v_1}, \tag{30}$$

where R is the usual "nuclear" radius.

Therefore, when the dependence on b of the absorption in Eqs. (28) and (29) is neglected, the differential cross section for the pion emission as NMCR in nuclear reactions is given by

$$\left(\frac{d\sigma}{d\omega}\right)_{\text{nonabs}}=\frac{V}{v_1}\frac{dN^M}{d\omega}(\omega), \tag{31}$$

where V is the collision volume $V=(4\pi/3)r_0^3(1+A_T^{1/3})^3$, $r_0=1.12$ fm, and A_T is the mass number of the target nucleus, since then

$$\frac{dN^M}{d\omega}(b,\omega)=\tau\frac{dN^M}{d\omega}(\omega)=\frac{2[R^2-b^2]^{1/2}}{v_1}\frac{dN^M}{d\omega}(\omega). \tag{32}$$

Absorption effects: Now, if absorption during the path x is taken into account in Eq. (29) in the standard way by

$$\left(\frac{dN^M}{d\omega}\right)_{\text{abs}}=\frac{dN^M}{d\omega}(\omega)\exp[-Bx], \tag{33}$$

then, from Eqs. (28) and (29), we can write

$$\left(\frac{d\sigma}{d\omega}\right)_{\text{abs}}=\left(\frac{d\sigma}{d\omega}\right)_{\text{nonabs}}F_{\text{abs}}, \tag{34}$$

where

$$F_{\text{abs}}=\frac{2\pi v_1}{V}\int_0^R b\,db\int_0^{\tau(b)}\exp[-Bx]dt. \tag{35}$$

The absorption coefficient is given by $B\equiv 2\,\text{Im}k(\omega)=2q\,\text{Im}n_M(\omega)=(\omega^2-m_\pi^2)^{1/2}\,\text{Im}n_M^2(\omega)/\text{Re}n_M(\omega)$, while $R=r_0(A_T^{1/3}+A_P^{1/3})$ is the radius of the target nucleus to which we have added the projectile radius.

A difficult problem in our NPICR investigation is to calculate the true absorption factor F_{abs}. However, if the distance x in Eq. (35) is approximated with $x=v_{M\text{ph}}(\omega)t$, then we get

$$F_{\text{abs}}=\frac{3}{X^3}\left[\frac{X^2}{2}-1+(X+1)\exp[-X]\right], \tag{36}$$

where $X=2BRv_{M\text{ph}}/v_1$. We note that in our quantum theoretical approach presented in Ref. [7], instead of F_{abs}, the following absorption factor was used:

$$\langle F_{\text{abs}}\rangle=\frac{1}{2R}\int_0^{2R}\exp[-Bx]dx=\frac{1-\exp[-2BR]}{2BR}. \tag{37}$$

Our general NMCR results can also be used as ingredients in different phenomenological models for coherent meson production in inclusive or exclusive nuclear reactions. But, then many other concepts such as form factors, distorted wave corrections, final state resonant interactions, etc., have to be added in a consistent way. Here we restricted ourselves to the estimation of NMCR as a nuclear medium effect in the M-Cherenkov-like limit [see Fig. 1(b), and Sec. III] that can be experimentally investigated. Hence, in all above considerations the influence of the nuclear medium on the propa-

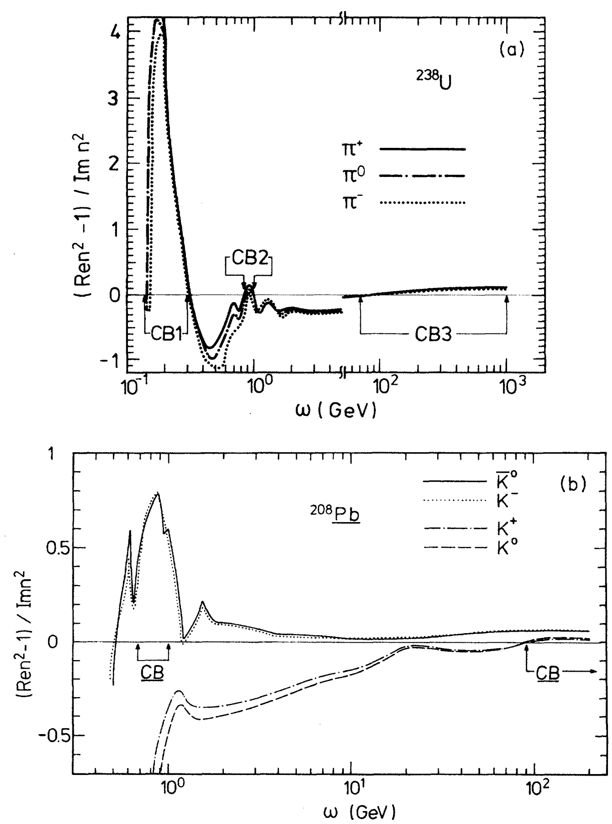

FIG. 3. (a) The refractive index of pions inside ^{238}U in the energy regions CB1, CB2, and CB3 (see text). (b) The refractive index of kaons inside ^{208}Pb in the energy region $\omega=(0.5-150)$ GeV (see text).

TABLE I. NMCR results for $T_N=500$ GeV for a nuclear medium of ^{208}Pb dimension and densities $\rho_0=0.17$ fm^{-3} and $\rho=5\rho_0$.

ρ(fm^{-3})	M-NMKR	ω(GeV)	T_{thr}(GeV)	$\frac{d\sigma}{d\omega}$ (μb/MeV)	Abs. factor (36)
$\rho=5\rho_0$	π^0-NPICR	0.157	1.274	0.970×10^{-2}	0.488
		0.168	0.512	4.355×10^{-2}	0.516
		0.181	0.354	9.464×10^{-2}	0.480
		0.195	0.293	0.167	0.399
		0.209	0.268	0.257	0.315
		0.225	0.259	0.348	0.236
		0.241	0.259	0.402	0.175
		0.258	0.268	0.358	0.124
		0.275	0.280	0.181	0.087
$\rho=5\rho_0$	π^0-NPICR	0.871	4.620	6.794×10^{-2}	0.0506
		0.890	3.077	0.142	0.0495
		0.910	2.562	0.181	0.0472
		0.930	2.362	0.174	0.0439
		0.950	2.382	0.126	0.0405
		0.969	2.564	6.718×10^{-2}	0.0376
		0.989	2.964	2.014×10^{-2}	0.0351
$\rho=5\rho_0$	π^0-NPICR	1.307	21.21	0.442×10^{-2}	0.0370
		1.327	21.75	0.434×10^{-2}	0.0363
$\rho=5\rho_0$	π^0-NPICR	80.00	396.5	7.656×10^{-2}	0.0563
		90.00	193.9	0.613	0.0562
		100.0	173.3	1.225	0.0562
		150.0	188.3	1.914	0.0561
$\rho_0=0.17$	K^--NKCR	100.0	470.3	0.346	0.2954
		150.0	416.7	0.907	0.2937
$\rho=5\rho_0$	K^--NKCR	0.857	5.207	0.113	0.0584
		0.898	4.464	0.142	0.0545
		0.940	4.319	0.159	0.0485
		0.983	4.398	0.170	0.0446
		1.027	4.467	0.180	0.0397
$\rho=5\rho_0$	K^--NKCR	50.00	104.8	1.252	0.0644
		100.0	152.9	3.408	0.0646
		150.0	201.5	3.408	0.0646

gation properties of the baryons B_1 and B_2 [see Fig. 1(c)] was neglected. This seems to be justified only for high primary energy E_1 and $E_2\simeq E_1$ [see M limit (a) in Sec. III], when the refractive indices of the B_1 and B_2 particles can be approximated by unity. For the final baryon B_2 the medium modifications can be very important even for the mesonic spectrum at very high meson energies ω ($\omega\simeq E_1-M_2$), beyond the mesonic Cherenkov band, where a Cherenkov-like transition in the nuclear medium can also take place. But in this case the transition can be identified with the process of single meson emission via *"baryonic" Cherenkov effects* [9]. This is proved in Sec. III (see Figs. 1 and 2) when one obtains the quantum baryonic Cherenkov coherence condition $v_{B\mathrm{ph}}(E_2)\leqslant v_1$. Moreover, the quantum theory for nuclear mesonic Cherenkov mechanisms must be developed in a more general way in order to take into account absorption effects as well as the finite size effects even in the definition of the density of final states (see Ref. [19]). All these improvements of the quantum theory of the mesonic Cherenkov mechanism should be worked out in future investigations.

V. NUMERICAL RESULTS

First, we recall that the pion-quasiparticle spectrum in the nuclear medium was considered by many authors (see Refs. [2–5] for details). The pionic excitations in nuclear matter at normal density $\rho=\rho_0=0.17$ fm^{-3} show (see Fig. 6 in Ref. [5]) three main spectral branches. These are (i) the Δ-*hole* excitations, (ii) the modified free pion branch, and (iii) the low-energy continuum of pionlike nucleon-hole states. Here, we restrict our discussion only to the nuclear pionic Cherenkov-like radiation (NPICR) which corresponds to the spontaneous emission of the modified free-pion branch (ii). The very special nature of the pion as a Goldstone boson of spontaneously broken chiral symmetry protects its mass against variations in the nuclear medium. The pion mass stays almost constant (see, e.g., Ref. [5]) at normal nuclear matter density and even beyond. The refractive properties of the modified free pion branch (ii) at normal nuclear densities can be estimated in the standard way by using the Foldy-Lax formula (1). So, we can investigate the signatures of the spontaneous pion production via a Cherenkov-like effect in-

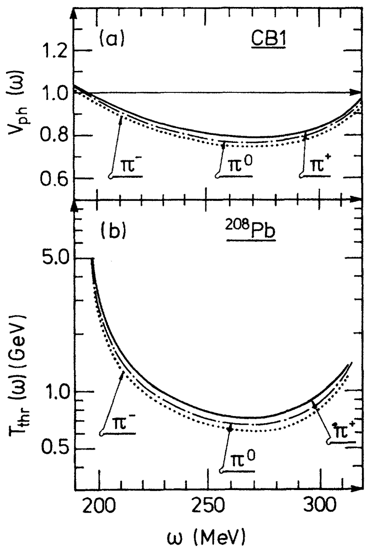

FIG. 4. (a) The pion phase velocity $[v_{\text{ph}}(\omega)]$ and (b) the NPICR thresholds (T_{thr}) for the CB1 energy region in ^{208}Pb.

side the nuclear medium since the elementary pion-nucleon forward scattering amplitudes are known with a high accuracy from the experimental data and also from the available standard amplitude analyses (see, e.g., Ref. [12]).

Unfortunately for the kaons the situation is not so simple as for pions. The physics of kaon-nucleon scattering inside of a nucleus has received considerable attention in the last time because of possible implications for the physics of neutron stars and also for relativistic heavy ion collisions. So, the possibility of the kaonic condensation as a new branch of the kaon quasiparticle spectrum inside of the nuclear medium was recently investigated by many authors [20–23]. It was argued that this new phenomenon can occur even at densities as low as 2.3 times normal nuclear density. The strongly attractive interactions between kaons and baryons, especially

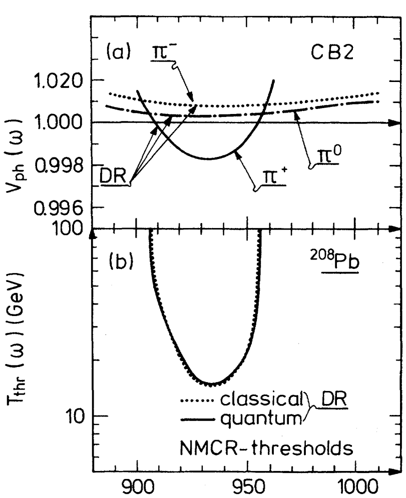

FIG. 5. (a) The pion phase velocity $[v_{\text{ph}}(\omega)]$ and (b) the NPICR thresholds (T_{thr}) for the CB2 energy region in ^{208}Pb.

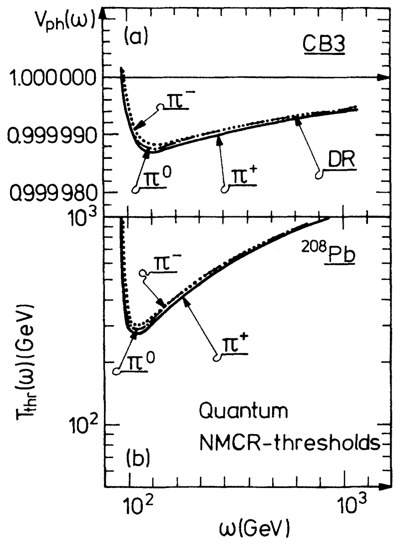

FIG. 6. (a) The pion phase velocity $[v_{\text{ph}}(\omega)]$ and (b) the NPICR thresholds (T_{thr}) for the CB3 energy region in ^{208}Pb.

from the so-called "sigma term," cause the effective kaon mass to drop even below the effective pion mass in dense baryonic matter. In fact the K^+ mass is found [5] to increase modestly with density, in accordance with K^+-nucleus scattering data. For the K^- the discussion is more involved due to the presence of the $\Lambda(1405)$ resonance just below threshold. Current studies are directed towards a better understanding of kaonic modes including the dynamics of the

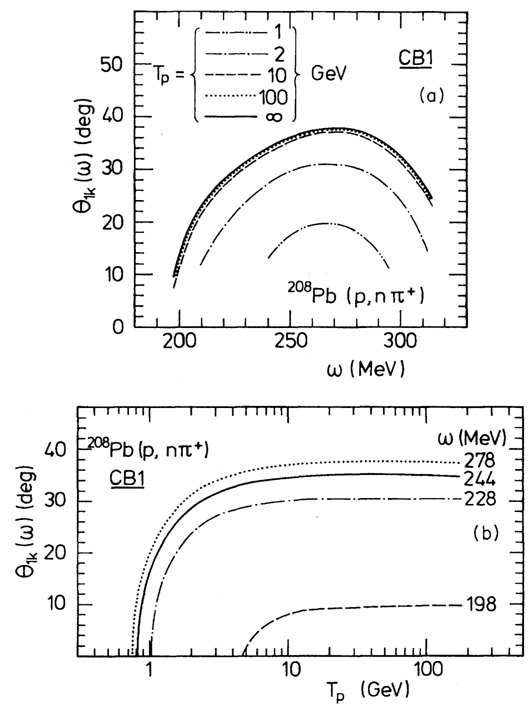

FIG. 7. (a) The (θ_{1k})-pion energy (ω) correlation of the NPICR pions in CB1 region. (b) The (θ_{1k})-projectile energy (T_p) correlation of the NPICR pions.

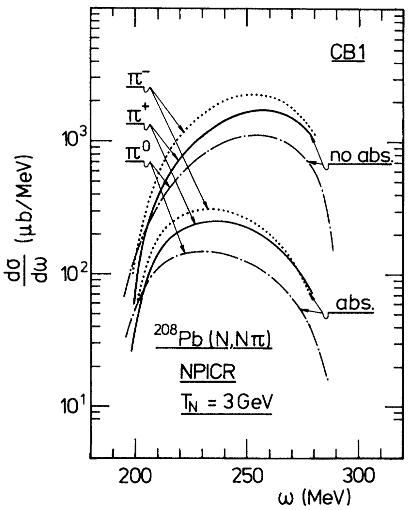

FIG. 8. The NPICR differential cross sections at $T_N=3$ GeV for CB1 band in ^{208}Pb. The absorption is calculated using $\langle F_{\rm abs}\rangle$ given by Eq. (37).

$\Lambda(1405)$. Here, for kaons, we restrict our considerations only to the excitation of the "quasifree" kaonic branch inside the nucleus (NKCR) which will be estimated in the standard way, similar to the pionic (NMCR) case.

Now, we present some numerical estimations for NPICR pions as well as for NKCR-kaons produced in nuclear reactions with high energy nucleons (or antinucleons).

Hence the basic ingredients for the NPICR as well as NKCR investigations are the refractive index of the pions and kaons inside the nuclear medium which for the modified free meson branch (ii) (see Sec. IV) can be calculated in a standard way by using a Foldy-Lax formula [10] [see Eq. (1)] where $\bar{f}_{MN\to MN}(\omega)$ is the averaged elementary meson-nucleon forward scattering amplitude:

$$\bar{f}_{MN\to MN}(\omega)=\frac{Z}{A}f_{Mp\to Mp}+\frac{A-Z}{A}f_{Mn\to Mn}\,. \tag{38}$$

Since our investigations are only exploratory, we restrict our discussion to the standard case of nucleons randomly distributed inside the nuclear medium where $C(\omega)=1$. Now, in Fig. 3(a), the refractive index of the (π^+,π^0,π^-) mesons inside ^{238}U is presented for all pion energies ω starting from the pion rest mass up to 1 TeV. Analogous results are given in Fig. 3(b) for kaons. These results are obtained by using the dispersion relations (DR) predictions for the elementary pion-nucleon [12] and kaon-nucleon [13] forward scattering amplitudes. In our standard estimations, the rest masses of pions, kaons as well as of nucleons and Λ baryons in the nuclear medium are considered to be the same as in the vacuum case. The small differences between the refractive indices for the different charged pion modes (or kaonic modes) are clearly connected with the differences in the elementary pion-nucleon (or kaon-nucleon) scattering amplitudes. From these results we see that three NPICR bands are expected to be experimentally observable in the nuclear reactions such as $^{238}{\rm U}(N\to N\pi^{\pm,0})^{238}{\rm U}$. These are as follows:

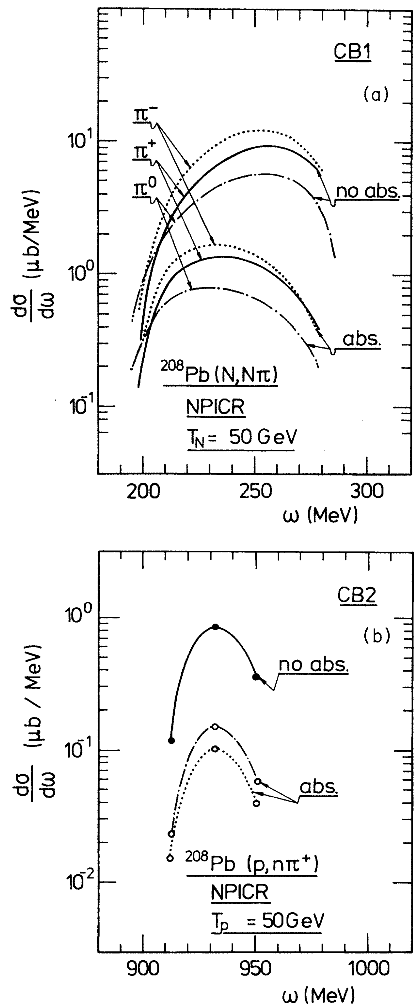

FIG. 9. (a) The NPICR differential cross sections at $T_N=50$ GeV for CB1 band in ^{208}Pb. The absorption is calculated using $\langle F_{\rm abs}\rangle$ given by Eq. (37). (b) The NPICR differential cross sections at $T_p=50$ GeV for CB2 band in ^{208}Pb. Absorption is calculated using $\langle F_{\rm abs}\rangle$ given by Eqs. (36) and (37).

$$\pi-{\rm CB1}-{\rm band}{:}\,\omega=(190-315)\ \text{MeV},\qquad \text{for }\pi^{\pm,0}; \tag{39}$$

$$\pi-{\rm CB2}-{\rm band}{:}\,\omega=(910-960)\ \text{MeV},\qquad \text{for }\pi^{+}; \tag{40}$$

$$\pi-{\rm CB3}-{\rm band}{:}\,\omega=(80-1000)\ \text{GeV},\qquad \text{for }\pi^{\pm,0}. \tag{41}$$

From Fig. 3(b) we see that it is possible to get NKCR kaons (or antikaons) at normal nuclear density in the following spectral regions:

$$K-{\rm CB2}-{\rm band}{:}\,\omega=(100-200)\ \text{GeV},\qquad \text{for }K^{\pm,0}; \tag{42}$$

$$K-{\rm CB1}-{\rm band}{:}\,\omega=(0.700-1.100)\ \text{GeV},\qquad \text{for }K^{-,0}. \tag{43}$$

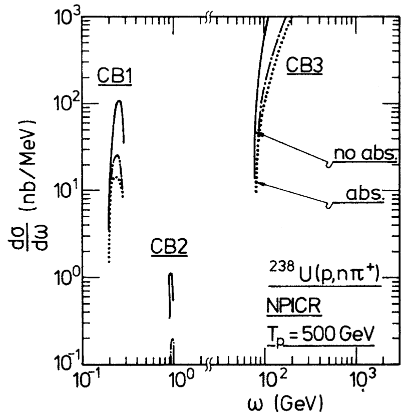

FIG. 10. The NPICR differential cross sections at $T_p = 500$ GeV for all CB1–CB3 regions in ^{238}U. The absorption is calculated using both Eqs. (36) (dotted curve) and (37) (interrupted curve).

Unfortunately, our detailed kinematical investigation shows that at normal nuclear densities such NMKR kaons can be emitted only in nuclear reactions with antinucleons, and only in the high energy kaonic band (42). At high densities, e.g., $5\rho_0$, both energy bands K-CB1 and K-CB2 can be emitted for antikaons (see Table I), but only CB-2 NKCR for the kaons $K^{+,0}$. However, the possibility of the $K^{+,0}$ production via NKCR is not excluded as an effect of the "correlations" [see the factor $C(\omega)$] in Eq. (1). But, then the study of the NKCR kaons emitted in the nucleon-nucleus interactions can become one of the powerful method for the accurate determination of such *coherence factors* (see Lax in Ref. [10]).

Next, it is important to note that a detailed discussion of the pion refractive index in nuclear media at low pion energies including the Δ-resonance region is presented in Ref. [2]. The consideration of a factor $C(\omega)$ in Eq. (1) via a Lorentz-Lorenz effect does not influence much our essential results and conclusions (see Ref. [7] for details) relative to the NPICR bands. However, some remarks have to be added. First, we emphasize that the scattering amplitude in Eqs. (1) and (38) must be evaluated at the wave number k inside the medium rather than at the free wave number $(\omega^2 - m_M^2)^{1/2}$. Second, more detailed considerations show that some conditions have to be fulfilled such that the scattering of a meson from one of the constituents of the nuclear medium can be treated as a two-body rather than a many-body problem. Such problems as well as the conditions for the adiabatic approximation for the motion of the nuclear scatterers have been systematically discussed by Feshbach [10] in connection with the justification of the nuclear optical potential. The reader should consult the literature quoted above and also the references in Goldberger and Watson [10], Bethe and Johnson [24], etc.

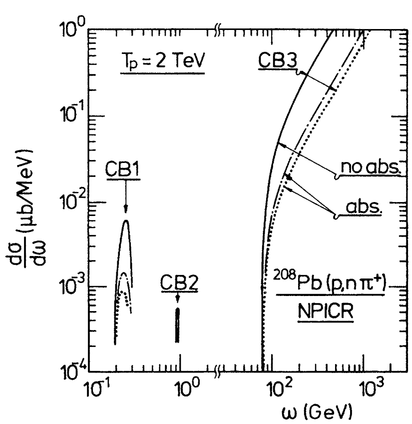

FIG. 11. The NPICR differential cross sections at $T_p = 2$ TeV for all CB1–CB3 regions in ^{208}Pb. The absorption is calculated using both Eqs. (36) (dotted curve) and (37) (interrupted curve).

The meson phase velocity and NMCR thresholds: In order to obtain a proof of the existence of the mesonic Cherenkov-like bands (39)–(43) we must calculate the meson phase velocity $v_{M\mathrm{ph}}(\omega)$ in the CB1-CB3 energy regions. Therefore, using the DR predictions [12,13] in Eqs. (1) and (38) and the relation (2) we obtain the $v_{M\mathrm{ph}}(\omega)$ in ^{208}Pb. The results obtained in this way together with the NPICR thresholds at normal nuclear densities are presented in Figs. 4–6. The NMCR thresholds are calculated by using Eq. (7) with $v_{1\mathrm{thr}} = v_{M\mathrm{ph}}$ for the classical NPICR threshold, and with Eqs. (16) and (17) for the threshold velocity $v_1^{\pi}(\omega)$. We note that $v_1^{\pi}(\omega)$ in Eq. (16) is the solution of Eq. (15) which preserves the classical coherence condition. The differences between the NPICR thresholds corresponding to the (π^+, π^-, π^0) emissions are induced by the corresponding differences between the refractive indices of these pionic modes. Similar results for the NMCR thresholds are presented as an example in Table I for the nuclear densities $\rho = 5\rho_0$.

Emission angle-energy correlations of the NMCR mesons: Other important signatures of the NMCR mesons are the strong correlations between the emission angle (θ_{1k}) and the meson (ω) and projectile (T_1) energies. These correlations can be calculated by using Eq. (13) as well as the above results for the pion (or kaon) refractive index and also for the meson phase velocity. For pions these characteristic features are in detail presented in Figs. 7(a) and 7(b), respectively. The (θ_{1k}, T_p) correlation [see Fig. 7(b)] as one of the important NPICR signatures is very similar to that of the Cherenkov photons emitted by a charged particle moving inside a dielectric medium.

NMCR differential cross sections: Now, the NPICR and NKCR differential cross sections are calculated using Eqs. (27), (31), and (34) as well as Eqs. (36) and (37) for the estimation of absorption effects. The values for the derivative $d(\mathrm{Re}k)/d\omega$ in Eq. (27) are obtained from a polynomial fit of Rek. The obtained results are displayed in Figs. 8–14 for all possible NPICR and NKCR bands and for projectile energies starting with the COSY energies 3 GeV up to 2 TeV. The essential signatures, such as the energy dependence (as T_p^{-2}) as well as the A dependence, and the density effects of the NPICR and NKCR cross sections are displayed in Figs. 12 and 14, respectively.

VI. CONCLUSIONS AND DISCUSSION

In this paper we investigated one of the fascinating new effects possible in nuclear physics, namely, the spontaneous pseudoscalar meson (π, K, etc.) emission as Cherenkov-like radiation (see also Refs. [3,6–9]). In fact, just as in normal

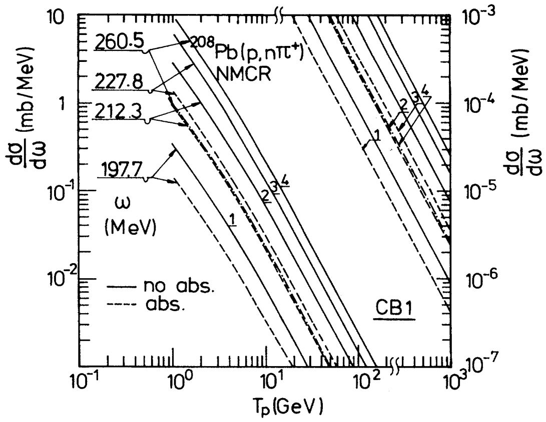

FIG. 12. The T_p dependence of the π^+- NPICR differential cross section in CB1 region. The absorption is calculated using $\langle F_{\text{abs}} \rangle$ given by Eq. (37).

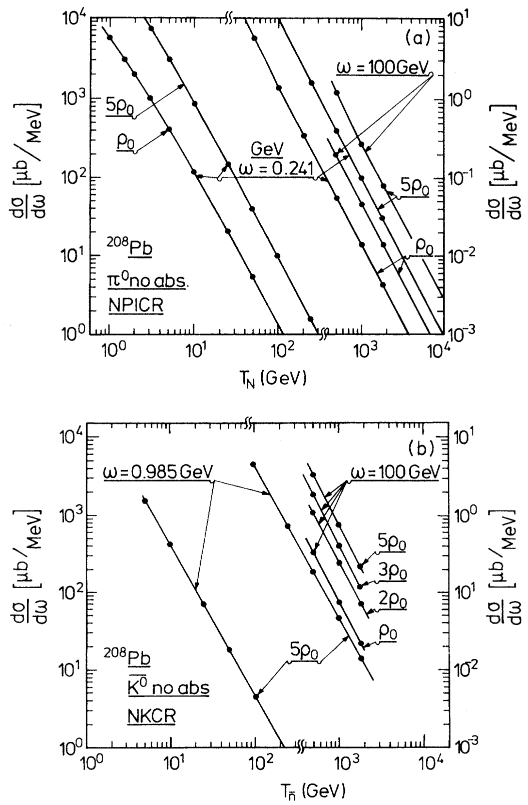

FIG. 13. (a) The T_p dependence of the π^0-NPICR differential cross section for $\omega = 0.241$ GeV and 100 GeV, respectively, for the normal nuclear density ρ_0 and for $\rho = 5\rho_0$, respectively. (b) The T_n dependence of the K^0-NKCR differential cross section for $\omega = 0.985$ and 100 GeV, respectively, for different nuclear densities up to $5\rho_0$.

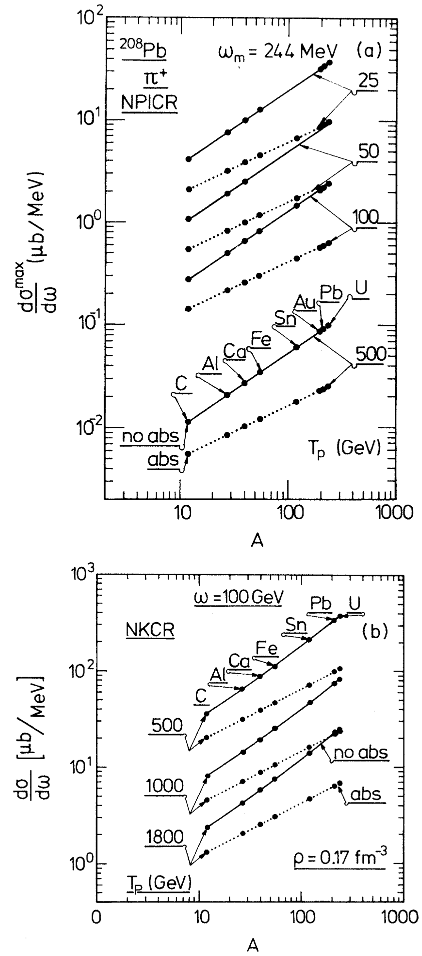

FIG. 14. (a) The A-target dependence of the π^+-NPICR differential cross sections at $\omega = 244$ MeV at normal nuclear density. The absorption is calculated using Eq. (36). (b) The A-target dependence of the K^0-NKCR differential cross sections at $\omega = 100$ GeV at normal nuclear density. The absorption is calculated using Eq. (36).

dielectric media, during the passage of a relativistic particle through a nuclear medium many collision processes involving the nuclear constituents can take place. These processess can mainly be divided into two classes, namely those in which the collision with a nuclear constituent can be considered as an individual event involving just only one bound nucleon, and those where a large number of nucleons of the nuclear medium are participating. This last class, called collective nuclear effects, are of great interest in heavy-ion collision at ultrarelativistic energies. One of the important consequences of collective effects is connected with the modifications of the propagation properties (e.g., phase velocity) of the particles and fields inside the medium. Consequently, many collective effects such as spontaneous meson production via Cherenkov-like effects can take place in the nuclear medium when the particle-sources are moving with velocities higher than phase velocities of radiation fields inside the medium.

The main results and conclusions of this paper can be summarized as follows.

(i) A genuine coherent pion emission as NPICR is possible in three energy bands in nuclear reactions such as ^{208}Pb$(N \to N\pi^{\pm,0})$ (see Figs. 3–6).

(ii) The NPICR pions must be coplanar with the incoming and outgoing projectile possessing a strong correlation between the angle of emission (θ_{1k}) and the pion (ω) and projectile (T_p) energies (see Fig. 7).

(iii) The NPICR-differential cross sections are peaked at the energy $\omega_m = 260$ MeV for CB1 band and $\omega_m = 930$ MeV for CB2 band when the absorption is neglected and the CB1-peak position is shifted down to $\omega_m = 244$ MeV when absorption is taken into account (see Figs. 8 and 9).

(iv) The energy dependence of the NPICR-peak position is as T_p^{-2} (see Fig. 12).

(v) The A-dependence of the NPICR-peak intensities is as shown in Fig. 14, when absorption is taken into account, and the peak intensity is increasing approximately as A^{+1} when absorption is neglected.

(vi) For the coherent pion production in the reaction $^{12}\mathrm{C}(p \to n\pi^+)^{12}\mathrm{C}$ [25] we find that the π^+-NMCR signal $d^2\sigma/d\Omega d\omega$ for $\theta_n = 0^0$ is of the order of magnitude of 0.04 μb/(sr MeV) at the energy $\omega \cong 244$ MeV. This allows us to conclude that π^+ NMCR is an effect competitive with the Δ-*hole* excitation mode in Ref. [14], even in light nuclei such as ^{12}C.

(vii) At normal nuclear densities such NMKR kaons can be emitted only in nuclear reactions with antinucleons, and only in the high energy kaonic band (42). At high densities, e.g., $5\rho_0$, both energy bands: K-CB1 and K-CB2 can be emitted for antikaons [see Table I and Figs. 13(b) and 14(b)]. However, the possibility of the $K^{+,0}$ production via NKCR is not excluded as an effect of the coherence factor $C(\omega)$ in Eq. (1). But, then the study of the NKCR kaons emitted in the nucleon-nucleus interactions can become one of the powerful methods for the accurate determination of such *coherence factors*.

Now it is important to note that the NPICR pions emitted in the CB1 region are just below the threshold ($\omega_{\Delta h} = 295$ MeV) of the "Δ-*hole*" excitation mode investigated in Ref. [14]. The NPICR pions are radiated spontaneously as real pions from projectiles while the "Δ-*hole*" branch can be described as "coherent excitations" of the medium, i.e., doorway states. A Cherenkov mechanism for this nuclear pionic mode can also be investigated when the Δ-*hole* branch of the pionic refractive index is known.

Therefore, we conclude that the NMCR cross sections as well as other signatures are sufficiently large in order to be experimentally measured. A special experimental technique based on coincidence measurements is necessary in order to extract the NPICR yields from the pion background produced by other processes. Such "coherent" techniques are well known in nuclear physics. First, the general set of pionic coherent events must be selected using as necessary condition that target as well as projectile preserve their ground state after pion emission. Secondly, the NPICR pions have to fulfill the NPICR coherence condition Eq. (1) as well as the *energy*(ω)-*angle*(θ_{1k}) correlations.

The general ideas and results obtained here can also be used as ingredients in different phenomenological analyses for the coherent or "diffractive" meson production in inclusive or exclusive nuclear reactions. For example, it would be interesting to compare the NPICR process discussed here with the well studied process of direct pion production in proton collisions at 500 GeV (see, e.g., Ferbel *et al.* [26]) or "coherent" pion production by diffraction dissociation of the nuclear target investigated at CERN (see Ref. [27]). Then, one can see that the $A^{1.07}$ dependence of the direct π^0-cross sections observed experimentally at Fermi Lab [26] is in a remarkable agreement with that of the NPICR pions given in Fig. 14(a). But, for the detailed comparisons many other ingredients such as form factors, distorted wave corrections, final state resonant interactions, etc., have to be added in a consistent way. However, we must underline that the NPICR mesons are emitted by projectiles and not by the nuclear target. Hence such NPICR coherent pions can be used as ingredients in phenomenologic analyses for description of pion production by projectile "dissociation" in nuclei. Moreover, in this case an idea of *multiple NPICR emission* must also be developed in a consistent way. All these extensions or improvements of the quantum theory of the mesonic Cherenkov mechanism should be worked out in our future investigations. Here we restricted our analysis to the estimation of NMCR as an important nuclear medium effect in the M-Cherenkov-like limit (a) (see Sec. II) that can be experimentally investigated not only by high energy proton beams, as that from Fermi Lab or from CERN, but also by low energy proton beam (e.g., at COSY energies), using the nuclear experimental "coherent" techniques.

ACKNOWLEDGMENT

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